BUSINESS STATISTICS

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BY

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SECOND EDITION
TENTH IMPRESSION

McGRAW-HILL BOOK COMPANY, INC.
NEW YORK AND LONDON
1938

PRINTED IN THE UNITED STATES OF AMERICA

PREFACE TO THE SECOND EDITION

The economic and other developments of the past few years have caused an increased interest in business statistics, and they have likewise caused new problems to arise in the application of statistical methods. In view of these conditions, this book on business statistics has been revised with the objective of bringing it more nearly in line with current business needs. The point of view throughout the revision is the same as that in the first edition, *viz.*, that of the business man, who judges the value of statistics according to their usefulness in securing practical results.

The book has been expanded particularly with regard to machine tabulation, index numbers, and the analysis of business cycles and long-time trends, and a chapter has been added on the normal curve and the problem of sampling. The illustrative material has been brought up to date where feasible, and new illustrations have been added where necessary. It was found advisable in a number of instances to retain illustrations from the period before 1930 for the reason that they are believed to be more representative of practical procedure for normal times than illustrations from later periods. In a number of instances, adapted charts from the first edition have been redrawn as of more recent dates.

The arrangement has been reorganized somewhat with the objective of better bridging the gap that so often exists between statistical methods and practical business problems. The material is still organized into two major parts, but they are not separated by a definite break. In the first part of the book the emphasis is placed upon studying the statistical methods that the authors and their associates have found to be useful in analyzing business problems. Typical problems and cases are freely employed to illustrate these methods and their uses. Then, in the later part of the book, the approach is deliberately changed, and the emphasis is placed upon solving practical problems as they actually occur in business, using as tools whatever technical methods or combination of methods may be necessary. That is, the study is centered upon the type of work that the statistician is called upon to do in actual practice, such as business forecasting, budgeting, production control, time and motion study, marketing analysis, real estate analysis, investment analysis, banking problems, and executive control.

For valuable assistance in revising material dealing with highly technical and specialized fields, the authors wish to express their apprecia-

tion to Theodore O. Yntema, Frank M. Weida, Wilford White, Woodlief Thomas, Albert H. Mowbray, Ada Lillian Bush, Jesse M. Cutts, G. I. Butterbaugh, and Walter G. Keim. Other valuable assistance for which the authors are very grateful was furnished by Arthur B. Fridinger, Esther K. Thompson, C. Howard Hunt, and Thomas B. Lewis.

Comments and suggestions from teachers, students, practicing statisticians, and others will be welcomed by the authors.

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PREFACE TO THE FIRST EDITION

This book has been written in the hope that it may encourage and assist in a more general use of statistical methods in business. It is intended primarily for use as a textbook in university and college courses in business statistics, especially in those which emphasize the actual application of statistics to business problems. It is the aim throughout the book to introduce the subject to future business men and women in such a way that they will appreciate the usefulness of statistical methods and employ them in their practical problems when they have entered upon their business careers. The book is designed to be of service also to practical business men who are interested in applying statistical methods to their problems.

If the objective of a course in business statistics is to train future business men to use statistics in making better analyses of their problems than they could without them, it is necessary to decide whether the course should be given from the point of view of the business man or from the point of view of the statistician. Since but few students in introductory courses in business statistics expect to become professional statisticians, it is thought best to emphasize the point of view of the business man who is to use the results of statistical work, rather than the point of view of the statistician. This approach should make a practical appeal to those who expect to go into business, without losing the interest of those who expect to become professional statisticians. Throughout the present text, the emphasis has been placed upon the point of view of the business man.

Experience in dealing with those who have had university courses in business or economic statistics has led the authors to believe that, in elementary courses, greater emphasis should be placed upon the practical usefulness of statistics. A knowledge of technical statistical methods is not sufficient in solving business problems. Solutions to these problems cannot be worked out with the mathematical exactness and precision of technical methods. Instead, the methods must be chosen with care, and they must be applied as aids in analyzing and solving problems rather than as complete solutions in themselves.

It is necessary, however, to begin the study from the point of view of technique in order to provide the working tools that are necessary in

statistical analyses. The first part of the course should include the technical methods used in the collection of data, tabulation, graphic presentation, frequency and other distributions, averages, index numbers, dispersion, correlation, and time series analysis with special emphasis on basic trends, seasonal variation, business cycles, and erratic movements. But in the last half or third of the course, the authors believe that the point of view should be changed to one in which the object is to consider business problems in which statistical methods are used. In this latter part, problems of business forecasting, budgeting, production control, marketing analysis, real estate analysis, investment analysis, banking statistics, management reports, and executive control systems may be considered. Thus, according to this plan, in the early part of the course the emphasis should be placed upon the necessary technical methods, using business problems as illustrations; in the second part of the course, the point of view should be directly changed and the emphasis placed upon solving practical problems, using the statistical methods as tools when necessary.

The foregoing plan has been carefully followed in developing the present textbook. The book is divided into two major parts. Part I is an introduction to statistical methods which the authors and their associates have found to be practical in analyzing business problems. Part II introduces the student to a number of specialized fields of practical application.

The object of the specialized chapters in Part II is not so much to teach the student the details of the various surveys and analyses as it is to indicate the general scope of the procedure in different kinds of investigations. It is impossible in the space that can be given in a general textbook to teach any great amount of technique in connection with the various specialized fields; in fact very highly specialized statistical procedure is found in some of these fields.

Part II contains much material that is not strictly statistical. This has been included deliberately, however, for the purpose of providing the necessary background. Although the chapters in Part II are somewhat interrelated, it is feasible to rearrange them if the instructor so desires. In courses where there is not sufficient time to cover the entire second part, it may be desirable to permit the individual student to select the chapters in which he is most interested. The fact should not be overlooked, however, that as a rule the business executive finds it important to know something of the possibilities of statistical investigation and analysis in all the fields included in the scope of the present work.

For the particular purpose previously discussed, this book represents an effort to bring together from many sources sufficient material to pro-

vide a practical foundation for a course in business statistics. The authors are greatly indebted to statistical textbooks and other publications of previous writers, and they have quoted freely from many sources.

In making their acknowledgments the authors should first express their indebtedness to M. C. Rorty, who read the entire manuscript, made many valuable suggestions, and furnished an introduction to the volume.

In connection with the planning and the early part of the preparation of the book, the authors wish to express their appreciation of suggestions made in the writings and statements of Leonard P. Ayres, and to acknowledge the influence of the teachings and writings of John M. Gries and Edmund E. Day.

Deep appreciation should also be expressed to George J. Eberle, who furnished many of the ideas used in the book, and to Floyd F. Burtchett and John C. Clendenin, who contributed several chapters to the preliminary mimeographed edition. Special appreciation should also be expressed for the great amount of work contributed by Lewis A. Maverick, Bradford B. Smith, and William F. French, who read the entire manuscript and made many valuable suggestions.

In a book that deals with so many rapidly developing and highly specialized fields, it is obvious that the only way to check the material and bring it up to date in accordance with the most approved ideas is to obtain help from the experts in these specialized fields. For valuable assistance in this connection, the authors are indebted to Ray B. Westerfield, Joseph H. Barber, Frederick C. Mills, L. M. Gilbreth, Irving Fisher, Frank M. Surface, A. L. Bush, John W. Riegel, Royal Meeker, John H. Cover, Frederick M. Babcock, Garfield V. Cox, Joseph L. Snider, Rufus S. Tucker, John M. Rae, C. Melvin McCuen, and Horace Secrist.

Other valuable suggestions and assistance in preparing and checking the manuscript and drawings for which the authors are very grateful were furnished by Paul Dodd, Robinson Newcomb, Arthur B. Fridinger, Gretchen Wells, Lloyd Rogers, and Anne S. Boyd.

The authors also wish to express appreciation to the publishers of books and magazines who have so liberally granted permission to use material from their publications. The authors are particularly indebted for permission to use material from "Management's Handbook."

Although the authors have endeavored to give full credit for all quotations, data, or ideas, it is impossible to acknowledge adequately at this point all the assistance that has been received. In many instances, further acknowledgments are made in the footnotes. It has been necessary to conceal the identity of the sources of some of the illustrations because of the confidential nature of the data presented.

While any merit that the present book may have may be due largely to the efforts and assistance contributed by contemporary statisticians and investigators, the authors alone are responsible for any errors or misstatements that may appear. Corrections or criticisms will be welcomed by the authors, and any suggestions regarding topics that should be omitted or added will be especially appreciated.

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January, 1932.

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CHAPTER I

STATISTICS IN BUSINESS

As competition increases and as business becomes more complex, it becomes increasingly important that the business man be equipped to analyze his problems with greater skill and accuracy. The ability to make skillful analyses of the problems in business can be developed by training; and it is for the purpose of contributing to this training that our university schools of commerce and business administration commonly require courses in business statistics. Although these courses in business statistics are relatively new, the value of special training in analysis has long been evidenced in the field of business. For instance, many of the most successful business men have been trained as lawyers or engineers. The training which they had in making careful analyses of specific problems, after gathering as many definite facts as possible, is commonly manifested in the thoroughness with which they analyze business problems later on. If business is to become a profession, careful analysis of definite facts must supersede much of the guess-work, which is now so common.

In the past, some business men have been successful because of superior energy, some because of sheer luck, and some because of the really careful analyses to which they subjected their problems. With the increasing competition, however, it will become relatively more essential that decisions shall be based upon accurate analyses of definite facts, rather than upon snap judgments and loose opinions. The primary purpose of statistical analysis in business is to make possible the use of definite facts where they apply and when they can be determined, and to permit personal or group judgment to be used when intangible factors must be balanced. All statistical analysis in business must aim at the control of action. The possible conclusions are:

1. Certain action must be taken.
2. No action is required.
3. Certain tendencies must be watched.
4. The analysis is not significant and either (*a*) certain further facts are required, or (*b*) there are no indications that further facts should be obtained.

In order to make the necessary investigations, to analyze the material gathered, and to draw sound conclusions, it is necessary that one have some knowledge of statistical methods and some experience in statistical analysis. It is the purpose of this book to describe the more important statistical methods and to show their practical application in business.

Application of Statistics in Business.—Much of the waste in business is due to lack of information. And when the information is available, waste often occurs because of lack of application or because of misapplication.

Sometimes the problem is principally one of securing the information, and the application may be relatively simple. For instance, a certain chain-store manager determines what a location is worth by the following method: From previous statistical surveys, he knows that, in his particular business, of the women passing his stores, 14 per cent will enter, 6 per cent will buy, and the average purchase will be 60 cents. He also knows that he can afford to pay 5 per cent of his annual sales for rent. He then counts the pedestrians passing a prospective location and, knowing the number of shopping days, determines the probable annual volume of sales. Five per cent of this figure is the amount that he can afford to pay for the location in yearly rentals. Of course, not all location problems can be solved as easily as this one, but the principle of the systematic approach and development is common to all.

In other instances, the problem may be principally one of application of information that is already available. Much of the work in business statistics might be called "the application of common sense." Yet business men often tend to be slaves to customs and fetishes, and they will cling to erroneous ideas because the crowd believes them to be true, when a careful consideration of a few simple and easily available figures would save them thousands of dollars. It does not seem reasonable that anyone would buy a house for investment, in a period of inflation, for $7,000 and rent it for $50 per month if it costs about $90 per month to cover interest, taxes, special assessments, depreciation, maintenance, insurance, vacancies, and management. Yet this was a common occurrence in certain localities during the boom years of the 1920's, and the buyer learned through costly experience what he could have determined at practically no expense by considering facts instead of loose opinions and fanciful ideas.

The application of statistics in a number of business fields will be discussed in some detail in the closing chapters of this book, but certain of the fields will now be considered briefly in order to introduce further the practical uses of statistics before taking up the study of technical methods.

Forecasting Business Conditions.—Bankers, manufacturers, merchants, and other business men must constantly consider future business prospects. In other words, they must forecast business conditions when they have to answer such questions as these: Should production be increased? Should inventories be accumulated? Should a loan be paid or renewed? Is it safe to extend more credit? Should building construction be delayed? Should wages be raised? Is the market sound? Is the investment safe? Are sales quotas fair? Should new lines of merchandise be added? Even where specific forecasting is not possible, a proper statistical analysis will *indicate* the plans that should be made in anticipation of booms and depressions.

Business executives must formulate their policies in accordance with the conditions that they expect to encounter later. If they err in judgment, they often sustain losses, and sometimes these losses prove disastrous to the business. When considering future policies, the business man should make a careful analysis of *internal* conditions (those within his business and over which he has control), as well as a thorough analysis of *external* conditions (those outside his business and over which he ordinarily has no control). External statistics usually are gathered and analyzed by trade associations, trade papers, economic services, government departments, and other agencies, and, unless the information is too highly specialized, sufficient information regarding external conditions may usually be obtained from these sources. The better the understanding which the executive has of both internal and external statistics, the better he can apply them in determining the outlook in his business.

The practical value of statistical work as a basis for determining business policies has been proved by the experiences of some of our more progressive business concerns. The American Telephone and Telegraph Company definitely plans its business on the basis of its statistical analyses. The Dennison Manufacturing Company, by being careful not to overexpand during periods of prosperity, is able to enjoy good business during periods of depression. The American Radiator and Standard Sanitary Corporation deliberately takes advantage of the business cycle, laying in inventories and expanding during slumps, and liquidating during the high prices of a period of prosperity. The Walworth Company bases its policies upon its statistical studies of conditions, and decides when to accelerate or retard buying, selling, manufacturing, and other operations on the basis of its own special statistical analyses. The uses of statistics by many other concerns will be discussed from time to time throughout the remainder of this book.

Not only are individual concerns making greater and greater use of statistics in determining their policies regarding the future, but coopera-

tive effort in both the collection and application of statistical data is increasing rapidly in importance.

Statistics in Manufacturing.—"Why are our costs higher than those of our competitor?" "Is our efficiency lower in general, or have we some

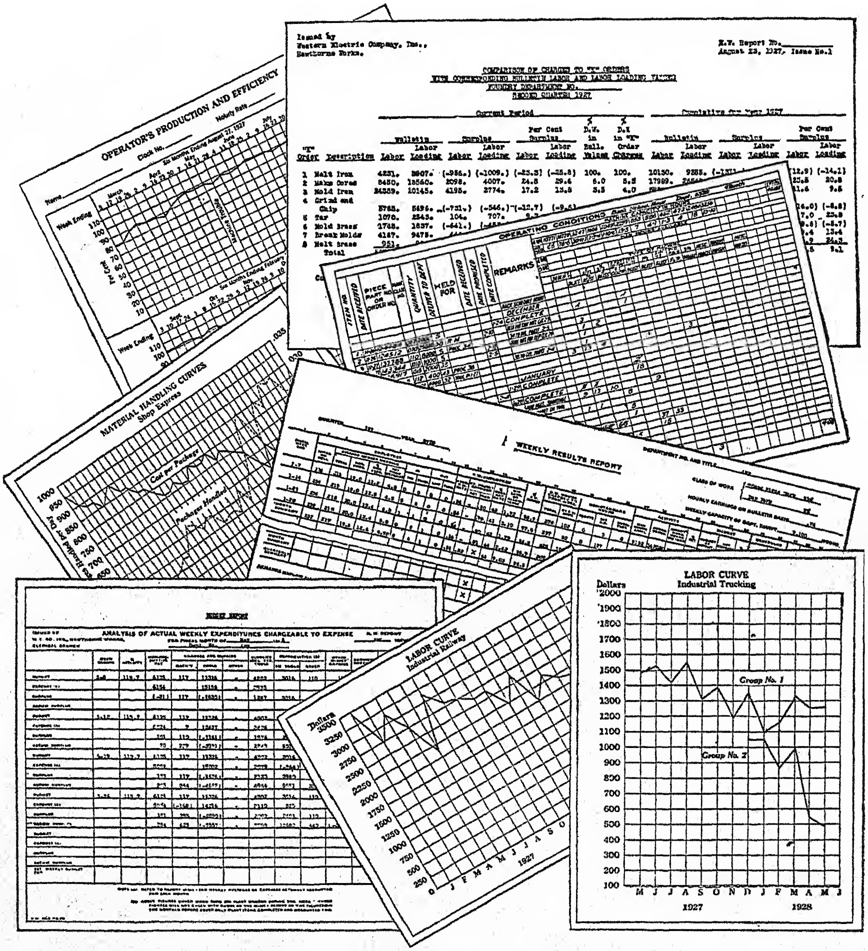

EXHIBIT 1.—Statistical records used by the Western Electric Company in the control of production and costs.

special disadvantage which may be corrected?" It is obvious that analyses based upon actual records of production and operation will be of utmost value in definitely determining the answers to such questions (Exhibit 1).

Statistics of labor conditions in a factory may include records of production by each laborer and by each group of laborers. These

records are compared with standards which have been established from time studies or past records. In eliminating waste, records may be kept of the materials used by each workman, and if, for instance, a cutter in a clothing factory wastes more than the average amount of cloth, the difficulty is easily detected. Often a high labor turnover indicates where other unsound conditions exist.

In order to indicate the efficiency of machine performance, records are kept of the production by each machine and of the number of hours operated. The relative condition is then determined by comparing the figures with established standards.

Where many operations are performed, or where many parts are made which will be assembled together later, as in an automobile plant, it is necessary to maintain a properly balanced production schedule. Records are kept which show currently the quantities of manufactured parts on hand, and this makes it possible to prevent shortages of some parts and overproduction of others. If production is properly balanced, the accumulations are reduced to a minimum.

Statistics in Marketing.—In building up a market, it is necessary first to find out what can be sold and then to work out the means of effectively placing the goods on the market. A skillful analysis of data on population, purchasing power, customs of people, competition, transportation costs, etc., should precede any attempt to establish a new market. Often such analyses will present difficulties which must be properly met before going to the expense of actually attempting to place the goods on the market, and often they will indicate markets where they were not thought to exist. For instance, a Los Angeles manufacturer believed that freight costs to foreign countries were prohibitive, especially to the more distant parts of the world. A statistical survey, however, showed that freight costs to such distant points as Buenos Aires, Argentina, and Sydney, Australia, were considerably less than to the principal cities in the neighboring states of Arizona and Utah.

In retail stores, in wholesale houses, and in the sales departments of manufacturing concerns, statistical records and analyses enable one to determine in advance, at a small cost, much that would be very costly if determined by actual experience.

In building up and maintaining any extensive market, it is important to keep accurate records on its present and potential geographic distribution (see Exhibit 2). Analyses of sales in relation to the distribution of population and purchasing power are especially important in establishing sales territories, routing salesmen, and in order to know where to advertise and where to push sales.

Statistical analyses are valuable not only in studying the market but especially in rating salesmen. A salesman may be compared with other

salesmen or with a standard. A salesman's current record may be compared with his previous records. Or a salesman's successes in selling different lines may be compared. In any of these comparisons, the weak points will be discovered, and such difficulties usually can be corrected when once it is known what they are and where they exist.

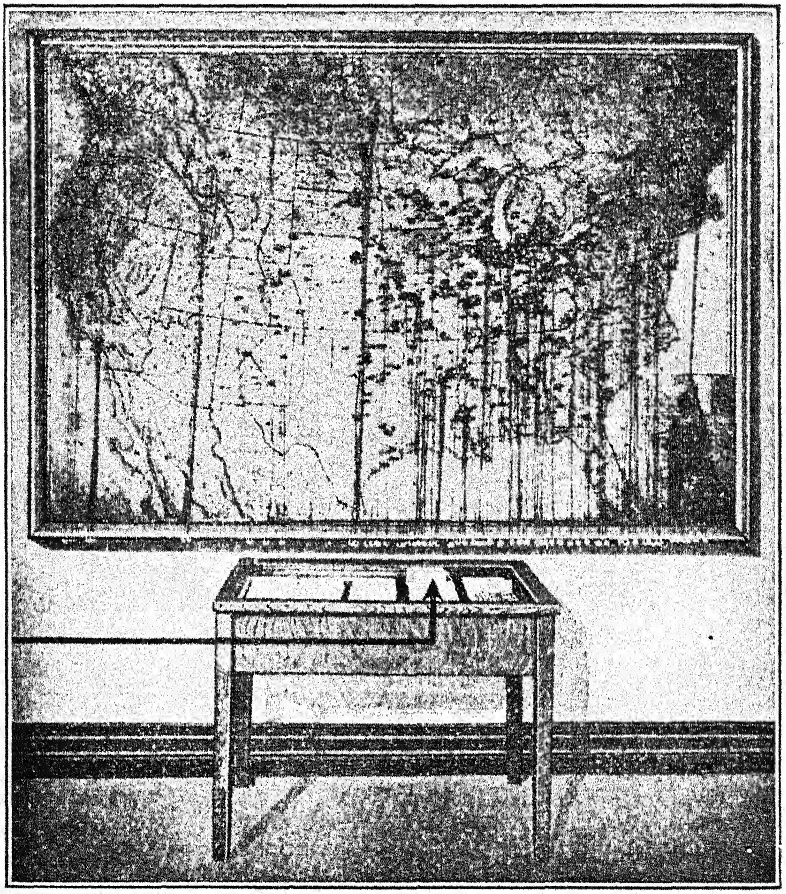

EXHIBIT 2.—Distribution map of the International Harvester Company with card-index system. (*Courtesy, Rand McNally & Company.*)

Statistics in Banking.—Banks have found it increasingly necessary to establish research departments within their organizations for the purpose of gathering and analyzing information, not only regarding their own operations, but on general economic conditions and on every line of business in which they might be directly or indirectly interested. Probably the bank, more than any other individual business, feels the direct effects of the conditions in every other type of business and needs to be constantly informed as to the trends in every line of activity. Its

reserves are influenced by money markets which are not local, but which are national or even international; its funds are influenced, not only by the business conditions in the immediate vicinity, but also by the conditions of business in areas far distant.

In making loans, banks have to be particularly careful that they do not lend too much money when business is dangerously inflated. In almost every period of hard times, many banks have failed because they did not correctly analyze the general business situation or the condition in individual concerns which had overexpanded.

The Federal Reserve System and the development of systematized influences over the credit and money markets of the entire country have not only made it desirable for individual banks to know something of the probable money conditions in the other banks in the immediate vicinity, but have made it necessary for them to consider the money situations in the other parts of the country. Also, the small bank compares its statements (or "position") with the statements of the large banks; the weak bank compares its credit and reserve position with that of the strong banks.

In all the foregoing problems, the bankers use the objective analysis furnished by statistics and then temper their decisions on the basis of qualitative information to the end that their conclusions are increasingly accurate.

Statistics in Investment.—Skillful analyses aid investors in selecting securities which are safe and which have the best prospects of yielding a good income or increasing in price. Such investigations assist in determining whether to buy, to sell, or to do neither. On the basis of these statistical guides, investors purchase securities when they are low, hold them for a few years until they are high, and then sell them and hold the proceeds until they can again buy at low figures. In this way, without any marginal purchases, but by buying outright high-grade, dividend-paying securities, investors have built up substantial fortunes with relatively little risk.

The investment banker is one of the greatest users of statistics. In his work he must accurately distinguish between good and bad securities. To do so, he must not only have a clear understanding of the present situation of the money and security markets and a definite knowledge of the actual conditions in the different industries, but he must also have a fairly clear conception of what will be the most probable future conditions in various industries. He must be able to detect in a concern's financial and other statements anything unusual or indicative of weakness in the securities of the company. The underwriting of a large bond issue requires information in great detail on the particular business involved, the immediate community, and on the businesses of similar nature in

other communities. The foregoing uses are of direct value to the investment banker in managing his operations. In addition to these, the various reports required by Federal and state governments necessitate the careful handling and expert application of statistical data.

Many other illustrations of the value of statistics in the investment field will be presented and analyzed in later chapters.

Value of Statistics Depends upon Practical Usableness.—The practical objective in business statistics is to provide various kinds of information which can be used as aids in making more accurate and more definite decisions than can be made without them. If this is to be possible, statistical work must be of the type that is appreciated and understood by the executives who are to use it. When one is doing statistical work for another, it must not be so involved, nor so mysterious, that the user cannot reason out, in a general way, how the results are obtained and what they mean. The approach has too often been from a technical point of view, with little regard for how the results can be used by the business man and with little consideration for his training and his problems. This often results in analyses so general, technical, or strange that only a few business men can apply them in their actual operations. Business men, as a rule, reason out the relative causes and effects in connection with the various factors considered in planning their operating policies, and are not prone to accept statistical conclusions if they do not readily appreciate their relation to their own businesses. The value of business statistics, therefore, depends upon their practical usableness, and it is the aim in this book to emphasize practical application, using technical methods merely as tools with which to increase the significance of statistical data.

Direct Value of Technical Statistical Methods in Business.—There are times when refined statistical methods are extremely valuable tools in analyzing business problems. For instance, it is often very much worth while to use these methods when trying to determine the relation between such factors as supply and demand, cost and price, interest rates and security prices, or wages and production efficiency. It is a most important point, however, that the user of statistical methods know when refined methods are practical tools and when they are not. This makes it necessary for him to know definitely both the uses and the limitations of data and of technique. Statistical methods are not magic mechanical devices which will take the place of judgment and experience.

There is a common tendency for practical business men and others to feel that the statistician "should be able to make better forecasts, for instance, from his statistical analyses alone than a business man can make without them or at least better forecasts than most well-informed business men can make without them. This is expecting too much from

statistics. The true test of practical usefulness of technical methods is that, when properly applied in connection with other information, they make possible the improvement of our forecasts."[1]

Whether or not refined statistical methods should be used depends upon the practical value of the results. Often it is easier to interpret the meaning of a series by collecting a few simple supplementary facts than it is to apply refined methods. But when the statistician does this, it must be because he knows that under the circumstances it is the most practical plan; and it must not be because he does not know the real value of technical methods.

Indirect Value of Technical Methods.—Aside from the direct value of mathematical methods as working tools in analyzing business problems, there is another value which is perhaps even more important; this refers to the attitude of mind developed by the training in rigorous thinking and disciplined imagination which accompanies mathematical study. A knowledge of mathematical methods teaches one "to distinguish between the particular and the general, to refrain from induction without adequate foundation, to test all operations by independent checks, and to approach a problem by more than one route. The practicing statistician, who is a trained mathematician, uses formal mathematical processes rather infrequently; but his mind is constantly guided by a mathematical sense which provides a general safeguard against unwise analysis and faulty reasoning."[2]

Simple Statistical Methods Usually Adequate.—While the business statistician should have at hand the proper statistical methods to use as tools in working out his problems, most of his work will require only the more simple technical equipment. Probably the most important requirements are to know what data are significant and where they can be found, and to know when data are adequate and accurate enough for the purpose at hand. Problems which are no more complicated than the following are representative of the bulk of statistical work: A large department store in a middle-western city of about 750,000 population engaged a new merchandise manager. He was told that normally sales declined about 30 per cent during July and August of every year because of people leaving the city. Upon analyzing the travel situation, he found that less than 12 per cent of the people left the city during the summer and that a large number of visitors came to the city during this period. It did not seem that a department store's general trade should fall off

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[1] See Holbrook Working, "Practical Applications of Correlation Studies of Prices," *Journal of Farm Economics*, April 1926.

[2] Crum, W. L., *American Economic Review*, Vol. XVII, No. 3, p. 454, September 1927.

so much under such conditions. A study of the geographical distribution of deliveries was made, and it was found that, except in July and August, three-fourths of the store's sales were made in the one-fifth of the delivery area which included principally families of high incomes, many of whom left the city during the summer. This was followed by an analysis of the store's advertising, which showed that the quality appeal had been so strongly emphasized that the great mass of people had become so impressed that this was a high-priced store that they patronized it but little. Steps were then taken to correct this impression and develop trade among those of moderate purchasing power.[1]

Though most practical statistical work in business is no more complicated or involved than that in the preceding illustration, it should again be emphasized that even in such cases a knowledge of statistical methods is of great value in the safeguard that it provides against inaccurate analysis and faulty reasoning. And, of course, it is very important that the statistician have the proper tools at hand (in the form of technical methods) when it is necessary to use them. Even in the simple problem discussed in the preceding paragraph, the concepts of seasonal changes and long-time growth tendencies, as considered from a technical point of view in later discussions of these subjects, are very valuable and almost required in making skillful analyses, though the actual computations may not be made.

Technique Studied through Practical Problems.—In this book the approach to the study of technical methods is made through the consideration of practical problems. A knowledge of statistical *methods* involves a study of collecting data, tabulation, graphic presentation, statistical distributions, averages, dispersion, index numbers, sampling, correlation, basic trend, seasonal variation, and cyclical movements.[2] After developing this background of technique, the methods will be used simply as tools, and the emphasis will be placed upon the practical application of statistics in answering such questions as: How can we increase efficiency in production? Where is the market for our products and how extensive is it? Are our investments secure? What is the general business outlook?

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[1] FREDERICKS, J. G., and F. M. FEIKER, *System*, p. 115, August 1912. Reprinted in M. T. Copeland, "Business Statistics," p. 207, Harvard University Press, Cambridge, 1921.

[2] Some of these methods involve mathematics, but only an elementary knowledge of algebra is presumed in discussing these methods in the body of this text. Any technical discussions, which involve higher mathematics, are presented only in the footnotes for the use of those who wish to study the subject from a mathematical point of view.

Questions and Problems

The purpose of the following introductory questions is to illustrate the necessity for careful analysis of statements which appear to be statistical in nature.[1]

1. Criticize the following statement: "Automobile accident figures for a large mid-western city show that in 1937 there were 320 different automobile deaths, only 80 of which were due to women drivers. This shows that women are more careful and safer drivers than men."

2. An advocate of the horse-and-buggy method of transportation argues that it is much safer than modern methods of travel. To prove his stand he quotes government figures which show that in a single year over 10,000 persons were killed in automobile accidents, 700 by train or street car, and 750 perished in boat or vessel, while only 157 were killed by horse and buggy. Do these figures prove his contention? Give reasons for your answer.

3. Two men, equally skilled as players and in the choosing of other men, are to act as captains of two baseball teams. Each is to choose 8 men from a group of 16 men who vary gradually in skill from the best to the poorest. They agree that the basis of selection should be such as to make the teams as equally matched as possible. One captain says that the fairest way is to toss a coin for the first choice and then choose alternate men thereafter; and the other captain says that this would not give equally balanced teams. Which is right and why?

4. The chances of throwing a six in one throw of a die (singular of dice) are one in six. You have already thrown two sixes in succession. What is the chance of throwing a six in the next throw? Give reasons for your answer.

5. In countries which have lotteries there is a common opinion that if one buys the same number for all of the different drawings he will have a better chance to win than if he buys a different number each time. (Numbered tickets are sold and at a certain time sales close and the winning numbers are drawn. Then new tickets are sold and the process repeated. Tickets are good only for the drawing for which they are sold.) Suppose that drawings are made every month and that Mr. *X* has bought the same number for 10 consecutive months. Suppose that 100,000 numbers were sold for each drawing. What were Mr. *X*'s chances to win the highest prize in the tenth lottery?

6. A western booster organization, in discussing a certain city, states that the mortality rate shows this city to be the most healthful city in the United States with a record of 9.78 per thousand. Are mortality rates a good index of health conditions? Why?

7. "The value of building permits in a certain city for 1908 was slightly under \$10,000,000. In 1923 this figure was slightly over \$200,000,000. Construction of building space in this city, therefore, increased over twenty times during this 15-year period." Point out an important error in this comparison.

8. A large paper company advertised as follows in a national magazine: "Do you know that checks are used in over 95 per cent of our business transactions? Do you know that last year the total bank clearings—practically all checks—amounted to over \$455,034,000,000, or ninety-five times the amount of money in circulation?" What is wrong with this comparison?

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[1] It is the aim of the questions and problems presented at the ends of the chapters in this text to provide material both for oral class discussion and written laboratory work—the particular material to be used to be selected by the instructor.

9. The following advertisement (name of car changed) appeared in a leading weekly publication: "Only in a car built throughout from standard specialized units—as the Apex Light Six is built—can value be definitely analyzed and proved by direct comparison. The average price of automobiles with the Continental Red Seal motor, for example, is $1,878. The Apex Light Six, with a 50-horsepower Continental Red Seal motor of the latest type, costs but $985. The average price of Timken-axled cars is $2,453, yet the Apex Light Six has Timken axles, both front and rear."

Comparative Values

Average Price of Cars with		
Continental Red Seal motor	$1,878	Apex Light Six has all of these at $985
Timken axles	2,453	
Durston transmission	1,495	
Auto-lite electrical equipment	1,674	
Spicer universal joints	2,388	
Gemmer steering gear	2,507	

Analyze the above advertisement from a statistical point of view.

10. Determine which of the following tables are statistical and which are non-statistical, and explain how the tables that are non-statistical at present might be made statistical.

A

Data

City	*Price*
Boston	$5.65
New York	6.12
Philadelphia	5.61
Cincinnati	5.25
Chicago	5.95
St. Louis	5.40

B

Production of Crude Oil in the United States (by States)

State	*Production in thousands of barrels*
Texas	44,366
California	20,365
Kansas	6,388
Louisiana	7,629
Michigan	1,316
New Mexico	3,463
Oklahoma	20,618
Pennsylvania	1,581
Wyoming	1,603
All others	3,582
Total	110,911

C RELATIVE PRICES OF THE POUND STERLING AND THE FRENCH FRANC
(Expressed in dollars)

Date	Pound sterling	French franc
March 20, 1938	5.02	.03300
March 27, 1937	4.88	.04593
April 17, 1937	4.92	.04464
April 10, 1937	4.90	.04556
April 3, 1937	4.89	.04600
April 18, 1936	4.94	.06592
April 20, 1935	4.85	.06593

D ANALYSIS OF RESIDENTIAL STRUCTURES IN CLARKVILLE

Type of structure	Number completed	Number under construction
Apartment house	18	7
Flat	63	8
Duplex	110	19
Single residence	978	21

E VOLUME OF AND REVENUE DERIVED FROM SALES OF NATURAL GAS IN THE UNITED STATES
1937

Month	Sales in millions of cubic feet	Revenue in thousands of dollars
January	129,312	47,847
February	134,025	48,995
March	126,633	45,234
April	124,723	42,667
May	104,242	34,159
June	94,494	28,733
July	91,777	26,440
August	94,201	26,329
September	94,959	26,738
October	102,651	31,012
November	110,724	36,911
December	129,341	46,673
Total	1,337,082	441,738

Source: Survey of Current Business.

CHAPTER II

COLLECTION OF DATA—PRIMARY SOURCES

The statistician of a certain manufacturing company received the following interdepartmental letter from the vice president in charge of manufacturing:

At the last meeting of the Executive Committee, the question was raised as to whether or not our wage rates are out of line with the wage rates in competing and similar lines of manufacture. You are requested to collect all information which you think will be necessary to answer this question as it applies to our several factories, and forward a report to me by the next regular meeting of the Executive Committee on November 17.

How should the statistician proceed to collect such information? Where should he begin the investigation, and just what steps are involved?

First, the problem must be defined, and the proper approach must be determined. Then a plan must be prepared, the statistical units must be defined, and it must be determined that the characteristics of the items upon which data are to be collected are alike—that is, that they are "homogeneous." Furthermore, the available sources of data must be determined and checked as to their accuracy, and the use of samples must be considered. *These steps are all preliminary to the actual assembling of data.* Each will be considered in the paragraphs that follow.

In short, a statistical investigation may be said to involve the following broad steps:

Definition of the problem.
Organization and planning.
Collection of the data (including both primary field work and use of secondary sources).
Tabulation and computation (preparation of working tables).
Presentation (including textual presentation, final tables, and charts or graphs).
Analysis and interpretation.
Preparation of report.

Obviously, there is some overlapping in the above items. A certain amount of analysis work, for instance, is carried on throughout the survey.

In general, the discussion of statistical methods in this book follows the order indicated by the above list. The present chapter covers the

collection of primary data, and the four following chapters cover the collection of secondary data, tabulation, and graphic methods, respectively.

Definition of the Problem.—In making statistical analyses in business, it is very important to define the particular problem to be analyzed as clearly as possible before beginning the investigation. This is necessary in order that the investigator can determine what elements are to be included in the survey and so that he can guard against the omission of factors which might be necessary in reaching a solution. It is also important to exclude the unnecessary factors when their inclusion would cause a waste of time and money.

Often the problem is presented to the statistician in rather general terms and he must take the responsibility for working out with the executives a more clear-cut definition before taking any further steps toward its solution. A problem which is apparently easy to state is not always clear. Suppose, for instance, that in connection with studying a market, an executive asks for a figure representing the population of New York City on January 1 of a certain year. Just what is the population of New York City? Does it include only those who live in New York City? Does it include those who work there and have their homes in the suburbs? What about tourists and travelers who are in the city on January 1? Should immigrants who are temporarily in the city be included? What about non-resident college students? What about those whose homes are there, but who are spending the winter in Florida or California? These are a few of the questions that would have to be answered in qualifying and clarifying the problem.[1]

The Approach to the Problem.—While a statistical problem should be defined as accurately as possible, it is usually impossible in business to work out an ideal definition in advance. Most statistical problems in business are somewhat confused and must be approached step by step without insisting that the entire solution be evident from the beginning. In such cases, it is always possible to select one or more facts, or whole series of data, which must obviously be assembled before any answer is possible. When these facts have been gathered together and analyzed, they will probably suggest the need for further facts. And, sooner or later, if this process is repeated, the direct road to a solution will present itself.

[1] The United States Census includes as the population of a city only its permanent residents or those whose actual homes are not elsewhere. Those whose actual homes are in New York, for instance, but who are in other parts of the country when the census is taken, are counted as of New York. When the transients or temporary residents in a city constitute a large number as compared with the permanent residents, it often happens, obviously, that the Federal Census population data alone are not adequate in making various kinds of market analyses, as they may represent only a part of the market.

Having defined his problem as carefully as possible, the statistician is ready to plan the investigation.

Planning the Investigation.—Before making an original investigation, the available existing information should be brought together in as much detail as is practical, covering as many as possible of the known questions. In many instances, it is advisable to carry on the investigation in such a way that the more important questions connected with the problem can be sorted out and answered first. Often a few of the outstanding facts will show that it is not advisable to consider the proposition further, and a considerable amount of money can be saved by not completing the survey.

As some surveys progress, the original problems may change to entirely different ones as a result of the answers or partial answers obtained to some of the major questions. For instance, a certain company, which planned to build an office building, started an investigation for the purpose of determining the most popular sizes of rooms and the most practical rentals. The high percentages of vacancies in competing office buildings, discovered early in the investigation, caused the purpose of the survey to be changed to that of forecasting how long the company should wait before considering construction at all. Of course, the survey could have been completed in accordance with the original purpose, but the results would not have been practical. It is highly important in business surveys that the statistician control his investigation so that it will be of the greatest value in relation to its cost.

Some types of investigations cannot be subjected to variations while in progress. For example, if it is desired to determine the effect on street traffic congestion of moving vans carrying household goods on a moving day, and it is known in a certain city that September 1 is the day on which families customarily move from one house to another, then the entire enumeration must be made on that day, and the complete plan must be carefully worked out in advance, as there would not be time to make changes while the survey is in progress, and any variations might change the representativeness of the data.

Statistical Units.—Before beginning the actual collection of the data, the statistical unit or units must be clearly defined, and these definitions must be carefully observed throughout the investigation. Fundamentally, the statistical approach is based on a count. If the definitions of the units are vague or inexact, the inclusion of things which should be omitted, and the omission of things which should be included, may not only cause confusion but may result in non-representative data. It is obvious that the units should remain the same throughout an investigation, and this requires a simple, fixed, and clear definition.

To the inexperienced, the definition of a unit appears to be a very simple matter, but the problem is not so easy as it seems. In taking a

population census, the unit is one person. This is an easily defined natural unit, and there is not much difficulty in determining what is and what is not a person.[1] But suppose that we are making a survey of the supply of dwellings in a city and that we are counting the single family houses. Just what is a single family house? Obviously, there would be no difficulty in classifying most houses, but what is a house with but one entrance which is occupied by two families? What is a rooming house? What is an abandoned house? What is one under construction? What are living quarters over a garage? What are living quarters over or in the back of a store? What is a tent in which people are living? What is a garage which is being used temporarily as a dwelling? What is an auto camp cabin? What is an automobile house trailer in which people are living temporarily? These are some of the questions which must be taken into consideration when defining the unit.

A unit itself may be difficult to define, but the problem of definition is often made much more difficult by attaching qualifications to the unit. For instance, if we are counting the vacant lots in a city, it is difficult enough to define the term "lot," but when we are considering only those that are vacant we must provide also for such questions as: What is a lot occupied by a temporary structure? What is an auto park? What is a temporary storage yard? What is a city block which is not subdivided? What is an abandoned stone quarry?

Usually it is not difficult to define the units of measurement which are in common use, such as the foot, pound, bushel, hour, dollar, and pound sterling. The actual use of these units, however, requires considerable care in order to prevent confusion. For example, the familiar unit "ton" is widely used, but in statistics of ocean shipping the following must be carefully distinguished: short ton (2,000 pounds); long ton (2,240 pounds); metric ton (2,204.6 pounds); displacement tonnage (weight of water displaced by floating vessel); gross tonnage (cubic feet of total vessel capacity divided by 100); net tonnage (cubic feet of vessel capacity available for cargo divided by 100); deadweight tonnage (maximum weight of cargo, passengers, and crew); measurement ton (40 cubic feet). In making original investigations or in compiling secondary data, such variations in the usages of terms must be carefully observed.

Pecuniary units (dollar, pound, lira, franc, yen, peso, etc.), which are used for measuring values, vary from country to country and from time to time, and these variations must be carefully considered in any analysis of international values. In certain other kinds of investigations,

[1] It should be noted that the difficulties previously mentioned on p. 15 in connection with a population census apply to the definition of the term "population." They do not refer to the definition of a "person," which is the unit.

it is necessary to use compound units, such as ton-mile, foot-pound, or kilowatt-hour.

Homogeneity or Uniformity.—Accurate deductions cannot be made from data which are lacking in homogeneity or uniformity. In other words, it is imperative that the characteristics of the elements under investigation be essentially alike in relation to the investigation, except in the differences that are being measured. For example, suppose that we are comparing building activity in two cities on the basis of building permits, and that one city includes street paving and subway construction while the other does not. Such data would be lacking in homogeneity. Or, suppose that a comparison is being made of the average wages in two department stores which are essentially alike, except that in one a much larger proportion of the employees are women than in the other. For the purposes of many investigations such data would be lacking in homogeneity.

Whenever an average is taken, the question of homogeneity must be considered. No average should be taken from cases which, for the purposes of the particular investigation, are lacking in homogeneity.

Homogeneity is especially important in studying business growth, price changes, and all other variations which take place over a period of time. Changes in conditions often make it very difficult to make valid comparisons of present data with those of previous periods. For instance, the use of bank clearings as an index of a city's business conditions is often hampered because of the lack of homogeneity due to the establishing of new banks and the consolidation of old ones.[1]

Adequacy or Representativeness.—The question of adequacy or representativeness is very important in business statistics because it is so often necessary to make investigations and to draw conclusions based on sampling. When the data are only partial they may not be representative, and care must be taken to see that they are adequate for the purpose at hand.

The requirement of adequacy or representativeness makes it necessary to have, not only a sufficiently large sample, but also one which is as free from bias as possible. This may be illustrated by a study which was made for the purpose of determining the proportion of vacancies in

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[1] A bank frequently cashes, for its customers, checks which are drawn on other banks. Every day each bank sends all of these checks to a central "clearing" place or directly to the banks upon which the checks have been drawn, and receives the checks drawn upon itself. Only the balances are settled in cash or its equivalent. These checks which are exchanged are the "clearings." When new banks are established, the volume of clearings will be increased even though there may be no increase in the total amount of business done in that community. The opposite is true when bank consolidations occur.

apartment houses at a time when apartment house owners, as a rule, were trying to discourage the building of additional apartment houses. It was found, in this instance, that a larger proportion of those whose apartment houses had high proportions vacant answered the questionnaire, because they felt it was to their interest to have this information brought to the attention of prospective builders in order to discourage them. Those whose houses were filled were not interested in the competitive situation so seriously, and relatively as many did not return questionnaires.

Accuracy.—The question of accuracy must be considered in all statistical investigations. When all the items can be counted, it is often possible to attain perfect accuracy, but, if the task is a large one, a count may not be worth the time and money that it would cost. Approximate accuracy may be sufficient. *Counts* can be perfectly accurate, but *measurements* are never perfectly accurate. In measuring, absolute precision is unattainable. The necessary degree of accuracy in counting or in measuring depends upon the practical value of the accuracy in relation to its cost.

Errors are due to various types of causes. Clerical errors or errors in computation can usually be eliminated by careful checking. Other errors result because of the conditions under which the investigation is made. Errors may be compensating or cumulative. Compensating errors tend to offset one another and leave the general result undisturbed, while cumulative errors tend to pile up with the result that the final data are not representative of the true values. In taking a federal population census, for instance, an enumerator may count some people twice who have moved from one place to another during the two-weeks census-taking period. He may also miss some people who move from a part of the city where the census has not yet been taken to a part where it has already been taken. At this point the errors are probably compensating. However, in checking the data, it is easier to eliminate the duplications than it is to add the omissions and, if the duplications are eliminated without adding the omissions, the final result is affected by a cumulative error. Referring again to the question of accuracy, it would be possible to shorten the census period and eliminate such an error as the one just mentioned, but it would not be practical to increase the cost so greatly for the purpose of correcting such a relatively small error.

Frequently it is possible to approach a problem from two distinct angles in such a manner that the bias, if any, is in opposite directions. In such cases, the main statistical operation may often be checked to advantage by means of moderate samples taken from the second angle of approach.

Reasonableness.—Conclusions in business statistics do not rest entirely on cold figures. Exterior tests of reasonableness, often of a

non-statistical character, must be applied. Frequently the conclusions that are arrived at are not the only possible conclusions, and too often they are not the most accurate conclusions that can be drawn from the given data. Careful exercise of judgment, based upon a wide experience in keeping in mind the reasonableness or probability of the conclusions, as a thing apart from the data, is of utmost importance in business statistics.

Primary and Secondary Data.—Data which are gathered originally during an investigation are known as *primary data*, whereas those which are used in an investigation, but which have been gathered originally by someone else, are known as *secondary data*. For instance, data gathered by the United States Bureau of the Census are primary data for the bureau but are secondary data when used by others. When one is using figures which he has actually secured by field enumeration, he is said to be using primary data, and when he is using figures which he has secured from someone else, he is using secondary data. Secondary data are removed one or more steps from the original enumeration.

In the same manner, a distinction is often made between *primary* and *secondary sources* of statistical data. The original compiler of the data is the primary source, but one who furnishes data that were originally compiled by someone else is a secondary source. Thus, the Bureau of the Census is the primary source of the decennial population census figures. And when these figures are published in the *Statesman's Yearbook*, this publication is said to be a secondary source of these population data.

Some statistical problems require only primary data, while some require only secondary data, but most require both kinds of data, and all require a knowledge of collection and use of both kinds of data. In order to use secondary data intelligently, it is necessary to understand the difficulties encountered in securing primary data. This is so important that the collection of primary data will be discussed in considerable detail before considering the collection of secondary data.

Collection of Primary Data.—Before beginning the actual collection of primary data, it is necessary to outline the procedure for solving the problem as completely as possible. In practical statistical work, one of the most important limitations upon the scope of an investigation is the cost. It may be necessary to confine the survey within narrow limits owing to the fact that only a certain amount of money is available for the particular problem. Of course, no investigation should be carried beyond the point where the additional refinements are no longer necessary.

Complete canvasses are so expensive that it is often impractical or impossible to make them in business statistics. In such instances, it is necessary to use the process of sampling, in which the analysis of a

part is taken as a basis from which to draw conclusions regarding the whole.

Sampling.—The manufacturer or trader who wishes to test out the market for a certain product can not interview all the potential buyers because of the cost involved. However, by securing the reactions of a representative sample, he will have an indication of how the product will be received by the market as a whole.

The basis of the sampling process is that *a moderately large number of items chosen at random from a very large number of items should have the characteristics of the larger group.* This does not imply that a perfect resemblance between the sample and the whole is necessary, but it is essential that the sample indicate the characteristics of the whole to a high degree. The importance of this statement will be better appreciated if careful consideration is given to certain fundamental principles of sampling.

Principles upon Which Sampling Is Based.—Although the principles of sampling are sometimes referred to as the "laws of sampling," they are not laws in the strict sense of the term, but are, rather, principles and tendencies which appear to operate universally.

The *principle of statistical regularity* may be briefly stated as: If a reasonably large sample is selected without bias from a mass of phenomena, the characteristics of this sample will differ but little from those of the entire universe. It follows that this principle is the very basis of sampling, for after having studied a portion of the phenomena bearing upon a given problem, it is possible by reliance upon this principle to depict fairly accurately the characteristics of the entire group. It is also clear that this principle makes possible a material reduction of the work necessary before accurate conclusions may be made concerning any large group of quantitative data. Thus, if one were desirous of studying the changes in the earnings of factory workers in the United States, it would not be necessary to include all factory workers, nor the workers in all factories. If the earnings records of 25 per cent of the workers in 25 per cent of the factories were studied, almost as good a view of the changes in the earnings of factory workers could be obtained as if all factory workers were included in the analysis. Such samples, however, must be properly chosen so that bias will be avoided, and so that there will be a fair distribution by areas and classes of factories.

The *principle of large numbers* is based upon the same reasoning as the principle of statistical regularity. A short statement of this principle is: Whenever an event may happen in only one of two ways and the event is observed to happen under the same essential conditions a large number of times, the ratio p of the number of times that it happens in one way to the total number of trials appears to approach a definite limit, let us say

P, as the number of trials increases indefinitely. Thus, in a bank with a large number of branches, the proportion of the number of customers having accounts with balances below $100 may vary greatly from week to week among the different branches, some branches having a smaller number than the preceding week, other branches having a larger number. For the bank as a whole, however, the *relative* number of accounts less than $100 may not change greatly from one week to the next. To use another common example, the fire losses in any city or group of cities selected at random may be considerably larger per month this year than last. It may be that other cities with large fire losses will be found, but, even though the number of cities with large losses increases in the year's count and is not entirely offset by decreases, the *relative* variation in *all* cities will decrease as more cases are considered, and therefore it may be stated that the *total* fire losses in *all* cities fluctuates much less than the fire losses in an individual city or in a small group of cities.

A related principle is the *principle of small numbers*, sometimes called "the principle of persistence of small numbers." In brief, it may be stated as follows: If, in a group of quantitative phenomena selected without bias, a small proportion of the group deviates sharply from the characteristics of the remainder of the group, this tendency will persist no matter how large the group may be made and irrespective of the number of samples selected. Thus, if one were to tabulate the length of service of employees of several concerns which had been operating for, say, forty years, one would be almost certain to find a small number of employees who had been in the service of the concerns for a considerable number of years, say twenty-five. From the principle of persistence of small numbers it would follow that other concerns which had been in operation an equally long time would probably exhibit nearly the same small proportion of employees with long service records. Again, if an analysis of the balances of savings accounts in a large number of branch banks is made, even though the branches are located in quite different sections of the community, it may be found that each branch will have a small number of savings accounts in which the balances are more than $10,000. Similar experiences will be found in the study of other business data, such as the amount of sales in a small department of a store, purchases of imported automobiles, and sizes of incomes. This law is the law of the distribution of events which happen rarely.[1]

An additional corollary of the principle of statistical regularity is the *principle of decreasing variation*, which is as follows: As a larger and larger proportion of a group of phenomena is selected by the cumulation of

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[1] For explanations of the mathematical basis the reader is referred to discussions of the Poisson exponential function which characterizes this law.

successive unbiased samples, the characteristics of each enlarged sample (namely the average, the deviation, the skewness, etc.) will tend to differ less and less from the characteristics of the preceding (smaller) sample and from the characteristics of the group as a whole. Intelligent application of this principle makes it possible to determine the size of the sample (in proportion to the whole of the data) which must be taken in order that the characteristics of the sample may, within prescribed limits of accuracy, be attributed to the whole group. Thus, if one should study samples representing increasing proportions of the checks presented at a bank for payment from day to day, one could approximate with increasing accuracy the distinguishing characteristics of all checks.

Selecting the Sample.—It is obvious that the sample must be representative, to a practical degree, for the problem at hand. In a strictly random selection, any item has as good a chance of being included as any other. If this is not the situation, a special effort will have to be made to select the sample in such a way that allowances can be made for any bias. Strictly *random sampling* is not commonly possible in business statistics. The most common form of sampling may be called *directed sampling,* since a distinct effort is made, by applying some system of control over the selection of data: (1) to increase the certainty of a proportional representation from the various related types and groups of elements in so far as those types or groups are known to exist or to have a bearing upon the problem at hand; and (2) to exclude (to save time, energy, and expense) those types or groups which are known to have no bearing upon the problem at hand.

To illustrate the above two types of sampling, let us suppose that one draws the prices of 50 commodities from a group of 1,000 prices which had been marked on small cardboard disks and mixed well. Such a selection might be said to be random in so far as the 1,000 prices were concerned. An index number computed from such a sample would not be nearly so likely to be reliable as a measure of the changes in the cost of living, however, as would a sample of prices that included the 50 most important articles in the budget of the average family which would be directed sampling.

Sometimes the chances of selecting fair samples are increased by dividing up the whole into classes and taking samples of each class. The relative importance of each class is known or is determined and the final result is weighted accordingly. Thus, in a sample for the purpose of determining the percentages of residences vacant, the degree of accuracy is greatly improved by recording the conditions separately by single-family houses, double houses, flats, apartments, etc., and then weighting the final figure for the situation as a whole in accordance with the proportions of these different types of dwellings. Within each class, of course,

some method of obtaining a random selection or its equivalent must be used. In this particular instance, the sample of single houses might be taken on a basis of regular intervals determined by drawing lines a mile apart north and south, and east and west, and then covering the streets nearest these lines. Again, suppose that we are estimating the total volume of unemployment on the basis of questionnaires addressed to certain employers. It adds greatly to the accuracy of the result to take separate samples for retail stores, factories, etc., and then make separate estimates of the total numbers unemployed and the percentages of unemployed in each group before combining the figures for grand totals. This principle is of almost universal application in sampling operations.

The degree of variation permissible between the sample and the whole cannot be stated arbitrarily since this depends upon the use that is to be made of the sample. There are various checks that may be made of the accuracy and adequacy of a sample. The more mathematical tests will be discussed in Chapter XI, but some other types of checks will be suggested in the section which follows. These tests alone are not complete and are not to be substituted for judgment and experience in selecting samples and interpreting the results of the sampling process.

Checking the Adequacy of the Sample.—The necessary size of the sample, or the necessary proportion represented by the sample, varies with different problems. In some instances, a sample which includes only a small percentage of the whole will indicate its characteristics to an extent which will be valuable. Such would be the case in determining whether or not a certain Cuban confection would have a market in the United States. If the confection were placed in a few representative stores in a few communities, it is probable that this test would be almost as satisfactory as one involving several thousand stores throughout the nation. In other instances, the sample must be relatively large in order to be significant. For example, if the adoption of a pension plan were under consideration by a large company, and the directors desired to know how many years it would be on the average before all employees reached the retirement age, it would not be adequate to consider the ages of only a few employees in a few departments.

The illustrations just cited indicate two practical rules or principles in determining the size of the sample: namely, (1) that the required number in a sample increases as the variation in the individual items increases, and (2) that the required number also increases as the required accuracy of the results increases. Thus, in testing the marketability of a product for which there apparently would be but relatively little variation in tastes within the nation selected, the required number of cases for the test would be relatively small. But in a large company with employees of many and widely varying ages, it would not be adequate

to consider the ages of relatively few employees when considering a pension plan. Also, in the latter illustration, a high degree of accuracy would probably be required because the adoption of the plan would be final, whereas in testing the marketability of a product, the decisions as a result of the sample might not involve sudden or widespread changes.

What has been called a "stability test" of sampling may be used as a rough check upon the adequacy of any sample. This test consists of: (1) division of the original sample into two equal samples by means of random sampling; or (2) selection from the original data of a new unbiased sample equal in size to the sample already taken. The next step consists of noting whether or not the characteristics of the divided sample (as shown by the two smaller samples), or of the newly selected sample, differ materially from the characteristics of the original sample. Thus, if we study the changes in the number of persons employed in factories by means of a sample containing, let us suppose, 25 per cent of all factories, then the adequacy of the sample could be roughly checked (1) by dividing the total sample into two groups and comparing the data of the two smaller groups with the results shown by the larger, or entire sample, or (2) by taking a second sample and comparing the results obtained from the two equally large samples. If a second sample is taken, it is not theoretically necessary to exclude from it all items which were included in the first sample. However, it should be unbiased.[1]

When a sample is considered to be inadequate, the problem of increasing the size of the sample arises. But the accuracy of the sample does not increase directly in proportion to the size; *it increases as the square root of the size of the sample increases.* Therefore, to double the accuracy, we must quadruple the size of the sample, and to treble the accuracy we must increase the number of items to nine times the former number. To illustrate, if there are 100 items in a sample and it is desired to double the accuracy of the sample, the square root of the size of the sample, 10, will be obtained. This will then be multiplied by 2, resulting in a product of 20, which will be squared, indicating that 400 items are required for a sample twice as accurate as 100 items. Similarly, for a sample three times as accurate as 100 items, the square root of 100 (10) will be multiplied by 3, and the result (30) will be squared, giving 900 as the number of items needed for a sample three times as accurate as 100 items.

Further explanation of the statistical technique and theory involved in sampling will be given in later chapters, especially in Chapter XI.

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[1] Sometimes it is desirable to divide the samples into more than two equal parts. If the sample is divided into, say, four equal parts, and the differences between the averages of the four groups are within the required limits of accuracy, then the average of the totals should be satisfactory.

Forms for Recording Original Data: Schedules and Questionnaires.—When statistical counts are made, or when other statistical data are gathered, the information is originally recorded on certain forms which are especially designed for the purpose. These forms are principally of two types:

1. The schedule. (Ordinarily filled out by a trained enumerator who gathers the information directly or questions the informant.)
2. The questionnaire. (Ordinarily filled out by the informant.)

Other original records are sometimes made (such as field notes and diagrams, and those made by means of mechanical dials, and instrument

OCCUPANCY SURVEY

STREET (ONE SIDE) Adams FROM NO. 2100 TO NO. 2142

CARD FILLED OUT BY L. B. Smith DISTRICT OR TRACT NO. 162

TYPES OF BUILDINGS	NUMBER OF UNITS OCCUPIED	NUMBER OF UNITS VACANT	NUMBER OF UNITS UNDER CONSTRUCTION	NUMBER OF EXTRA FAMILIES
SINGLE HOUSES	2	2	0	2
HORIZONTAL DOUBLE HOUSES	6	2	2	1
ROW HOUSES	0	0	0	0
VERTICAL TWO-FAMILY HOUSES	0	0	0	0
FLATS	3	1	4	1
APARTMENTS	10	4	12	1
LIVING QUARTERS OVER STORES	0	0	0	0
STORES (NEIGHBORHOOD)	0	2	1	
OFFICES (NEIGHBORHOOD)	0	0	0	

DATE June 15

EXHIBIT 3A.—A simple schedule used by field enumerators. (*Courtesy, United States Department of Commerce and National Conference on Construction.*)

tapes and graphs), and combinations of the two above mentioned major types are often used, but such variations will be understood without difficulty if the principles underlying the two main types are understood. Each of these two major types will now be discussed in some detail.

Schedules for Field Enumerators.—As used in this text, the term "schedule" refers to a blank form which is used by a field or other enumerator to facilitate the recording of data. An example of a schedule is shown by Exhibit 3A. The schedule for a field enumerator should be drawn up in such a way that it will be convenient for use in both the field and the compiling office.

RESIDENTIAL OCCUPANCY SURVEY

INSTRUCTIONS TO ENUMERATORS

1. Fill out a card for each side of every block. (Do not use the same card for two sides of a street.)

2. Record in the space provided at the top of the card the name of the street, and the numbers of the first and last houses in the block. Record a corner house according to the street on which it faces or according to the street on which the main entrance is located. (The house number or street address will ordinarily indicate the main entrance.)

3. Be sure to record the number of the district or census tract on each card in accordance with the map furnished for this purpose.

4. All counts are to be made on the basis of units. A "unit" is defined as the quarters occupied by, or intended to be occupied by, one family. It is one family capacity or one family accommodation. Thus, a single house contains one unit, a double house two units, etc.

5. In recording the data, give careful consideration to the different types of buildings. The different types of buildings may be defined according to typical characteristics as follows:

Single house. The typical characteristics of a single house are: single, free-standing building for occupancy by one family.

Horizontal double house. The typical characteristics of a horizontal double house are: two family units standing side by side; each family occupies its half from cellar to roof, and in many cities the two halves are owned separately.

Row house. The typical row house is one of several units built together in a row with individual outside entrances; each family occupies its unit from cellar to roof, and in many cities the units are owned separately.

Vertical two-family house. The typical characteristics of a vertical two-family house are: two units with one over the other.

Flats. The typical characteristics of a flat are: heat not furnished—no janitor service—frequently individual outside entrances—ordinarily three to six unit capacity and two stories, but sometimes a greater number of units and as many as three or even four stories.

Apartment house. The typical characteristics of an apartment house are: heat furnished—janitor service—one or more joint entrances—usually has elevator service (but not always).

Living quarters over stores. Residential quarters over stores (usually one story over a store) where the structure is principally a store building rather than an apartment house that has stores on the first floor.

6. Include residential units not readily defined as above in the most nearly related classifications.

7. Include rooming houses with the nearest related houses structurally. That is, if a rooming house is in a single house include it with single houses, and if it is in a double house include the unit in the double house classification, etc. (See notation at end of Item 8.)

8. Extra families. Where a unit intended to be occupied by one family is shared with one or more extra families, temporarily or permanently, record the number of "extra" families. (Do not include rooming houses in the count of extra families.)

9. A unit is to be counted as "occupied" if it is being used and is not available to another user. For example, a northern home might be closed for the winter while the owner is in Florida, but if it is not available to another resident, it should be counted as "occupied."

10. Empty space that is available or on the market should be counted as "vacant." An apartment which is under lease to some one who is not living in it, but who is trying to sublet it, should be counted as vacant. Unoccupied buildings which are temporarily unfit for occupancy should be classified as "vacant."

11. Units in a building should be considered as "under construction" any time between the beginning of the use of building materials and the time when the building is ready for occupancy.

EXHIBIT 3*B*.—Instructions to enumerators. (To accompany the form in Exhibit 3*A*.) (*Adapted from recommendations of the United States Department of Commerce and the National Conference on Construction for making a standard occupancy survey.*)

If the data are confidential, the schedule should be so marked. If the identity of the informant is indicated on the schedule, it is sometimes advisable to place this information on a stub bearing a duplicate number which can be removed upon the receipt of the schedule at the office. This is especially important when the data are not solicited with mandatory power.

The schedule should be drawn up with a full appreciation of the type of informants to be interviewed by the enumerator. In an interview, it is often necessary to record the information step by step as a method of leading up to the more important items. The terms used on the schedule should be very carefully worded, and if there is any question of clarity they should be rigidly defined. If this is not done, there may be but little homogeneity in the final results. If only one investigator is collecting the information, it is likely that the data would conform to one standard even though the terms were not completely defined; but when a number of enumerators are collecting information, they should be relieved as much as possible of the necessity of making interpretations.

Exhibit 3*A* is designed so that one card is used for one side of the street for one block. The card was designed in part so that when regular postmen did the enumerating, they could follow their regular routes and then the cards could be sorted readily by census tracts. Obviously, if additional details were to be recorded, the enumeration would have to be made by individual houses instead of being summarized by one side of a block. A schedule should always provide for making the original field record. It ordinarily never should be such that the use of scratch pads, etc. are required before making the original entry, especially if it is a count. Yet it should not be so detailed or so simplified that useless or unnecessary compilation work would be required.

Space should be provided in many instances for coding and computation in the office. Ordinarily, computation should be done in the office rather than in the field by the enumerator. As far as possible, the work in the field should be enumeration rather than computation.[1]

Instructions to Enumerators.—When data are collected by enumerators in the field, the enumerator gathers the information directly or questions the informant and records the information on the schedule. Consequently, he must thoroughly understand the items to be enumerated. He must also have adequate instructions regarding the time of enumeration, the territory to be covered, and any other special information needed in making the investigation.

If the schedule is a complicated one, the enumerator should be given considerable training and practice before he enters the field. In any

[1] Further examples of schedule forms are presented in Appendix I (pp. 636–638).

case, the instructions should be printed or typed in detail so that all enumerators will decide questions in the field as nearly alike as possible. Exhibit 3*B* presents the set of enumerators' instructions that accompany the schedule shown in Exhibit 3*A*.

Checking the Enumerator.—In making statistical surveys, it is usually necessary to provide for some kind of a check on the work of the enumerator. This is ordinarily done by field supervision, by having more than one enumerator gather the same data, by recording the data in such a

(LETTERHEAD)

July 1, 1938.

Mr. John Smith, Manager,
John Smith Apartments,
New York, N. Y.

Dear Sir:

In order that you and others may know the true apartment-house situation, we are requesting apartment-house owners to give us the information asked for in the question list below. You will receive the results of this survey through the *Apartment House Journal*. Please fill out the answers to the questions and return them in the enclosed envelope (which requires no stamp) before July 4. Your answers, of course, will be held in strict confidence.

Very truly yours,

THE APARTMENT HOUSE JOURNAL

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1. How many apartments are there in your apartment house?__________
2. How many apartments were occupied on July 1, 1938?__________
3. How many apartments were vacant on July 1, 1938?__________

Name of apartment house__________

Address of apartment house__________

Name__________

PLEASE RETURN YOUR ANSWERS BEFORE JULY 4.

EXHIBIT 4.—Simple questionnaire and letter combined. (The letter from which this exhibit was adapted was multigraphed and filled in with a typewriter.)

manner that it can be checked up later, by including data which serve as cross-checks, and by checking with previous reports and other sources.

Field supervision, where the supervisor watches each enumerator for a time to see how the work is being done, is especially necessary when the enumeration is being made by inexperienced men.

Questionnaires.—The questionnaire (Exhibit 4) is often used to take the place of the field enumerator and his schedule. The desired information is asked for directly on the questionnaire, and a space is provided for the answer. The questions may be simple or complicated, according to

the character of the investigation. However, the questions must be stated clearly because the informant must record the information without any assistance from the inquirer.

The questions on the questionnaire should be worded in such a way that they can be answered easily by "yes," "no," or a definite number when possible. In distributing a questionnaire by mail, it is customary to enclose with it a letter and a self-addressed envelope. Formerly, the envelopes were ordinarily stamped, but now the postage can be paid by the addressee when the questionnaire is returned. Sometimes it is best to include a personal letter, while in other instances it is best to use a form letter. When possible, it is well to indicate some advantage which will make it worth while for the informant to fill out and return the questionnaire.[1]

Requisites of a Good Schedule or Questionnaire.—As far as possible, schedules and questionnaires should be made to meet the following requirements:

1. There should be as few questions as possible.
2. Each question should be simple and easy to understand. It should not be ambiguous.
3. Each question should cover exactly the information desired, *i.e.*, it should be sufficiently inclusive but unnecessary information should not be requested.
4. As far as possible, the questions should be such that they can be answered by "yes" or "no" or simple numbers.
5. The questions should be such that they will be answered without bias.
6. The questions should not be unnecessarily inquisitorial or arouse antagonism in any way.
7. The printing, typing, paper, letterhead, etc., should be appropriate.

If the questions on a questionnaire are few in number and easy to understand, and if they can be easily answered by "yes," "no," or a simple number, a much larger proportion will be returned than there would be if the questions were ambiguous or if the answers requested were difficult or complicated. Likewise, enumerators working with schedules will find that the same principles apply. Unnecessary questions should not be included, as such a procedure makes it more difficult for the informant to give the information and increases the expense of compilation.

The requirement of simplicity must be considered in relation to the particular type of informant. Questionnaires, for instance, which might be appropriate for lawyers, might be so complicated for laborers that none would be returned. It is extremely important to see that the terms used are clear and that they are not subject to

[1] Further examples of questionnaires are presented in Appendix I, pp. 639–646.

double interpretation. For example, in a certain automobile and truck traffic survey, the questionnaire included the following items: "Registration fee paid," and "City or town." It was desired that the informant enter after the first item the amount (in dollars and cents) of the registration fee that he had paid, and after the second item the city or town in which he lived. However, a considerable number of the returned questionnaires had the entry "yes" after the first item and "city" after the second item.

It is also very important that the questions on questionnaires and questions asked by enumerators be worded and explained in such a way that the answers will not be biased. In the traffic survey referred to above, considerable difficulty was experienced in connection with obtaining representative data on overloading trucks owing to the fear that the information might be used as evidence of a violation of the law.

When information of a private or personal nature is being requested, care should be taken to ask the questions in such a way that they will not appear to be unnecessarily inquisitorial, and the point should be emphasized that the answers will be held strictly confidential. Otherwise, the informant may resent being questioned and refuse to give the information.

The form of a schedule and especially of a questionnaire should be appropriate for the purpose at hand. Sometimes high quality, engraved stationery and printed forms should be used, while in other instances mimeographed letters and forms are appropriate. Often return postal cards can be used for simple questionnaires. The arrangement on the page should be such that it will be practical from the points of view of both the informant and the compilers in the office.

The business statistician observes the above requisites in order that his questionnaires may achieve their primary purpose, which, simply stated, is "to get the necessary information." One of the most important safeguards in preparing questionnaires is to require all concerned in the investigation to prepare blank tables which will indicate what data are finally wanted. Frequently, it is desirable to fill in dummy data where one has a rough idea of what the investigation will show. Then the questionnaire must be prepared in such a way that there will be no question but that all of the desired information will be requested.

As a preliminary check upon the clarity of classifications and terms, it is well to have several persons in the office, including those who helped with the preparation of the questionnaire or schedule, try out the form in the field and then compare results. In a certain survey of automobile retail and service facilities, for instance, a schedule supplemented by a questionnaire was used and both required the classification of establishments as "retail garage," "maintenance garage," "service station,"

"filling station," "storage warehouse," "parking garage," and "parking lot." Six men, including three who had taken an active part in preparing the wording, tested the schedule by filling out the form for a section of a certain street. The results varied so greatly that a new set of instructions and definitions was prepared. This method can be used, not only as a preliminary check on clarity, but also as a check to see whether enumerators are well instructed and prepared to carry on the field work.

Methods of checking the data on a questionnaire and of taking the data from it should also be worked out and tested before the blank forms are finally printed. When a survey involves a wide distribution of a questionnaire, it is advisable to try it out on a small proportion of the mailing list before going to the expense of sending it to the entire list. Such trials often indicate important weaknesses that were not thought to exist, and the correction of these greatly improves the accuracy and completeness of the data when the entire list is used.

Care of Returns.—As schedules or questionnaires are received at the central office, they should be carefully and systematically filed in preparation for the next step in the work. In a large survey, this provision is particularly important in order to prevent confusion and loss.

Before describing the next step in handling field schedules and questionnaires (i.e., compiling the data from them), it is desirable to discuss the collection of data from secondary sources which is the subject of the next chapter.

Questions and Problems

1. What are the important steps that should be considered in planning a statistical survey?

2. What is meant by "defining the problem?" Illustrate by defining in detail such problems as:

a. A magazine survey to determine duplication of magazine subscriptions.
b. A house-vacancy survey.
c. A survey of dealers in a certain branded article to determine their interest in the product.

3. What precautions should be taken in collecting data as regards statistical units, homogeneity, and representativeness?

4. What are compensating errors? Cumulative errors? Why must they be considered in the collection of data?

5. Are analyses of business statistics based solely on figures or do other elements enter into such analyses?

6. Distinguish between primary and secondary data; primary and secondary sources.

7. What is meant by "sampling"?

8. Explain the principle that "a moderately large number of items chosen at random from a very large number of items should have the characteristics of the larger group."

9. Explain the following: the principle of statistical regularity; the principle of large numbers; the principle of small numbers; the principle of decreasing variation.

10. Compare random and directed sampling. Which is most often used?

11. How may one check the adequacy of a sample?

12. State and explain the requisites of a schedule. How is a schedule used?

13. State and explain the requisites of a good questionnaire.

14. (Planning, schedule making, and collection of primary data.) Assume that the class represents the statistical department of a large furniture store. The director of the statistical department receives the following request from the sales manager: "In connection with analyzing the market for furniture in our vicinity, we wish to know what the proportions of vacancies are in residential buildings by classes. Will you kindly make a field investigation (on the basis of a representative sample), which will give us a reasonably accurate idea of the proportions vacant in apartments, single-family houses, and other classes of residences? Your report should be in my hands not later than two weeks from today."

You then receive the following instructions from the director of the survey: "You have seen the letter from the sales manager requesting information on residential vacancies. Kindly prepare a detailed plan for the field work, including a schedule and a set of instructions for enumerators (similar to Exhibits 3*A* and 3*B* on pages 26 and 27), for the purpose of obtaining information on the proportions of family capacities vacant, occupied, and under construction, by the different classes—apartments, single houses, etc. After the schedule is approved, you will collect the desired field information from a sample district which will be assigned by the director of the survey. After the schedules have been filled out, assemble them and see that they are ready for editing." (This problem will be continued in Question 22 on page 74.)[1]

15. (Planning, schedule making, and collection of primary data.) Assuming that the class is the statistical department of the Royal Drug Company, the director of the department has received a letter from the president asking him which of several near-by corners has the heaviest traffic. The information is to be used in locating another store. The director of the department will have the traffic checked on certain days or at certain periods to determine the number of persons passing the various corners classified as to pedestrians and automobile traffic, as to men and women, and as to sides of the intersecting streets.

According to the above requirements prepare a detailed plan for the field work, including a schedule and a set of instructions for enumerators, for the purpose of obtaining the desired information. After the schedules have been approved, collect the desired information at the time and place assigned by the director of the survey. When the schedules have been returned to the office, see that they are assembled in preparation for editing. (This problem will be continued under Question 23 on page 75.)[1]

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[1] It is the purpose of Problems 14 and 15 on this page and Problems 22 and 23 on pp. 74 and 75 to provide actual practice in connection with the following topics discussed in Chaps. II and IV in the text: Planning a survey; first-hand personal enumeration; schedules; questionnaires; sampling; editing returns on schedules; taking of data from schedules; tabulation; preparation of final tables; checking accuracy in tables; and explanation and interpretation of tables.

CHAPTER III

COLLECTION OF DATA—SECONDARY SOURCES

Much of the work in business statistics involves the use of data which have already been collected. They must be compiled from other sources in such a way that they will be useful for the investigation in hand. In fact, many investigations are based entirely upon secondary data which are only indirectly related to the question under investigation, because the cost of gathering primary data makes the collection of figures directly related to the problem impossible.

Schedules and Forms for Secondary Data.—In gathering secondary data, it is usually best to prepare a form or schedule similar to the schedule previously described for field enumerators. Such a schedule not only facilitates the recording of the data, but it also facilitates later compilation and tabulation.

If the data are reasonably complete, this schedule may take the form of a blank table. These blank tables can be made in quantities at the central office by some duplicating process, or they can be drawn up individually in the field if only a few are needed. The use of coordinate paper facilitates the laying out of such forms and enables the enumerator to line up the data in such a way that it is easy to make computations directly from these records. (Tables are described in detail in Chapter IV, pages 63–71, inclusive.)

Tendency to Show Favorable Facts.—In most efforts to make projects successful, there is a general tendency to disregard the facts that would be unfavorable. Business men on the whole, however, wish to know both sides of the question, but it is usually much more difficult to get adequate information on the unfavorable side of the question. Though the actual statistics used in pushing a project may be very accurate, it is the common practice to use only the favorable data, and to make comparisons only on the bases that are favorable to the project.

As an illustration of the above difficulty, suppose that a New York manufacturer wishes to locate a branch at the leading Pacific Coast port. Suppose that he writes to Los Angeles, San Francisco, Seattle, and Portland and inquires regarding the standing of each. He probably would find that each one claims to be the greatest port. "When an attempt is made to answer this question, one immediately runs against the problem as to what basis should be taken for computation—whether the volume

of the commerce or the value of the commerce should be considered. Also, should one consider only foreign commerce (as do some eastern ports) or should domestic commerce also be taken into consideration? If the latter, is it a question of actual ocean commerce, or should local movements by barge or river steamers likewise be included? Instead of figuring actual commerce, should one take shipping as a basis, and make one's computations on the number of vessels entering into and departing from a port in a year, together with their net tonnage? As a matter of fact, each port has its own favorite way of figuring—depending upon which method gives it the best standing. Thus, Los Angeles figures it is the second port in America because more tonnage of ocean cargoes is handled through her harbor than through any other American port with the single exception of New York. San Francisco claims pre-eminence among Pacific ports by reason of the value of her commerce rather than on a basis of volume. Seattle's showing for first place is based on entrances and clearances of vessels through the custom house, because of the daily commerce with Vancouver, Victoria, and other British Columbia ports. Portland makes a very high showing in exports to foreign countries because of the tremendous quantities of wheat and lumber exported."[1] The situation just described is common in certain types of records of business data as it is natural to present one's case in the most favorable light. As a practical matter such presentations need not be criticized, but the business man must be on guard to see that he has adequate information on both sides of a question before drawing conclusions and making decisions.

Bogus Statistics.—The enthusiastic use of statistics to prove one side of a case is not open to criticism providing the work is honestly and accurately done, and providing the conclusions are not broader than indicated by the data. This type of work must not be confused with the unfair and dishonest use of both accurate and inaccurate data, which too commonly occurs in business. Dishonest statistical work usually takes the form of: (1) deliberate misinterpretation of data; (2) intentional making of overestimates or underestimates; and (3) biasing results by using partial data, making biased surveys, or using wrong statistical methods.

An illustration of unfair use of statistics is found in a report distributed to the public in a certain city in 1928 by a concern whose prosperity depends upon a high degree of activity in real estate trading. In order to show that people should buy in 1928, the concern forecasted a peak year in construction activity in 1929. This was "proved" by a chart which

[1] Clarence H. Matson in Los Angeles Chamber of Commerce *Bulletin* 104 (abstracted in part).

showed that peak years in building permits were five and six years apart and that it would be six years from the 1923 peak to the 1929 peak. In this chart, the curve of actual values was drawn in very lightly, while every five or six years a heavy bar was drawn in to represent peak years, whether they were there or not. As a matter of fact, a scientific study of the cycles of building activity in this particular city indicates that they are ordinarily much longer than six years in length.

The use of statistics is now being regulated to a certain extent and undoubtedly this regulation will increase. The Federal Trade Commission, the Securities and Exchange Commission, the Better Business Bureau, and various trade and civic organizations have already done much to discourage dishonest use of statistics.

Importance of Original Sources.—Ordinarily, in the use of secondary data one should select a source which is as near the original source as possible. In general, there is a tendency for reprints to be less reliable than the original tabulations, because of errors in transcription and the practice of abbreviating or summarizing the data and the explanations. The term "original source" as here used refers to the first compilation and its revisions. (It does not follow, of course, that the first compilation should be used instead of the revisions.)

It will be noted that the rule above is not stated arbitrarily. Sometimes the secondary data are more reliable than the primary data. This is true when those who are responsible for a secondary compilation check and analyze the data so carefully that errors in the original tabulation are discovered and eliminated, or important qualifications are indicated.

Important Sources of Secondary Data.—A number of the more important secondary sources of statistical information will be introduced at this point.[1] The most readily available and widely used data are those of the Federal and state governments, commercial and trade associations, trade papers and magazines, daily papers, private services, research bureaus, and yearbooks—whether published by government agencies or private organizations.

For the purposes of this discussion, the sources of general business statistics will be divided into four principal groups—namely, (1) governments, (2) commercial and trade associations, (3) trade papers and technical periodicals, and (4) private organizations. However, the line of demarcation between these groups of sources is not always well defined, as each one uses data obtained from the others.

Government Sources.—Federal, state, and city governments are important sources of statistical data which are issued currently. As a complete list of Federal and other government sources of statistics would

[1] For a more complete list, see Section II of Appendix IV to this book (pp. 693 to 712) and *Market Research Sources,* compiled by the United States Department of Commerce, published by the Government Printing Office, Washington, 1938.

require several volumes, only those of greatest general interest can be included in this discussion. The Federal Government sources will first be considered.

The Department of Commerce, Bureau of Foreign and Domestic Commerce, publishes the *Survey of Current Business* which is one of the most complete reports available on current statistics. This publication each month presents the principal statistical facts regarding the most important lines of trade and industry. Detailed tables are published in special issues, giving for each item monthly figures for the past several years, and yearly comparisons are given back to 1913 when possible. In addition to the monthly and special reports, supplements are issued every week, which give such information as has been received during the previous week. The compilation consists largely of secondary data. The object is to bring together data which are scattered in hundreds of different publications and other sources, and to make them available to business men and others in a convenient form at a very low cost.

Another important publication of the Bureau of Foreign and Domestic Commerce is *Commerce Reports*, which is a weekly survey of foreign trade and gives various data regarding business conditions in foreign countries. The *Monthly Summary of Foreign Commerce* is also an important source of data on foreign commerce. *Domestic Commerce* contains much valuable statistical information on domestic marketing. The Bureau of the Census compiles various monthly reports, such as those on coal stocks, cotton consumption, and cotton ginned, in addition to the regular biennial, decennial, and other census reports.

The Department of Agriculture publishes each month *Crops and Markets*. This is a compilation of the most recent data regarding current and prospective crop conditions.

The Department of Labor publishes the *Monthly Labor Review*, which contains special information concerning labor conditions as well as data on prices and costs of living.

The Board of Governors of the Federal Reserve System publishes each month the *Federal Reserve Bulletin*, which contains a variety of statistical information regarding financial conditions, production, stocks, shipments, and prices in different countries.

The monthly letters of the twelve Federal Reserve banks show the financial conditions of the banks in each section and present many other types of statistics which are of value in analyzing district business conditions.

Many other publications, which contain statistics of interest to business men, are published by the Federal Government. Some of them are published at regular intervals, and others are single reports presenting the results of special investigations. The Government Printing Office is the largest printing establishment in the world, and there are but few, if any, business investigations in which at least some of the government

publications would not be of value. Because of limited space, it is possible to mention only a few of the most widely used publications in this text. Lists of publications on given subjects may be obtained from the Government Printing Office or Federal departments concerned.[1]

The state governments publish various types of statistical reports. The annual reports of certain departments often contain statistical data that are valuable in particular cases, and much of the information published in these reports can be obtained currently. Not only do the various state departments publish data on their own activities, but many of them compile outside data. Instances of this are the data on weekly earnings and unemployment published by the Industrial Commission of New York, building and employment data published by the Massachusetts Department of Labor, the mineral industry reports of the State of California, and the highway traffic statistics of the state highway commissions.[1]

City governments are also important sources of statistical information. While most of the data ordinarily have to be gathered for the purpose at hand directly from the city records, many cities publish certain types of statistics at regular intervals. One of the most important of these is monthly data on building permits.

Commercial and Trade Association Reports.—The term "trade association," as here used, refers to the various organizations of business men and businesses which have been developed for the purposes of improving conditions in certain trades or lines. Many of these organizations have as one of their purposes the collection and dissemination of statistical information. Some of the important trade journals are closely connected with trade associations. Much of the information collected by trade associations is not published, although it would be of great value to business men generally in analyzing business conditions.

Some of the more important associations which publish reports currently are as shown in the table on page 39. This list is intended to be a representative one of national interest, although it does not include all the important commercial and trade sources. Many other valuable sources of this class will be found by referring to the list published in certain issues of the *Survey of Current Business.* The United States Bureau of Foreign and Domestic Commerce publishes a directory of trade associations which can be obtained from the Government Printing Office. A number of these associations compile valuable statistics that are not regularly published, but which will be furnished to interested parties under certain conditions. The United States Department of Commerce makes much of this material available through the *Survey of Current Business.*

[1] For more complete lists of Federal and state government sources, see references in footnote on p. 36.

Among the more important local associations furnishing statistics are chambers of commerce, real estate boards, and the bank clearing-house associations. Although, in many instances, chambers of commerce are essentially booster organizations, they ordinarily are able to furnish valuable statistical information concerning the community. The reliability of the data depends upon the individual chamber of commerce,

Name of association	Data	Where published	Frequeney of publication
American Iron and Steel Institute	Steel production	Trade papers	Monthly
American Paper and Pulp Association	Paper and pulp production	*Monthly Report*	Monthly
American Petroleum Institute	Gasoline consumption, etc.	*Bulletin*	
Associated General Contractors of America	Indexes of construction cost and volume	*The Constructor*	Monthly
Association of American Railroads	Car loadings	Printed statements	Monthly
Chicago Board of Trade	Wheat, corn, and oats receipts, etc.	Trade papers	Daily
Cotton Textile Institute	Cotton yarns and cloth printed	*Weekly Report*	Weekly
National Association of Real Estate Boards	Index of real estate sales	*Bulletin*	
National Board of Fire Underwriters	Fire losses	Trade magazines and newspapers	Monthly
National Electrical Manufacturers Association	Electric motors, etc.	*Nema Survey*	Monthly
Portland Cement Association	Pavement contracts awarded	*Concrete Highways*	Monthly
Rubber Manufacturers Association	Consumption of rubber	Printed reports	Monthly
United Typothetae of America	Printing activity	*Typothetae Bulletin*	Monthly

but in cases where the degree of reliability is not known, the data should be checked in some manner, if possible. Real estate boards may have vacancy data or other statistical information on real estate situations. Bank clearing associations issue reports on bank clearings. Ordinarily, one can also find many other organizations in a locality which compile statistical information. These will vary from one community to another. If the data are in existence and one does not know to what organization to go, he can first inquire where he thinks the prospects are best, and he will ordinarily be directed to the organizations which have the data.

In addition to the above, which are strictly local organizations, statewide organizations, such as the California State Chamber of Commerce,

or regional organizations, such as the New England Council, are important sources of statistical information.[1]

Trade Papers and Technical Periodicals.—Much valuable statistical information is published regularly in trade papers and technical periodicals. This is especially true for the period since the World War. Some of the more important periodicals which publish statistical data of value in analyzing current conditions are presented in the list shown above.

Publication	Typical data	When data listed are published
American Metal Market	Composite pig-iron and steel prices	First or second week of month
The Annalist	Stock sales, prices, etc.	First weekly issue of month (Fridays)
The Bond Buyer	State and municipal bond issues and yields	First weekly issue of month (Saturdays)
Coal Age	Mine price of bituminous coal	Weekly (Thursdays)
Commercial and Financial Chronicle	Cotton, visible supply	Weekly (Saturdays)
	Interest rates on call loans and commercial paper	Weekly (Saturdays)
	New corporate securities	Last weekly issue of month (Saturdays)
Electrical World	Sales of electrical energy, central stations	First weekly issue of month (Saturdays)
Engineering and Mining Journal	Prices of copper, lead, and zinc	
Engineering News-Record	Construction cost index	First weekly issue of month
Iron Age	Pig-iron production, furnaces in blast, etc.	First weekly issue of month (Thursdays)
New York Journal of Commerce	Dividend and interest payments and new incorporations	First week of month, or tenth of month
	Fire losses	Tenth of month
Northwestern Miller	Grain receipts, etc.	Weekly (Wednesdays)
Oil and Gas Journal	Gasoline prices	Weekly
Oil, Paint, and Drug Reporter	Price indexes of drugs, oil, etc.	Weekly (Mondays)
Printer's Ink	Magazine advertising	Second week of month
Railway Age	Railway-equipment orders	
Statistical Sugar Trade Journal	Sugar stocks, receipts, meltings, and Cuban statistics	Weekly (Fridays)
The Wall Street Journal	Stock and bond sales and prices	Daily

[1] For more complete information on commercial and trade association statistics, see references in footnote on p. 36.

While this is a representative list of the more important technical periodicals for general purposes, it is not complete, and many other periodicals are published which furnish valuable statistical information. Brief descriptions of these publications will be found in Crain's "Industrial Marketing."[1]

Private Organization Statistical Reports.—Many concerns are now issuing various kinds of statistical data. Some of these data are external, collected in the course of the concern's business, while others are internal, such as sales, which are often of general interest to the public. Among the more important sources of data of this type are the following:

Name of organization	Data	Where published	Frequency of publication
Aberthaw Construction Company	Building costs	Trade papers	As issued
American Appraisal Company	Construction costs	*American Appraisal News*	Monthly
Best, A. M., & Company	Insurance data	Reports	Monthly
Dodge, F. W., Corporation	Construction contracts awarded	*Dodge Statistical Research Service*	Monthly
General Motors Corporation	Sales of cars	Financial papers	Monthly
Lloyd's	World ship construction	Printed release	Quarterly
Kresge, S. S., Company	Chain store sales	Financial papers	Monthly
National Bureau of Economic Research	Purchasing power, etc.	Special reports	As issued
Polk, R. L., & Company	New passenger-car registrations	Automobile trade papers	Monthly
Sears, Roebuck & Company	Mail-order sales	Financial papers	Monthly
United States Steel Corporation	Unfilled orders, earnings, wages	Press releases	Monthly or quarterly
Woolworth, F. W., & Company	Chain-store sales	Financial papers	Monthly

The National Bureau of Economic Research listed above was formed for the purpose of carrying on "impartial investigations in the field of economic, social, and industrial science." Its statistical compilations and analyses in connection with national income, business cycles, unemployment, economic changes, and other business and economic problems are of great value to the serious student of business.

Economic and Statistical Services.—In addition to the above, there are several economic and statistical service organizations which make

[1] Published by Advertising Publications, Inc., 100 East Ohio Street, Chicago. See also references in footnote on p. 36.

it their business not only to collect but to analyze data for the general or special use of business men. Among the best known of these are: Babson's Statistical Organization; The Brookmire Economic Service; Moody's Investors Service; and Standard Statistics Company. The work of these services will be discussed in some detail in Chapter XVII on Business Forecasting.

A description of many other statistical services of general and special natures will be found in the 1938 edition of the "Handbook of Commercial and Financial Information Services" issued by the Special Libraries Association.[1] This handbook describes statistical services available from commercial sources. It includes all of the well-known services and a brief description is given of the facilities of some of the smaller agencies which are less known, but which furnish special information of value.

University Bureaus of Business Research.—Many universities have established bureaus in recent years which compile and analyze business statistics. Among the universities which have such bureaus or carry on business research are: Harvard University, University of Chicago, New York University, Northwestern University, University of Pennsylvania, University of Pittsburgh, University of Wisconsin, Leland Stanford University, Ohio State University, University of Denver, Brown University, University of Iowa, University of Michigan, University of Oregon, University of Texas, and University of Illinois.[2]

As the work of these bureaus is usually directed and carried on by highly trained staffs, it is ordinarily carefully and accurately done, and many valuable contributions have been made. Some of the results of this work are distributed gratis, but usually a charge is made. Much of this work is highly specialized and direct inquiries must be made in order to secure accurate descriptions of it.

Yearbooks and Other Reference Books.—The preceding publications are principally of a current nature. That is, they are published monthly, weekly, daily, or at irregular intervals as soon as the data are available. In addition to these, there are several yearbooks that are valuable sources of statistical information, especially that which extends back over a considerable period of time. Some of the yearbooks are of great value in that they furnish convenient sources of summarized information over long periods, and if more recent or more detailed data are required, references are given to the proper sources.

The brief description given below includes the more widely used government and private statistical yearbooks.

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[1] Further information relative to this handbook may be obtained from local libraries. See also the references in the footnote on p. 36.

[2] For a more complete list, see second reference in the footnote on p. 36.

One of the best yearbooks on American economic statistics is the *Statistical Abstract*. This publication presents a digest of statistics relating to business collected by all statistical agencies of the national government, as well as those of a considerable number of state and private agencies. It includes some 850 pages of data on area, population, immigration, public land, climate, government finances, money and banking, wealth, prices, wages, communication, electric light and power, public roads, motor vehicles, railways, commerce, farm products, fisheries, minerals, manufactures, and other subjects of interest to the business man. It is compiled by the Bureau of Foreign and Domestic Commerce, and bound volumes are sold by the Superintendent of Documents.

For many years, the *Statesman's Yearbook* (published by The Macmillan Company, New York) has been one of the best general-reference yearbooks. It covers a wide range of subjects, and is recognized as a very reliable source of information.

The well-known *World Almanac*, published by the *New York World-Telegram*, covers a wide range of subjects, with special emphasis on New York data. It is a popular yearbook because it is issued promptly on January 1 each year, and is sold at a low price. It is reasonably accurate, although it does not present a dignified appearance.

The United States Department of Agriculture's *Yearbook of Agriculture* (before 1936) and (beginning in 1936) the separate publication *Agricultural Statistics* contain several hundred pages of statistical data on farm production and prices. The yearbook can ordinarily be obtained gratis from one's senator or representative.

The annual *Minerals Yearbook* covers production, shipments, prices, etc., in the mineral industry. It is compiled by the United States Bureau of Mines, and can be obtained from the Superintendent of Documents.

Of particular interest to sales and advertising managers is the *Sales Management Reference Number*. This is a statistical report published by *Sales Management*, which presents data on United States commerce, measures of buying power, extent of the industrial market, distribution of goods, advertising appropriations, advertising media, and the foreign market.

In addition to the above yearbooks, there are many other valuable yearbooks and similar publications which may be found in well-supplied libraries.[1]

Among the reports that are issued regularly, but less frequently than yearbooks, are a number of publications compiled by the Bureau of the Census. This bureau takes a census of population every 10 years, agriculture every 5 years, and manufactures every 2 years. In the 1930

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[1] For additional information on yearbooks, see references in footnote on p. 36.

census, a census of distribution was included. A census of business was taken in 1933 and 1935. These and other reports are enumerated in "A List of Publications of the Department of Commerce" which is available, without charge, from the Superintendent of Documents.

For those who wish to make long-term studies of monthly data and analyses which involve the use of such data as daily stock prices, the Standard Statistics Company's *Base Book* is one of the most comprehensive and complete compendiums of business and financial statistics available. It contains long-term monthly statistical records and graphs of prices, production, foreign trade, consumption, stocks on hand, stock and bond price indexes (including some daily figures), classified monthly data on corporation financing, and other data of interest to business men. A number of series are presented by months as far back as 1900 and some are given for even earlier periods.

A statistical reference book of great value in making market analyses is the *Consumers Market Data Handbook*, published by the United States Bureau of Foreign and Domestic Commerce. It is probably the most complete report available covering certain types of information on consumer markets. Data are given by counties on population, manufacturing, agriculture, mining, income tax returns, telephones, electric consumers, automobile registrations, periodical circulation, number of trade outlets, number of farms, farm values, types of industry, and other items of special interest in analyzing commodity markets.

The *Market Guide*, published annually by the Editor and Publisher Company, New York, provides a helpful summary of statistical data pertaining to small towns and cities, and includes state summaries.

Market Research Sources, published biennially by the United States Department of Commerce, Bureau of Foreign and Domestic Commerce, is a guide to statistical publications that is of great value in many kinds of industrial financial market, real estate and other business analysis.[1]

Other Sources of Information.—It has been the purpose of this discussion to introduce the student to some of the more important sources which publish reports regularly. Familiarity with these will indirectly acquaint one with many other general and specialized sources which will be of value in making statistical investigations. Many other good sources of statistical information will be found by referring to the various directories of publications. Some of the most important of these directories have been mentioned in the preceding discussion.

Often an investigator has a general idea that the information for which he is searching appears in published form, but he does not know the names of the publications in which the information appears or where

[1] See second reference in footnote on p. 36.

to find them. Ordinarily, the best procedure in searching for such sources is to approach statistical organizations and others who are most likely to know where the desired information can be found, and if they do not know, they may be able to refer the investigator to someone who does. This method may involve a long series of calls—that is, the investigator first calls upon Jones, who tells him that Smith may know, and Smith, in turn, refers him to Brown, etc. However, by following up these leads, the investigator is ordinarily able to determine the source of the information, or that it is not available.

Estimation and Indirect Sources.—It often happens in business surveys that secondary data are not available and that the cost of making a complete count or even of taking a sample is prohibitive. Consequently, it may be necessary to use careful estimates instead. If the estimates are made independently and without bias by skilled estimators, they are often of as great practical use in determining policies as would be an exact count. If, for instance, in making a market analysis, it is necessary to know the distribution of the purchasing power in a given city, a good idea of the distribution can be gained by estimating the value of the occupied dwellings and (after determining by means of a sample the average relation of dwelling value to income) converting the estimated values to income figures. Such an estimation does not measure the distribution, but it may be said to indicate it. In large cities, figures are often available on school enrollment, water services, and other series which have a reasonably constant relation to the population. Consequently, population growth can be estimated from these bases. Great care must be taken, however, when making such estimates to make all necessary corrections and to make allowances for any bias or cumulative errors.

Cost is not the only factor that makes it impossible to make surveys in certain instances. It frequently happens that, as a practical matter, the information can not be secured at *any* cost. In such instances, it may be desirable to make a complete or extensive enumeration of certain basic items, or to secure the data from secondary sources if they are available, and then tie certain other data to these basic data by means of special investigations made by other skilled workers. For example, it may be impossible to secure figures directly as to the total sales made by a given class of manufacturers, though at the same time these manufacturers would give full information as to employees and pay roll. In this case, skilled investigations might readily determine average ratios between sales and pay rolls by using adequate samples, and estimates of the desired total sales figures might be made which would be sufficiently accurate for the purpose at hand.

Assembling and Presenting the Data.—After the schedules, questionnaires, and other primary or secondary records in a particular investiga-

tion have been accumulated at the central office, the next major step is to combine the data in various groups or classifications (which, in a well-planned survey have been predetermined as far as possible). That is, the data from the schedules, questionnaires, and other records must first be assembled in the form of detailed master working tables, and then the final presentation may be in the form of finished tables, running text, or graphic charts. To explain in some detail the assembly and presentation of statistical data is the objective of the next three chapters.

Questions and Problems

1. Why are secondary data often used instead of primary data?

2. What kinds of forms are used in compiling secondary data?

3. Why is it easier to find favorable data than unfavorable data?

4. When using secondary data, why must one be on guard against deliberate misrepresentation of data; intentional overestimates or underestimates; and biased results due to use of partial data, biased surveys, or wrong statistical methods?

5. When are primary data more reliable than secondary data? When are secondary data more reliable than primary?

6. What important current sources of business statistics are published by the Federal Government?

7. What types of information are published in the *Survey of Current Business?*

8. What governmental agencies other than those of the national government publish statistical information that is valuable in business analysis, and what are some examples of the data collected and published?

9. Name several associations that publish trade papers. What local bodies or organizations collect data in cities or adjacent areas?

10. Name several typical periodicals of a technical sort and mention typical data published by each.

11. What are several private organizations that collect and publish information on their own or related business activities? Where do they publish these data?

12. Name several national economic or statistical services. Where can one find information on other similar services?

13. What type of work is done by university bureaus of business research?

14. Describe several yearbooks containing economic statistics. What types of data are published?

15. When one does not know the sources of the information desired, how can he proceed?

16. When Census data on population are out of date and the cost of taking a new census is prohibitive, what figures can be used as bases upon which to make population estimates? What kind of source would this be, and what kind of count would be secured?

17. (Introduction to secondary sources.) Familiarize yourself with the statistical reports, yearbooks, magazines, journals, and services listed below. Only a general knowledge will be required at the present time.

Statistical Abstract.
Statesman's Yearbook.
Foreign Commerce Yearbook.
Monthly Labor Review.
Fifteenth Census of the United States.
Yearbook of the Department of Agriculture.
Federal Reserve Bulletin.
Survey of Current Business.
Commercial and Financial Chronicle.
Engineering News-Record.
Monthly Review—Federal Reserve Bank of your district.
Commerce Reports.
Review of Economic Statistics (Harvard).
Brookmire Economic Service.
Babson's Reports.
Standard Statistics Company's Service.
Moody's Investors Service.
Poor's Investment Service.

18. Compile figures for the United States for the last 12 years for the following:

Pig-iron production (*Iron Age*).
Bank debits, excluding New York City (*Federal Reserve Bulletin*).
Construction contracts awarded in 37 states (*Survey of Current Business*).
Unfilled orders of the United States Steel Corporation (*Survey of Current Business*).
Bureau of Labor Wholesale Price Index (*Monthly Labor Review*).
New York Stock Exchange Transactions (*Annalist*).
Prices of industrial stocks (Dow-Jones Averages in *Wall Street Journal* or other newspapers).
(Other sources and other subjects may be suggested by the instructor.)

19. Using *Market Research Sources* (United States Department of Commerce),[1] compile a list of Federal and state government, trade association, chamber of commerce, trade paper, newspaper, private statistical service, and university bureau sources of specified types of data that you believe would be helpful in analyzing the following:

a. A national market for high-grade radios.
b. A bond of a manufacturer of automobile tires having a national market.
c. Cost-of-living and wage conditions from the point of view of establishing a shoe factory in a new location.

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[1] See second reference in footnote on p. 36.

CHAPTER IV

TABULATION

After the schedules, questionnaires, or other records of original data have been assembled together at the central office in sufficient quantities, definite plans should be made for combining and arranging the data in the form of detailed working tables. Later these working tables will be revised and presented as finished tables, text, or graphs, but the preliminary detailed working tables must be compiled first to facilitate the preparation of the finished forms. Before any hand or mechanical tabulating work can proceed, however, the schedules, questionnaires, or other records must be edited, and the system of classification must be checked and reconsidered.

In broad terms, the steps in tabulation are as follows:

Editing (and coding if necessary).
Checking and reconsideration of classifications.
Sorting according to classes.
Counting or compiling.
Preparation of working tables.
Preparation of final tables.

To discuss these steps is the purpose of the present chapter.

Editing the Returns.—The first step after the schedules or questionnaires have been received at the central office is to examine them in detail for the purpose of determining whether or not they are complete and accurate as far as may be indicated by internal checks. This work of editing the returns requires a high degree of skill and care, for it involves making additions and alterations, but there must be no falsification of the original returns.

If statements are mutually contradictory, it is desirable to determine which is correct. If this cannot be done, the return will probably have to be discarded. Returns always should be corrected and used when possible, since hasty decisions to throw out doubtful returns would result in the loss of a large amount of information. When the editor enters any corrections, they should be entered in a distinctive ink which is readily distinguishable from the original. Any original data which are in error should be canceled in such a way that they can be used again if necessary. Data should never be destroyed by erasure.

PURCHASES OF THE WESTERN ELEVATOR COMPANY—1937
By Kinds of Grain

Grain	*Dollars*
Corn	22,029,288.40
Wheat	18,882,247.20
Oats	13,217,573.04
Barley	5,664,674.16
Rye	1,258,817.50
Flax	1,258,815.46
Buckwheat	629,408.24
Total	62,940,824.00

A.—Classification by commodities.

PURCHASES OF THE WESTERN ELEVATOR COMPANY—1937
Origin by States

State	*Dollars*
Iowa	12,588,164.80
Illinois	11,329,348.32
Wisconsin	9,441,123.60
Michigan	8,811,715.36
Minnesota	8,182,307.12
Missouri	6,294,082.40
Nebraska	5,035,265.92
South Dakota	1,258,816.48
Total	62,940,824.00

B.—Classification by states.

PURCHASES OF THE WESTERN ELEVATOR COMPANY—1937
By Months

Month	*Dollars*
January	3,892,560.12
February	3,087,532.08
March	2,123,567.00
April	3,002,781.02
May	1,578,832.01
June	1,206,878.00
July	1,789,862.21
August	4,987,700.04
September	9,882,830.12
October	13,078,320.04
November	12,078,506.14
December	6,231,455.22
Total	62,940,824.00

C.—Classification by month of purchase.

EXHIBIT 5.—Three different classifications of one concern's purchase data.

If data are to be segregated by descriptive classes or class intervals, it is always necessary in machine tabulation, and sometimes desirable in hand tabulation, to give these items or classifications numerical designations. This work, known as coding, is usually carried on in conjunction with editing. Since an understanding of coding depends to a certain extent upon a knowledge of machine tabulation, further study of this subject will be reserved until after some consideration has been given to machine methods.

Final Classification.—In a well-balanced statistical investigation, the scheme of classification is commonly determined as far as possible before the data are gathered. However, a complete classification can seldom be worked out for working tables until after the data have been collected and edited, or for final tables until after the working tables have been

OFFICE SPACE RENTAL RATES

RENTAL PER SQ. FT. PER YEAR	TALLY	TOTAL
$ 1.00-$ 1.49	//	2
$ 1.50-$ 1.99	~~////~~ ///	8
$2.00-$2.49	~~////~~ ~~////~~ ///	13
$2.50-$2.99	~~////~~ ~~////~~ ~~////~~ ~~////~~ ////	24
$3.00-$3.49	~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ /	31
$3.50-$3.99	~~////~~ ~~////~~ ~~////~~ ///	18
$4.00-$4.49	~~////~~ ~~////~~	10
$4.50-$4.99	~~////~~ //	7
$5.00-$5.49	/	1

EXHIBIT 6.—A tally sheet.

finished. It is often necessary to collect the original data in great detail in order that the final classification may be determined in a satisfactory manner. Then the individual items can be grouped systematically according to their common characteristics, and the relationships between items and groups can be brought out by the arrangement.

Often the same data are classified in a number of different ways. Exhibit 5 illustrates three different classifications of the purchases of the Western Elevator Company in 1937. The subject of classification is discussed in greater detail in Chapter VII on Statistical Distributions.

Hand and Machine Tabulation.—Ordinarily in a statistical survey the decision will have been made, before the field work has been started, as to whether hand or machine tabulation methods will be used. It is important that this decision be made in drawing up the original plans of procedure because the different tabulation methods usually require

different kinds of schedules, questionnaires, etc., to obtain the greatest efficiency in sorting and tabulating.[1]

In hand tabulation, the schedules or questionnaires are sorted by hand into groups according to the smallest classification break-down required. Then the necessary counts and additions are made (using an adding machine when feasible) and the results are entered on the master working table. The work of counting or "tallying" may be facilitated by the use of the tallying sheet, illustrated in Exhibit 6.

In machine tabulation, the original data from the schedules or questionnaires are punched on cards. These cards are sorted by machine according to the desired classifications, then the counts or additions are made by machine and the results are automatically recorded in tabular form.

The procedure in both hand and machine tabulation will now be illustrated in detail by describing the steps necessary in sorting and compiling the occupancy survey data by census tracts gathered on the schedule illustrated in Exhibit 3*A* on page 26.

Hand Tabulation.—Referring to Exhibit 3*A* on page 26, it will be noted that the census-tract numbers appear on the schedule cards. Let us assume that the editing work has been completed and that the census-tract numbers have been checked. Also let us assume that the cards have been filled out by postmen and that they still are arranged according to postmen's routes, which, of course, may cut across several census tracts.[2] The first step is to sort the schedule cards according to census tracts. If the tract numbers have been carefully entered on the cards, this sorting is a routine matter, yet it requires considerable care if errors are to be prevented.

The next step is the compiling of the various classes of data according to each tract. Using an adding machine, the figures are added directly from the original field cards and the totals are recorded on a master working table, such as the one illustrated in Exhibit 7, on the following page. That is, one would take the pack of cards for one tract and add up the items for, say, number of units occupied in single houses, check the adding machine tape back with the cards, and record the total in the proper column opposite the proper tract number on this working table. Then one would follow the same procedure for the next item in the single family house column (units vacant) and so on. The adding machine tapes should be kept in order and when a tract has been completed the entries that have been made on the master working table should

[1] The terms "tabulating" and "tabulation" when used in this sense should not be confused with the same terms when used in connection with the making of tables.

[2] A census tract is a relatively small area usually including, in the larger cities, a population of from roughly 2,000 to 10,000.

District	Tract Number	Population 1930	Single Houses								
			Number of Units				Percentage of Units				
			Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Number of Extra Families
Northwest	1										
	3										
	7										
	9										
	11										
Grant	2										
	10										
	12										
	13										
	14										
West Park	4										
	5										
	6										
	8										
	16										
Cedar Grove	15										
	17										
	18										
	19										
	20										

District	Tract Number	Horizontal Double Houses								
		Number of Units				Percentage of Units				
		Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Number of Extra Families
Northwest	1									
	3									
	7									
	9									
	11									
Grant	2									
	10									
	12									
	13									
	14									
West Park	4									
	5									
	6									
	8									
	16									
Cedar Grove	15									
	17									
	18									
	19									
	20									

District	Tract Number	Row Houses								
		Number of Units				Percentage of Units				
		Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Number of Extra Families
Northwest	1									
	3									
	7									
	9									
	11									
Grant	2									
	10									
	12									
	13									
	14									
West Park	4									
	5									
	6									
	8									
	16									
Cedar Grove	15									
	17									
	18									
	19									
	20									

District	Tract Number	Vertical 2-Family Houses								
		Number of Units				Percentage of Units				
		Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Number of Extra Families
Northwest	1									
	3									
	7									
	9									
	11									
Grant	2									
	10									
	12									
	13									
	14									
West Park	4									
	5									
	6									
	8									
	16									
Cedar Grove	15									
	17									
	18									
	19									
	20									

District	Tract Number	Flats								
		Number of Units				Percentage of Units				
		Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Number of Extra Families
Northwest	1									
	3									
	7									
	9									
	11									
Grant	2									
	10									
	12									
	13									
	14									
West Park	4									
	5									
	6									
	8									
	16									
Cedar Grove	15									
	17									
	18									
	19									
	20									

District	Tract Number	Apartments								
		Number of Units				Percentage of Units				
		Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Number of Extra Families
Northwest	1									
	3									
	7									
	9									
	11									
Grant	2									
	10									
	12									
	13									
	14									
West Park	4									
	5									
	6									
	8									
	16									
Cedar Grove	15									
	17									
	18									
	19									
	20									

District	Tract Number	Quarters over Stores								
		Number of Units				Percentage of Units				
		Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Number of Extra Families
Northwest	1									
	3									
	7									
	9									
	11									
Grant	2									
	10									
	12									
	13									
	14									
West Park	4									
	5									
	6									
	8									
	16									
Cedar Grove	15									
	17									
	18									
	19									
	20									

District	Tract Number	Total of All Types								
		Number of Units				Percentage of Units				
		Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Occupied	Vacant	Under Construction	Total Occupied, Vacant, & Under Construction	Number of Extra Families
Northwest	1									
	3									
	7									
	9									
	11									
Grant	2									
	10									
	12									
	13									
	14									
West Park	4									
	5									
	6									
	8									
	16									
Cedar Grove	15									
	17									
	18									
	19									
	20									

Exhibit 7.—Master working table for tabulating data from the schedule of Exhibit 3*A* on page 26. (Size reduced two-thirds.) (*Courtesy, United States Department of Commerce and National Conference on Construction.*)

again be checked with the totals on the tapes. Other orders of procedure may be followed, and additions may be made without machines or by machines without tapes, but whatever the methods may be, they must provide for the foregoing entries and checks.

Considerable care should be taken in drawing up the master working table form. All of the classifications to be used should be included, and the tract numbers should be arranged in geographical groups for convenience in use (see Exhibit 7). The sheet should be as large as is necessary for properly recording the data—it may have to be two or three feet long in order to accommodate the necessary classifications. It will be noted that the table in Exhibit 7 provides space for entering percentages which are to be computed after the counts have been entered.

In arranging the census tracts on tables, it is advisable to group them so they can be combined readily into larger statistical districts. It may be advisable, for example, to combine, say, one hundred census tracts into, say, ten statistical districts. If the census tracts can be combined so that each group is reasonably homogeneous, it will be more satisfactory from certain points of view to have a city divided into, for instance, ten parts rather than one hundred. The original data should be recorded by small tracts, however, so that combinations can be made in accordance with the various requirements, and so that the more detailed types of studies can be made of the developments and distributions within the city.

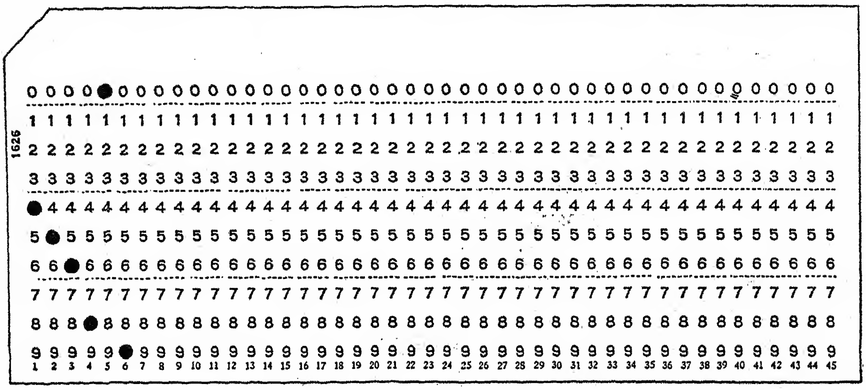
After the number of units occupied, vacant, and under construction, and the number of extra families, have been compiled, by different types of buildings, and recorded on the master working table, the totals of the different unit items for the city can be found by adding the columns vertically, and the total units by types of buildings can be determined by adding the unit items horizontally. The percentages occupied, vacant, and under construction, can now also be computed and entered in the proper places on the master working table. The base for these percentage computations should ordinarily be the total of all units occupied, vacant, and under construction.

After the master working table has been completed, it can be broken down or rearranged into whatever final tables will best meet the requirements of the survey.

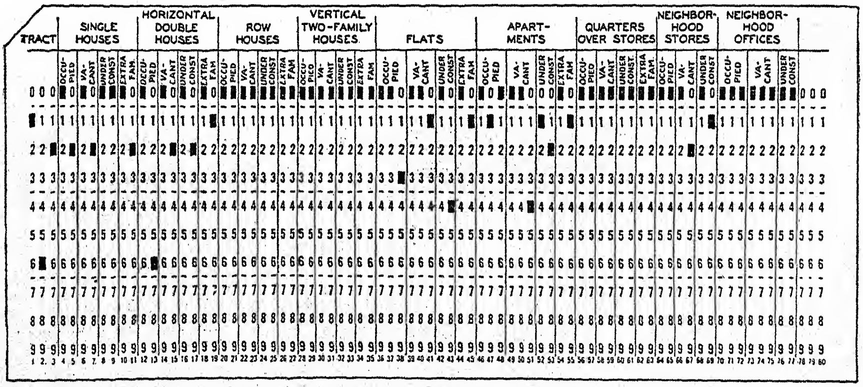
Machine Tabulation (Occupancy Survey Example).—The problem of sorting and tabulating the data from the occupancy survey schedule shown in Exhibit 3*A* on page 26, which was used to illustrate hand tabulation, will now be used to illustrate machine tabulation.

Mechanical sorting and tabulating equipment and supplies are furnished by two concerns: The Electric Bookkeeping and Accounting Machine Division of the International Business Machines Corporation,

and the Powers Accounting Machine Division of Remington Rand Inc. The Machines of the former company are commonly referred to as "I.B.M." or "Hollerith" machines and those of the latter are referred to as "Powers" machines. Each set of equipment consists of one or more key punches, a sorting machine, and a tabulating machine.



A.—A plain 45-column tabulating card punched to represent the number 456,809.



B.—An 80-column occupancy survey tabulating card punched to represent the data shown on the enumerator's field schedule presented in Exhibit 3*A* on page 26.

Exhibit 8.—Examples of tabulating cards (full size $3\frac{1}{4} \times 7\frac{3}{8}$ inches). (*A*, *courtesy, International Business Machines Corporation; B, courtesy United States Department of Commerce and National Conference on Construction.*)

The basis of mechanical sorting and tabulating is a card known as a "tabulating card," "punch card," or "code card" (see Exhibit 8), to which the data are transferred from the field forms by punching holes. The position of a hole in the card represents a figure. The hole makes it possible for an electrical contact to be made in the case of the I.B.M. equipment or a mechanical plunger to pass through the card in the case of the Powers equipment, and on the basis of the position of the contact or plunger, the cards can be sorted and tabulated mechanically.

Tabulating cards are of uniform size, namely, $3\frac{1}{4} \times 7\frac{3}{8}$ inches, and those in common use have 45, 80, or 90 columns of digits. Blank tabulating cards can be secured upon which there are no captions of any kind and upon which there are no lines ruled between the columns of numbers (see Part *A*, Exhibit 8). This basic card can be adapted to the particular purpose at hand by inserting column captions for groups of columns as desired. In large jobs, the captions and rulings are usually printed on the card. The headings for an occupancy survey tabulating card corresponding to the field schedule illustrated in Exhibit 3*A* are shown in Part *B* of Exhibit 8.

Any number can be recorded in any group of columns by punching (by means of a "key punch") the successive digits in the columns that correspond with the positions of these digits in the written number. For example, if the number 456,809 were to be placed on the tabulating card in Part *A* of Exhibit 8, the digit 4 would be punched in the first column by means of the key punch, the digit 5 would be punched in the second column, the digit 6 in the third, the 8 in the fourth, the 0 in the fifth, and the 9 in the sixth column as illustrated.

The numbers punched upon the code card in Part *B* of Exhibit 8 give all of the essential information recorded on the field schedule shown in Exhibit 3*A*, page 26. On this card the tract number (162) has been punched in the first three columns, the number of single houses occupied (2) has been punched in the columns which are so designated, and so on. (It will be noted that zeros are punched ahead of the figures when they are not large enough to use all of the columns allotted. This is done to move the key punch into position for the succeeding columns and to indicate that the column was not passed in error.)

While the field schedule shown in Exhibit 3*A*, on page 26, was designed to facilitate sorting by tracts in hand tabulation, it is convenient and otherwise satisfactory for machine methods. However, when it is planned to use machine tabulation, it is often preferable to use a larger schedule, which, in the case under consideration, would provide one line for one side of a block instead of a whole card. The headings on such a schedule would be similar to those on the master working table of Exhibit 7, omitting, of course, the percentage and total columns under each major heading. Since one line on such a schedule provides the data for one punch card, no difficulty is encountered if a schedule includes data for more than one tract, since the mechanical sort of the punch cards is for the same purpose as the hand sort of the schedule cards.

After the tabulating cards have been punched, the next step is to sort them according to the desired classifications. The "sorting machine" sorts the cards according to the holes punched in one column only at a sorting. To sort 5,000 cards by 275 tracts, for example, the machine

DATE		INVOICE NUMBER	CUSTOMER			CREDIT RATING	YEARS AS CUS-TOMER	BRANCH	SALES-MAN	YEARS AS SALES-MAN	QUAN-TITY	UNIT	COMMODITY	NET SALES PRICE	COST
DAY	MONTH		NAME	TOWN	STATE										
05	06	12418	107	052	03	4	7	1	12	8	0134	1	1216	006432	004717
5	June	12418	J.R.Brown&Co.	Phila.	Pa.	100,000	12.5	Phila.	T.H.Little	24.2	134	lbs.	castings	$64.32	$47.17
06	06	12419	189	093	03	0	4	1	38	7	0005	8	4521	022800	018000
6	June	12419	Smith&Co.	York	Pa.	5000	4	Phila	J.Stone	14	5	tons	4" iron pipe	$228.00	$180.00
07	06	12420	304	076	02	1	6	2	42	5	0100	1	0871	000800	000754
7	June	12420	Black&Co.	Albany	N.Y.	10,000	7	N.Y.	D.Wells	6	100	lbs.	No. 6 wire	$8.00	$7.54

EXHIBIT 9.—An illustration of coding procedure as applied to a special record of individual sales compiled from invoices, cost sheets, and other sources. (The lightweight figures are in red on the original.) Once the principles of coding illustrated above are understood, the intermediate form might not need to be prepared, as the invoices, with the extra information added to them, could be used directly by a specially trained punch operator. But, in the more complicated problems, information should be recorded and coded on a field schedule or intermediate form meeting the general requirements of the one illustrated above.

will be set to sort the entire 5,000 cards according to the third digit (reading from the left) under "tract." When this sort has been made, the cards will be taken in order from the compartments of the sorter and re-sorted according to the holes punched in the second column. Next, the cards will again be taken in order from the machine and re-sorted according to the holes punched in the first column on the left. The result of this operation is an arrangement of the cards by tract numbers in sequence, which constitutes a complete sort. The procedure will be described in greater detail in connection with the example in Exhibit 9.

When the cards have been sorted, they are run through the tabulating machine, which automatically adds the numbers punched on the cards. Two types of tabulating machines are available: One both tabulates and records the numbers punched; the other only tabulates without recording the numbers and the totals are read from the machine. If recorded totals of each of the columns under "single houses" are desired for each tract, for instance, the recording tabulating machine will be set to add and record the information in all of these columns. In one operation the numbers, represented by the holes punched in the columns headed "occupied," "vacant," "under construction," and "extra families," can be tabulated to give a subtotal for each tract and a grand total for each building classification.

Machine Tabulation (Sales Record Example).—In sorting and tabulating the occupancy survey data as described in the preceding paragraphs, the machine method has no general advantages over the hand method. This is due to the fact that only one sort is required, that by tracts, and that this sort is largely provided by the field-survey procedure. That is, with the field cards arranged by postmen's routes, it is a relatively simple matter to hand sort them into tract groups. And to add up the counts on an adding machine is a simpler process than punching the cards. But if the data are to be tabulated with the groups arranged to show different relationships to each other, requiring several sorts of the same data, then the machine method has a great advantage over the hand method, for once the data are punched on the cards, they can be sorted and tabulated mechanically in as many combinations as are desired. To illustrate such a problem, the procedure in tabulating the sales data in Exhibit 9 will be described.

For purposes of illustration let us assume that the desired compilations include the following classes of data:

Sales by commodities by customers by months.
Sales by commodities by salesmen by months.
Sales by commodities by size of sale by months.
Sales by salesmen by size of sale by months.

Other combinations would also be compiled but the above are sufficient to illustrate the procedure when the same original entry is used in several different relationships.

Coding.—A punch card with headings corresponding to the captions of the table in Exhibit 9, and with the proper number of columns assigned to the different items, is presented in Exhibit 10 on page 60. But before the tabulating card can be punched, it is necessary to "code" the sales data shown in Exhibit 9. That is, the non-numerical data on the record must be given numbers and the manner of punching the numerical data must be indicated. This has been done in Exhibit 9 by entering code numbers on each of the light lines to represent the corresponding sales items on the heavy lines below.

Coding is done according to a definite set of instructions. For instance, the instructions to the coder for filling in the code numbers on Exhibit 9 included the following:

On the light lines on the schedule, above the lines on which the entries have been made, place the proper code numbers in red for all items. In the following instruction list the items under the caption "Headings" refer to both the sales record sheet and the punch card, and the entries under "Punch-card columns" refer to numbers at the bottom of the punch card.

Headings	Punch-card columns	Instructions
Day	1–2	Code day of month directly according to date.
Month	3–4	Code as follows: 01 Jan. 04 April 07 July 10 Oct. 02 Feb. 05 May 08 Aug. 11 Nov. 03 Mar. 06 June 09 Sept. 12 Dec.
Invoice number	5–9	Code invoice number directly.
Customer	10–12	Look up code number in alphabetical card file A-106.
City	13–15	Look up town or city number in alphabetical list TS-16.
State	16–17	Look up State number in alphabetical list TS-16.
Credit rating	18	Code as follows: 0 under–$ 5,000. 5 $ 200,000– 500,000. 1 $ 5,000– 20,000. 6 500,000–1,000,000. 2 20,000– 50,000. 7 1,000,000–2,000,000. 3 50,000– 100,000. 8 2,000,000–5,000,000. 4 100,000– 200,000. 9 5,000,000 and over.
Years as customer	19	Code as follows: 0 under .5 year 5 5–7 years 1 .5–1 year 6 7–10 years 2 1–2 years 7 10–15 years 3 2–3 years 8 15–25 years 4 3–5 years 9 25 years and over

Similar instructions are given for the rest of the items to be coded.

On the basis of the instruction list the coder proceeds to code the sales data entered on the heavy lines in Exhibit 9, as shown on the light lines above each entry. The fifth day of June is coded as 05 under the first heading and 06 under the second heading. As indicated previously, 0 is punched in the columns when the figure is not large enough to require their use.[1]

According to the above set of coding instructions, the invoice number is re-entered on the code line (see Exhibit 9). This procedure is followed here to simplify the illustration, but it is feasible only when the work of transferring the figure to the code line is not great and it is desired to simplify the work of the punch operator. When a considerable number of large figures are to be punched, some other procedure is followed in order to avoid unnecessary copying, such as encircling the figures to be punched with the same colored pencil as used in entering the code numbers.

To code the name and address of the customer the coder turns to the supplemental code lists referred to and finds that the code number for J. R. Brown & Co. is 107, that for Philadelphia is 52, and that for Pennsylvania is 3, and these numbers are entered accordingly (Exhibit 9).

Obviously, the credit rating could be punched directly in dollars as entered on the sales record but to do so would require several columns. By grouping the information into classes, as listed in the foregoing instructions, the groups can be coded, and sufficient information can be obtained by punching only one column. This method greatly increases the efficiency of coding, punching, sorting, and tabulating, and it is also often used to save space when there is difficulty in placing the desired information on one card. A similar method is used in coding the number of years that the purchaser has been a customer.

Sometimes the work of editing and coding is combined, and sometimes the work is divided among several persons, the more complicated editing and coding being done by the more experienced and skillful workers. On simple schedules the work of coding can sometimes be omitted, and the information can be punched directly by a specially trained operator.

Punching from Coded Records.—After the information on a schedule, questionnaire or other record has been coded and checked, the next step is to punch the information on the tabulating cards. The record in Exhibit 9 is designed so that the information on one line is transferred to one card as previously indicated. The punch operator, therefore, simply punches a card for each coded line. That is, the information on the top line of Exhibit 9 is punched on the card in Exhibit 10. It is obviously important, in securing the greatest efficiency in punching, to have the

[1] By over-punching November and December in the margin of the card, it is possible to punch 12 months in one column.

items on the record and on the punch card arranged in exactly the same order (see Exhibits 9 and 10).

It should be appreciated that Exhibit 9 has been designed to provide a simple illustration of the general procedure in coding, and that in actual practice, since nearly all of the information in this particular example is taken from the invoices, it would ordinarily be feasible to code the extra information directly on the invoice sheets and have a specially trained operator punch directly from these invoices. As a general rule, in the more complicated surveys, however, the information should be

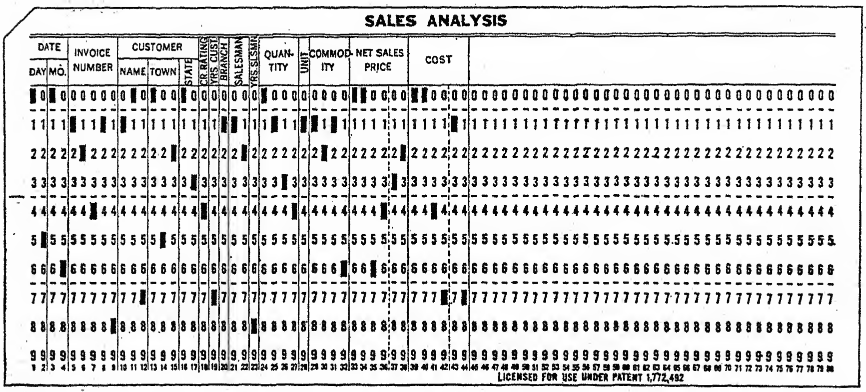


EXHIBIT 10.—Tabulating card punched to represent the data on the top line of the sales analysis record of Exhibit 9. (*Courtesy, International Business Machines Corporation.*)

recorded and coded on a field schedule or intermediate form meeting the general requirements of Exhibit 9.

Sorting and Tabulating the Sales Record Data of Exhibit 9.—After punching the code cards, the next step is sorting according to the classifications by which the information is desired. In order to illustrate the sorting procedure, let us refer again to the four classes listed on page 57. As all four of the classes are by months the first sort would be by months. As three of the classes are by commodities, the next step would be to break down each month by commodities. Then each commodity group for each month would be sorted by customers for the first class, salesmen for the second class, etc.

In order that we may appreciate more clearly the details of the sorting operation, let us consider again the first class mentioned above, viz., sales by commodities by customers by months. Suppose that we wish to sort the cards for bronze castings for October by customers. All of these cards have been punched 0 in the month column and 1,216 under the commodity heading, and the problem is to group all like numbers together that represent customers. As previously mentioned, the machine sorts according to the holes punched in one column only at a sorting. It may

help to understand the principle of the sorting operation if we suppose that we wish to sort all of the sales cards for J. R. Brown & Co. (customer 107) from the entire set of cards for the year. This sorting might be done as follows: First, set the machine to sort all of the cards according to the first digit on the left. Then take the cards from the compartments of the sorter, and re-sort the cards punched 1 in the first column according to the holes punched in the second column. Then re-sort the cards having 0 punched in the second column according to the holes punched in the third column, and those in the compartment of the machine for the figure 7 will be all of the cards for customer 107.

Now, let us suppose that instead of selecting the cards for a single customer, we proceed to sort the entire pack of cards for bronze castings by customers. The reader is undoubtedly familiar with the usual procedure in hand sorting, and if he were to sort these cards, according to 3-digit numbers, by hand, he would ordinarily sort first according to the figures in the hundreds place, then he would sort each pack separately according to the tens place, and then each of the new packs according to the units place. Likewise, by the machine method, the sorter could be set to sort the cards according to the first digit of the 3-digit code. Then the cards could be taken from the compartments of the sorter and each pack re-sorted according to the next column on the right. Then each of these sorts, in turn, could be re-sorted according to the third digit, which would complete the sorting by individual customers. This procedure, however, would require 111 runs for a completely used 3-digit code, whereas by a less familiar procedure it is possible to sort the entire pack in 3 runs.

To sort the entire pack of bronze castings cards by customers in 3 runs, the procedure is as follows: First, sort on the third digit (counting from the left). Then keeping the cards in order as they are taken from the pockets and placing them in the machine to be run in ascending order, sort on the second digit. Now all of the 1's in the third place will be arranged according to the sequence of the second digit. Then repeat the process for the third digit, and the entire pack will be arranged according to the sequence of the customer code numbers at the end of the third run.

After the cards have been sorted in the above manner according to sales by commodities by customers by months, and tabulated, the process is repeated for sales by commodities by salesmen by months, and for sales by commodities by size of sale by months. Then to sort by sales by salesmen by size of sale by months, the commodity break-down is ignored and each month is sorted by salesmen and then by size of sale.

The above brief descriptions of a few of the sorting operations involved in compiling the data from the sales record of Exhibit 9 are believed to

be sufficient to illustrate the basic principles of the sorting operation. Sorting instructions, however, as prepared for the machine operator, should be complete for a given survey and cover all of the sorts that are to be made.

To perform the counting or adding operations required, the packs of cards are run through the tabulating machine and the counts or additions are automatically made, as was described previously in connection with Part *B* of Exhibit 8. The printing tabulator can be set to add all of the columns of data at the same time, and to record automatically the subtotals each time the customer's number, for instance, changes. At the end of a run, it will record the grand total. The machine can also be set so that instead of printing, for example, only the totals of each customer's purchases, it will list each individual purchase and then record the total for each customer. The non-printing tabulator operates similarly, stopping when the customer's number changes so the operator can copy the totals on his tabulation sheet from the dials.

Formal Presentation of the Data.—After the data have been assembled in a preliminary way in the form of detailed working tables, the question arises as to the forms in which the final or finished presentations should be made. Ordinarily the methods of presenting statistical data may be grouped into three broad classes—textual, tabular, and graphic. The method or combination of methods to be used depends upon the purpose to be served. The same data may be presented in the three ways and from different points of view.[1]

The textual method refers to the presentation of data in paragraph form. Textual discussion frequently accompanies a statistical table or graph for the purpose of emphasizing the outstanding points. When the data are introduced into textual discussion, they are used as required and often are not arranged in any systematic order. The following is an example of textual presentation:

> The indicated domestic demand for motor fuel in December 1937 was 39,457,000 barrels, or a daily average of 1,273,000 barrels. This demand for December 1937 was 8 per cent larger than for the same month in 1936. Stocks of gasoline increased 6,362,000 barrels in December, reaching a total of 76,990,000 barrels at the close of 1937, compared with 62,914,000 on hand at the close of 1936.

Presentation by tabular methods will be the subject of the remainder of the present chapter, and presentation by graphic methods will be the subject of the two following chapters.

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[1] At this point, the discussion is confined to the technical methods of presenting numerical data. For suggestions on how to prepare a statistical report, see Appendix V, pp. 713 to 717, inclusive.

Tables.—Ordinarily in a statistical survey there will be several working tables to every one that is prepared for formal presentation. This discussion, however, will cover chiefly the principles of making finished tables, for the reason that if one knows how to make a finished table he will have no difficulty in applying whatever principles are necessary to the making of working tables.

Tables are constructed in various ways, depending upon the use to which they are to be put, and, therefore, it is very important, before beginning the construction of any one in particular, to decide as definitely as possible upon the purpose of the table.

General- and Special-purpose Tables.—When determining the purpose of a table, a distinction must be made between general-purpose tables and special-purpose tables. General-purpose tables are ordinarily a matter of record, and are designed to bring together a wide range of data regarding a certain subject. Special-purpose tables are usually drawn up to aid in statistical analysis, or to show the results of statistical analysis, by throwing into relief the relationships which are of special significance. Summary tables are usually special-purpose tables. The large detailed tables in the Federal Census records are general-purpose tables. When such data are used, they are ordinarily taken from the general-purpose tables and presented as special-purpose tables, which emphasize the relations the user wishes to stress.

Parts of a Table.—Making a good statistical table involves a thorough understanding of the details of developing each part so that it will be clear in presentation, efficient in analysis, economical in space, and attractive in appearance. With this purpose in mind, the title, captions, stubs, ruling, spacing, arrangement of items, emphasis, and footnotes will now be considered in detail.

The Title.—It is the purpose of the title to indicate the subject and scope of the table quickly and clearly. Ordinarily the title should tell *what* is represented and *where* and *when* the data apply. The foremost requirement of a title is that it be clear. The next most important consideration is that it be short. Clearness, however, should not be sacrificed for the sake of brevity. Long titles are not so easy to read as short titles, but sometimes it is necessary to use a long title to make a meaning clear to the reader. The title should be very carefully worded so that it will permit only one interpretation. Sometimes, when a title would be very long, it is best to present it as a short main title with a subtitle which supplements the main title.

Titles are commonly in the form of a series of phrases rather than complete sentences (see Exhibit 11). Certain elements of the table may be emphasized in the title when they are of particular importance in relation to the special purpose of the table.

The title lettering should be the largest and most prominent of any lettering on the table when it is hand lettered or printed with type. If

Major title
Subtitle
Reference to source
Designation of units
Column Caption
Stub items or stubs
Stub column or stub
Footnote

NEW YORK STATE SALES OF
AMAZON INDIA RUBBER CORPORATION – 1937
TIRES AND OVERSHOES BY MONTHS
FROM ANNUAL REPORTS OF THE CORPORATION
IN DOLLARS

MONTH	TIRES	OVERSHOES			TOTAL (TIRES AND OVERSHOES)
		MEN'S	WOMEN'S	TOTAL	
JANUARY	8,428	2,248	1,892	4,140	12,568
FEBRUARY	6,534	2,521	1,682	4,203	10,737
MARCH	7,821	1,890	626	2,516	10,337
APRIL	10,845	1,089	542	1,631	12,476
MAY	9,832	861	504	1,365	11,197
JUNE	11,016	808	283	1,091	12,107
JULY	12,526	2,234	1,842	4,076	16,602
AUGUST	11,654	8,636	8,721	17,357	29,011
SEPTEMBER	10,859	14,508	10,324	24,832	35,691
OCTOBER	9,834	8,640	4,852*	13,492*	23,326
NOVEMBER	8,728	4,337	1,686	6,023	14,751
DECEMBER	6,863	2,864	1,842	4,706	11,569
TOTAL	114,940	50,636	34,796*	85,432*	200,372

*INCLUDES A SPECIAL ORDER FOR $650.00 WORTH OF TENNIS SHOES

EXHIBIT 11.—The parts of a table.

NEW YORK STATE SALES OF

AMAZON INDIA RUBBER CORPORATION - 1937

Tires and Overshoes by Months

From Annual Reports of the Corporation

(In dollars)

Month	Tires	Overshoes			Total tires and overshoes
		Men's	Women's	Total	
January	8,428	2,248	1,892	4,140	12,568
February	6,534	2,521	1,682	4,203	10,737
March	7,821	1,890	626	2,516	10,337
April	10,845	1,089	542	1,631	12,476
May	9,832	861	504	1,365	11,197
June	11,016	808	283	1,091	12,107
July	12,526	2,234	1,842	4,076	16,602
August	11,654	8,636	8,721	17,357	29,011
September	10,859	14,508	10,324	24,832	35,691
October	9,834	8,640	4,852*	13,492*	23,326
November	8,728	4,337	1,686	6,023	14,751
December	6,863	2,864	1,842	4,706	11,569
Total	114,940	50,636	34,796*	85,432*	200,372

*Includes a special order for $650.00 worth of tennis shoes.

EXHIBIT 12.—The data of Exhibit 11 set up in typewritten form. (Note that proper spacing and a small amount of underlining makes ruling and boxing unnecessary on such tables.)

the lettering is made on the typewriter, title lettering can be emphasized by underlining. It is the common custom to present titles in capital letters, although it is not incorrect to use lower case letters, capitalizing

only the important words. Emphasis in the title by size of lettering is illustrated in Exhibit 11, and a method of distinguishing between the main title and the subtitle when using the typewriter is indicated in Exhibit 12. The clearest titles are those which are arranged as in Exhibit 11 or 12. It will be noted that the arrangement of the phrases in this title is such that they stand out and are easy to read. The phrases are not divided at the ends of lines, but are centered, and each line represents a definite idea or group of ideas. The titles to many published tables, however, are not arranged in this way, the lines being broken without any regard for the breaking of ideas at the ends of lines. Titles made according to the second method are not so easy to read as those of the first method, but they are widely used because of the time and space saved.

The preceding discussion applies to titles that are made up for final tables, such as those prepared for publication. However, in practical work, for every final table there are many working tables on which the titles and other means of identification need only to be clear to those who are concerned with using them. The point should be emphasized, nevertheless, that the titles should be complete and clear even if they are not in finished form. Probably the most important item to put on working tables is the date of preparation. This enables one to retrace his procedure chronologically, and it is of great assistance in relating different pieces of work to each other and in determining whether one table supersedes another.

Captions or Columnar Headings, and Definitions of Units.—At the top of each column in a table is a designation explaining what the column represents. This is called a "caption" (see Exhibit 11). If the different columns are expressed in different units, the definitions of the units should be included with the captions. If the unit of measurement is the same throughout the table, it is usually described at the top of the table, as in Exhibit 11. The statement of the unit should be unmistakable, as "height in feet," "price in dollars," etc. The expressions of the unit should be accompanied by all necessary explanations. If such explanations are long or involved, they may be given in a footnote. When a number of digits have been dropped, ordinarily this fact should also be explained in the caption at the head of the column, or at the top of the table as illustrated in Exhibit 13. An explanation such as "In millions of dollars" or "000,000 omitted" shows that six places have been dropped from the values represented.

The captions may be shown in smaller letters than the main part of the table in order to save space. In general, the space for each column should be determined by the width of the largest numbers to be represented, and the captions should be made to fit, even though it may be necessary to divide words and make several lines in the captions.

Sometimes a simple caption, such as the one at the top of the first column of figures in Exhibit 11, is all that is necessary. In other instances, a more complicated caption, with part of the caption being common to more than one column, is more satisfactory, as shown in the second, third, and fourth columns of figures in Exhibit 11. Note in the typewritten table of Exhibit 12 that the relation of major to minor column captions is made clear by indicating with underlining.

Obviously, the proper relation of the title to the captions and stubs should be maintained. If the title is expressed in detail, the captions may be simple. And when a short title is used, it may be necessary to express the captions and stubs in greater detail.

Stubs or Row Designations.—Each horizontal row of data in a table is designated as illustrated in Exhibit 11. The designations of the rows are commonly termed "stubs" or "stub items," and the complete column of these designations is known as the "stub." It will be noted in Exhibit 11 that the stub column also bears a caption. The stub items should be complete and clear. Any irregularity should be explained in a footnote if there is not room for a clear statement in the stub.

The lengths of the stub items determine the width of the stub column. If possible, the items should be condensed so that they can be included in one line. In published tables, the stub items are usually set in the same size type as the numerical data.

Ruling and Spacing.—A rough draft of the table should ordinarily be made before the figures are entered permanently. This is necessary in order that the table may be made to fit a sheet or a certain space, and in order that the proper widths can be determined for the columns. Space must also be allowed for footnotes, references, or any other matter which is to be included in the table.

When tables are drawn by hand, and sometimes when they are set up in type, the columns and headings are separated by various kinds of rulings. Custom varies in the use of rulings, but in all cases the major divisions are separated with heavier or double rulings, whereas the minor divisions are set off with light or single rulings. In many published tables, and in tables made with a typewriter, spacing is done in such a way that vertical ruling is unnecessary (see Exhibit 12). Wide spaces take the place of heavy rulings and narrow spaces are used instead of the light rulings; that is, related columns are placed closer together than those where the relation is not so close.

The reading of a table is often facilitated by grouping the items in the stub by 5's or 10's. When months are included in the stub, similar divisions are made on a basis of 6's and 3's. This grouping is illustrated in Exhibits 11 and 12.

Arrangement of Items.—In all tables, the captions and stub items should be arranged in some systematic order. This not only makes the table easier to read, but it allows the more important items to be emphasized.

Ordinarily a larger number of items will be placed in the stub than in the captions, as it is more convenient to place a greater number of words or

FOREIGN TRADE OF THE UNITED STATES
Imports and Exports—1928–1937
(In millions of dollars)

Year	Imports	Exports	Total
1928	4,091	5,128	9,219
1929	4,399	5,241	9,640
1930	3,061	3,843	6,904
1931	2,091	2,424	4,515
1932	1,323	1,611	2,934
1933	1,450	1,675	3,125
1934	1,655	2,133	3,788
1935	2,047	2,283	4,330
1936	2,423	2,456	4,879
1937	3,084	3,345	6,429

A.—Ordinarily this is a much better arrangement than that in Part *B.*

FOREIGN TRADE OF THE UNITED STATES
Imports and Exports—1928–1937
(In millions of dollars)

	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937
Imports............	4,091	4,399	3,061	2,091	1,323	1,450	1,655	2,047	2,423	3,084
Exports............	5,128	5,241	3,843	2,424	1,611	1,675	2,133	2,283	2,456	3,345
Total............	9,219	9,640	6,904	4,515	2,934	3,125	3,788	4,330	4,879	6,429

B.—Usually a poor arrangement.

EXHIBIT 13.—Illustrations of table arrangement.

figures in a vertical column than in a horizontal row (see Exhibit 13). Numbers which are to be compared over a period should be arranged, when possible, so that the principal comparison can be made by following the figures up or down the vertical column rather than along the horizontal row. When columns are to be compared, they should be arranged, as far as possible, so that the principal comparisons can be made between adjacent columns.

Totals of columns are commonly placed at the bottoms of the columns, and totals of rows are commonly placed in the column on the extreme

right, but sometimes, for emphasis, totals are placed at the tops of the columns and in the first column on the left.[1] In many tables, the totals serve primarily as checks, and in such cases should be placed in the customary positions at the bottom and on the right.

It is often impossible or undesirable to include the classifications for which there are only relatively small numbers of items. In such instances, these few and scattered items are usually thrown together in a "Miscellaneous" group. The proper position for the miscellaneous group is ordinarily in the last column or in the bottom row, or just preceding the totals if they are on the right or at the bottom.

The principal bases for arranging items are alphabetical, chronological, geographical, order of size, order of causal relationship, order of interest, according to custom, and according to the desired emphasis.

The *alphabetical* order refers to the arrangement according to the alphabet, which is very common in general-purpose or reference tables.

The *chronological* order is of particular value in presenting historical data. The chronological arrangement ordinarily proceeds over the period from top to bottom or from left to right. Sometimes, however, this order is reversed for purposes of emphasis.

The *geographical* arrangement is used when data are presented by certain territorial units, such as countries, states, counties, and sales districts. In this arrangement some one unit is selected as a starting point and the others are arranged according to their physical nearness to one another.

The order of *size* is commonly used as a basis for arranging items according to their importance. The arrangement may be in ascending or descending order, but usually the large items will be placed in the top row or in the first column.

Some items may be arranged according to *causal relationship*. In tables which are set up primarily for purposes of analysis, this is often the best arrangement. In such a table, the items will be grouped so that the required comparisons can be made much more readily than if they were arranged according to any one of the other orders.

Another basis for arranging items is according to the order of *interest.* Under such a plan the most important items are placed at the top of the table and in the first column on the left.

In some instances, the order in which it is *customary* to mention items is the basis for their arrangement in a table. For example, it is customary to arrange the classifications men, women, and children in the order named.

In special-purpose tables the matter of *emphasis* is of primary importance. Sometimes certain items are placed at the top of a table and in

[1] The United States Census Bureau has adopted the practice of placing the totals at the tops of the columns and in the first column on the left.

the first column of data in order to emphasize them. When an item or point cannot be satisfactorily emphasized on the basis of arrangement, it may be desirable to emphasize it by underlining or by using bolder type.

Footnotes and References.—When any irregularity occurs in a table, or when any explanation is not complete, it may be advisable to give a full explanation in a footnote. This provides a method for using a textual explanation for the purpose of clarification. A table ordinarily should be complete in itself and should not depend upon the accompanying text for any qualifications which would change the interpretation of the table itself if it were detached.

Reference to the source is often desirable, especially in tables of secondary data. This reference, when given, should be complete so that if the user wishes to check the data with the original source or add later data from the same source, he will have complete instructions as to where to look for the information. Since this is merely a reference, it should be inconspicuous, and it should be placed where it will not interfere with the other uses that are to be made of the table. In published tables, it should be set in small type. A common position for this reference is at the top of the table, as illustrated in Exhibit 11, or in the lower left corner of the table.

On working tables, it is especially important that the names of the compilers and checkers, with the dates of their work, be included for reference purposes (see Exhibit 14).

Requisites of a Good Table.—Although the varying purposes on tabular presentation prevent a complete standardization, the following general requisites of a table are commonly recognized.[1]

1. A table should be complete within itself, containing all of the explanations necessary to make the meaning of the items perfectly clear.
2. A table should be a logical unit, containing only data which are closely related to each other.
3. The items in the captions and stubs should be arranged in such a way that a general-purpose table will be most convenient and a special-purpose table will secure the desired emphasis.
4. A title should be included which is clear and concise. If necessary, a subtitle should be included.
5. Captions and stub items should be brief, unambiguous, and self-explanatory, footnotes being added, when desirable, to make them clear.
6. Relationships between the column and row headings should be indicated by the arrangement or boxing in the captions, and by grouping and indentation in the stub.
7. Variations in column widths, interlinear and intercolumnar spacings, widths of lines, and faces of types (bold face, italics, etc.) should be employed to facilitate the use of the table.

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[1] See Edmund E. Day, "Statistical Analysis," p. 404, The Macmillan Company, New York, 1925.

8. When it is desired to facilitate reference to specific parts of a table, columns and rows should be lettered or numbered.

9. Units should always be carefully and completely stated, such as "price in dollars," or "weight in pounds."

10. Sources of the data and responsibility for the table should be included when desirable.

Accuracy in Tabulation.—Since any error in a table may cause one to doubt the accuracy of the work as a whole, it is very important to see that a table is accurate. The data should be carefully checked to make certain that the original entries have been made correctly. When this has been done, the table itself should be checked to see that the totals are correct. In tables which have both vertical and horizontal totals, it will be noted that the horizontal total of the totals of the columns and the vertical total of the totals of the rows will be the same (see Exhibit 11).

It is extremely important that tables of data be carefully checked whenever data are transferred from one record to another. Ordinarily, the best method of checking such work is to have one person read aloud the original table while another person reads (silently) the new copy. When an experienced person and an inexperienced person work together, it is usually best to have the inexperienced person read the original data, and the experienced person follow the new copy. In this way, the experienced person will check any mistakes made by the inexperienced person in reading, for the chances are that an error made in reading will not also appear in the copy. It is very important in this connection that the one who reads the original copy be someone other than the one who copied the figures, for the person who copied the figures is more likely to repeat a mistake when reading the original data.

If the process of checking is to be both accurate and rapid, the method of reading is important. When the words "millions," "thousands," "hundreds," etc., are read, considerable time is taken, not only in reading these words, but also in following, especially if there are several zeros in the numbers. One efficient method of reading figures which enables one to follow easily what is read, is simply to read the digits, from left to right, pausing at the commas, and saying "point" at the decimal points. That is, the figure 796,032,090.2 would be read "seven nine six" (pause), "oh three two" (pause), "oh nine oh point two."

In tables which show a complete percentage distribution, the percentages should be added together to see if their sum is 100. If the fractions in such a percentage distribution prevent the sum from equaling 100, it is often well to adjust these fractions so that the sum equals exactly 100.

Use of Coordinate Paper in Tabulation.—When compiling secondary data or when setting up the first draft of a table of primary data, coordinate paper is convenient in lining up rows and columns and in counting

out spaces (see Exhibit 14). Ordinarily, the figures should be placed one in a space. In this way, it is easy to keep the figures in line, which greatly facilitates computation and transcription. It is not necessary to place the letters of the caption one in a space. If the tables are to be copied on the typewriter, coordinate paper can be used which bears a convenient relation to the spacing on the typewriter. For instance, if coordinate paper which is ruled six lines to the inch is used, it is a great

UNITED STATES
AUTOMOBILE PRODUCTION
1937
PASSENGER CARS AND TRUCKS
BY MONTHS

MONTH	PASSENGER CARS	TRUCKS	TOTAL
JANUARY	309,637	70,418	380,055
FEBRUARY	296,636	67,359	363,995
MARCH	403,879	90,398	494,277
APRIL	439,980	96,359	536,339
MAY	425,432	91,487	516,919
JUNE	411,394	85,917	497,311
JULY	360,403	78,568	438,971
AUGUST	311,456	82,874	394,330
SEPTEMBER	118,671	52,532	171,203
OCTOBER	298,662	31,214	329,876
NOVEMBER	295,328	64,727	360,055
DECEMBER	244,385	81,849	326,234
TOTAL	3,915,863	893,702	4,809,565

SOURCE– SURVEY OF CURRENT BUSINESS MARCH, 1938.

COMPILED BY L.G.M. 5/3/38.
CHECKED BY A.J.L. + C.R. 5/5/38.

EXHIBIT 14.—Use of coordinate paper in making working tables. It is especially important that the names (or initials) of compilers and checkers, with the dates of their work, be included on the working tables. (Full size, $8\frac{1}{2} \times 11$ inches.)

convenience to the typist when copying tables on a typewriter with elite type, which has twelve letters to the inch.

Examples of Other Tables.—The illustrations of tables which have been used in this chapter represent the simpler types. Examples of the more intricate forms will be given later in this text, and their use will be illustrated in connection with the various presentations and analyses under consideration.

Questions and Problems

1. What is meant by the "classification" of data?

2. Explain the principal steps that should be covered in a plan for tabulating the data of a statistical survey.

3. Describe hand tabulating methods, using a residential occupancy survey as an example.

4. In what kind of statistical work does mechanical tabulation have advantages over hand-tabulation methods? In what kind are hand methods best?

5. Explain "coding" and illustrate its use.

6. Describe the purpose of the key punch, the sorter, and the tabulating machine.

7. Describe the process of sorting figures in sequence, using 3 digit figures as an example.

8. Distinguish between general- and special-purpose tables.

9. What is the purpose of the title of a table? What requirements must be met in presenting the title as regards length, wording, prominence and emphasis, position and arrangement?

10. Define "caption," "unit of measurement," "stubs," and "rows."

11. State where totals of columns and rows may be placed and give reasons for so doing. How should columns be ruled or spaced?

12. Explain the principal bases for arranging the items in a table.

13. Describe three methods of obtaining emphasis in a special-purpose table.

14. What are the requisites of a good table?

15. How should a table be checked for accuracy?

16. Discuss the use of coordinate paper in tabulation.

17. Follow carefully instructions given on pages 63 to 71 (particularly the requisites of a good table) and observe the special instructions given at the end of this problem. Tabulate the data given in the following paragraph upon coordinate paper (see Exhibit 14):

In 1935, the Johnstown Manufacturing Company employed 400 non-union and 1250 union employees. Of these, 220 were females of which 140 were non-union. In 1936, the number of union employees increased to 1,475 of which 1,300 were males. Of the 250 non-union employees 200 were males. In 1937, 1,700 employees were union members and 50 were non-union. Of all employees in 1937, 250 were females of which 240 were union members.

Use coordinate paper that is ruled six spaces to the inch.

Place one figure in a square. (Use pencil.)

Prepare a title that is simple and clear.

Print all titles and other lettering carefully and correctly. (See page 668 for letter forms.)

All lettering should read horizontally if possible. (When impossible to letter horizontally, the lettering should read from bottom to top—never from top to bottom.)

Be sure to designate units.

Give reference or source.

Check for accuracy.

Place your name in the lower right-hand corner and the date in the lower left-hand corner.

18. Present the following in the form of a table. (Your work will be judged on the basis of the number of desirable features of tabulation which you have incorporated.) See instructions at the beginning and end of Problem 17.

The total number of employees hired by the Allen Manufacturing Company during 1937 may be summarized by marital status of native and foreign males and females as follows:

American males, married 72.

American males, single 94.

American males, marital status unknown 8.

American females, married 81.

American females, single 47.

American females, marital status unknown 7.

Of the foreign males, 73 were married, 104 were single, and 11 were of unknown marital status. Of the 153 foreign females, 97 were married, and 44 were single and the rest were of unknown marital status.

19. (The purpose of this problem is to illustrate the value of systematic arrangement and organization of data.) Make two retabulations of the data shown in the table below giving particular attention to the requirements listed on pages 69 and 70.

CARGO TONNAGE OF WATER-BORNE EXPORTS IN THE TEN LEADING TRADE REGIONS OF THE WORLD (1935)
(Long tons)

United Kingdom	3,943,832	West India	1,747,240
North Atlantic and Baltic Europe	1,306,335	East Coast South America	1,340,221
Havre-Hamburg Range	4,762,081	Australia & Antarctic	985,038
South Atlantic Europe	948,105	East Asia	7,313,824
West Mediterranean	2,390,716	Canada	10,265,121
		Total	35,002,513

Source: Statistical Abstract of the United States, 1936.

20. See instructions at beginning and end of Problem 17. Tabulate the total population figures from the following paragraph for all the census years from 1850 to 1930 and show the 10-year increases.

According to the United States Census, the population of the continental United States in 1930 was 122,775,046 persons. This is roughly five times the population of 1850 which was 23,191,876, and roughly twice as great as the figure for 1890, which was 62,947,714. The rate of growth shows a steadily decreasing tendency. The increase shown for 1860 over 1850 was 8,251,445, and for the next 10 years the increase was 7,115,050. By 1880, the total population figure had increased to 50,155,783, which was more than twice as great as the figure for 1850, 30 years earlier. The increase shown in 1890 over 1880 was 12,791,931, and by 1900 the total population figure had reached 75,994,575, which was roughly three times as great as the figure for 1850. While the 1850 figure was more than doubled in the 30 years that followed, the 1880 figure had increased at the end of the next 30-year period (1910) to only 91,972,266, and did not approximately double until the end of a 40-year period when the figure reached 105,710,620 in the year 1920. In comparing either the rate of amount of increase in 1930 with that shown for the decade 1910–1920, proper allowance should be made for the fact that the period between the censuses of 1910 and 1920 was less than a full decade, and that between the 1920 and 1930 censuses was more than a full decade.

21. See instructions at beginning and end of Problem 17. Arrange the data below in the form of a table showing the value of raw materials, the value added by manufacture, and the total value of products manufactured. Arrange the stub items according to the order of importance of total value of products manufactured. Compute any missing items.

According to the *Biennial Census of Manufactures*, 1933, published by the United States Bureau of the Census, the most important major class of manufacturing industries in 1933 was that of making food products. The total value of the group of products was $6,604,036,429. The value of the raw materials of this class of products was $4,210,015,149, which is a relatively high proportion of the total as compared with

other groups. A very important class from the viewpoint of the value added by manufacture was that of publishing and allied products where, out of a total of $1,733,437,-479, $1,355,592,892 was the value added by manufacture. This group was followed closely in the same respect by the machinery group where $789,239,157 went for raw materials and $1,280,179,766 went for manufacturing costs, giving a total value to the products of the group of $2,069,418,923. The second largest group from the point of view of value added by manufacture was the textile industry. Here $2,278,069,713 was the value added by manufacture. The total value of this group was $4,811,237,-757. The Iron and Steel industry was a very important group from the viewpoint of the cost of raw materials. Raw materials in this industry totalled $1,400,829,861. Here only $1,062,170,982 was added by manufacture to give a total value of $2,463,000,843. Transportation equipment closely followed iron and steel for the cost of raw materials. Here a total value of $2,058,195,466 was made up of $1,292,-289,578 cost of raw materials and $765,905,888 value added by manufacture. This was closely followed by petroleum and coal products where raw materials cost $1,285,-560,676 and the value added by manufacture was $585,933,159. In 1933 the rubber-products industry had fallen to the smallest major group with a total value of $472,-743,587, of which $211,396,716 consisted of the cost of raw materials and $261,346,871 was the value added by manufacture. This is probably explained by the decreased production of new automobiles. Closely approaching the rubber-products industry and reflecting the failure of the railroads to maintain their equipment in first class condition is the railroad repair shops group where $191,845,201 cost of raw materials and $314,186,631 value added by manufacture gave a total value of $506,031,832. The chemical- and allied-products group did not decrease as much proportionally during the years 1929–1933 as did most of the groups. Here a total value of $2,117,-513,188 compares with a total value for 1929 of $3,702,672,063. Of the 1933 total, $968,472,553 went for raw materials. Quite the reverse is true in the stone, clay, and glass products group where a total value in 1929 of $1,561,414,590 fell to $608,699,486 for 1933. Of the 1933 total, $212,154,614 was the cost of raw materials and $396,544,-872 was the value added by manufacture. In the forest products group, of a total value of $1,127,404,693, $509,181,862 or less than one-half went for the cost of raw materials. In the paper and allied products group $654,646,541 went for the cost of raw materials out of a total value of $1,172,742,984, leaving $518,096,443 as the value added by manufacture. Another group still of major importance is that of leather and leather products. Here a total value of $996,772,988 consisted of $544,737,292 cost of raw materials and $452,035,696 value added by manufacture. In 1933 the non-ferrous metals group accounted for a total value of $1,068,753,411, of which $427,-626,848 was added by manufacture. In addition to the manufactures covered by the preceding groups, there was a miscellaneous group amounting to $1,677,357,437. Of this total $998,313,583 was cost of raw materials and $679,043,854 was value added by manufacture. The total value of products of the 16 groups was $31,358,840,338. Of this amount the value of raw materials was $16,820,821,977 and the value added by manufacture was $14,538,018,361.

22. (This problem is continued from Question 14 on page 33.) Edit the schedules filled out in Question 14 on page 33, compile the data and compute percentages which will show the proportions vacant, occupied, and under construction by classes of residences. Arrange these data in the form of tables. Check very carefully for accuracy. Write a one-page explanation and interpretation of the principal points shown by the data.[1]

[1] See footnote on p. 33.

23. (This problem is continued from Question 15 on page 33.) Edit the schedules in Question 15 on page 33, compile the numerical information, and prepare percentage tables to assist in the interpretation. Both the numerical and percentage figures should be set up in the form of tables. Carefully check for accuracy. Write a one-page report explaining and interpreting the tables.[1]

24. Assuming that you are to use I.B.M. or Powers tabulating equipment in making a survey similar to that referred to in Problem 22 on the preceding page, prepare the following:

1. A detailed plan for making the survey.
2. A set of blank tables indicating the data required.
3. A field schedule.
4. A punch card.
5. Instructions for coding (or punching if coding is not necessary).
6. Instructions for tabulating.
7. Instructions for preparing and presenting final tables.

25. Assuming that you are to use I.B.M or Powers tabulating equipment in making a survey similar to that referred to in Problem 23, prepare the items listed in Problem 24 above.

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[1] See footnote on p. 33.

CHAPTER V

GRAPHIC METHODS

The cold facts of business experience which are used as the bases for new actions are commonly presented on the printed page, but such pages of cold figures as a balance sheet or a census report speak in clear tones only to the expert who has the time to read them with care. For the ordinary business man, or even for the busy expert, the typed page should have its supplemental language—the language of graphics.

The distinction between the language of graphics and the methods of statistics is as well marked as that between the mere ability to read and write, and the special skill of a trained journalist. And just as the ultimate star reporter must begin his career by learning the alphabet, the vocabulary, and the grammar of the language in which he is to write, so also should the beginner in statistics, and, in many cases, the trained or naturally gifted statistician, study the alphabet and grammar of graphics as a thing apart from statistical technique.[1]

Necessity for Clear Presentation.—When business data have been collected and tabulated, it is often difficult to appreciate their interrelations; or if the data have been carefully studied, it may be almost impossible to show another person that the conclusions reached are correct and sound. It often happens in a meeting of a committee or board of directors that a sound and carefully prepared plan is rejected simply because the facts presented are not clear. Though it is very important to have accurate and significant data, one cannot get the most valuable results unless the methods of presentation are effective.

Use in Analysis.—One of the greatest values of the graphic chart is its use in the analysis of a problem. Ordinarily, the chart brings up many questions which require careful consideration and further research before a satisfactory conclusion can be reached. A properly drawn chart gives a cross-section picture of the situation. While charts may bring out hidden facts in tables or masses of data, they cannot take the place of careful analysis. In fact, charts may be dangerous devices when in the hands of those unwilling to base their interpretations upon careful

[1] The two introductory paragraphs of this chapter are abstracted from M. C. Rorty's Introduction to John R Riggleman, "Graphic Methods for Presenting Business Statistics," 2d ed, pp. xi–xiii, McGraw-Hill Book Company, Inc., New York, 1936.

study. This, however, does not detract from their value when they are properly used as aids in solving statistical problems.

Importance of Graphic Methods.—Engineers and technical men have long realized the importance of graphic methods, and more recently their value has been recognized by business men. Graphic charts have often been thought to be tools of those alone who are highly skilled in mathematics, but one needs to have a knowledge of only eighth-grade arithmetic to use intelligently even the logarithmic or ratio chart, which is considered so difficult by those unfamiliar with it.

In many business organizations at the present time, the executives and managers value their charts as highly as engineers value their drawings. When daily reports are presented in graphic form for comparison, one can see at a glance the state of affairs in the business. What the executive desires is the presentation of statistical material in a form that will permit him to grasp easily and quickly the essential points, and graphic charts are a means to this end.

If graphic methods are to be most effective, those who are unfamiliar with charts must give some attention to their fundamental structure. Even simple charts may be misinterpreted unless they are thoroughly understood. For instance, one is not likely to read an arithmetic chart correctly unless he also appreciates the significance of a logarithmic chart.

Graphic methods are used for presenting all kinds of statistical data. They may be used extensively by a well-developed statistical department, or to a limited extent where a few important facts are recorded by a clerk. There is almost no limit to the use of charts in analyzing the conditions within a firm and in making comparisons with outside conditions. Many such uses will be suggested to the reader throughout this text.

Drawing and Lettering.—A knowledge of the actual drawing of statistical charts is an advantage to all students of business statistics because it gives valuable training in quick and accurate reading of data presented in the graphic language.

In the selection of drawing equipment, one should secure the best that can be afforded, for success in charting depends to a considerable extent upon the quality of the instruments and materials used.

For rough work or in making preliminary studies, charts may often be made on coordinate paper without the use of special equipment, but for the better class of work the chart must be drawn on plain, white paper (using drawing equipment).

As far as appearance is concerned, there is no part of the drawing more important than the lettering. A good drawing may be ruined, not only in appearance, but also in effect, by poor or careless lettering, as any evidence of carelessness or lack of skill tends to destroy confidence

in the work. Almost any consistent style of plain, neat lettering is acceptable in the ordinary course of chart work. Ornate lettering is not appropriate for business charts.

For the use of those who wish to study the mechanical part of chart making, an appendix on the technique of drawing and lettering is included at the end of this book (pages 659 to 670).[1]

Planning the Chart.—There are several requirements, common to all of the different types of charts, which must be considered before work is begun on any particular chart. One of the most important of these is to decide definitely upon the purpose of the chart. Other requirements are to decide upon the type of chart to be used, its size, proportions, form, and the preliminary if not the final title.

In planning the construction of charts, the reader's point of view must always be kept in mind. When there is a question as to which of several methods should be used, the method should be chosen that will enable the reader to grasp the essential points the most easily and clearly. The size of the chart should be determined by considering its convenience in handling and ease of reading.[2]

Use of Printed Forms.—Various kinds of ready-ruled coordinate paper can be used in chart making, but for many business graphs it is easier to rule a special form to fit the problem than it is to fit the problem to a ready-ruled form. This coordinate paper, however, is convenient for working charts and other rough forms of presentation although it is rarely suitable for work requiring a finished appearance such as that prepared for publication. Some common forms of coordinate paper used in making working charts are illustrated in Exhibit 15.

The principles of charting described in this discussion are applied for the most part to finished drawings prepared for publication. It should be appreciated that for every chart prepared for publication in a research project, there are many that are used as working charts upon which it would not be practical to do the amount of work required to make finished drawings. The principles, however, are applied to the more finished work in this discussion for the reason that if one understands the principles involved in the finished drawings, it is easy to modify them for working-chart purposes.

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[1] For a more complete discussion, see John R. Riggleman, "Graphic Methods for Presenting Business Statistics" 2d ed., pp. 217–244, McGraw-Hill Book Company, Inc., New York, 1936.

[2] Usually a chart which is drawn for publication is made somewhat larger than the reproduction. The reduction tends to eliminate minor irregularities, and gives the chart a much more finished appearance than is possible when charts are reproduced in the exact size of the original.

Accuracy in Charting.—In charting business statistics, a reasonable degree of accuracy is, of course, necessary, but the problem differs from that of plotting technical data where measurements are made from the charts with finely graduated instruments. If the chart is accurate to the degree that visual comparisons will not be impaired, it is accurate

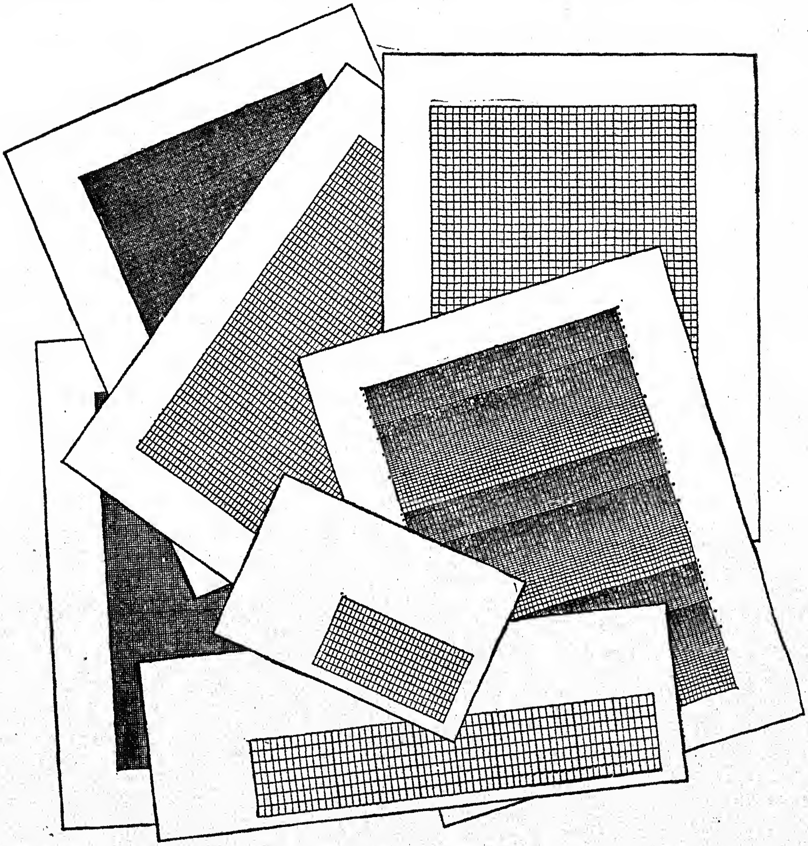

Exhibit 15.—Coordinate paper for making working drawings and preliminary sketches in business charting. (*Courtesy, Educational Exhibition Company.*)

enough for most practical purposes. Care should be taken, however, that the chart be accurate to this degree.

The Title.—Titles have been placed in various positions on business charts in the past, but at present the weight of authority agrees in placing the title at the top of the chart as illustrated in Exhibit 16. This is convenient and natural from the viewpoint of the business man, since his bills, tables, books, accounts, schedules, and other forms are commonly labeled at the top. The title lettering should be the largest and most prominent on the chart.

The title is one of the most important parts of a business chart, especially if it is prepared for publication. It should give the reader the subject of the chart clearly and completely. This point is often overlooked, and, though the chart may be clear in its other parts, it may not be read at all unless the title is complete and clear. All of the requirements that were given for titles to tables on pages 63 to 65 apply also to titles on charts.

The titles of working charts, like the titles of working tables, need not be so carefully worked out, but they should be complete and clear.

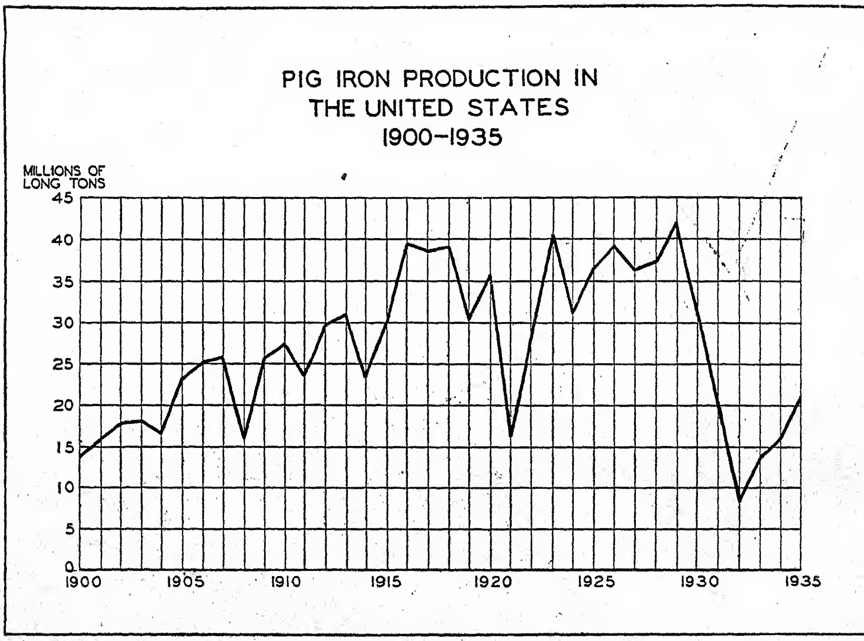

Exhibit 16.—The correct position for the title is at the top of the chart.

All that was said regarding the value of having the date upon a working table is of equal importance in connection with working charts. The importance of requiring that the date of preparation be placed upon all working charts cannot be too strongly emphasized; it is often the principal means of determining the relation of one piece of work to another. The source of the charts, *i.e.*, the department, company, etc., and at least the initials of the compiler or compilers should also ordinarily be shown.

General Appearance and Proportions.—Confidence in a chart is much greater if it is neat and accurate rather than slovenly in appearance. The chart must be well proportioned, it must not be crowded, and ample margins must be left. It is of utmost importance in securing an appearance of high quality to have the title and all other lettering done neatly

and carefully. Usually a border line should be drawn around the chart to give it a finished appearance.

Kinds of Charts Considered.—In this discussion of graphic methods (Chapters V and VI), the following basic kinds of charts will be considered:

1. Simple comparisons of size. (Size only.)
2. Arithmetic time curves. (Sizes compared over a period.)
3. Logarithmic time curves. (Rate of change over a period.)

In number 1 above, one size is compared with another, but not in relation to any other variable. That is, we may compare the height of the Empire State Building with the height of the Eiffel Tower. We are not concerned with their relation to a period of time, with their geographic

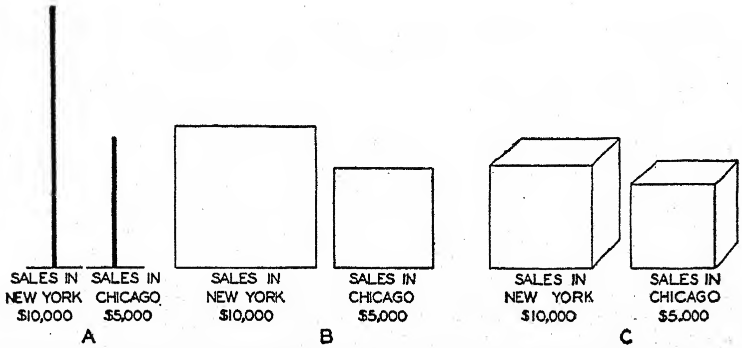

EXHIBIT 17.—Use of different geometric forms in comparing sizes (sales). *A*. Lines (or bars) are easy to compare. *B*. Areas are difficult to compare. *C*. Volumes are very difficult to compare.

distribution, or with their relation to any other variable. We are simply comparing heights (or sizes).

The arithmetic time curve (number 2 in the above list) compares sizes over a period of time. Thus, we may consider the size of automobile production by months.

In number 3 (logarithmic time curves) not size, but rate of change (over a time period) is considered. Such a curve would show, for instance, how rapidly a city's population is growing.

The form and use of three other kinds of charts (frequency graphs, correlation charts, and statistical maps) will be illustrated from time to time throughout the text. The methods of constructing them will not be described, however, as they will be fairly obvious after studying the construction of the first three kinds listed above.

Simple Comparisons of Size.—The most simple type of graphic chart is that comparing magnitudes where the sizes do not vary in relation to any other variable—that is, there is no consideration of time variations,

geographic distributions, or other variables. We may wish to compare, for instance, simply the size of the *Queen Mary* with the size of the *Normandie*. Such comparisons are commonly made by using certain geometric forms (lines, squares, rectangles, cubes, etc.) which vary in their sizes in proportion to the different magnitudes represented.

The first step in making a simple chart is to select the geometric form which is to be used. The form selected must be easy to understand and it must give a correct impression. These are the two fundamental requirements which must be recognized in selecting a geometric form for use in graphic presentation.

It is always easier to compare the sizes of forms which vary in one dimension than it is to compare those which vary in two or three dimensions. In other words, it is much easier to compare the lengths of lines

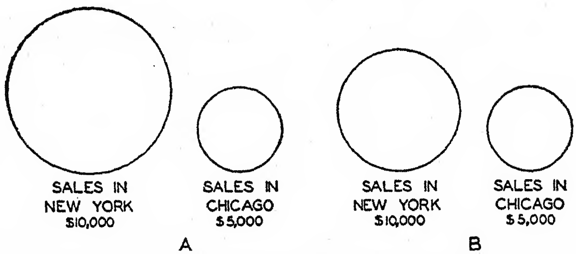

EXHIBIT 18.—*A*. Circles with diameters proportional to numbers. *B*. Circles with areas proportional to numbers.

or bars than it is to compare areas and especially volumes. These three forms are illustrated in Exhibit 17, and the difficulties involved in comparing areas and volumes will be readily appreciated by reference to this illustration.

Occasionally circles are used to represent size, but it is difficult to compare them, and they should not be used when it is possible to use bars. When it is necessary to use circles, as in making certain kinds of statistical maps, they should be compared on an area basis, rather than on a diameter basis, as the diameter basis is very misleading (see Exhibit 18).

Pictorial Forms.—The principles just discussed in connection with simple lines, areas, and volumes must be kept in mind when using pictorial forms. Many pictorial charts are incorrectly drawn. Suppose, for instance, that figures of persons are used to represent the populations of New York and Chicago, and that they are so drawn that the heights of the persons are proportional to the numbers represented. Such a chart would be supposed to show that New York is roughly twice as large as Chicago, while, as a matter of fact, the effect would be that New York is much more than twice as large. Charts of this kind usually give

false impressions, as the reader tends to compare the figures of persons on a volume basis, which greatly exaggerates the actual differences in the numbers represented. If it is desired to use figures of persons in popular presentations of this type, it is far better to represent numbers, not by the heights of persons, but by rows of persons of the same size, evenly spaced, with the lengths of the rows proportional to the numbers represented, as in Exhibit 19. Then the comparison would be made upon the basis discussed in connection with Part *A* of Exhibit 17.

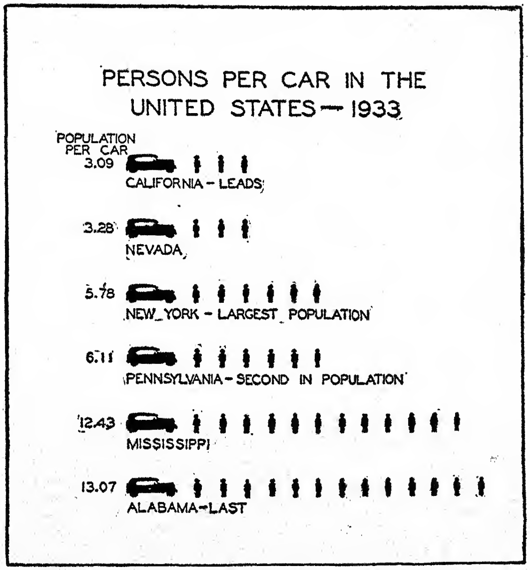

EXHIBIT 19.—Simple pictorial statistical chart.

There are many figures that can be used in popular presentations which may be made to vary in only one dimension without disproportioning the figures, as is the case when varying only the heights of persons. For instance, illustrations of office buildings, in which height would be the only dimension changed, could be used to make comparisons in accordance with the principle discussed in connection with Exhibit 17.[1]

Horizontal- and Vertical-bar Charts.—The most common method of representing simple comparisons of size graphically is by some form of *bar chart*. When using the bar chart in presenting several items for careful analysis, the chart must include certain details, such as a properly constructed scale, explanation of units used, background of guide lines, and title.

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[1] See John R. Riggleman, "Graphic Methods for Presenting Business Statistics," 2d ed., pp. 14–25, for further discussion of pictorial charts.

In making simple comparisons of size, there is no particular rule for choosing between horizontal and vertical bars, except that those should be used which will enable the reader to understand the chart the most readily.[1]

In cases where the things represented vary naturally in either a horizontal or vertical direction, the bars are usually made to vary in the same direction. Convenience in reading the lettering is another point that must be considered when choosing between horizontal and vertical bars. It is easier to read the lettering in Exhibit 20, for instance, if it is lettered horizontally, than it would be if it were lettered vertically (see also Exhibit 25). For this reason, many statisticians prefer to use the

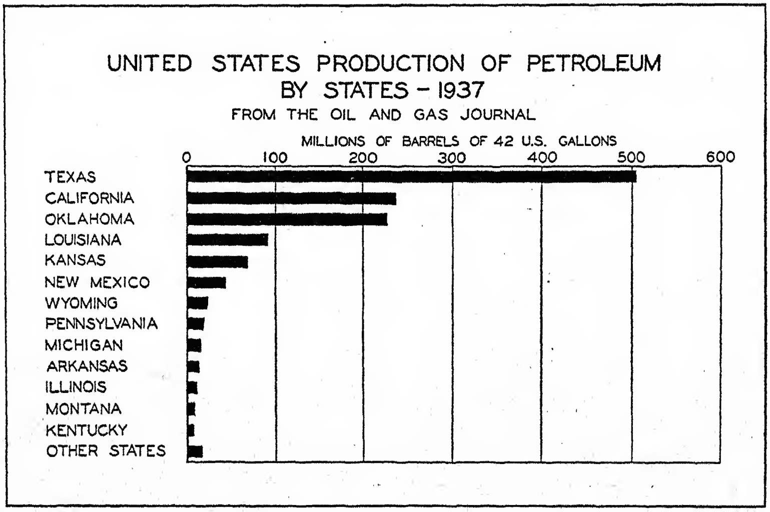


EXHIBIT 20.—A horizontal-bar chart.

horizontal-bar chart when making simple comparisons of the type illustrated in these Exhibits.

Bar-chart Scales and Bar Designations.—Except in cases of rough comparisons or popular presentations, it is important to include a scale on a bar chart. A scale enables one to read the chart much more easily and accurately than when it is not used. It is a fundamental rule in making bar charts which show the ordinary comparisons of size that the scale begin with zero (Exhibits 20 and 21). This rule is sometimes broken by those who are inexperienced, and the values are inaccurately represented. Note that in Exhibit 21 copper is about twice as large as silver. If, for instance, the scale began with 10 instead of 0 (Exhibit 22), the bar

[1] It should be clearly understood that this statement applies only to *simple comparisons of size.* In showing *time series,* only vertical bars should be used (see pp. 96 and 97).

for copper would be about ten times as long as the bar for silver, and the graphic impression would be very inaccurate.

Scales should be laid out in round numbers of convenient intervals, such as 2's, 5's, or multiples of 10, or halves and quarters of 100 (25, 50, 75, etc.). Too many numbers in the scale or too many guide lines in

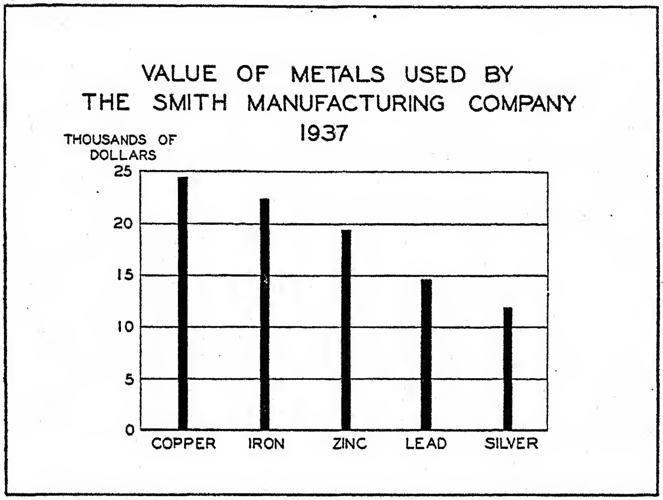


EXHIBIT 21.—A vertical-bar chart (correctly drawn).

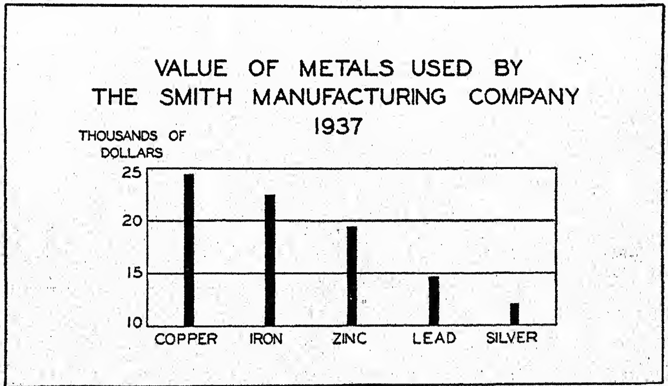


EXHIBIT 22.—Vertical-bar chart (incorrectly drawn, as scale is cut off). Compare with Exhibit 21.

the background hinder rather than aid the reading of the chart. Only enough intervals should be recorded on the chart, and only enough guide lines should be drawn, to serve as an aid in making comparisons through visual inspection. More scale numbers than are shown in Exhibit 21,

for instance, would not aid in making comparisons, and they would make it much more difficult for the eye to carry a scale value across the chart.

The scale on a horizontal-bar chart is commonly placed along the top just above the bars as in Exhibit 20. It may be placed along the bottom also, if it will add to the clearness of the chart. The scale of Exhibit 20 represents number of barrels, and this fact should be indicated as shown by the designation just above the scale (see also the word "dollars" in Exhibit 23). If there is a scale below the chart, the designation of units would be shown just underneath the scale.

The commonly accepted position for the scale of a vertical-bar chart is on the left and outside the background of guide lines (see Exhibit 21). Each number should be exactly opposite the line which it represents (Exhibit 21). If it is desirable to have a scale on the right of the chart,

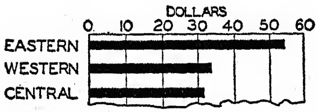


EXHIBIT 23.—Position of scale-unit designation on horizontal-bar charts.

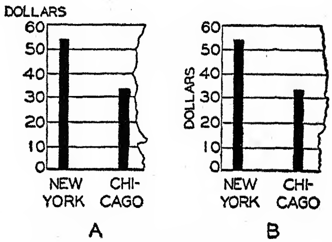


EXHIBIT 24.—Positions of scale-unit designations on vertical-bar charts. *A*. Best practice for business charts. *B*. Also correct, but not so easy as *A* for the business man to read.

one may be placed there in addition to the one on the left, but the left scale should in no case be omitted.

The units represented by the scale should be indicated clearly as shown in Exhibit 24. The preferred method is the one shown in Part *A* (Exhibit 24) which shows the scale designation lettered horizontally at the top of the column of figures. Another common practice, lettering the designation vertically along the scale, is illustrated in Part *B*, Exhibit 24. This is not incorrect but it is more difficult for the business man to read.

Expressions such as "one inch equals \$100" should not be used on statistical charts. The scale on the graph is much more convenient and gives all the aid necessary in measuring the different bars. Furthermore, if the chart is enlarged or reduced by photographic reproduction processes, a scale laid out on the graph is also enlarged or reduced in direct proportion.

On charts for careful reading, it usually is best to include a background of guide lines which assist in comparing the bars or in reading from the scale. As previously mentioned, too many lines should not be included,

as it is difficult for the eye to follow them across the chart if they are close together. As guide lines should appear to be behind the bars, they should be drawn between (not through) the bars when shaded, colored, or outlined bars are used (see shaded bars in Exhibit 29 on page 90).

Designations explaining what is represented by horizontal bars are placed on the left as illustrated in Exhibits 20 and 25.

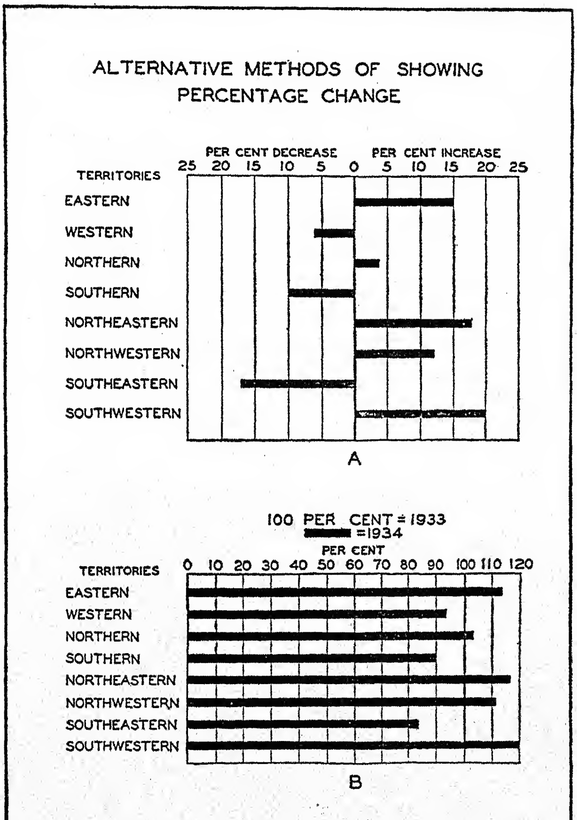


EXHIBIT 25.—Showing loss and gain with horizontal bars.

Designations explaining what vertical bars represent usually are placed underneath the bars as shown in Exhibits 21 and 26. Words should be divided if they are too long for the spaces allowed, and they should be lined up with reference to the zero line as shown.

Presenting Gains and Losses.—When it is desirable to show decreases or losses along with increases or gains, they may be shown by means of a horizontal-bar chart as illustrated in Exhibit 25, or a vertical-bar chart, as illustrated in Exhibit 26. In these charts, the horizontal bars indicate decreases when they extend to the left from the zero line, and the vertical

bars indicate losses when they extend downward from the zero line. Sometimes one sees such bars used to indicate positive data, but, as a rule, the practice of extending horizontal bars to the left of the zero line and vertical bars below the zero line should be reserved to show decreases, losses, and other negative items.

Order of Items.—The items represented on any chart should always be arranged according to a systematic order. The most common order in size comparisons is the order of size, but it is often desirable to use

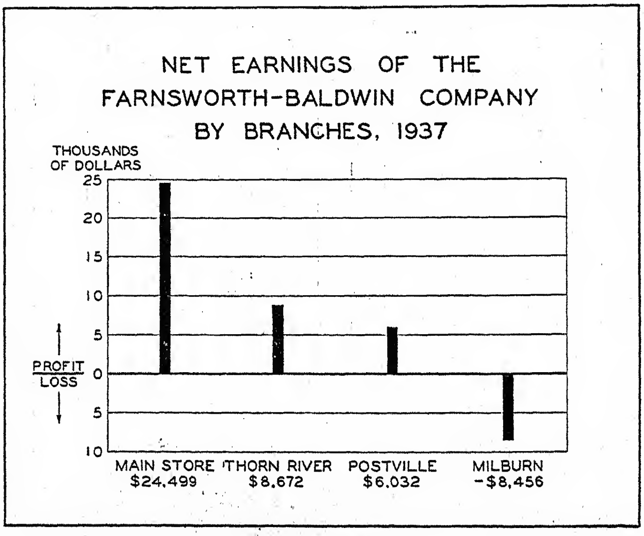


EXHIBIT 26.—Showing loss and gain with vertical bars.

another order, such as the alphabetical, the geographical, or the chronological. In any case, some systematic order should be used, for it increases the effectiveness of the chart.

Tabulating Data on Charts.—When it is desirable that the numerical data represented be tabulated directly on the chart, they may be shown on horizontal-bar charts as indicated in Parts *A* and *B* of Exhibit 27, and on vertical-bar charts as indicated in Exhibit 28. Tabulations should be made in such a way that they will not interfere with the reading of the charts. In Part *C*, Exhibit 27, the figures have the effect of increasing the lengths of the bars, which results in a wrong impression of the relative values indicated. This principle applies to both horizontal and vertical bars.

Comparing Groups.—Bar charts are often made to show, not only differences between the sizes of various individual items, but also to compare the differences between certain groups. Exhibit 29 on commercial failures makes a comparison of assets and liabilities in each

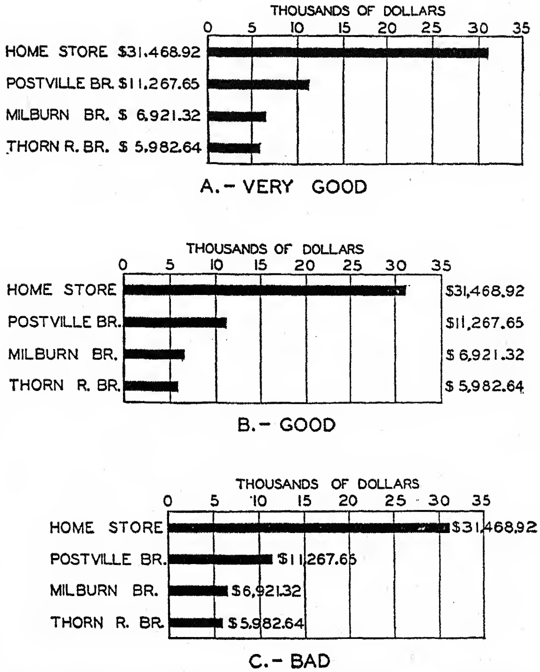


EXHIBIT 27.—Tabulating data on horizontal-bar charts.

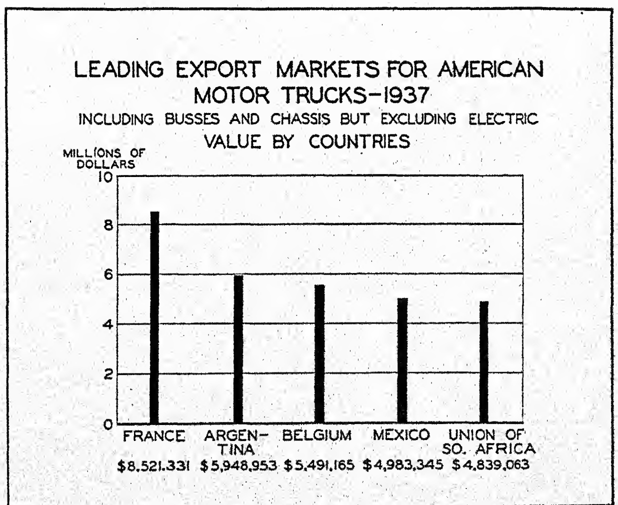


EXHIBIT 28.—Tabulating data on vertical-bar charts. (Note that on charts like this one, the data should be lined up with reference to the zero line as shown.)

section, and a comparison of the differences within each pair of assets and liabilities by sections for the group as a whole. These differences can be easily seen and compared throughout the eight sections. Either vertical or horizontal bars may be used in making such comparisons. It is necessary to differentiate the bars within a pair or group by shading, and there should be more space between the groups than between the bars within the groups, in order that the groups will stand out distinctly.

Necessity for Common Unit.—It is necessary to use a common unit in making all comparisons of size. It means nothing to make a chart

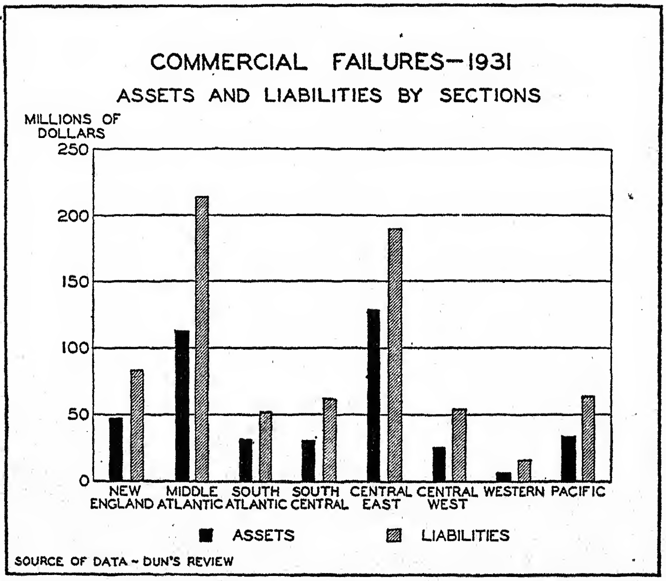


EXHIBIT 29.—Comparing different groups by means of bars.

comparing 10 dozen oranges and 50 pounds of prunes. The unit of measurement should be one which is common to both in this type of comparison.

Comparison of Component Parts.—Up to this point, the discussion has considered comparisons of separate items and comparisons of differences between groups of items. Another important problem in size comparisons is that of comparing the component parts of an aggregate—one with another or one with the whole.

The method illustrated in Exhibit 30, commonly known as the "pie chart," probably is most generally known because of its wide use in popular presentations. The pie chart permits a convenient arrangement of the designations of the different components, as well as the percentages and the actual values represented. The wording should be placed inside the sectors when there is sufficient room, and when possible it should be so

placed that it reads horizontally (see Exhibit 30). The wording should never be placed in such a way that it reads radiating from the center, or following the circle, because of the difficulties and confusion in reading.

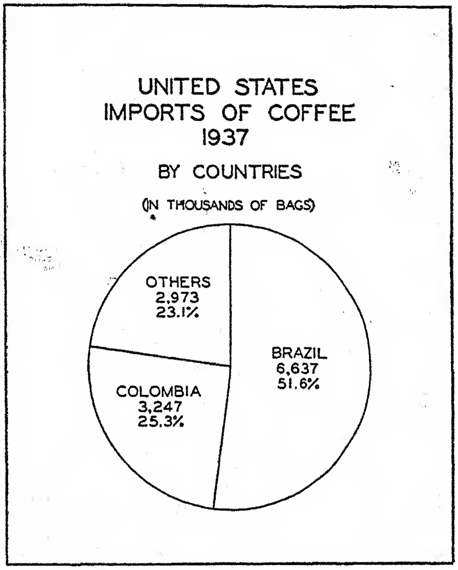


EXHIBIT 30.—A pie chart.

Pie charts can often be made more effective by coloring or shading the sectors. The sectors of a pie chart should be arranged according to some systematic order. Usually they are arranged in order of size, and the

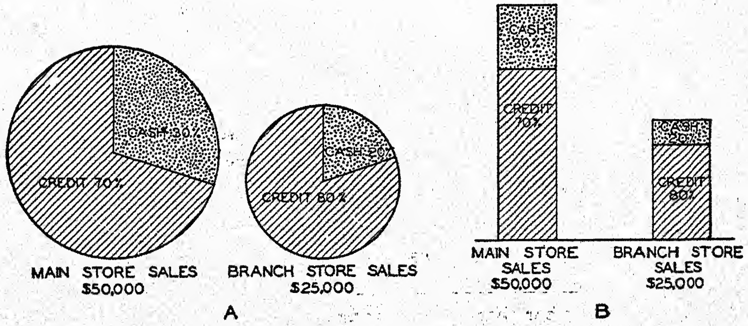


EXHIBIT 31.—Comparison of aggregates showing component parts. *A*. Poor practice as sizes of circles are difficult to compare. *B*. A good method as it is easy to compare lengths of bars.

most generally followed order is to arrange them from large to small in a clockwise order beginning at the top as in Exhibit 30. Of course, a sector representing a combination of several small items and designated as

"all others," would be given last place, even if larger than some of the other sectors.

An excellent form for showing the component parts of a whole is the component bar (see Part *B*, Exhibit 31). Component bars may

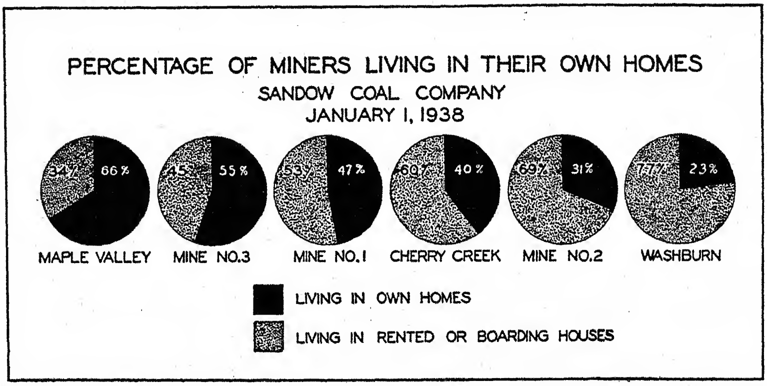


Exhibit 32.—Use of pie charts in comparing the percentage distributions of the component parts in different aggregates.

extend either vertically or horizontally, but that direction should be chosen which will make the chart easiest to read. When comparing different aggregates made up of component parts, the component bar

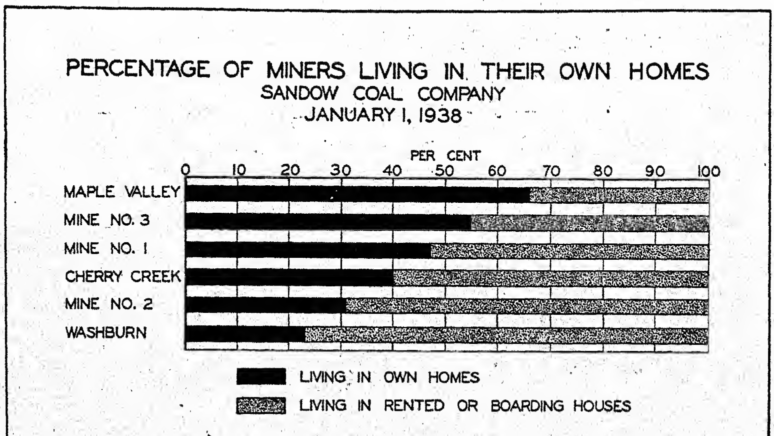


Exhibit 33.—Use of the bar chart in comparing the percentage distributions of the component parts of different aggregates.

is much more satisfactory than the pie chart (see Exhibit 31), because of the difficulty of comparing circular areas. In Part *B*, Exhibit 31, the component-bar chart is a very simple one; if a chart is more complicated, scales should ordinarily be added.

In comparing different aggregates on the basis of the percentage distribution of their components, the pie chart and the bar chart are about equally effective. This is illustrated by Exhibits 32 and 33 in which the two different methods are used to present the same data.

Checking List.—The following list includes the most important points to be observed in making the charts described in this chapter, and can be used when checking a chart to see if it is complete:[1]

CHECKING LIST FOR SIMPLE SIZE CHARTS

Did you check for accuracy?

Is the title simple and clear? Did you place it at the top of the chart?

If you have used shadings or colors to make any necessary distinctions, have you included a legend or key to the meanings of the shadings or colors?

Have you placed a convenient scale and a background of lines upon the chart to aid in measuring sizes?

Are the faces of the bars free from these lines?

Is the zero line heavy?

Are the other scale lines light?

Have you designated the scale units?

Have you given the reference or source?

Does all lettering read horizontally? (When impossible to letter horizontally, lettering should read from bottom to top—never from top to bottom.)

Are your letters formed according to a commonly accepted style of plain lettering?

Have you ruled in a border line? (Optional but advisable.)

If desirable, have you included your name (lower right) and date (lower left)?

Are pencil lines erased?

Is the size of the chart appropriate for the purpose intended?

Will your chart be clear to the reader?

Questions and Problems

1. Discuss the importance of graphic charts in business statistics.

2. What are some of the limitations of graphic methods?

3. How does a neat and orderly appearance increase the value of a chart?

4. What requirements must be considered in planning a chart?

5. What are the disadvantages of using printed forms for graphic presentation? When is it advantageous to use them?

6. What degree of accuracy is required in charting business statistics?

7. State the requirements of the title of a chart as to position, prominence, and content.

8. Discuss the use of geometric forms varying in one, two, and three dimensions in graphic presentation of statistics.

9. How do the requirements of charts for publication differ from those used as "working charts"?

10. What factors should be taken into consideration in choosing between horizontal- and vertical-bar charts?

11. Where should the scale be placed on a horizontal-bar chart? On a vertical-bar chart? How should the scale units be designated on a horizontal-bar chart scale? On a vertical-bar chart scale?

[1] A checking list for curve charts is given on p. 122.

12. In connection with the arrangement of the scale, what should be done to make the chart easily readable?

13. Where should the bar designations be placed on a horizontal-bar chart? On a vertical-bar chart? If figures are included, where should they be placed?

14. Should the items of a chart be arranged in any definite order? Why?

15. How can groups of items be compared by means of bars?

16. Can 100 tons of coal and $50,000 worth of iron be shown in a simple comparison of size? Why?

17. What is meant by the comparison of component parts?

18. Describe two methods of presenting component parts. What requirements should be followed in the use of these methods?

19. In making a simple chart, what are the requirements of the geometric form selected? In this connection, what precautions should be taken in using pictorial or popular presentations?

20. Present as a horizontal- or vertical-bar chart:

ESTIMATED COST OF NEW RESIDENTIAL BUILDING, JUNE 1937

Geographic division	*Estimated cost*
New England	$ 4,025,264
Middle Atlantic	14,439,473
East North Central	20,159,483
West North Central	5,806,617
South Atlantic	8,274,974
East South Central	1,434,501
West South Central	3,622,061
Mountain	1,793,802
Pacific	11,315,415
Total	$70,871,590

Source: Commercial and Financial Chronicle, July 31, 1937, p. 673.

Check your work carefully according to the checking list on page 93.

21. The cost of new residential building data for June 1937 given in problem 20 expressed as a percentage change from June 1936 was:

Geographic division	*Percentage change from June* 1936
New England	+16.9
Middle Atlantic	−56.5
East North Central	+49.7
West North Central	+57.4
South Atlantic	− 4.2
East South Central	+41.8
West South Central	+21.9
Mountain	+27.4
Pacific	+24.9

Present the percentage-change data given above on a horizontal-bar chart. Check your work carefully according to the checking list on page 93.

22. Present as a horizontal- or vertical-bar chart:

SALES OF THE NATIONAL PRODUCTS COMPANY
By Branches

New York	$4,410,168	Detroit	$348,400
Chicago	2,085,747	Cincinnati	233,464
Los Angeles	975,358	Cleveland	132,557
Philadelphia	553,442	Seattle	130,356
Washington	448,828	Baltimore	127,824

Check your work carefully according to the checking list on page 93.

23. Present as a component-bar or as a pie chart:

THE PALAIS ROYAL DEPARTMENT STORE
Distribution of a Dollar of Sales—1937

	Per cent
Cost of merchandise	72
Operating expenses	24
Profit	4
Total	100

24. Present as a component-bar or as a pie chart:

NUMBER OF BUSINESS FAILURES IN 1935 (UNITED STATES)

Group	Number	Total liabilities (thousands of dollars)
Manufacturing	2,842	72,912
Trading	8,035	95,318
Agents, brokers, etc	1,002	61,891
Total	11,879	230,121

Source: Statistical Abstract of the United States, 1936, p. 296.

25. Present as a component-bar or pie chart:

THE HOMESTEAD FIRE INSURANCE COMPANY ASSETS ON DECEMBER 31, 1936

Cash in banks	$ 303,441.93
Mortgage loans	13,950.00
Bonds and stocks	2,951,761.00
Premiums uncollected	172,542.42
Accrued interest	7,718.30
Other admitted assets	250.00
Total assets	$3,449,663.65

Source: Annual report of company.

CHAPTER VI

GRAPHIC METHODS (*Continued*)

Some of the most important work in business statistics is that in which phenomena are compared at different points of time. Charts are very valuable tools in analyzing these changes, and it is the purpose of the following discussion to describe the graphic methods of presenting data with reference to definite dates.

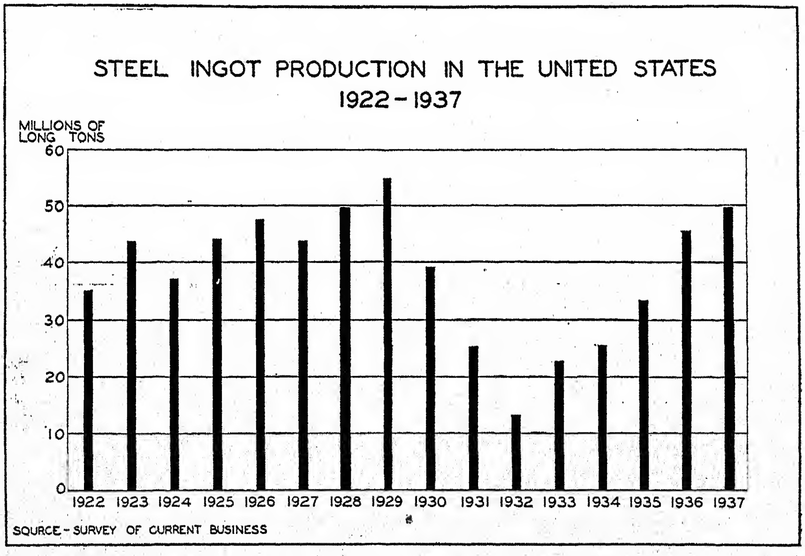


EXHIBIT 34.—The vertical-bar chart is the only form of bar chart that should be used in presenting time series. (The horizontal-bar chart should not be used for this purpose.)

Bar Charts for Time Series.—Bar charts similar to those described in the preceding chapter are often used to show times series. In showing time series, however, only the vertical-bar chart is used (Exhibit 34).[1] The reason for this is that the chart of vertical bars is read in exactly the same manner as the time curve, which is by far the most common way of presenting historical data. When the vertical-bar chart is used for showing a series over a period, the horizontal time scale should read from

[1] This rule ordinarily applies when data are presented in relation to two variables; but it does not apply in making simple comparisons of size, where both vertical- and horizontal-bar charts may be used, as described on pp. 83–85.

left to right as shown in Exhibit 34. This scale should be laid out in such a way that equal spaces represent equal time intervals over the period. The vertical scale is laid out in the same manner as described previously.

Curves of Size Changes in Time Series.—The most common method of showing time series graphically is by means of a curve.[1] In Exhibit 35, the arithmetic time curve (Part *C*) tells the same story as the bar chart (Part *A*) and the similarity is illustrated (Part *B*). The curve in Part *C*

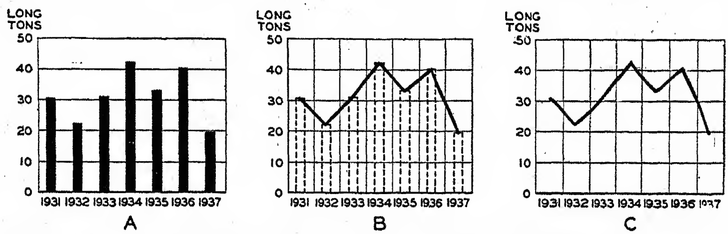
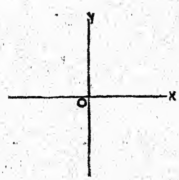


EXHIBIT 35.—Comparison of bar and curve methods of showing time series.

represents the ends of the bars, but by being continuous it indicates the direction of the change more clearly than a bar chart.[2]

The Vertical Scale on Time Charts.—The vertical scale for an arithmetic time curve is laid out in the same way as the vertical scales of the bar charts previously mentioned. The correct position is on the left of the first ordinate as shown in Part *A*, Exhibit 36. If desirable, a scale may also be placed on the right of the last ordinate, but in no case should it be omitted on the left.[3] The scale figures should be exactly

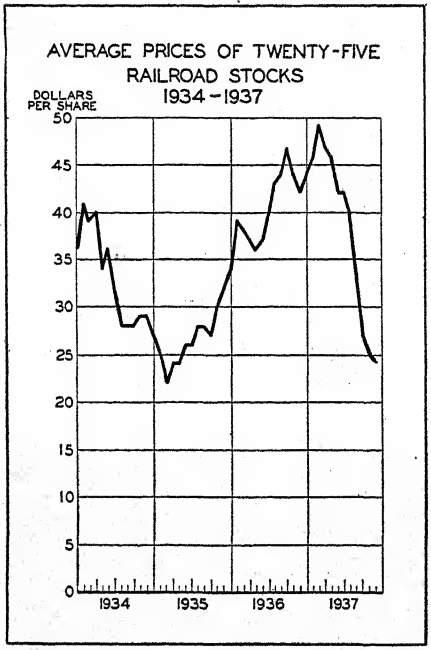
[1] The plotting of curves in graphic presentation of business statistics is based on the use of coordinates in determining the positions of points in a plane. Coordinate axes are two straight lines at right angles to each other. Their point of intersection *o* is called the origin. The horizontal line is called the axis of x, or the axis of abscissas. The vertical line is called the axis of y, or the axis of ordinates. The exact position of any point can be located by measuring from the x-axis and from the y-axis.



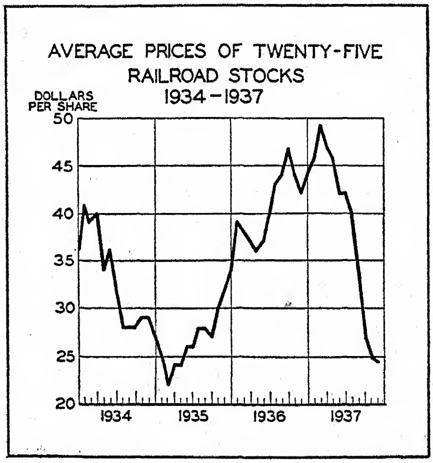
[2] By convention, the *independent variable* is plotted on the horizontal scale and the *dependent variable* is plotted on the vertical scale in statistical charts. The horizontal scale defines the question and the vertical shows the answer. One determines the independent variable when he decides upon the terms of his problem. Readers of a chart like Part *C*, Exhibit 35, for instance, will recognize years as the basis of the study, but will fix their immediate attention upon the dependent variable or tonnage measured upon the vertical scale.

[3] Ordinarily when it is necessary to place two or more different vertical scales on a chart, they should be placed on the left of the graph, as in Exhibit 94*B* on page 192, rather than on both sides of the graph. There are instances, however, of charts with two scales where there is a good reason for not following this rule. An illustration of this will be found in Exhibit 200 on p. 483, in which a second scale is placed on the

opposite the lines which they represent (see Exhibit 36). A caption explaining the units used should be included, preferably at the top of the scale, as illustrated in Exhibits 36 and 38, though lettering vertically along the side is not incorrect. When the problem is one of *comparing sizes* over a period of time, the zero line and all of the scale must be included on the graph. If the lower part of the scale is not shown, the



A



B

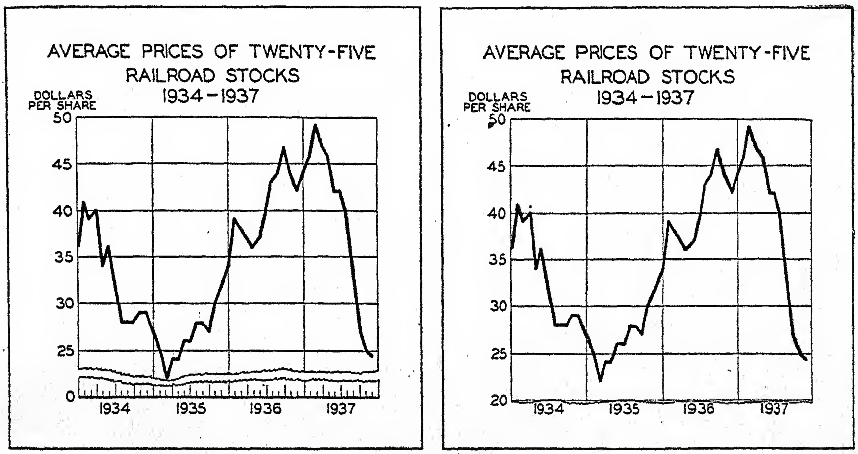
EXHIBIT 36.—*A*. Correct method of showing *size* over a period of time, as the vertical scale is complete. *B*. Incorrect for showing *size* because lower part of scale is cut off. (*Data from Survey of Current Business.*)

reader is likely to be given a very inaccurate impression of the relative size changes (Part *B*, Exhibit 36). That is, the graphic impression of Part *B* is, for example, that prices were over five times as high at the peak of 1937 as they were at the end of the year, while the graphic impression of Part *A* is that they were approximately twice as high. The principle here is the same as that discussed previously in connection with bar charts, and a correct impression of size changes is given graph-

right of each graph for the reason that it is much easier to read the curves which are plotted to this scale, because they do not extend across to the left sides of the graphs. Ordinarily, however, all of the vertical scales should be on the left, as there is less confusion, and one is not likely to make the mistake of reading a certain curve from one scale on the left and a different scale on the right.

ically only when the entire scale is included. (See also Exhibit 47 on page 106.)

Sometimes it is desirable to show time changes without considering size or rate of change. In such cases, when the entire scale does not appear, attention should be called to the fact by showing a horizontal break across the graph just above the zero line, as in Part *A*, Exhibit 37, or simply by showing a break at the bottom of the graph as in Part *B*. Either of these warnings should prevent the reader from attempting to compare sizes for the different dates.



A *B*

EXHIBIT 37.—When the entire scale is not shown, a break should be shown across the chart as in *A* or the bottom of the graph should be a broken line as in *B*. (See Exhibit 36.)

When a curve is plotted in percentage relatives,[1] with the 100 per cent line as the base above or below which the curve is plotted, the zero line is commonly omitted if it is a relatively long distance from the 100 per cent base line.

The Horizontal Scale on Time Charts.—The generally accepted position for the horizontal scale is along the bottom of the graph as shown in Exhibits 36 and 38. If desirable, a scale may be placed along the top in addition to the one along the bottom, but the one along the bottom should not be omitted. The position along the bottom of the chart is the most convenient position for making comparisons from the zero base

[1] Percentage relatives of values, at given times, are the percentages that these values are of a value for a certain period taken as 100. Thus, the imports of coffee from Brazil into the United States (in round numbers) in 1913 were 5,000,000 bags; in 1935, they were 8,700,000 bags; in 1936, they were 8,000,000 bags; and in 1937, they were 6,600,000 bags. The percentage relatives for these coffee imports for 1935, 1936, and 1937, on a base of 100 for 1913, would be: 174, 160, and 132, respectively. (For further explanation of percentage relatives, see pp. 191–194.)

line. It is not necessary to explain in which units the time scales are expressed, except when hours, days, or some unit is used which is not clearly recognized. Years or months are always clear without a designa-

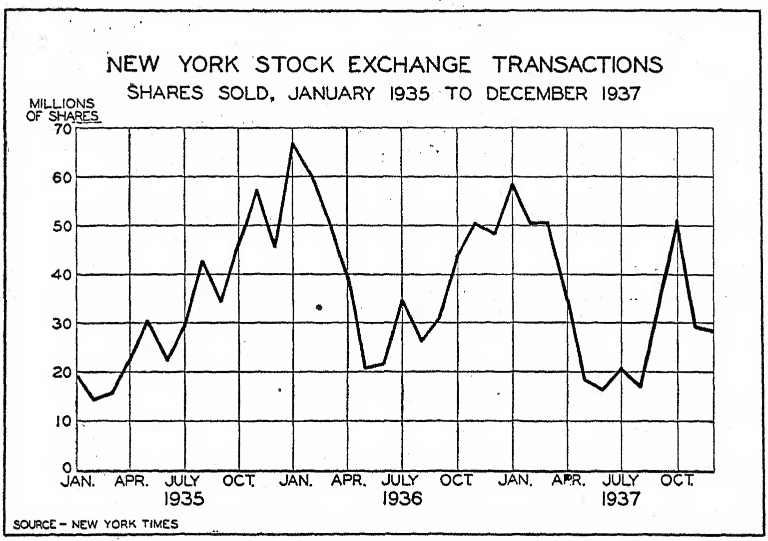


EXHIBIT 38.—Chart of monthly data designated by quarters.

tion. It is usually best to designate months by their common abbreviations rather than by their first letters. If there is not room to letter all the abbreviations horizontally, the chart will usually be clearer if only one month in each quarter is designated, as in Exhibit 38, in which the

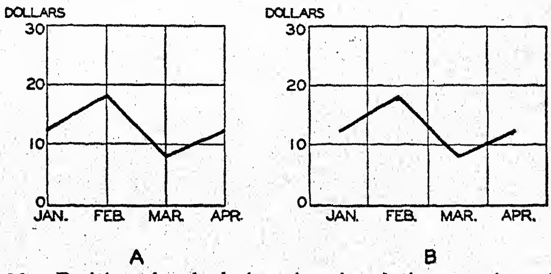


EXHIBIT 39.—Position of scale designations in relation to points plotted.

data are plotted monthly whereas the rulings and designations are on a quarterly basis. When several years are designated by months, the year designation should be shown under the months at the middle of the year as in Exhibit 38. No arrows, dimension lines, or other guides or boxes are necessary to show the relation of months to years. In fact, no lines should be included which are not necessary in reading the chart.

Whether the points should be plotted on the lines as in Part *A*, Exhibit 39, or between the lines as in Part *B*, is ordinarily a matter to be decided by personal preference. In either case, the designation of the month (or other period) should be placed exactly below the point plotted, in order that there may be no confusion. Thus, in Part *A*, each name of the month is directly under the vertical line on which the point is plotted. In Part *B*, each name is directly under the space in which the point is plotted. While the method shown in *A* is the most commonly used, the method shown in *B* is preferable in such instances as when it is desired

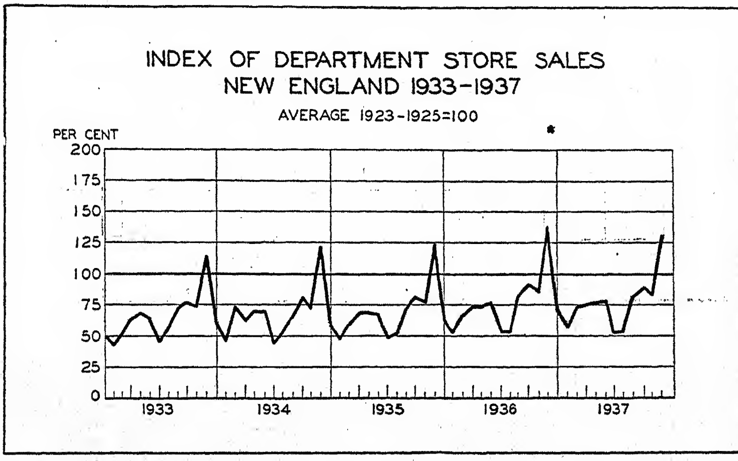


EXHIBIT 40.—The zero line and the 100 per cent line should be emphasized. The curve should be the heaviest ruling on the chart.

to plot yearly averages for past years, and then continue the curve on a monthly basis for the current year.

Emphasis of Coordinate Lines and Curves.—The zero line should be made heavy in order that it may be sharply distinguished from the other coordinate lines. Usually it is desirable to emphasize the 100 per cent line, or any other line that is used as a base (Exhibit 40). In the horizontal scale of a time graph, it is usually best not to emphasize the first ordinate since, as a rule, it is no more important than the other ordinates. If it is a base or zero line it should be emphasized. Any line may be emphasized for the definite purpose of adding clearness to the graph, but no lines on the graph proper should be emphasized merely for decorative purposes. Ordinarily, there is no natural break in a series at the end of a year, but often a clearer chart will result if arbitrary breaks are made on charts, at such points as ends of years, by heavier vertical lines. As a general rule, however, the first and last vertical lines and the top horizontal lines in a chart like Exhibit 40 should not be emphasized, because they are no more important than the other coordinate lines.

The curve on a graph should be heavier than the coordinate ruling, and it should stand out clearly on the background. The curve and the zero line should be prominent since they define the limits of the sizes

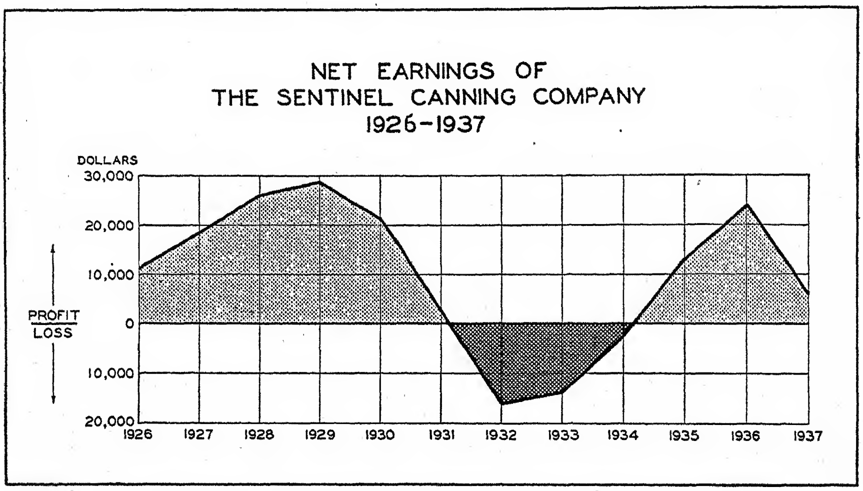


EXHIBIT 41.—The use of shading to emphasize variations.

represented; that is, in a chart like Exhibit 38 the zero line defines the bases of the vertical sizes to be represented and the curve defines the tops. Since our interest is centered chiefly in the curve, it should, as a rule, be made heavier than the zero line. The background of coordinate lines should be very light as compared with the curves and zero lines. In instances where it is desired to emphasize the variations, the areas between the curve and the zero line may be blocked in as in Exhibit 41.

Smoothed, Broken, and Step Curves.—In charting non-cumulative data, the original points plotted should be connected by straight lines. All that the lines between the points signify is the direction from one point to another. In Exhibit 42, for instance, each ordinate represents a month. The first one represents the entire month of January, the second represents February, etc. There is no time represented between the lines, and there is no value on the curve between the plotted points. The point on the February ordinate represents the total for that month, but half way on the curve between January and February would not represent the sales for the middle of the month. All that the curve does is to point the way from one total to the next.

Exhibit 43 shows the data of Exhibit 42 plotted on a cumulative basis. The space from one month to the next now represents a period of time and every point on the curve is now indicative of a value. For instance, the point half way between January 31 and February 28 indicates the sales accumulated from the beginning of the year until the middle of

February. In data like sales, however, such an indication might not be highly accurate within the month. In other kinds of variables, such as

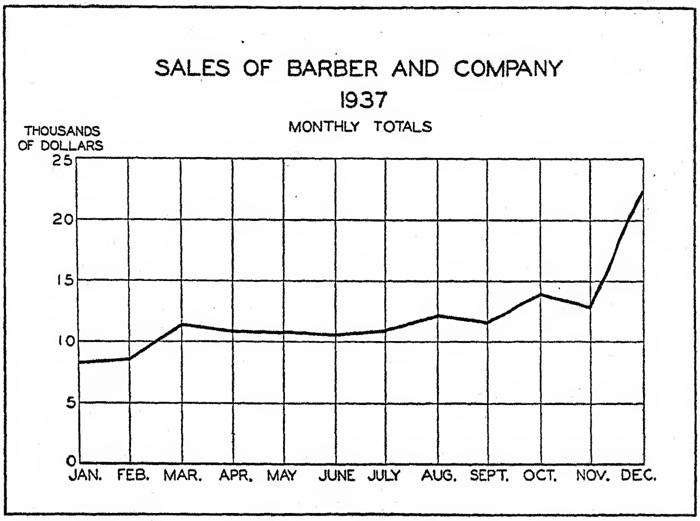


Exhibit 42.—Non-cumulative curve. (Compare with Exhibit 43.)

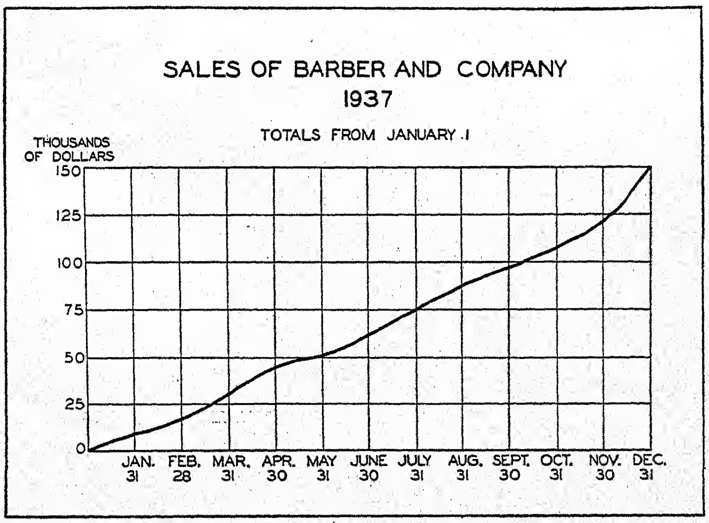


Exhibit 43.—Cumulative curve of the data presented in Exhibit 42. (Compare with Exhibit 42.)

population, changes rarely occur suddenly, and sharp angles nearly always would be absent from such a curve. Consequently, in plotting cumulative data or a continuous series, ideally the points should be connected by a smoothed curve, and not by straight lines. In smoothing

a curve the maximum possible radius of curvature should be maintained constantly, and all breaks should be avoided unless such breaks are actually known to have occurred.

The step curve, shown in Exhibit 44, is usually not good practice. It does not show the direction of the change over a period as well as the broken curve, and it does not bring out the relations at individual points

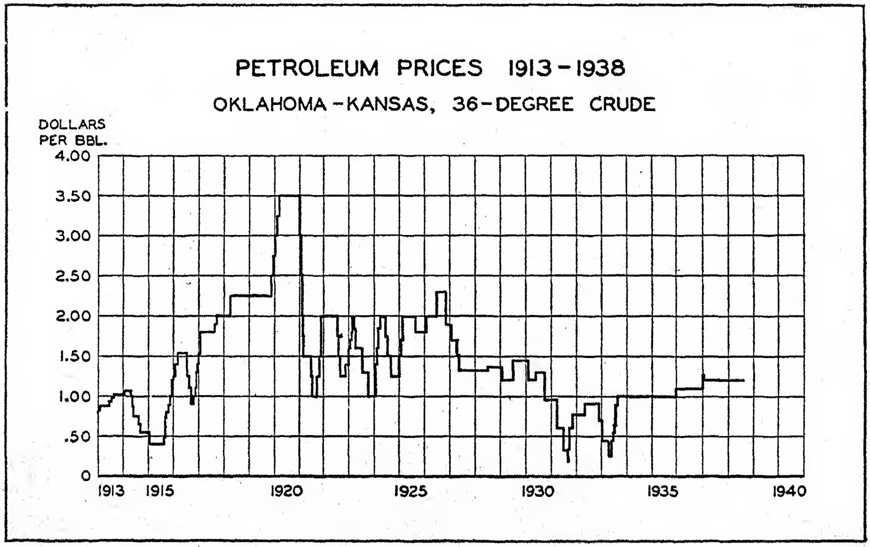


EXHIBIT 44.—Step curves should be used only when changes take place abruptly as in the above chart. They should seldom be used when ordinary broken or smoothed curves can be used.

as well as a bar chart. If two or more step curves cross one another, they may be quite confusing, since either (or both) vertical or horizontal parts may coincide. This form of curve should be used, however, when changes take place abruptly, as was the case in Exhibit 44.

Comparison of Size Differences in Time Variables.—Up to the present point in this discussion of time charts we have considered only one variable. Probably the greatest use of graphic methods in business statistics is not only in showing how the size of a single variable changes, but in comparing the sizes of two or more variables over the same period.

Choice of Forms When Comparing Two or More Series—Bars and Curves.—Bar charts can be used when showing two series over a period of time, but when several series are shown it is almost imperative that curves be used. When bar charts are used, it is necessary to differentiate the bars by some kind of shading or color throughout the comparison. The use of bars for comparing two series is illustrated in Exhibit 45, and the use of curves to show the same data is illustrated in Exhibit 46.

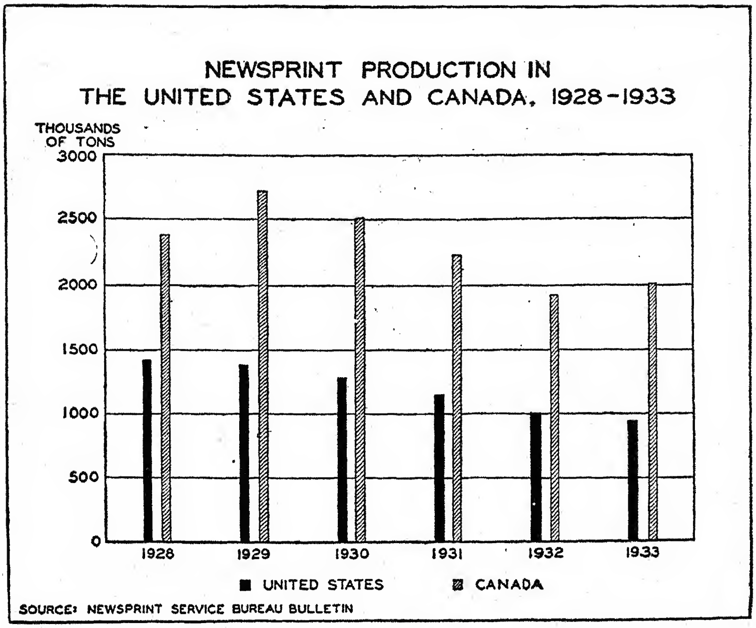


EXHIBIT 45.—Use of a bar chart in comparing two series over a period of time. (*Adapted from Newsprint Service Bureau.*)

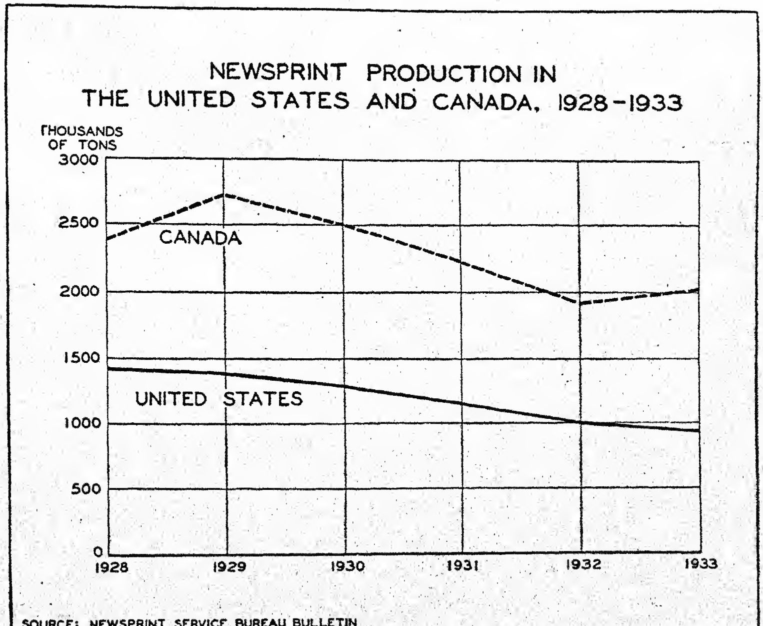


EXHIBIT 46.—Data of Exhibit 45 presented as curves. (*Adapted from Newsprint Service Bureau.*)

Common Units Necessary.—In making comparisons of size of the kind illustrated by Exhibit 46, it is necessary that the scale used be common to both series. We cannot directly compare quantities of oil in barrels with tons of coal, but by reducing them to pounds, monetary values, heat units, or other units common to both, we can plot them to the same scale and make direct comparisons. It should be noted that the preceding statement applies to *comparisons of size.* The statement does not mean that series in different units should not be included on the same chart for convenience in special cases.

The importance of including the entire scale, which was discussed on page 98 in connection with analyzing a single series, should be emphasized

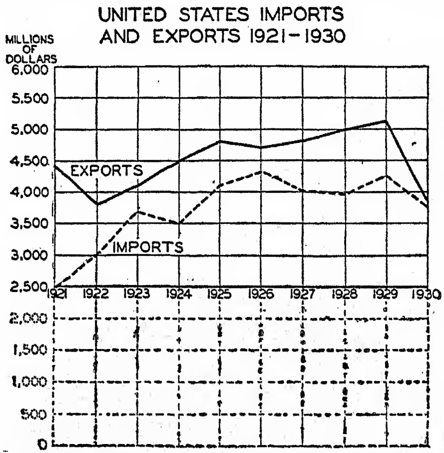


EXHIBIT 47.—In comparing *sizes* over a period of time, it is important to include the entire scale. (Note the wrong graphic impression that is given if the dotted portion of the chart is omitted.)

again in connection with making comparisons of more than one series. *When comparing sizes over a period of time,* the true relation will be shown graphically only when the chart is drawn to include the entire scale. One often sees such charts published without the lower part of the scale, and, consequently, an inaccurate graphic impression is given. In Exhibit 47, if the dotted portion of the chart were omitted, imports would appear to be about half as large as exports in 1924. When the entire scale is shown, the impression at this same point is that imports were more than three-fourths as large as exports.

It should be clearly understood that the preceding discussion applies to the problem of *comparing sizes* over a period. Whether or not the entire scale should be included in other cases depends upon what one wishes to show. There are occasional instances when it is not necessary to include the entire scale, but, when it is omitted, the reader should be warned as discussed in connection with the single-curve chart of Exhibit

37 on page 99. The reason for emphasizing the importance of including the entire scale is that so often it is omitted by the inexperienced when the object is to make comparisons of size.

Differentiating and Designating Curves.—When two or more variables are represented on a graph, it is necessary to differentiate the curves so that they will not be confused. There are two common methods of doing this. One is by using different colors; the other is by using different kinds of dot-and-dash combinations as illustrated in Exhibit 48. The latter method is the one usually used when the charts are to be reproduced by any kind of photographic process.

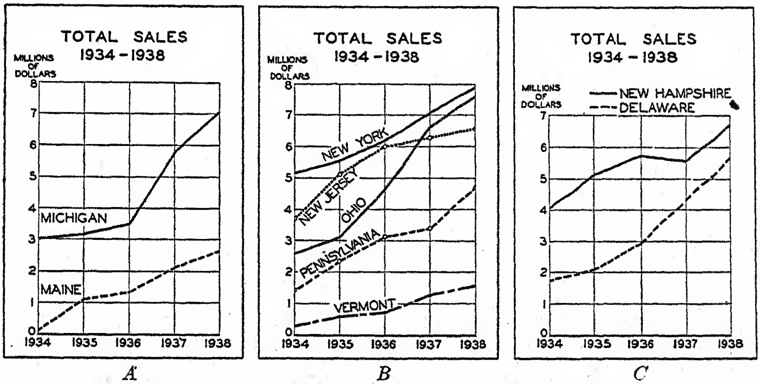


Exhibit 48.—Differentiating and designating curves.

When two or more different curves are used, it is necessary to designate them with the names of what they represent. This designating is done in two different ways, namely: (1) placing the names on the graph near the curves that they represent (Parts *A* and *B*, Exhibit 48); and (2) showing a separate key or legend elsewhere on the chart with a sample of each kind of curve (Part *C*, Exhibit 48). Of these two methods, the one illustrated in Parts *A* and *B*, Exhibit 48, is recommended as best for general use, since it is the most convenient from the point of view of the reader. The lettering should be done horizontally when there is room (Part *A*) but, if the space is limited, the lettering should follow the curve as in Part *B*. When the curves are very irregular or crowded, it is frequently necessary to tie the name to a particular curve by means of an arrow as illustrated in Parts *A* and *B*, Exhibit 130, on page 277.

Comparing Many Curves.—The number of curves that can be placed upon a single graph depends chiefly upon the way in which they are distributed over the field, but in no case should so many be used that they will run together in such a manner that they can be distinguished only with difficulty. An exception may be made to this rule, however, when curves are plotted together to show correlation.

When it is necessary to plot a large number of curves, it is usually better to break the chart up into a number of small graphs which should be drawn to exactly the same scale, as illustrated in Exhibit 49.

Components of Time Variables.—It is often desirable to show the component parts of a variable over a period of time, and this may be

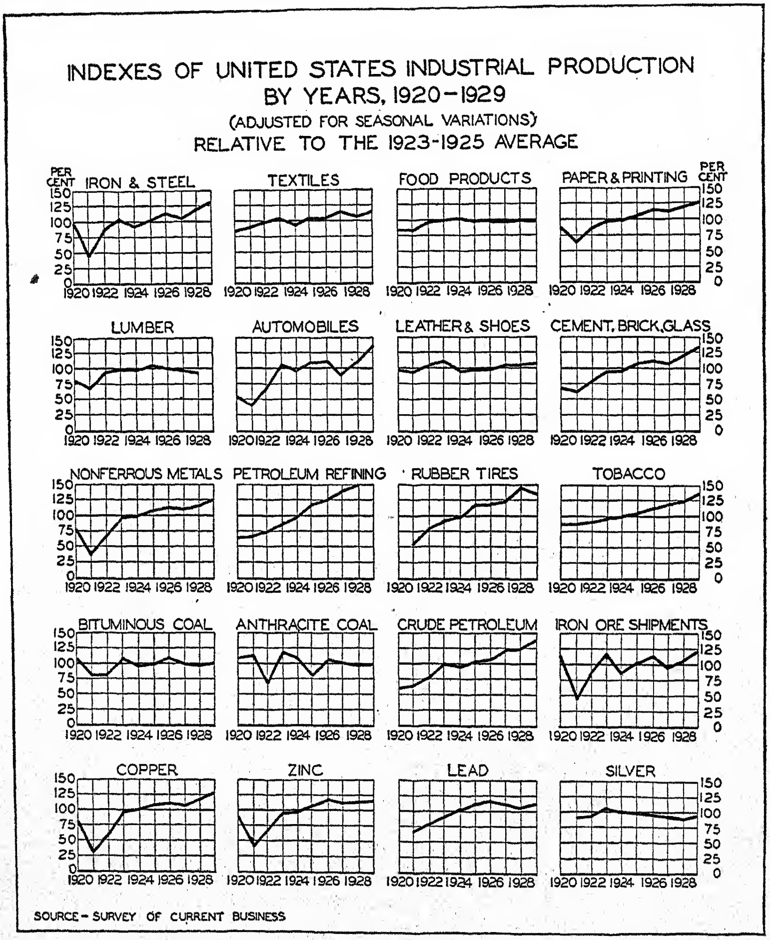


EXHIBIT 49.—A good method of showing a large number of curves.

done by using bars, curves, and other devices. Component parts on the basis of their actual sizes are clearly shown over a time period in Exhibit 50. In such a chart, the "belts," as the areas between the curves are called, are shaded in order that they may be more clearly indicated. The shaded areas are identified by placing the designations on the belts, as shown in Exhibit 50, or by using an outside legend or key that explains the meaning of each kind of shading.

Rates of Change—the Logarithmic Curve.—So far, in the study of graphic presentation of time series, we have considered only variations in *size* over the period, and the charts were constructed with arithmetic scales. If, instead of showing the actual size changes, we wish to show the *rate* of change, we can do so by using a logarithmic scale. The purposes of arithmetic curves and logarithmic curves are entirely distinct. One cannot be used instead of the other, and, in fact, a clear understanding of the arithmetic curve is practically impossible unless one understands also the logarithmic curve. Although in the past there has been a considerable amount of prejudice on the part of the business man against

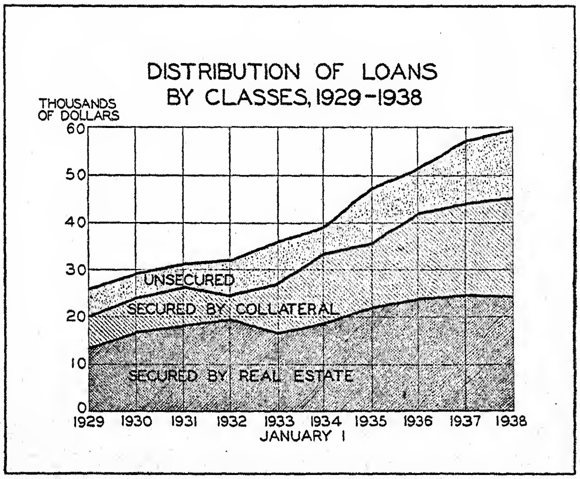


EXHIBIT 50.—Method of showing component parts over a period of time. (*Data adapted.*)

the logarithmic chart, he is now using it widely, since he appreciates its importance and realizes its simplicity.

The preceding discussion of time series considered principally the different *sizes* in time variables, as for instance in Part *A*, Exhibit 51, in which the *size* of the population for any one point of time can be compared with the *size* for any other date. That is, we can see that the population of 1930, for instance, is about twice that of 1890. We cannot, however, directly compare the rate of increase of one part of the curve with that in another. Although the curve is steeper from 1920 to 1930 than it is from 1820 to 1830, this does not mean that the population was increasing at a more rapid rate in the later period. It means simply that the actual difference in size is greater. If the *rate* of change is to be shown, the logarithmic vertical scale is used, as in Part *B*, Exhibit 51, in which the slope of the curve indicates the *rate* of growth. It is readily seen that this *rate* is falling off slightly, since the curve is less steep in the latter part of the period. It must be clearly understood at the out-

set that the two parts of the chart in Exhibit 51 have two different and distinct purposes. Part *A* shows *differences in size*, but does not show directly the *rates of change;* Part *B* shows the *rates of change*, but does not show directly the *differences in size.*[1] Both charts are simple, but the arithmetic scale (*A*) has been used more widely in charting business

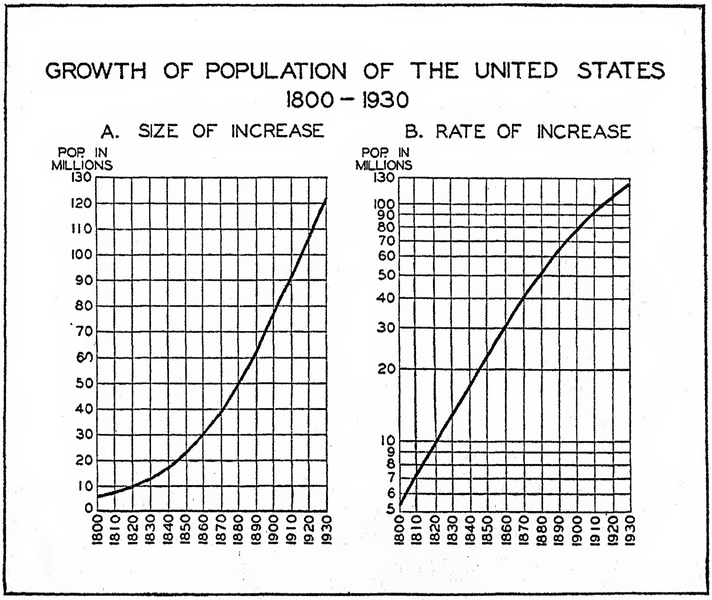


EXHIBIT 51.—Arithmetic and logarithmic scales.

statistics than the logarithmic scale (*B*). It is not necessary, however, to know anything about logarithms in order to make or use the logarithmic chart.

Scales for Logarithmic Curves.—In Part *B*, Exhibit 51, the horizontal scale is ruled arithmetically while only the vertical scale is ruled logarithmically. Curves made in this combination are commonly referred to in business usage as logarithmic curves.[2]

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[1] It should be appreciated that not all problems are those of differences in size or those of rates of change. Sometimes, while one may not really wish to show rates of change, it is very desirable to show a wide range of values within a small space, and still leave the lower values readable. In such instances, the logarithmic scale may be used for the purpose of showing high- and low-value series on the same chart without using double scales or losing legibility, even though the study may not be strictly one of rates of change. The ratio characteristics, however, of such a curve must not be misinterpreted.

[2] In practical business use, this chart is commonly called a "logarithmic chart" or "logarithmic curve." It is sometimes suggested that the term "semi-logarithmic chart" should be used since only one scale is logarithmic, leaving the term "logarith-

The arithmetic horizontal scale allows the uniform spacing for the periods of time, such as months or years, while the vertical logarithmic scale, which is geometrical progression, shows directly the true relative changes.[1] On the arithmetic chart, equal intervals on the vertical scale

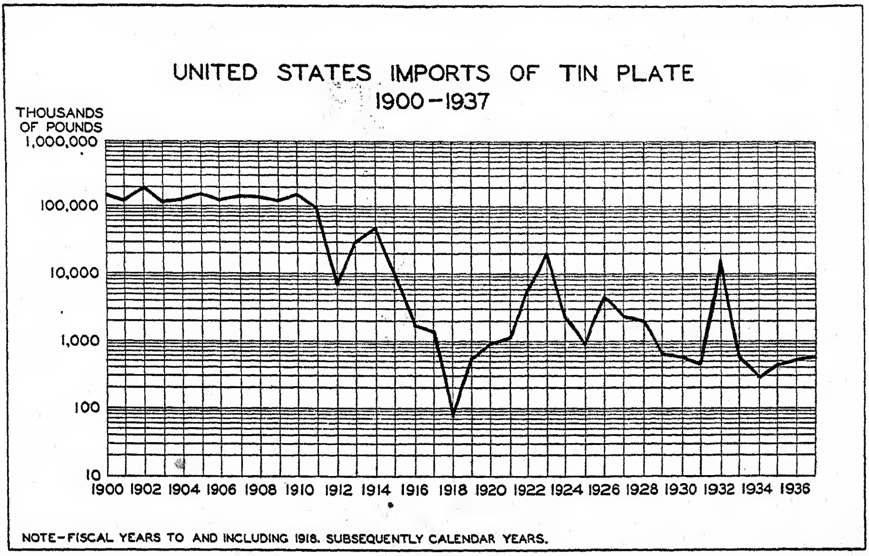

EXHIBIT 52.—Logarithmic chart of five decks.

represent equal *differences* as in Part *A*, Exhibit 51, where each vertical spacing represents ten million persons throughout the scale. In the logarithmic chart, equal intervals on the vertical scale represent equal *ratios*. Thus, in Part *B*, Exhibit 51 the space from 5 to 10 is the same as from 40 to 80, or 10 to 20 or for any other twofold or 100 per cent increase. In other words, all twofold or 100 per cent increases have the same spacing.

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mic" to apply to charts in which both scales are logarithmic. A chart with two logarithmic scales is so seldom used in business statistics, however, that it has become common practice for the business user to attach the modifying term to this chart, calling it a "double logarithmic chart," and to apply the less awkward term "logarithmic" to the single-scale logarithmic chart which is in common business use. The terms "semi-logarithmic" and "arithlog" should be used, however, when ordering the coordinate paper that is on the market for use in making these charts, as a precaution in preventing errors or confusion.

[1] A geometric progression is a series of numbers in which each successive term is multiplied or divided by a constant factor thus, 1, 2, 4, 8, 16, 32, etc.; or 16, 8, 4, 2, etc. An arithmetic progression is a series of numbers in which each successive term is increased or decreased by a constant amount, as 1, 2, 3, 4, etc.; or 2, 4, 6, 8, etc.; or 15, 12, 9, 6, etc.

Ordinarily, it is not necessary for a logarithmic chart to show any more horizontal lines than are shown in Exhibit 51. If the curve extends through several decks, it may be necessary to number the vertical scale as in Exhibit 52, which shows a scale of from 10,000 to 1,000,000,000.

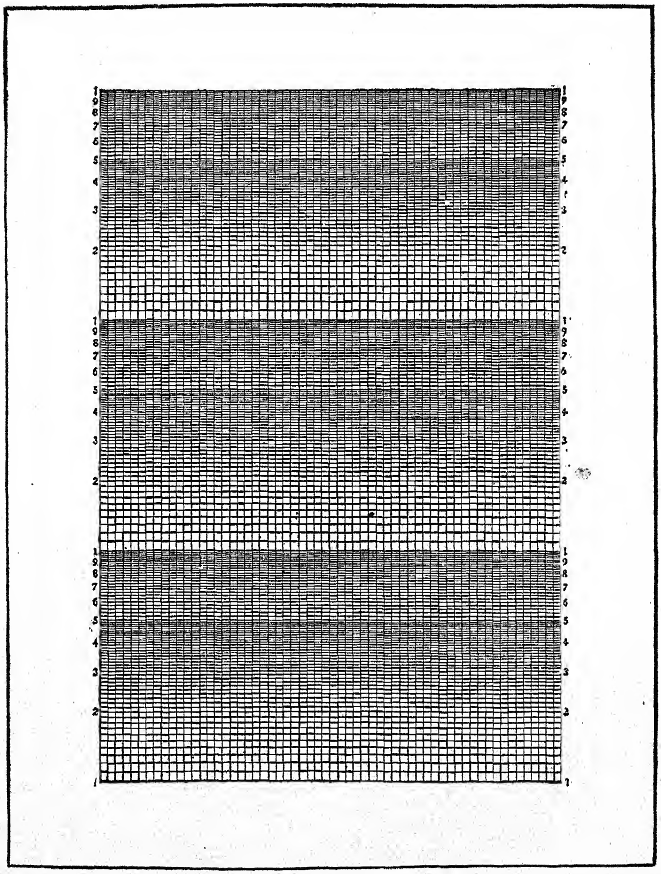


EXHIBIT 53.—Letter-size logarithmic paper. (Reduced $2\frac{1}{2}$ times linear.) (Sometimes called semi-logarithmic since only the vertical scale is logarithmic.)

When it is desired to plot a logarithmic curve, special coordinate paper may be obtained, or the scale may be taken from a statistician's scale or slide rule, and the coordinate lines drawn on plain paper.[1] One complete cycle or deck on a logarithmic scale is usually numbered from 1 to 10, and the decimal point may be placed so that the scale will be suitable for the curve to be plotted—that is, a cycle or deck may be .01 to .1, .1 to 1, 1 to 10, 10 to 100, 100 to 1,000, etc. In all cases each deck

[1] See p. 661 for a statistician's scale; and see pp. 664–665 for methods of reducing or enlarging a logarithmic scale.

or cycle represents figures 10 times as large as those represented by the deck immediately below.[1] On semilogarithmic coordinate paper (see Exhibit 53), there are more subdivisions ruled in for convenience in plotting than are shown in Part *B*, Exhibit 51, but ordinarily no more are necessary for reading than are given in Exhibit 51. It will be noted that the lines are closer together as the scale ascends and finally some must be omitted.[2]

A simple test to apply to the logarithmic scale in determining whether or not it is laid out correctly is to take a scale, the dividers, or an edge of paper, and see if the spaces from numbers to their doubles are the same—that is, the distance from 1 to 2 should be same as from 3 to 6; 500 to 1,000; 4,000,000 to 8,000,000; etc.

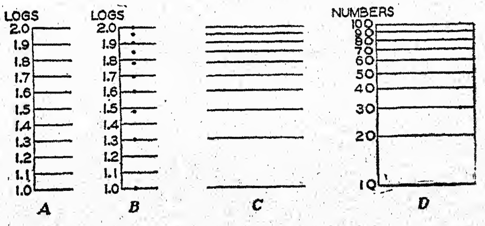
Since the logarithmic scale is laid out in geometrical progression, obviously we can approach zero indefinitely, but we can never reach it.

[1] See John R. Riggleman, "Graphic Methods for Presenting Business Statistics," 2d ed., pp. 109–115, McGraw-Hill Book Company, Inc., New York, 1936.

[2] The divisions of a logarithmic scale are proportional, not to the numbers themselves, but to their logarithms. That is, the scale is divided logarithmically, and the designations are not the logarithms but the numbers corresponding to the logarithms. Thus, for instance, 4 is over halfway between 1 and 10 because the logarithm of 4 is 0.6021, which is more than half as large as the logarithm of 10 which is 1; but instead of using the logarithm 0.6021 in the scale, the corresponding number 4 is used.

If semi-logarithmic paper or a logarithmic scale is not at hand, a ratio chart may be made by reducing the data to logarithmic terms, and then, using the ordinary arithmetic scale and paper, the logarithms of the given quantities will be plotted instead of the quantities themselves (Exhibit 60, Part *G*, page 119).

If no logarithmic scale is at hand, and it is desired to make a chart on a semi-logarithmic coordinate ruling, the logarithmic scale may be laid out as follows: Suppose that it is desired to lay out a logarithmic scale in round numbers from 10 to 100 (10, 20, 30, 40, 50, 60, 70, 80, 90, 100). Look up the logarithms of these round numbers, which would be as follows: 10, 1.00; 20, 1.30; 30, 1.48; 40, 1.60; 50, 1.70; 60, 1.78; 70, 1.85; 80, 1.90; 90, 1.95; 100, 2.00. Next, lay out, arithmetically, a scale on which to plot these logarithms as in *A* as illustrated below. Then plot the fore-



going logarithms as shown by the points in *B*. Next, rule lines spaced according to the points plotted in *B*, as in *C*. Then number the lines according to the natural numbers as in *D*, not according to the logarithms. As explained above, the divisions of a logarithmic scale are proportional, not to the numbers themselves, but to their logarithms; that is, the scale is divided logarithmically and the designations are not the logarithms, but the natural numbers.

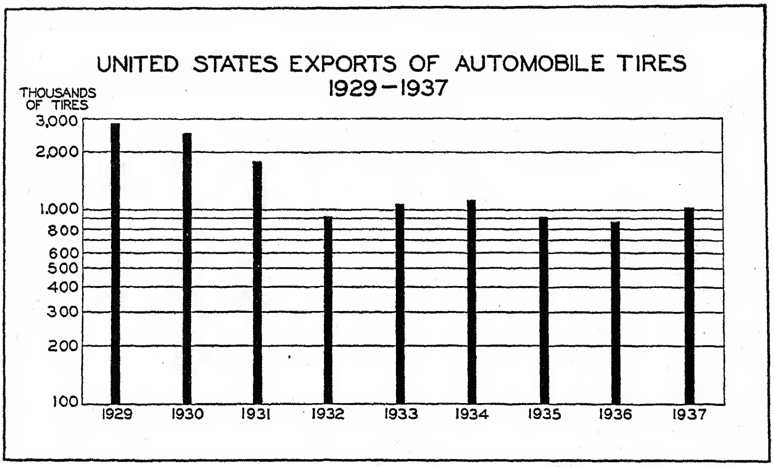
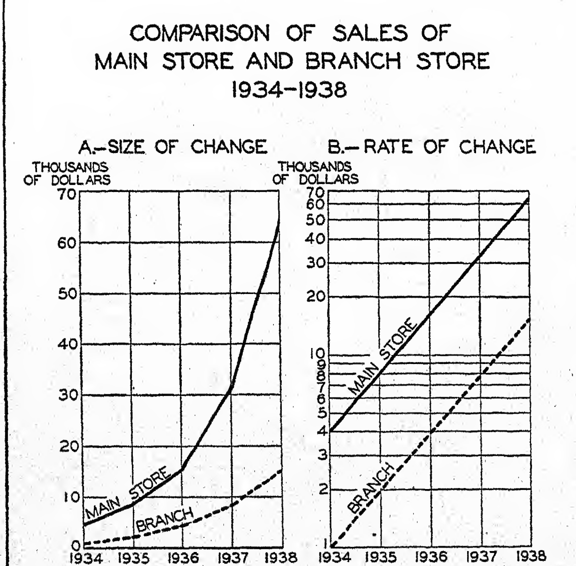


EXHIBIT 54.—Wrong. Such a use of bars on a logarithmic scale is incorrect.



	MAIN STORE	BRANCH
1934	$4,000	$1,000
1935	8,000	2,000
1936	16,000	4,000
1937	32,000	8,000
1938	64,000	16,000

EXHIBIT 55.—Simple comparison of sizes of change (*A*), and rates of change (*B*).

In other words, there is no zero on a logarithmic scale. Furthermore, there is no base line on a logarithmic scale, for any value is as much base value as any other. The scale may begin with 1, 3, or any other number. It is necessary only to choose a scale sufficiently long to include the data to be plotted, as in Part *B*, Exhibit 51. Since there is no fixed base, the bottom line of a logarithmic chart should not be emphasized as is the zero line for an arithmetic scale.

Meaning of Slopes of Curves.—In logarithmic charts the index of the rate of change is the slope of the curve. If a curve has a steeper slope

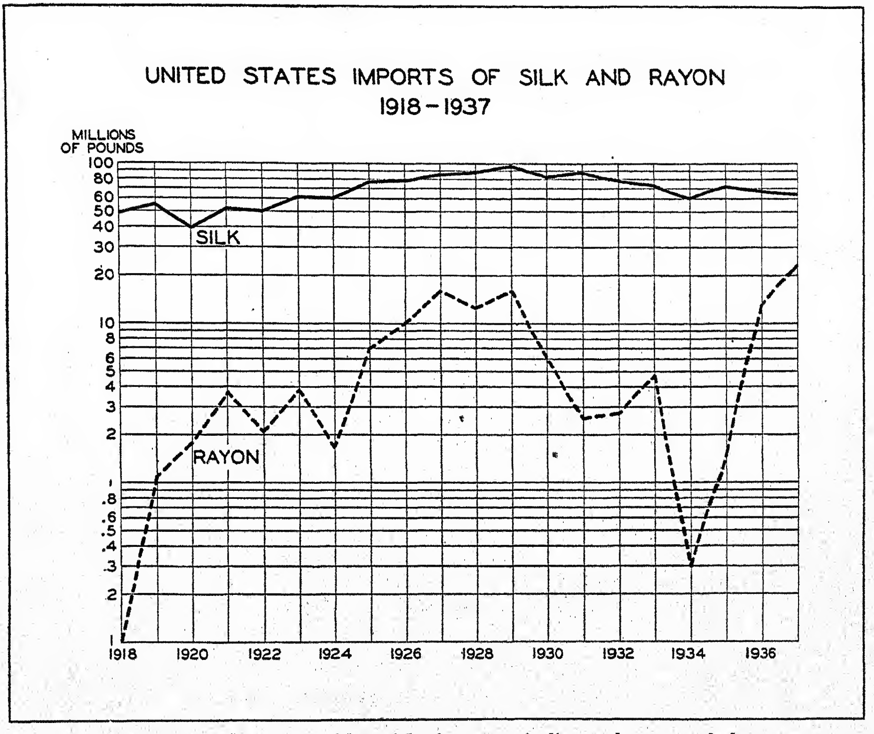


EXHIBIT 56.—The slopes of logarithmic curves indicate the rates of change.

in one part than in another, it means that the steeper part is changing at a higher rate. Thus, in Exhibit 51, population increased more rapidly during the first half of the period shown than during the second half. A straight line over a period would represent a constant rate of change. The area under a curve, which is so important on an arithmetic graph, is meaningless on a logarithmic chart.

Occasionally one sees a chart drawn like Exhibit 54, which is distinctly incorrect. The bars have no significance in this chart in repre-

senting relative changes, and they cannot represent sizes, as there can be no zero base on a logarithmic chart.

Comparisons of Rates of Change.—It must be kept in mind that the purposes of the arithmetic curve and the logarithmic curve are distinctly different. The two cannot be combined, and one cannot take the place of the other. This point will be brought out more clearly by a consideration of Parts *A* and *B*, Exhibit 55. In *A*, we can compare the size of one store with that of the other, and it is easy to see that the main store is about four times the size of the branch. We cannot, however,

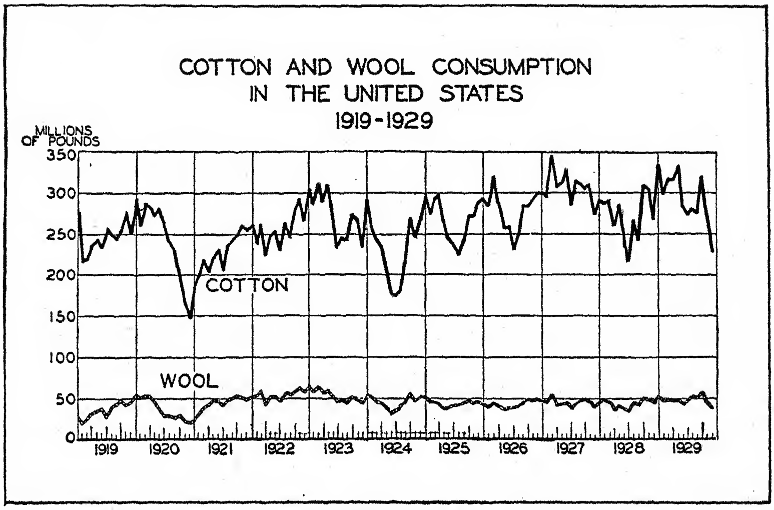


EXHIBIT 57*A*.—Comparing *size* changes. (See Exhibit 57*B*.)

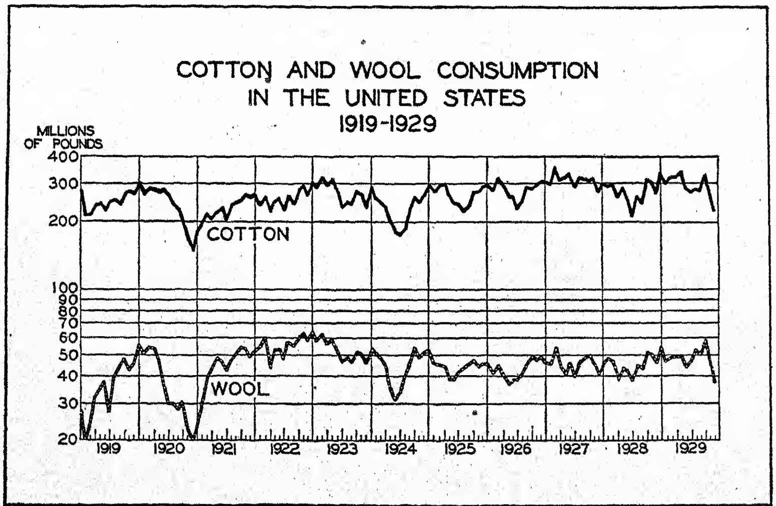


EXHIBIT 57*B*.—Comparing relative fluctuations or *rates* of change. (See Exhibit 57*A*.)

directly compare the rates of change in the two curves—that is, we cannot tell directly which store is growing the faster. To determine this we can use the logarithmic curve as illustrated in Part *B*, Exhibit 55, which shows that the rates of change are the same.

Exhibit 56 further illustrates the comparison of rates of change between two different series. It is easy to see in this chart that imports of rayon into the United States have tended to increase much more rapidly during the last few years than imports of silk.

Business men often attempt to compare the rates of change between arithmetic curves without realizing how greatly they are being misled. For instance, a number of executives of a large eastern bank were studying the 1920 drop in cotton and wool consumption (see 1920 in Exhibit 57*A*) from an arithmetic chart which appeared in the *Federal Reserve Bulletin*. After a discussion, they agreed as to the reason why cotton consumption is more speculative than wool consumption, when, as a matter of fact, cotton does not fluctuate relatively as much as wool (Exhibit 57*B*). The arithmetic curves which these men were studying showed *sizes* of fluctuations, whereas the conclusions that they were drawing should have been based upon *relative* fluctuations, which are shown directly by the logarithmic chart. Such instances indicate that it is very important to understand the nature and use of the logarithmic curve in order that one may use correctly the arithmetic curve.

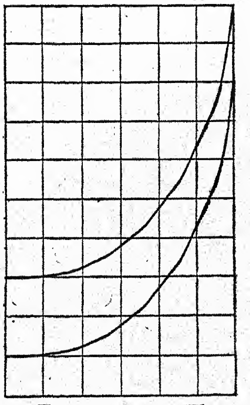


EXHIBIT 58.—These curves are parallel in the sense that the vertical distances between them are the same.

It has been pointed out that the rates of change of the two series in Part *B*, Exhibit 55, are the same. Whenever, in a logarithmic chart, two curves run parallel in the sense that the vertical distance between them remains unaltered, the rates of change are the same. In this connection, it should be appreciated that curves which are parallel in the sense that the vertical distance remains the same, may not be separated by equal direct distances. Exhibit 58 shows two curves that are parallel in that the vertical distance between them does not vary, but the direct distance between the two curves changes. This effect is pronounced only when curves are far from straight, but in such cases care must be taken to consider the vertical distances between the curves.

Common Units Not Necessary in Comparing Rates of Change.—Rates of change in variables may be compared on a logarithmic chart, even though they are not expressed in common units. In arithmetic charts, we cannot directly compare the sizes of coal and oil productions when coal is expressed in tons and oil in barrels. In logarithmic charts, however, different units do not prevent comparing rates of change, for if oil

production increases over a given period from 200 to 400 million barrels, and coal increases from 300 to 600 million tons, the production has doubled in both instances, and hence the rates of change are the same.

Comparing Different Logarithmic Charts.—When it is desired to make logarithmic charts in such a way that they can be compared directly, they should be drawn with the same spacing in the horizontal scales, and the vertical scale should be so drawn that equal spaces will represent

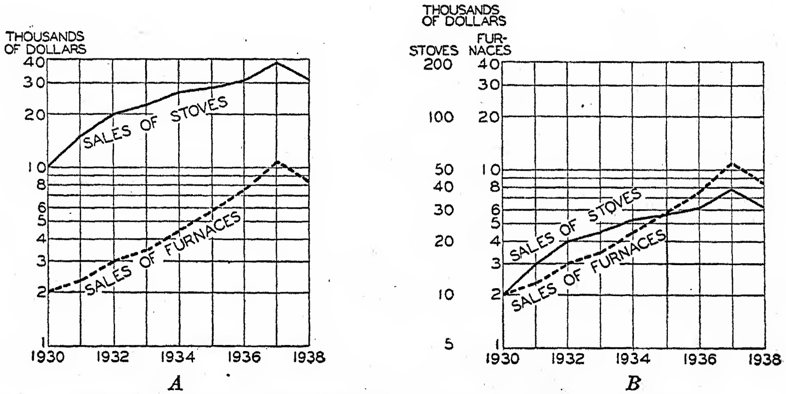


Exhibit 59.—Shifting logarithmic curves to facilitate comparison.

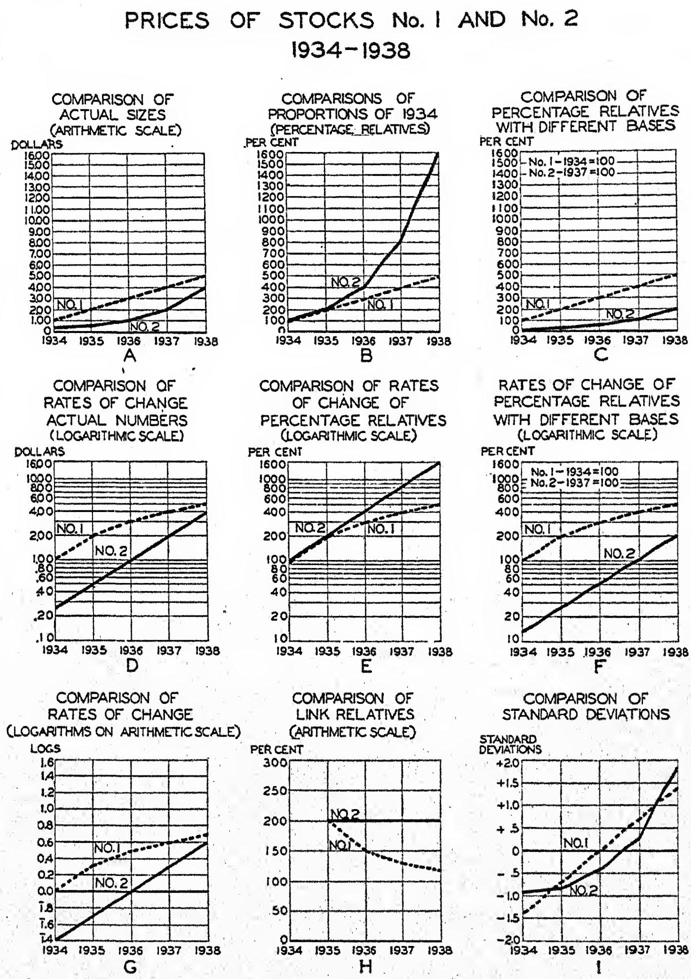
equal ratios. That is, on all of the logarithmic scales the spaces from all numbers to their doubles should be the same.

Shifting Logarithmic Curves to Facilitate Comparison.—Since, on a logarithmic chart, the index of the rate of change is the slope of the curve, it is sometimes desirable to bring the curves closer together in order to facilitate comparison. Thus in Part *A*, Exhibit 59, the curves are too far apart to see easily the differences in the rates of change. So in Part *B*, these same curves are placed together and the differences can be noted more easily. Both curves may still be read from the scales if they are included as in Part *B*, Exhibit 59. If it is desired to plot the data directly, one scale may be raised or lowered in position along the other by multiplying or dividing it by a constant. Thus, in Part *B*, Exhibit 59, every number in the left-hand scale is exactly five times the opposite number in the right-hand scale.

Another method of shifting curves to facilitate comparison is through the use of percentage relatives.

Comparison of Percentage Relatives.—Percentage relatives are used when it is desired to compare not absolute sizes of the quantities or rates of change over the period, but the changes that have taken place in terms of percentages of the value at some particular period. In determining percentage relatives, the data for one period or average of a period are taken as the base, usually indicated as 100, upon which the data for the

following or preceding periods are computed as percentages. Thus, in Exhibit 60, Chart *A* compares the actual sizes of the two prices over the



PRICES OF STOCKS NO. 1 AND NO. 2

YEARS	DOLLARS		PERCENTAGE RELATIVES 1934=100		PERCENTAGE RELATIVES		LOGARITHMS		LINK RELATIVES(%)		STANDARD DEVIATIONS	
	NO. 1	NO. 2	NO. 1	NO. 2	NO. 1 1934=100	NO. 2 1937=100	NO. 1	NO. 2	NO. 1	NO. 2	NO. 1	NO. 2
1934	1.00	0.25	100	100	100	12.5	0.00	$\bar{1}$.39			-1.42	-.95
1935	2.00	0.50	200	200	200	25	0.30	$\bar{1}$.69	200	200	-.71	-.76
1936	3.00	1.00	300	400	300	50	0.47	0.00	150	200	0.00	-.40
1937	4.00	2.00	400	800	400	100	0.60	0.30	133	200	+.71	+.33
1938	5.00	4.00	500	1600	500	200	0.69	0.60	125	200	+1.42	+1.79

EXHIBIT 60.—Various methods of comparing time series.

period, while Chart *B* compares the changes that have taken place by showing what per cent of the 1934 values the prices are for the other dates

—that is, for 1936 the percentage relative on the base of 1934 for stock No. 2 is 400, which means that it is 400 per cent of the price for 1934.

The use of the percentage-relative chart must not be confused with that of the logarithmic chart. The two are distinctly different. While we can see from Chart *B*, Exhibit 60, that the proportional increase from 1934 to 1938 is greater in No. 2 than in No. 1, we cannot compare the rates of change for the different periods in either curve. *Rates of change* are compared in Charts *D*, *E*, *F*, and *G* (Exhibit 60). When the curves are plotted on a logarithmic scale, it will be noted that the slopes of the curves are the same whether the actual numbers or the percentage relatives are plotted.

If percentage relatives of the same base year are plotted as in Chart *E*, Exhibit 60, the curves are automatically shifted so that they are close to each other, which facilitates the comparison of logarithmic curves as has been discussed previously. Another good instance of reducing the data to percentage relatives for the purpose of shifting the curves to facilitate comparison is the chart shown in Exhibit 175 on page 370. The slight spread between these curves is very significant. Unless they are close together, it is difficult for the reader to appreciate the differences in the slopes.

If two series of percentage relatives, having different base periods, are to be studied, they cannot be compared directly on an arithmetic scale without distortion (see Chart *C*, Exhibit 60). Different base periods, however, make no difference in the shapes of the curves plotted on the logarithmic scale; this is illustrated by Chart *F*, Exhibit 60. These different methods of handling the data do not alter the rates of change, and hence there is no difference in what is shown by Charts *D*, *E*, and *F*, Exhibit 60.

If a logarithmic scale is not at hand, a logarithmic chart may be constructed as shown in Chart *G*, Exhibit 60. In this graph, the logarithms of the given quantities are plotted (on an arithmetic scale) instead of the quantities themselves.[1] Rates of change are shown by the slopes of the resulting curves. It will be noted that the shapes and slopes of the curves in Chart *G* are the same as in Charts *D*, *E*, and *F*.[2]

Cumulative and Non-cumulative Series.—It is important that the distinction between cumulative and non-cumulative data be clearly appreciated in the field of business statistics as a whole, as well as in the special field of graphic presentation. Otherwise, considerable confusion is likely to result due to attempts to make direct comparisons of cumulative with non-cumulative series. This is true whether amounts of change or rates of change are being compared. When a chart similar to that presented in Part *A*, Exhibit 61, was being considered by a certain group

[1] See first two paragraphs of second footnote on page 113.

[2] Charts *H* and *I*, Exhibit 60, will be referred to later; see footnotes pp. 224 and 322.

of business men, a number of those present raised the question as to why building activity declined so materially while population continued to increase. Though several possible reasons were advanced, the chief difficulty was that the building permit series was non-cumulative, while the population series was of a cumulative nature. As a general rule in such instances, either the annual building permit figures should be

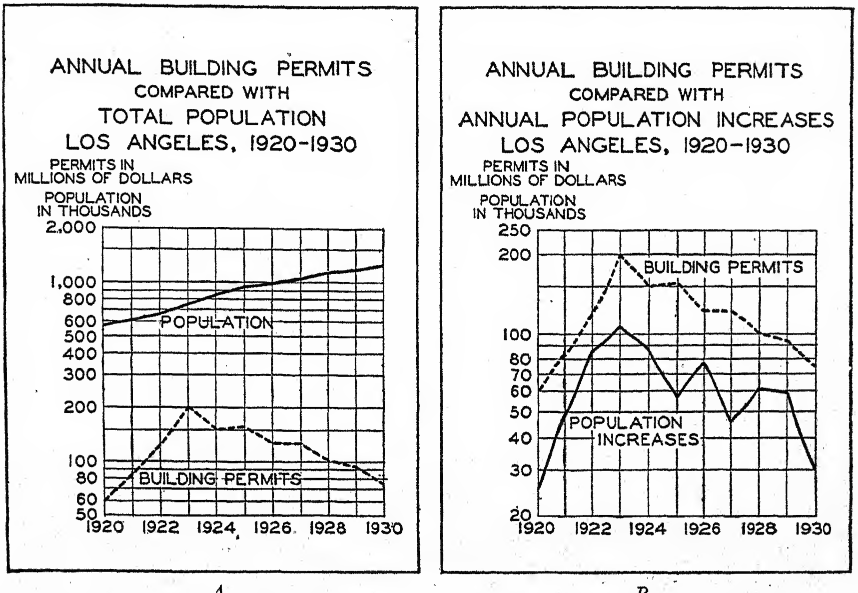


Exhibit 61.—In comparing series (graphically or otherwise), it is important to distinguish between cumulative and non-cumulative data. *A*. Non-cumulative series compared with cumulative series—a difficult comparison to analyze. *B*. Both series of Part *A* shown on a non-cumulative basis; the comparison is much easier to analyze.

compared with the annual increases in population, or the total population figures should be compared with a similar net cumulative series such as total building in existence.[1]

The two series of Part *A*, Exhibit 61, are both shown on a non-cumulative basis in Part *B*. That is, the annual building figures are compared with the annual increases in population. It is obvious that the type of comparison illustrated in Part *B* is ordinarily of greater significance and easier to interpret than that in Part *A*.

Other Types of Statistical Charts.—This discussion of graphic methods has considered only the methods for showing simple comparisons of size (bar charts, pie charts, etc.), arithmetic curves, and logarithmic curves. In addition to those, charts of frequency curves, correlation

[1] It is also often necessary to make other adjustments and corrections, such as eliminating effects of price changes, when making such comparisons. These will be discussed later. It is the object here only to point out the importance of distinguishing between cumulative and non-cumulative series.

curves, and statistical maps are used in business statistics. For discussion of the technique involved in constructing these charts, the reader is referred to the more specialized books on graphic methods. Examples of these charts will be given, and their use will be illustrated later on throughout this text in connection with the various applications and interpretations of statistical methods and data.

Checking List for Curves.—The following list includes the most important points to be observed in making curve charts; it can be used when checking arithmetic and logarithmic time charts, frequency charts, and correlation charts to see if they are complete:

CHECKING LIST FOR CURVE CHARTS

Have you placed your vertical scale (dependent variable) on the left of the graph, and does it read from bottom to top? (If desirable to show the scale on the right, it may be placed there in addition to the scale on the left.)

Have you placed your horizontal scale (independent variable) along the bottom of the graph, and does it read from left to right? (If desirable to place the scale at the top, it may be placed there in addition to the scale at the bottom.)

Have you centered a caption over the vertical scale?

If necessary, have you placed a designation under the horizontal scale?

If you have used two or more curves, have you designated them, and can they be distinguished clearly?

Are your curves heavy?

If you have shaded or colored bars or areas, have you included designations or a key or legend? Are the faces of any such bars free from coordinate or scale lines?

If the vertical scale is arithmetic, is the zero line or 100 per cent base line heavy? (The bottom line of a logarithmic chart should not be emphasized.)

Is your background of coordinate lines light?

Is your title your largest lettering?

Is the title clear and easy to read?

Have you given the reference or source (if desirable)?

Does all lettering read horizontally? (When impossible to letter horizontally, lettering should read from bottom to top—never from top to bottom.)

Are your letters formed according to a commonly accepted style of plain lettering?

Did you check for accuracy?

Have you ruled in a border line?

Have you given your name (lower right) and the date (lower left) if desirable?

Are pencil lines erased?

Will your chart be clear to the reader?

Summary of Graphic Methods.—In the description of methods as applied to the charts in the last two chapters, certain fundamental rules of practice have been developed. These rules apply to other kinds of charts, and a thorough knowledge of them on the part of the reader will be assumed when other kinds of charts are used later on in this book. For convenience, both in review and in reference, these fundamental rules have been summarized and presented graphically in Exhibit 62.[1]

[1] This summary of rules is adapted in part from the "Report of the Joint Committee on Standards for Graphic Presentation," *Quarterly Publications of the American Statistical Association*, Vol. 14, pp. 790–797, 1915.

YEAR	TONS
1900	270,588
1930	555,071

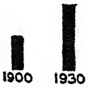


Fig. 1a

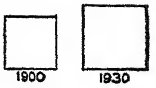


Fig. 1b

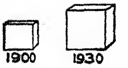


Fig. 1c

1. When possible, linear magnitudes should be used (Fig. 1a) rather than areas (1b) or volumes (1c).

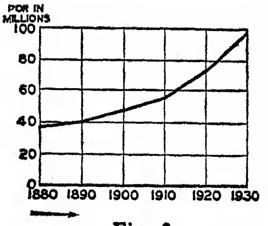


Fig. 2

2. The general arrangement should proceed from left to right (Fig. 2).

3. If the graph is to show sizes directly, the complete scale and zero line should be included (Fig. 3).

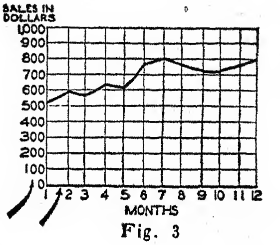


Fig. 3

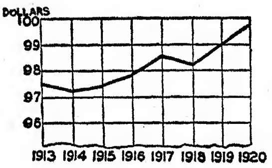


Fig. 4a

4. If the complete scale is not included, the fact should be emphasized by a horizontal break as in Fig. 4a or 4b.

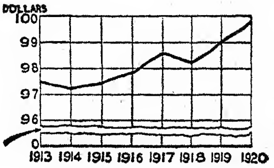


Fig. 4b

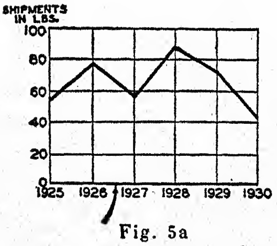


Fig. 5a

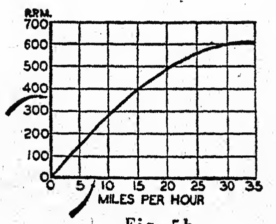


Fig. 5b

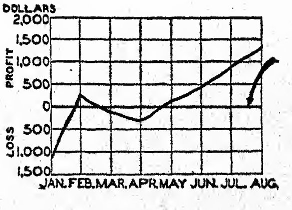


Fig. 5c

5. The zero lines of curve scales should be sharply distinguished from the other coordinate lines (Figs. 5a, 5b and 5c). In showing time series, the first vertical line should not be emphasized as a zero line since it does not represent the beginning of time (Fig. 5a).

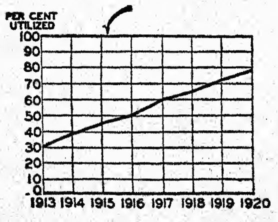


Fig. 6a

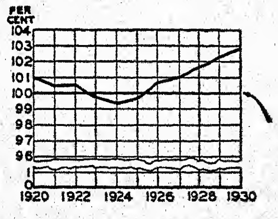


Fig. 6b

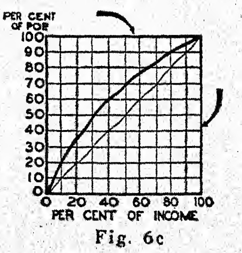


Fig. 6c

6. When the 100 per cent line is used as a basis of comparison, it should be emphasized (Figs. 6a, 6b and 6c).

EXHIBIT 62.—Summary of rules for graphic presentation of business statistics. (*Continued on pages* 124 *and* 125.)

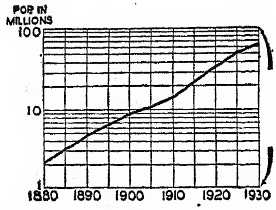


Fig. 7a

7. When a logarithmic chart is made, the top and bottom limiting lines should each be at some power of ten if convenient (Fig. 7a). If not convenient, as when only a small portion of the range is used (Fig. 7b), any other lines may be used as top and bottom limits.

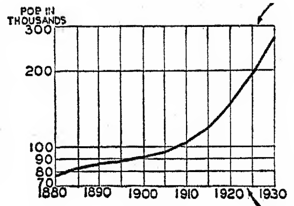


Fig. 7b

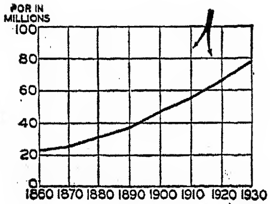


Fig. 8a

8. It is advisable to show no more coordinate lines than are necessary to guide the eye in reading the diagram. Fig. 8a is easier to read than Fig. 8b.

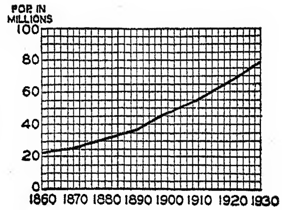


Fig. 8b

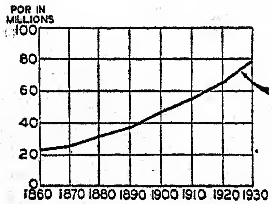


Fig. 9

9. The curves should be sharply distinguished from the background of coordinate lines (Fig. 9). Since the attention is centered on the curve, it should be made more prominent than the zero line as shown in Fig. 9.

10. When lettering cannot be made to read horizontally as in Fig. 9, it should be made to read from bottom to top as in Fig. 10—never from top to bottom. That is, if it cannot be placed like this: ABC, it should be placed like this: ABC, but never like this: ABC.

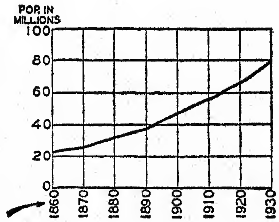


Fig. 10

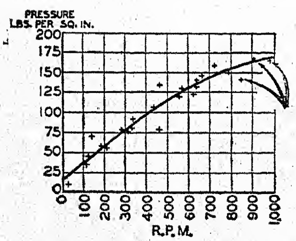


Fig. 11a

11. When irregular or scattered data are plotted, it is often desirable to indicate the points representing the different observations as in Figs. 11a and 11b.

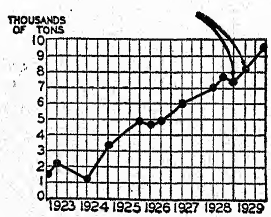


Fig. 11b

Exhibit 62 (*Continued*).—Summary of rules for graphic presentation of business statistics.

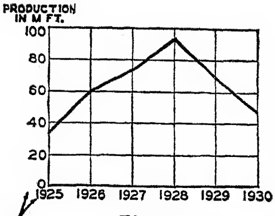


Fig. 12a

12. The scales should be placed at the left and at the bottom of the graph as in Figs. 12a and 12b. If desirable they may be placed at the top and on the right also, but they should never be omitted on the left and on the bottom.

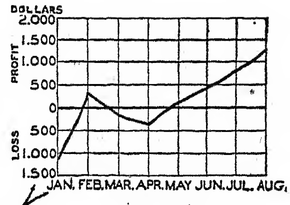


Fig. 12b

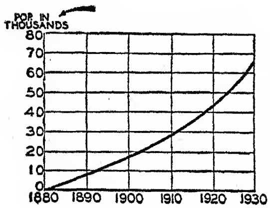


Fig. 13a

13. A caption describing the units used in the vertical scale should be placed at the top of the scale as in Fig. 13a. (Another method is to place the designation along the side but this is less preferable.) When desirable, the horizontal scale should be designated as in Fig. 13b. (Ordinarily it is not necessary to designate years or months.)

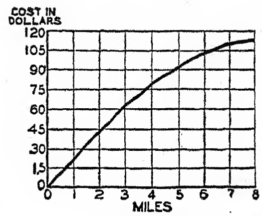


Fig. 13b

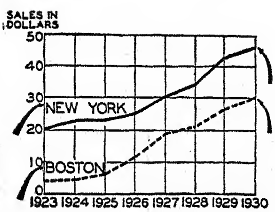


Fig. 14a

14. When two or more curves are presented, they should be differentiated (solid, dashed, dotted lines, etc.) and properly designated (see Figs. 14a and 14b). The method of placing the designations along the curves (14a) is preferred.

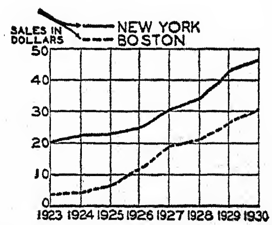


Fig. 14b

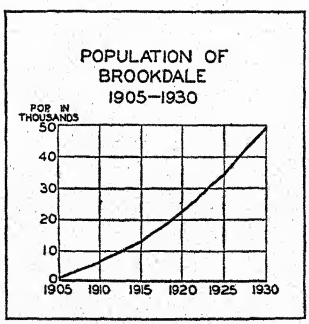


Fig. 15

15a. The title of a chart should be placed at the top as in Fig. 15. It should be as clear and complete as possible. Subtitles, descriptions and footnotes should be added if necessary to insure clearness.

15b. Usually it is desirable to place a border line around the entire chart to give it a finished appearance (Fig. 15).

EXHIBIT 62 (*Concluded*).—Summary of rules for graphic presentation of business statistics.

Questions and Problems

1. Can time series be presented by bar charts? What kind of bar charts? Describe a more common way of presenting time series graphically.

2. What are coordinate axes? What is the axis of ordinates? The axis of abscissas? The axis of x? The axis of y?

3. Should the dependent variable be plotted on the horizontal or vertical scale in an arithmetic time chart? What figure should constitute the base and with what exceptions?

4. On what scale should the independent variable be plotted?

5. Should points be plotted on or between the vertical lines? What are the requirements regarding the location of the horizontal scale figures or words?

6. How should the vertical scale be arranged and designated?

7. What lines of a curve chart should be emphasized? Why?

8. How should points plotted on curve charts be connected?

9. Why is it seldom good practice to use step curves? When should they be used?

10. What kind of chart is best for presenting several series of data over a period of time? Why are bar charts not feasible for this kind of presentation?

11. Do curve charts have the same requirements as bar charts as regards the necessity for common units? Why?

12. When several series are shown on a curve chart, how should they be distinguished?

13. What can be done if the number of series becomes unwieldy for one graph?

14. What are "belt" charts? When should they be used? How are they designated?

15. "The purposes of the arithmetic curve and logarithmic curve are entirely distinct." Explain.

16. What does the steepness of an arithmetic curve signify? The steepness of a logarithmic curve?

17. What is the difference between arithmetic and logarithmic scales?

18. Are there any requirements regarding the base figure of a logarithmic scale? Discuss.

19. Arithmetic curves of the sales of two stores would tell us what regarding their records? Logarithmic curves would tell us what regarding their records?

20. In comparing series on logarithmic charts is it necessary to have common units? Why?

21. Present the following data in the form of a vertical-bar chart (see Exhibit 34):

MOTOR VEHICLE REGISTRATION IN THE UNITED STATES, 1925–1937

Year	*Registration in thousands*
1925	19,937
1926	22,001
1927	23,133
1928	24,493
1929	26,501
1930	26,545
1931	25,833
1932	24,115
1933	23,844
1934	24,952
1935	26,231
1936	28,166
1937	29,705

Source: Automobile Facts and Figures.

Check your chart according to the checking list on page 122.

22. Below are given the factory sales of passenger cars in the United States and Canada for cars in two price classes.

Year	Number sold	
	$501 to $750	$2,001 to $3,000
1925	660,520	54,814
1926	1,149,301	73,738
1927	957,987	50,064
1928	1,219,266	55,304
1929	1,317,116	47,587
1930	680,352	27,266
1931	413,929	12,714
1932	260,831	8,679
1933	237,099	8,725
1934	715,989	6,879
1935	1,444,529	5,413
1936	1,677,558	4,326
1937	2,392,415	4,061

Source: Automobile Facts and Figures.

Present the above data on an arithmetic time curve (see Exhibit 46).
Check your work according to the checking list on page 122.

23. *The Annalist* index of business activity in relation to a computed normal is:

Month	Year and index as a per cent of computed normal									
	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937
January	98.0	105.5	95.0	74.4	73.4	65.7	79.6	87.2	92.2	104.2
February	99.7	106.1	94.2	76.2	71.4	66.1	83.2	86.7	88.9	105.7
March	99.4	104.3	91.2	78.0	69.8	62.5	84.6	84.4	89.4	106.8
April	99.9	108.8	95.0	80.8	66.8	69.2	85.9	82.8	94.1	107.0
May	101.3	110.1	90.0	78.1	64.3	77.3	86.4	81.8	95.8	110.0
June	98.7	108.9	89.0	76.5	63.9	87.5	83.8	82.0	97.6	107.1
July	100.5	109.9	86.4	78.2	62.9	94.0	78.0	82.7	102.3	
August	102.1	108.1	83.1	73.5	64.4	87.5	75.1	84.9	102.5	
September	102.4	107.3	82.4	70.8	68.5	82.0	71.4	86.1	102.9	
October	105.0	105.7	79.5	66.2	69.8	78.5	74.6	89.1	103.3	
November	103.7	96.9	76.1	65.0	69.2	75.3	76.0	92.0	107.1	
December	102.0	92.1	76.1	65.8	68.8	77.5	82.4	96.7	110.5	

Source: Survey of Current Business and *The Annalist.*

Present the per cent deviations from normal of the above data on an arithmetic plus and minus time curve and shade the proper area for emphasis (see Exhibit 41).

24. Monthly sales of two units of Jay and Co., a chain-store retail merchandising company, for the three years 1935–1937 were as follows:

Year and month	Sales, in thousands of dollars		Year and month	Sales, in thousands of dollars	
	Unit A	Unit B		Unit A	Unit B
1935			1936		
January	24.1	418	July	41.8	542
February	26.2	429	August	49.6	551
March	25.1	431	September	43.5	562
April	27.4	464	October	50.2	566
May	25.7	456	November	48.7	582
June	24.2	455	December	64.3	636
July	26.0	461	1937		
August	24.9	458	January	60.1	590
September	31.8	449	February	59.7	598
October	27.6	470	March	62.3	622
November	32.4	484	April	50.3	608
December	40.3	521	May	72.4	610
1936			June	70.1	621
January	34.1	502	July	69.2	624
February	36.0	508	August	67.4	625
March	35.8	525	September	74.3	632
April	43.2	541	October	81.2	648
May	34.1	524	November	78.2	654
June	40.0	531	December	94.1	682

a. Present the data as two curves in the form of an arithmetic time chart.

b. Present the data as two curves in the form of a logarithmic time chart.

c. Does the chart presented in (*a*) clearly compare the "amount" of sales of the two units over the period? Does the chart presented in (*b*) clearly compare the "rates of change" of sales of the two units over the period?

Analyze your charts and present a brief report of your analysis.

25. Monthly sales of Sears Roebuck and Co. for the years 1934–1936 were (sales in thousands of dollars):

Month	Sales		
	1934	1935	1936
January	21,971	23,776	27,672
February	20,594	23,668	27,580
March	25,280	31,980	36,081
April	25,165	34,073	39,011
May	30,137	35,190	44,923
June	27,064	35,131	46,359
July	21,496	29,594	39,634
August	25,219	29,553	38,903
September	29,904	34,301	47,617
October	34,430	44,048	58,135
November	33,694	40,867	49,014
December	41,947	51,338	66,433

Source: Survey of Current Business.

a. Present the above data in the form of an arithmetic time curve.

b. Present the above data in the form of a logarithmic time curve.

c. Analyze your curves and write a brief report of your analysis.

26. The sales of Sears, Roebuck and Co. and of Montgomery Ward and Co. for the years 1934–1936 were as follows (sales in thousands of dollars):

Month	1934		1935		1936	
	Sears, Roebuck and Co.	Mont-gomery Ward & Co.	Sears, Roebuck and Co.	Mont-gomery Ward & Co.	Sears, Roebuck and Co.	Mont-gomery Ward & Co.
January	21,971	14,779	23,776	17,418	27,672	18,508
February	20,594	15,422	23,668	17,905	27,580	17,855
March	25,280	18,312	31,980	22,783	36,081	24,845
April	25,165	20,872	34,073	25,571	39,011	30,403
May	30,137	20,935	35,190	22,915	44,923	30,295
June	27,064	19,266	35,131	23,822	46,359	30,330
July	21,496	15,891	29,594	20,293	39,634	25,636
August	25,219	18,915	29,553	22,849	38,903	27,422
September	29,904	23,093	34,301	25,173	47,617	33,357
October	34,430	29,704	44,048	35,879	58,135	45,455
November	33,694	26,901	40,867	30,910	49,014	36,979
December	41,947	34,684	51,338	39,475	66,433	51,789

Source: Survey of Current Business.

a. Present the data in the above table as an arithmetic time chart. Does the chart clearly compare the amount of sales of the two stores over the same period?

b. Present the data in the above table as a logarithmic time chart. Does the chart clearly compare the rates of change of sales of the two stores over the period?

c. Analyze your charts and hand in a brief report of your analyses.

27. *a.* Present the data on the number of commercial failures in the form of an arithmetic time curve and in the form of a logarithmic time curve (see Exhibit 51).

b. Using 1913 as 100 express the number of failures in other years as a per cent of the number of failures in 1913 and present these percentages as an arithmetic time curve.

Year	Number of failures	Year	Number of failures	Year	Number of failures
1913	16,032	1921	19,656	1929	22,908
1914	18,276	1922	23,676	1930	26,352
1915	22,152	1923	18,720	1931	28,284
1916	16,980	1924	20,616	1932	31,824
1917	13,848	1925	21,216	1933	19,860
1918	9,984	1926	21,768	1934	11,724
1919	6,456	1927	23,148	1935	11,508
1920	8,880	1928	23,844	1936	9,185

28. Present the following data as a belt chart (see Exhibit 50):

MIDVALE DEPARTMENT STORE
COST OF GOODS, OPERATING EXPENSES, AND NET PROFIT, 1928–1937

Year	Cost of goods sold	Operating expenses	Net profit	Total sales
1928	$3,940,172.40	$1,869,432.47	$387,132.17	$6,196,737.04
1929	4,132,116.44	1,932,186.41	401,327.42	6,465,630.27
1930	2,013,412.80	1,132,114.10	104,309.11	3,249,836.01
1931	981,431.41	493,132.11	42,116.31	1,516,679.83
1932	642,118.92	321,168.42	10,214.60	973,501.94
1933	983,114.17	302,114.07	41,172.15	1,326,400.39
1934	1,113,417.86	414,618.35	120,112.74	1,648,148.95
1935	1,842,191.56	984,361.12	114,182.43	2,940,735.11
1936	3,143,861.34	1,489,361.42	294,132.14	4,927,354.90
1937	4,132,171.64	2,167,804.90	321,432.81	6,621,409.35

29. (Percentage distribution of components over a period.) Reduce cost of goods, operating expenses, and net profit in the table of Problem 28 to percentages of total sales for each year, and present as a percentage belt chart. (Your chart will be similar to Exhibit 50, except that the top of the upper belt will be a straight horizontal 100 per cent line.

Prepare a brief analysis of your chart.

30. (Simple comparison of arithmetic and geometric progressions.)
Assume that the following represents the growth of two investments from 1935 to 1938:

Year	Investment No. 1	Investment No. 2
1935	$1	$1
1936	2	2
1937	3	4
1938	4	8

Present the above data graphically as follows (see first six graphs in Exhibit 60):
a. Actual data on arithmetic chart.
b. Percentage relatives (1935 = 100 for both series).
c. Percentage relatives (1935 = 100 for No. 1, and 1938 = 100 for No. 2).
d. Same as *a*, but on semi-logarithmic chart.
e. Same as *b*, but on semi-logarithmic chart.
f. Same as *c*, but on semi-logarithmic chart.
Analyze your curves and explain what each graph shows.

CHAPTER VII

STATISTICAL DISTRIBUTIONS

To facilitate interpretation and analysis, unorganized statistical data must be arranged or grouped in an orderly manner. When thus arranged or grouped, the data constitute a statistical distribution.

The most common bases of arrangement or classification are: (1) kind, (2) size, (3) location, and (4) time of occurrence. Distributions according to these bases will now be considered and illustrated.

Distribution According to Kind.—When the data represent kinds of objects or kinds of characteristics, the distribution may be made on the basis of *kind*, rather than upon size, location, or time. In Chapter IV, Part *A* of Exhibit 5 (page 49) presented the purchases of the Western Elevator Company according to the kinds of grain. Similarly, the kinds of people in the United States are shown in the *Federal Census Reports*—kinds being stated by race, by nativity, by sex, etc. Many classifications by kind will be found in business data, such as distributions by kind of consumer, by kind of product, by kind of raw material, or by kind of workers.

Distributions by Size or Degree of Variation.—Some data are collected in the form of varying sizes of one variable quality. Thus, if earnings of laborers are compiled, the data will present varying sizes or variations in degree of one variable, namely earnings.

There are two principal types of distributions by sizes of the items. One, *the array*, does not group the data into classes, but merely presents them in an order of size arrangement. The other, the *frequency distribution*, presents the data according to occurrence by size classes.

The Array.—If we were to collect data on the incomes of families living in New York City, listing each amount of income in the order observed, our collection would appear as an unorganized mass of details. Each family income would have no size relationship to the adjacent incomes listed, for the adjacent amounts might be slightly or very much higher, or lower, depending upon chance circumstances.

However, if the incomes were arranged in order of size from the smallest to the largest (or *vice versa*), each item being listed even though incomes of given amounts were repeated one or more times, the result would be an orderly presentation of the incomes of New York families. Such a distribution is called an *array; it is a distribution in which the varying*

sizes[1] *are shown in the order of size, with every value listed.* A tabular illustration of an array is given in Exhibit 63.

The array, because it shows the value of every observed case, is not always a practical form for presenting a large number of items. In showing a small number of observations, however, it is often useful. What is the population of each of the 10 largest cities in the United States? In answering this question, it is natural to present the cities according to the variation in the attribute, population, in order of size, and the result is an array. Furthermore, if we wish to show the annual sales of certain department stores in a city, it is probable that we would list the stores in the order of size of sales. Whether the varying sizes of sales are shown in the order from the largest to the smallest or from the smallest to the largest is immaterial; in either case, the result is an array. It may be added that the largest size will ordinarily be given first, if this size is important for the purpose of the distribution, but if greater interest or importance is attached to the smaller cases, these will be shown first.

The Frequency Distribution.—When a large number of variate values are to be presented, as, for example, the incomes of all families in New York City, a condensation of the data may be advisable. A condensed presentation of data is obtained by use of the frequency distribution. The nature and use of the frequency distribution will be discussed in the following pages.

Nature of the Frequency Distribution.—If one should read through an array of the family incomes in New York City, would he be able to tell whether many earned large, medium, or small incomes? The

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[1] *Varying sizes* are called "variate values" in statistical terminology, but only variables have variate values. A variable is anything which can be quantitatively measured, and the measurements of a variable are its variate values. Thus, income is a variable; income can be and usually is quantitatively measured, and the quantities of income are usually stated in numerical rather than in general terms. Likewise wages, sales, and population are all variables, and each is commonly measured in numerical terms.

On the other hand, some qualities are not variables for they cannot be quantitatively measured. Religion is not a variable; neither is sex; nor is nationality. While it is to be recognized that these qualities or attributes *vary* in different persons, they do not meet the test of a *variable* in statistical language, for we cannot measure them in any other terms than kind (rather than degree) of variation.

Occasionally a quantitative measurement is arbitrarily used in describing a quality that is not essentially capable of direct measurement. To illustrate, knowledge ordinarily is not a variable according to strict statistical terminology for we do not commonly think of numerical quantities of knowledge. But if we apply a system of numerical measurement to this quality, such as a percentage grading system, it then becomes a variable and the numerical measurements used in describing it are its varying or variate values.

largest income might be noticed and the smallest also might be observed. But, if one were to attempt to obtain from an array the significance of the distribution of the numerous cases between the largest and the smallest, he would find it difficult to get a clear idea of the incomes by reading through the details. In other words, from a mere reading of the data in an array, one would have difficulty in determining how many

SMITH MOTOR CAR COMPANY
SALES OF AUTOMOBILES BY AMOUNTS OF INVOICES
Week ended September 17, 1938

Invoice No.	Name	Amount
910	R. L. Owens	$3,050.00
899	B. L. Smith	3,020.00
896	T. W. Evans	2,750.50
891	K. C. Wright	2,750.00
893	D. D. Griffin	2,750.00
908	G. H. Benson	2,500.00
892	F. L. Barber	2,450.00
903	W. W. Wheeler	2,375.00
895	E. S. Fisher	2,000.00
904	C. S. Bradford	1,875.50
907	R. S. Craig	1,800.00
909	B. I. Singer	1,800.00
894	T. W. Stone	1,750.00
897	H. K. Little	1,750.00
902	G. P. Fisk	1,750.00
905	F. H. White	1,750.00
898	T. F. True	1,725.00
911	A. L. Thrift	1,500.00
906	E. W. Hill	1,500.00
900	W. A. Lyons	1,475.00
901	L. M. Parker	1,450.00
	Total	$43,771.00

EXHIBIT 63.—An array according to amount of invoice.

received approximately $1,000, or how many received approximately $2,000, as compared with other significant amounts.

To grasp the significance of a large number of varying values, these values must be grouped. When they are grouped into size classes and these classes are arranged in the order of magnitude, the result is a frequency distribution. The grouping into size classes may be obtained (1) by showing the number of times each size occurs, or (2) by showing the number of times sizes occur within size intervals. Stated in another way, one type of frequency distribution shows the frequency of occur-

rence of given sizes, while the other type shows the frequency of occurrence of sizes within certain class intervals.

To illustrate the first method, let us suppose that it is desired to obtain a frequency distribution of family incomes in New York City from the data previously suggested. We might present the incomes by classes of income, each class being a size of income found in the data. Opposite each class we would then list the number of times that particular size of income is found. For example, in the $2,000 class of income we might find 50,000 families receiving exactly that income. If the next highest income is $2,001 and 450 families are found receiving exactly that income, these amounts would be presented respectively as the next class and the frequency of occurrence for the class. Then, if it is found that the next highest family income is exactly $2,002 and 700 families are receiving that income, the class $2,002, and the frequency, 700, would be shown. In other words, by this method the number of times a given size occurs (that is, the frequency of sizes) is presented. In the ordinary tabular presentation, the sizes of the variable will be listed from the largest to the smallest, or *vice versa*, in a column on the left of the page; and the frequency of occurrence of each varying value will be listed in an adjacent column to the right. Where there are many varying sizes, the number of classes under this method will be too large to give a clear picture of the whole.

By the second method mentioned above, a further condensing may be obtained, for by this plan certain value limits are determined upon, and all sizes found within the interval between these value limits are listed as the frequencies of occurrence. Thus, in showing family incomes, class limits of $1,500 and $2,000 might be taken, and the number of families found to have incomes within these limits would be shown as the frequency of occurrence for this class. The same thing would be done for the class, $2,000 to $2,500, and so on.

Non-continuous and Continuous Series.—To understand when to group the cases according to each individual value and when to group them according to an interval or class of values, we must recognize two types of varying values. The measurements for some variables are expressed in units which must necessarily differ from each other by definite equal amounts, while in other series it is possible for the varying sizes to differ from each other by infinitesimally small amounts. The former type of data—namely, where the varying values differ from each other by definite equal amounts—is known as a *non-continuous*[1] series. The latter type—namely, where the varying values occur in sizes which

[1] Non-continuous series are also called "discrete" or "broken" series.

differ from each other by infinitesimal amounts—is known as a *continuous* series.

THE PARIS STORE
SIZES OF MEN'S SHOES SOLD IN 1937

Size	*Number of pairs*
$4\frac{1}{2}$	5
5	10
$5\frac{1}{2}$	20
6	25
$6\frac{1}{2}$	75
7	150
$7\frac{1}{2}$	300
8	475
$8\frac{1}{2}$	410
9	375
$9\frac{1}{2}$	220
10	125
$10\frac{1}{2}$	75
11	20
$11\frac{1}{2}$	15
12	5
Total	2,305

EXHIBIT 64.—Frequency distribution according to individual sizes. (Non-continuous series.) (*Data adapted.*)

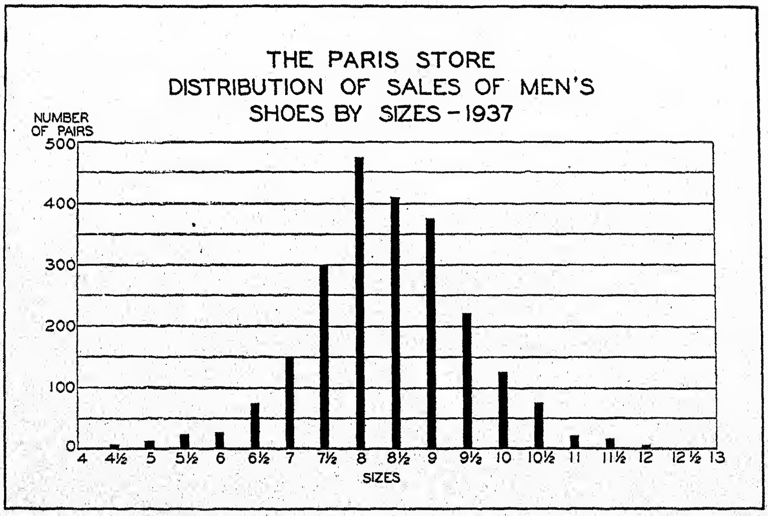

EXHIBIT 65.—Data of Exhibit 64 presented as a frequency bar chart. (Compare with Exhibit 66.)

Classification by Each Value or Measurement—Non-continuous Series.—A frequency distribution of sizes of men's shoes sold by The

Paris Store is presented in tabular form in Exhibit 64 and in graphic form in Exhibits 65 and 66.[1] This distribution is of a non-continuous nature. Although sizes are quoted in half units, the differences between these half units is relatively large, for the entire range or spread from the

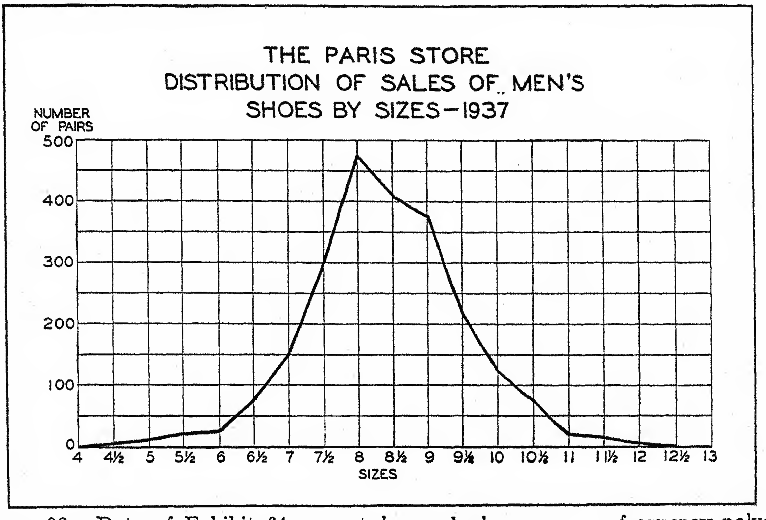

EXHIBIT 66.—Data of Exhibit 64 presented as a broken curve or frequency polygon. (Compare with Exhibit 65.)

smallest to the largest is only from $4\frac{1}{2}$ to 12.[2] Therefore, in this table there are only 16 class sizes and the classification shown is according to each size or measurement. This type of frequency distribution is preferable in presenting non-continuous data[3] when the number of sizes is small enough to condense the presentation adequately.

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[1] Only vertical bars should be used when presenting a frequency distribution in the form of a bar chart. The bar chart in Exhibit 65 is read in the same manner as the curve in Exhibit 66. In a chart like Exhibit 66, which shows a non-continuous series, the plotted points should be connected with straight lines. Ideally, the curve for a continuous series should be smoothed, but in practice, it is usually shown as points connected with straight lines also.

[2] Obviously if the spread of sizes were from 1 to 1000, a difference between each size quotation of $\frac{1}{2}$ an integer would be relatively small; but where the spread is from $4\frac{1}{2}$ to 12, a difference of $\frac{1}{2}$ is relatively large.

[3] Sometimes there is difficulty from a practical point of view in determining whether a series is continuous or non-continuous in nature. Naturally it is hard to draw a line between these two classes, for it is difficult to decide when data are not sufficiently continuous to be classed as continuous and, conversely, when non-continuous data become too continuous to be called non-continuous.

In determining the character of data, statisticians consider that the nature of the attribute measured, and not the particular sizes of the units found in a given case, determines whether the data are continuous or non-continuous. Thus, the unit in

Classification by Class Intervals.—Exhibit 67 presents the form of frequency distribution in which the classes are not stated in certain sizes, but as intervals within which various sizes may occur.

DISTRIBUTION OF EMPLOYEES IN PLANT 6 BY WEEKLY EARNINGS
Week ending July 23, 1938

Weekly earnings in dollars	*Number of employees*
10.00–11.99	3
12.00–13.99	6
14.00–15.99	10
16.00–17.99	15
18.00–19.99	24
20.00–21.99	42
22.00–23.99	75
24.00–25.99	90
26.00–27.99	79
28.00–29.99	55
30.00–31.99	36
32.00–33.99	26
34.00–35.99	19
36.00–37.99	13
38.00–39.99	9
40.00–41.99	7
Total	509

EXHIBIT 67.—Frequency distribution according to class intervals.

If the actual earnings of the 509 employees in Plant 6 had been shown in the form of an array, 509 items would have been listed. If there had

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which wages may be measured is relatively small and, generally speaking, wages are classed as continuous data. Likewise sales, incomes, production, and many other commonly quoted business data are considered to be continuous in nature.

Interest rates, however, are non-continuous in nature, as the interest rates for most types of loans are expressed in units varying but ¼ of 1 per cent with a total probable variation of from about 2 per cent to 8 per cent. Therefore, a variation of ¼ of 1 per cent in interest rates is relatively large.

Lastly, it will be well to note that in a given case continuous data may appear to be non-continuous and may be treated as such in determining the form of frequency distribution to be used. Thus, while wages of workers can be expressed to a fraction of a cent and are continuous in nature, in a given factory wages may be non-continuous. If a wage scale with relatively wide steps or gradations has been adopted in a given factory, the wages of workers in this instance will be non-continuous. However, it is to be noted that the wage scale must have relatively large steps between different wage rates if it is to be classed as non-continuous. In many factories, the wage scale has relatively small variations and does not result in a non-continuous series.

been listed each weekly wage amount and the number of employees who received that weekly wage, a frequency distribution showing somewhat less than 509 classes would have resulted (assuming that some of the workers received exactly the same weekly wage). By grouping the employees by classes of earnings, as shown in Exhibit 67, the 509 items are collected into 16 classes, and a condensed picture of the whole distribution in Plant 6 is obtained.

The Range—Class Intervals—Class Limits—Mid-points.—There are several terms which should be thoroughly understood in studying frequency distributions. The nature of the *range* may already be familiar; it is the difference between the smallest and the largest case. The range within a class or group of items, however, is called the *class interval*—that is, it is the interval between the smallest and the largest possible cases in a given class. The smallest and the largest possible cases within a given class are the *class limits*—that is, they are the dividing points between classes. The point mid-way between the class limits is called the *mid-point*.

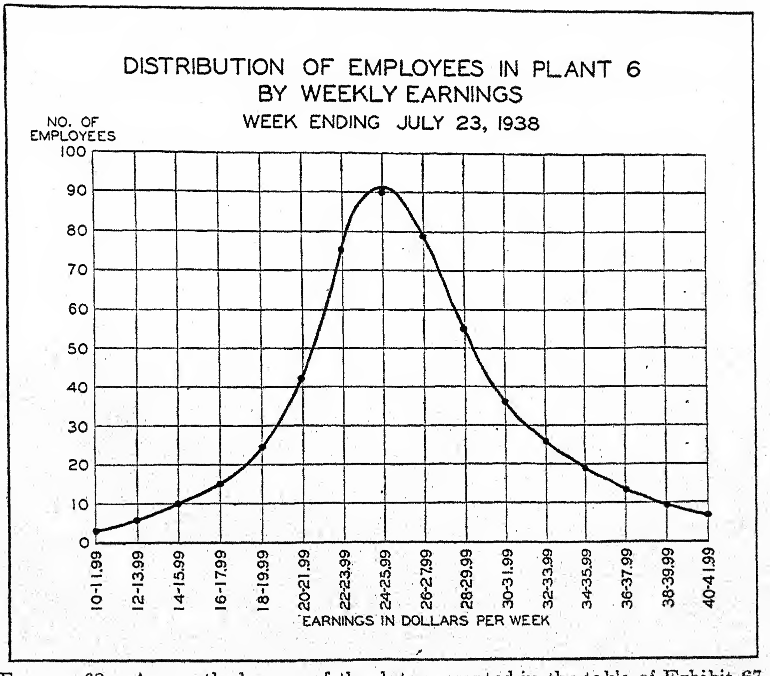

EXHIBIT 68.—A smoothed curve of the data presented in the table of Exhibit 67.

In the frequency distribution presented in the table of Exhibit 67 and in the chart of Exhibit 68, the range is from $10.00 up to (but not including) $42.00, or it is $32.00. Perhaps the actual range of wage

earnings in Plant 6 was not exactly $32.00, because there may have been no one earning as low as $10.00 nor may there have been any worker receiving as high as $41.99 per week. However, the range of the *frequency distribution* which includes these class limits is $32.00.

The class intervals in Exhibits 67 and 68 are $2.00 between the class limits of $10.00, $12.00, $14.00, etc. A difficulty arises in stating class limits because in reality one class begins an infinitesimal amount above where the next lower class ends. Therefore, when class limits are stated as $10.00 and $11.99, it must be realized that the upper limit ($11.99) means up to $12.00, and $12.00 is included in the next higher class ($12.00 to $13.99). In business statistics, it is sometimes both practical and adequate to express the classes in terms of the simpler limits, namely $10.00 to $12.00, $12.00 to $14.00, etc. The dividing points must be clearly kept in mind, however, and a consistent viewpoint must be maintained in classifying the data. Such a statement of the limits is taken to mean from and including $10.00, up to but not including $12.00, etc. The exact meaning of such a statement of the limits must be clearly understood because it is essential to keep in mind the classes as they actually exist.

The mid-points of the above classes are $11.00, $13.00, etc. A mid-point is a point located equidistant from the limits of the class in which it is situated. In determining the mid-point of a class, we must recognize that there is in reality but one dividing point between classes. Therefore, the mid-point is found by adding together the class limits and dividing the sum by two. For example, we add $10.00 and $12.00 together and divide by two to get the mid-point $11.00, just mentioned. Although the limits are expressed as $10.00 to $11.99, we find the mid-point by considering the $11.99 as $12.00, as the expression $11.99 represents the point as infinitely near $12.00 as it is practicable to measure it.

Determining the Class Interval.—There are four main considerations in determining the size and location of the class interval. These are:

1. The range.
2. The number of classes.
3. The position of the class limits.
4. The representativeness of the class mid-point.

The range of the data and the number of classes tend to determine the size of the class interval. Although there are important exceptions, it is generally considered that a frequency distribution should not have more than 25 classes, nor should there be less than 10. The reason for this is that more than 25 classes results in an unwieldy table, while less than 10 usually will not show the distribution in sufficient detail. The size of the class interval can be obtained by dividing the number of classes into the range. Thus, where the range is large the class interval

will tend to be large, and where the range is small the class interval will tend to be small.

It is well to have the class division points expressed in round numbers when convenient, such as in multiples of 5, 10, 50, 100, 500, or similar numbers which are easily comprehended. When fractions are used it is preferable to state them as decimals in multiples of .05, .10, .25, etc. Also, the class limits should be located so that there is a tendency toward a balanced distribution of cases within each class. Thus, if the earnings represented by Exhibit 67 happen to be for the most part near the lower limit of each class throughout the entire distribution, then the class interval shown has not been well chosen.

The mid-point of the class is important because it is sometimes taken to represent the entire class to which it belongs. When cases are grouped within given class limits, the exact sizes of these cases are not given. All that is shown concerning the sizes of the cases falling in a given class interval is that there is a certain number of cases which may vary in size between the class limits. When it is necessary to use one figure instead of the class interval, the mid-point of the class is taken as the value of all cases within the interval.[1] For this reason, the class interval should be chosen so that on the whole the mid-point will be reasonably representative of the values which actually fall within the class limits.

It will be seen that the problems of choosing the proper size and of locating the position of the class interval are interdependent. The size of the interval is dependent upon the range and the number of classes, and yet it is also dependent upon obtaining the desired position of class limits and of the class mid-point. The weight to be given to each of these factors will naturally depend upon the purpose of the frequency distribution and upon the uses for which it is being prepared.

Smoothing a Frequency Curve.—In order to explain further the nature of a frequency distribution of a continuous series, some of the more important points involved in smoothing a frequency curve will be discussed.

When smoothing a frequency curve of a continuous series, care should be taken to keep the area under the curve the same as if a column diagram (Part *A*, Exhibit 69), or a broken curve or frequency polygon (Part *B*, Exhibit 69), were constructed. In the diagram of Part *A*, the total area of the columns represents the magnitude of the total number of observations, and a part of this area represents the magnitude of a certain part of the observations. For instance, if two spaces on the horizontal scale represent $10, and two spaces on the vertical represent

[1] In Chap. VIII, methods of analyzing and condensing frequency distributions will be considered and the importance of having mid-points expressed in amounts which are easy to use in further analysis will become evident.

100 occurrences, 100 × \$10, or \$1,000, is the value of the occurrences represented by an area of four spaces. The total area under the curve in Part *B* of Exhibit 69 is the same as the area under the column tops in Part *A*. This is illustrated by Part *C*, which shows that the areas cut off of the columns when a broken curve is drawn are the same as the areas added by the curve to the adjoining columns. Nevertheless, while the total area is correct in both *A* and *B*, the *area over any one class* is not correct for a continuous series in the frequency polygon of Part *B*. For instance, the middle class has lost without gaining anything (see Part *C*). This must be considered when smoothing the curve, and usually the best method is to draw the curve so that the areas gained and lost *by each column* will be equal (see Exhibit 70).[1] Thus, the top of the curve should slightly overtop the highest plotted point, as in Exhibit 70.

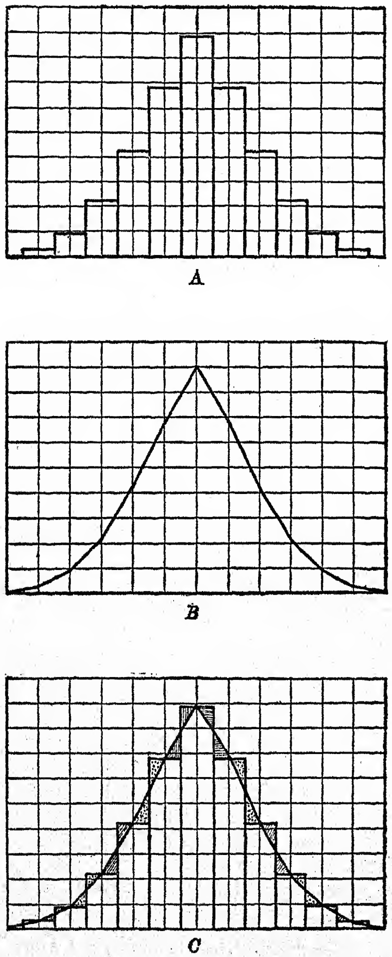

EXHIBIT 69.—The relation between a column diagram and a broken curve.

In actual practice, the column diagram is not ordinarily drawn in as a basis for drawing the smoothed curve, but the proper allowances are made by keeping in mind the principles of the area distribution as above described. The smoothed curve then represents the normal tendency or approximate nature of the distribution indicated by the plotted points of the sample, and it is ordinarily desirable to have the smoothed curve overtop the highest point to represent the probable distribution of the cases within the class of highest frequency. The purpose of smoothing a curve such as that of Exhibit 68 is to indicate the normal tendency within the distribution, and within each class, using the particular data as a sample, and

[1] Note that the area cut off of a column and the area added to compensate are shaded in a manner to indicate the compensating relation in Part *C*, Exhibit 69, and in Exhibit 70.

this tendency will be misstated unless the curve overtops the point of highest frequency as illustrated. That is, the data plotted are only a sample, but the smoothed curve is designed to indicate the nature of what the distribution of the universe would be if all of the values were plotted instead of a sample grouped by classes. Such a curve is sometimes confusing, but it will cause no difficulty if it is clearly appreciated that it represents the type or form of the distribution indicated by the sample rather than the actual values in the sample by classes. When a curve is smoothed from a sample in this manner, the points representing the actual values in the sample ordinarily should be indicated on the chart.

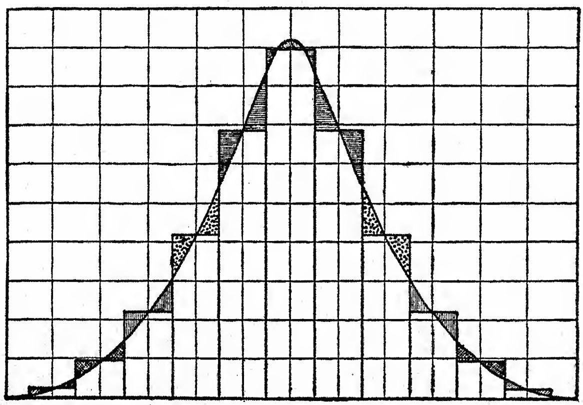

EXHIBIT 70.—Smoothing a frequency curve.

If the series begins and ends with an infinitely small number of occurrences, the curve should begin and end on the zero line as illustrated in Exhibit 70.

Ideal conditions are probably never found in analyzing business frequency distributions. Very often one has only partial data from which to study the tendencies of the distribution. It must be remembered, however, that business data are full of actual irregularities, and that smoothing must not obscure any except the minor irregularities which are not thought really to exist. If an irregularity is such that it would not be eliminated by further sampling, it should not be smoothed out. If smoothing is freely indulged in, it is well to indicate the points from which the curve is made (see Exhibit 71).[1]

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[1] The principle of smoothing illustrated in Exhibit 71 should not be confused with that illustrated in Exhibit 70. In Exhibit 71, the object is to show the tendency indicated by the irregular data, and the low frequencies for the class interval 45 to 50 and 55 to 60 would prevent the curve from overtopping the highest point at the 50 to 55 class interval.

Absolute rules cannot be laid down for smoothing frequency curves, for each problem must be considered according to the representative character of the data, and the curve should be plotted which will best

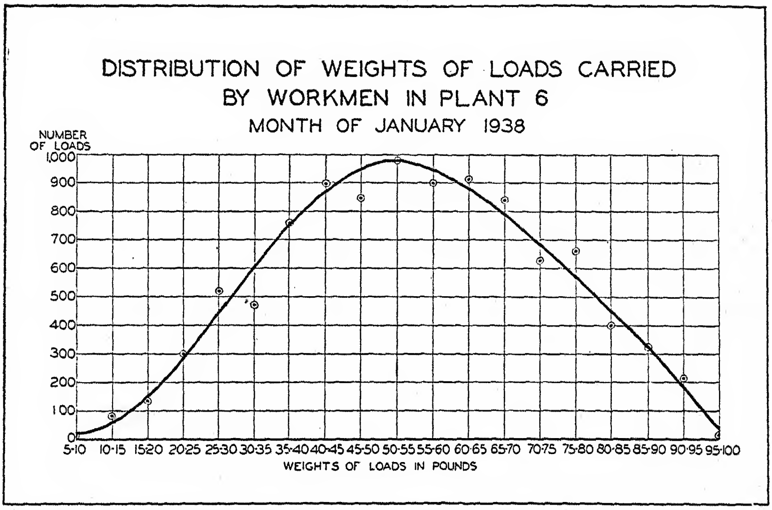

EXHIBIT 71.—Indicating tendency by freely smoothing curve. (On this type of chart it is important to indicate the plotted points.)

suit the purpose for which the graph is made. The principles just discussed, however, are fundamental, and a thorough understanding of them is of great value in analyzing and interpreting frequency distributions.

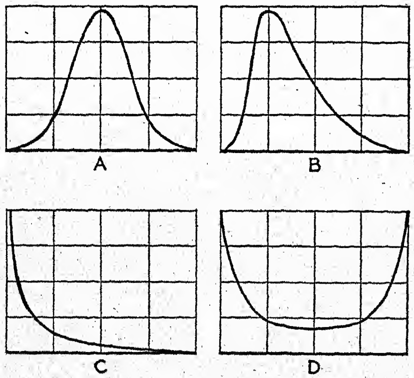

EXHIBIT 72.—Fundamental types of frequency distributions.

Fundamental Types of Frequency Distributions.—In general, frequency distributions may be classified under four fundamental simple types—namely, the symmetrical (Part *A*, Exhibit 72), the moderately asymmetrical (*B*), the J-shaped (*C*), and the rarely occurring U-shaped (*D*). While the "bell-shaped curve" is known as the normal frequency curve, business data more commonly describe forms similar to Parts *B* and *C*, Exhibit 72. An appreciation of the fundamental nature of the normal frequency curve, however, is very important to an understanding of the problem of sampling, and the subject will be discussed at some length in a later chapter. The form illustrated by Part *C* is very commonly found when studying such distributions as sizes of stores, sizes

of towns and cities, and the like, where the general tendency is that the greatest frequencies are in the small sizes and the lowest frequencies are in the large sizes.[1]

Uniform and Non-uniform Class Intervals.—In the type of frequency distribution just discussed—namely, where class intervals are shown—each class interval ordinarily should have the same class range as the other intervals in the distribution. But there are exceptional cases where this rule may not be a practical one, as in certain population or income distributions.

Where a uniform class interval would result in the placing of nearly all cases in a few groups and a scattering of a few remaining cases among the remaining intervals, it is often permissible to adopt class intervals of varying sizes. Thus, although the range of incomes that will be found in any large city is very great, it is probable that 98 per cent of the cases will fall within a very small range and, in certain instances, the class intervals within this small range should be much smaller than those outside of it. Occasionally, the rule of equal class intervals is not a practical one, for other reasons. For example, in a marketing survey it may be advisable to classify consumers according to unequal class intervals of income. The income interval from $1,750 to $2,000 may be just as important a range as the interval between $4,000 and $5,000, because a dollar means more to the consumers in the first group than to those in the second. A person in the first group will probably try as hard to save $1 as one in the second group will try to save $4. Therefore, the class intervals, $1,750 to $2,000 and $4,000 to $5,000, may actually be about equal from the standpoint of the consumer's desire to save.

However, as we shall see in the following chapters, where further analyses of the distribution are to be made, it may be advisable to have all intervals of the same numerical size. The reason for this is that it is difficult to compare accurately the cases presented in a class interval of one size with the cases presented in a class interval of a different size.

Classes must not be omitted within the range of the entire distribution. Even though no cases occur with a given class value or within a given class interval, the class or the class interval must be shown if it is within the range. In Exhibit 64, for example, if there had been no shoes sold of size $5\frac{1}{2}$, this size should, nevertheless, be shown and the number of pairs, or frequency, should be shown as zero. Also in Exhibit 67, if there had been any class interval within the range in which no actual earnings were found, this class interval should be shown and the number of employees should also be shown as zero. An illustration of this practice will be found in Chapter VIII, Exhibit 85, page 165.

[1] See G. U. Yule and M. G. Kendall, "An Introduction to the Theory of Statistics," Chap. 6, pp. 82–111, Charles Griffin & Company, London, 1937.

Sometimes the class interval at either extreme of the table is stated in indefinite terms. Thus, in Exhibit 67, the lowest class might have been stated as "under $12," and the highest class as "$40 and over." Such a table is called an *open-end table*, and usually should be avoided where possible. It will be obvious that unless definite limits are given to all class intervals, no mid-point value can be obtained and, therefore, no representative size can be assumed for the cases falling within the unlimited intervals.

Comparison of Classification by Each Value and Classification by Class Intervals of Values.—It will be noted that in Exhibit 64 none of the details of the data on sizes[1] of men's shoes sold in 1937 by The Paris Store have been lost by our frequency distribution form of presentation. In other words, where data can be presented in a frequency distribution with classes in terms of the individual values, the value of each and every case is presented. However, where a class interval is the basis of the classification, as in Exhibit 67, the values of the individual cases represented in the frequency distribution are not shown. Thus, it may be said that by the method of Exhibit 64 none of the characteristics of the original data are lost, but by Exhibit 67 only an approximation of the original data is shown.

It is not to be considered that either method is always the better; the former should be used for a distribution of non-continuous series with a small number of sizes, and the latter method should be used for continuous series and for non-continuous series with a large number of sizes. A presentation of non-continuous series in a frequency distribution having class intervals is to be avoided when the number of size variations is small, for it is difficult to arrange the class intervals so that there is a balanced distribution of cases within each interval, and so that the mid-point of the class interval is representative of the non-continuous series.

Cumulative Frequency Distribution.—Up to this point, only simple frequency distributions have been considered, such as the series presented in Exhibit 67, page 138, which is repeated in the first column of data in the table of Exhibit 73. In addition to the simple frequency distribution, it is often desirable to present series in the cumulative forms shown in the last two columns of the table in Exhibit 73. Frequency series are made cumulative when successive frequencies are added together so that each class includes all the lower or upper classes, depending upon the end of the range from which the cumulative process begins. Curves of the last two columns in Exhibit 73 are presented in Exhibits

[1] The sizes of shoes given in Exhibit 64 do not consider variations in width within each size.

74 and 75. A cumulative frequency curve is often referred to as an "ogive" or "S-curve."

A decision as to which end shall be the starting point of the cumulating process depends upon the purpose of the data. If cumulating proceeds from the lesser to the greater, the corresponding frequencies are read "less than" as in the next to the last column in Exhibit 73 and in the graph in Exhibit 74. If it proceeds from the greater to the lesser, the frequencies are read "more than" as in the last column of Exhibit 73 and in the graph in Exhibit 75. In a table like the one in Exhibit 73, when cumulations are read "less than" they refer to the upper limits of the classes and when they are read "more than" they refer to the lower limits. It should be noted also that the class limits are included when intervals are used such as those in Exhibit 73 (see footnote to the table).

Distribution of Employees in Plant 6 by Weekly Earnings
Week ending July 23, 1938
Cumulative frequencies

Weekly earnings in dollars	Number of employees		
	Simple frequency	Cumulative frequency	
		Less than*	More than*
10.00–11.99	3	3	509
12.00–13.99	6	9	506
14.00–15.99	10	19	500
16.00–17.99	15	34	490
18.00–19.99	24	58	475
20.00–21.99	42	100	451
22.00–23.99	75	175	409
24.00–25.99	90	265	334
26.00–27.99	79	344	244
28.00–29.99	55	399	165
30.00–31.99	36	435	110
32.00–33.99	26	461	74
34.00–35.99	19	480	48
36.00–37.99	13	493	29
38.00–39.99	9	502	16
40.00–41.99	7	509	7

* In this table, "Less than" applies to and includes the upper limit of the class, while "More than" applies to and includes the lower limit.

Exhibit 73.—The data of Exhibit 67 cumulated on both "Less than" and "More than" bases.

Referring to the charts in Exhibits 74 and 75, it will be noted that a curve of the "less than" form extends from the lower left to the upper

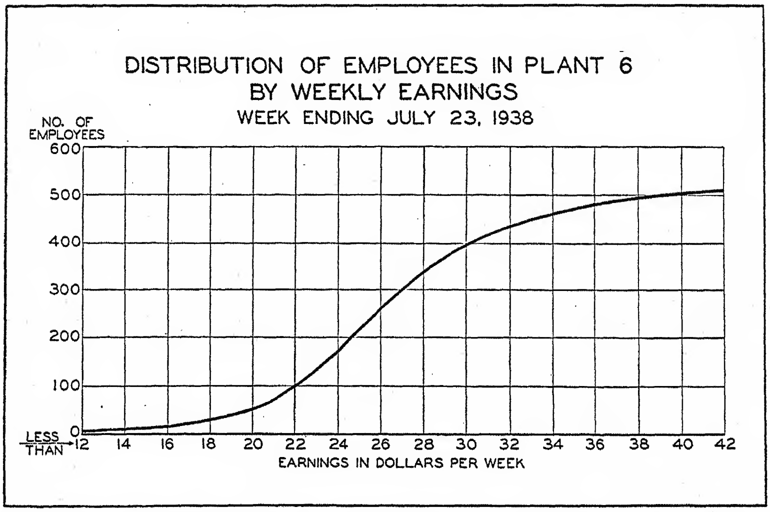

EXHIBIT 74.—Data of Exhibit 73 presented as a cumulative curve on a "less than" basis. (Compare with Exhibit 68, which shows the same data presented as a simple frequency curve, and with Exhibit 75, which shows the same data as a "more than" curve.)

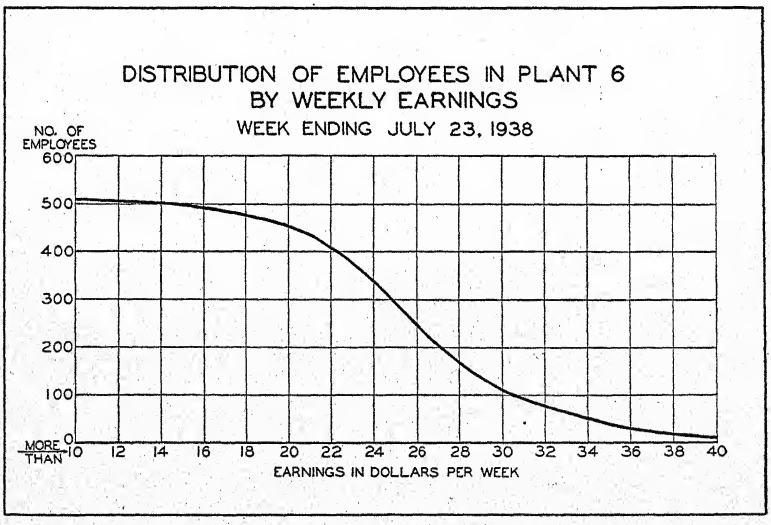

FIG. 75.—A "more than" curve based on the same original data as the "less than" curve of Exhibit 74.

right, and that a "more than" curve extends from the upper left to the lower right. In either case as the curve approaches the vertical, relatively higher frequencies are represented.

This form of distribution is used in special cases as, for instance, where serial bonds have been issued which are secured by equipment, and it is desired to show how much of the equipment will be in use at each maturity date of the bonds. Thus, by this form of table, the remaining security can be compared with the remaining bonds. In other words, this table would show the value of the equipment which lasted one year or longer, two years or longer, three years or longer, four years or longer, etc.

Distribution of Male Employees in Plants 2 and 3 by Ages
Week ending July 9, 1938

Age to nearest year	Plant 2		Plant 3	
	Number	Per cent	Number	Per cent
15	17	0.1	0	0
16	120	1.0	0	0
17	426	3.4	0	0
18	763	6.2	0	0
19	1,020	8.3	0	0
20	1,380	11.2	7	.2
21	1,500	12.2	83	2.1
22	1,410	11.4	167	4.3
23	1,139	9.2	278	7.3
24	980	7.9	302	7.9
25	860	7.0	421	11.0
26	719	5.8	562	14.7
27	600	4.9	607	15.9
28	516	4.2	411	10.7
29	380	3.1	301	7.9
30	231	1.9	243	6.3
31	106	.9	199	5.2
32	72	.6	149	3.9
33	61	.5	70	1.8
34	26	.2	26	.7
35	0	0	2	.1
Total	12,326	100.0	3,828	100.0

Exhibit 76.—Percentage distributions of frequency series. (See Exhibits 77 and 78 for charts of above data.)

Comparing Frequency Distributions—Percentage Tables and Charts. Frequency distributions may be compared on the basis of numbers, as illustrated in Exhibit 77. It will be noted in this chart, however, that the differences in the altitudes of the curves interfere with the comparisons of the distributions. Hence, it is often advisable to place the large and small groups on an equal basis by reducing the frequencies to percentages, as shown in Exhibit 76. These percentages are shown

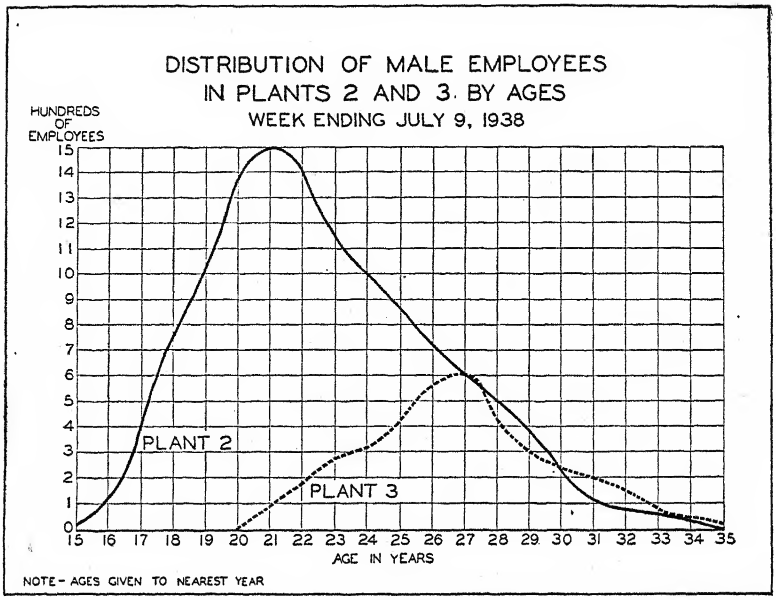

EXHIBIT 77.—Comparison of frequency distributions on a number basis. (Data from Exhibit 76.) (Compare with Exhibit 78.)

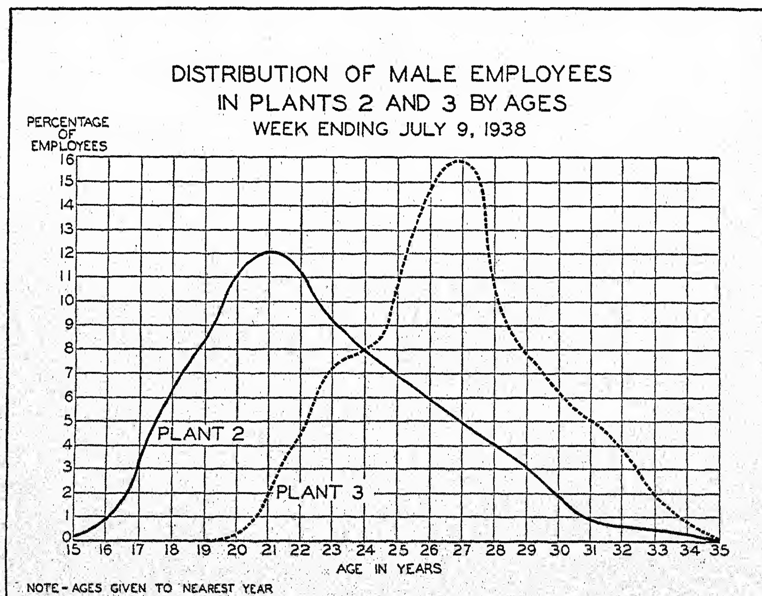

EXHIBIT 78.—Comparison of frequency distributions on a percentage basis. (Data from Exhibit 76.) (Compare with Exhibit 77.)

graphically in Exhibit 78. In these Exhibits (76 and 78), the total number of employees in each plant is taken as 100 per cent and the percentage for each class represents the proportion the corresponding number is of the total.

In analyzing the curves in Exhibit 77, it would be easy to assume that the proportion of employees of 21 years of age in Plant 2 is greater than the proportion in Plant 3 that are 27 years of age. That the opposite is true, however, is shown directly and accurately by the percentage curves in Exhibit 78.

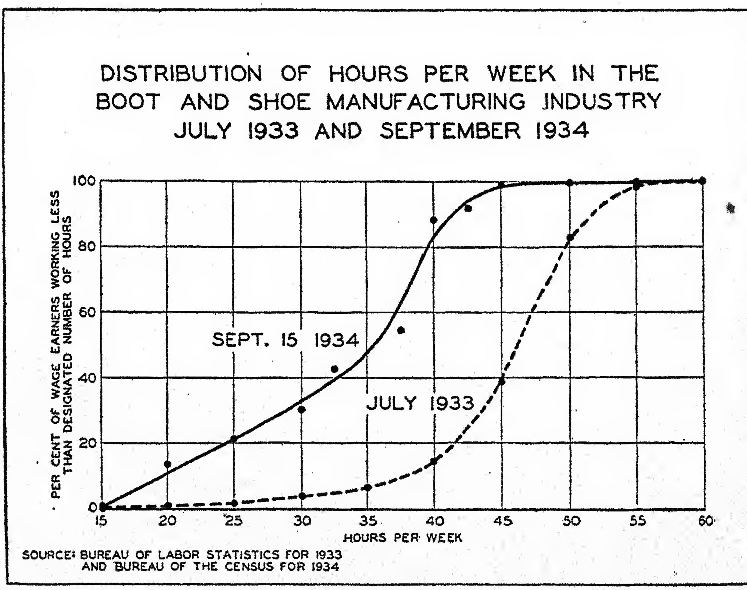

EXHIBIT 79.—Comparison of percentage cumulative frequency curves.

When cumulative curves are being compared, better results can be obtained in many instances if the frequencies are reduced to percentages, since proportional size changes of the frequencies and the regularity of the distribution can be appreciated more readily. The change in the proportions of employees working designated lengths of weeks in the boot and shoe manufacturing industry from July 1933 to September 1934 is clearly shown by the percentage cumulative frequency curves presented in Exhibit 79. The dots on the chart show the actual frequencies and the curves are smoothed to show tendencies.

Summary of Distributions According to Size.—It has been shown that statistical distributions setting forth varying sizes are of two general types—(1) the array, and (2) the frequency distribution. The array shows the numerical value of every case, even though some of the values

may be repeated; it presents these values in an orderly arrangement proceeding from the smallest to the largest, or from the largest to the smallest. The frequency distribution classifies the varying sizes according to the number of times a given measurement occurs, or according to the number of times measurements are found within certain class limits. A third type of frequency distribution is sometimes used, namely one in which the cumulative frequencies, that is, the accumulated totals of the frequencies, are presented.

There remain to be considered statistical distributions according to location, called geographical distributions, and statistical distributions according to time of occurrence.

Geographical Distributions.—In such problems as analyzing markets, appraising real estate, and determining locations for stores, banks, or factories, it is important to consider the distribution of many factors throughout the areas under consideration. Such distributions are ordinarily referred to as "geographical distributions."[1] Sales of commodities are often analyzed according to states, counties, cities, or parts of a city. Any one of these geographical distributions of sales is often useful in considering such problems as the location of branches or warehouses. Such studies are also of great value in measuring the relative efficiency of different divisions of the sales department. Such geographical distributions as those showing vacancies in residential structures are of practical interest to landlords, tenants, realtors, construction companies, lumber companies, grocers, furniture retailers, investors, and countless other business men whose profits are directly or indirectly affected by such situations.

Numbers or Sizes for Particular Areas.—One of the simplest kinds of geographical distributions is that in which simple *numbers* or *sizes* are considered in relation to certain specific areas. Such a distribution is illustrated in Exhibits 80 and 81, which show the number of automobiles for each state in the United States on January 1, 1930. The purpose of this particular type of map is simply to show numbers for the particular areas.

In Exhibit 80, the area of each dot is proportional to the number represented. This practice is commonly followed, but the numbers are not so easily read as in Exhibit 81, in which the dots are uniform in size and the numbers of automobiles are represented by placing the proper number of dots in each state. In this map the dots are carefully lined up and evenly spaced, which makes it easy to determine the approximate number of cars in each state. The purpose of the maps in Exhibits 80 and 81 is simply to show numbers for the particular areas; such maps

[1] Geographical distributions are also referred to as "spatial distributions."

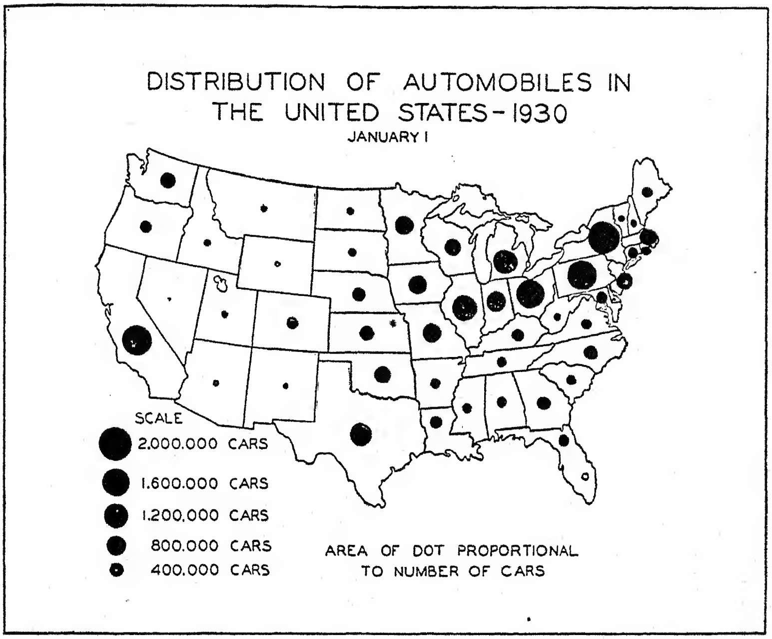

EXHIBIT 80.—Use of circular areas in showing geographical distribution of numbers or sizes. (Compare with Exhibit 81.)

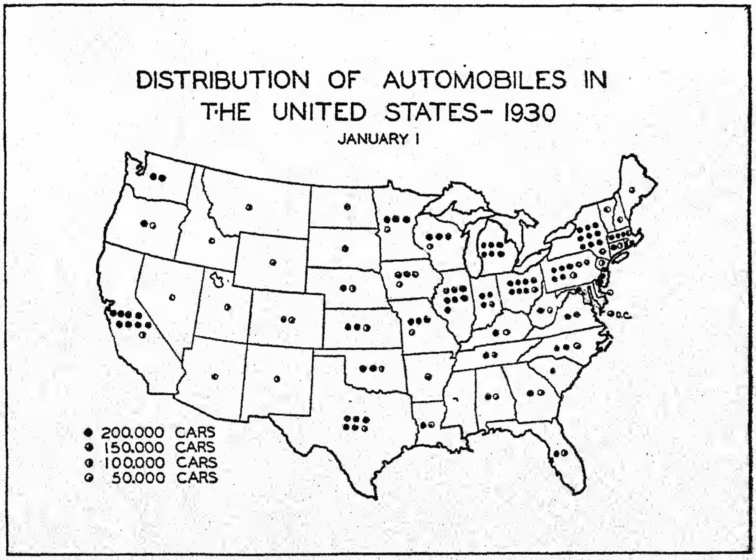

EXHIBIT 81.—Ordinarily a better method of showing simple distributions of sizes than that of Exhibit 80. In this type of map, the primary purpose is to show numbers or sizes for particular areas. (Such a presentation should not be confused with Exhibit 82 where the primary purpose is to show density rather than numbers or sizes.)

should not be confused with Exhibit 82, where the primary purpose is to show density rather than size.

Densities.—If the problem is to study *densities* rather than *numbers* (or *sizes*), the data may be presented as in Exhibit 82. Ordinarily, in studying geographical distributions, the densities are more important than the numbers. The important questions are: Where are the points showing a high degree of concentration? or, Where are the sparse distributions? Numbers are not well shown by the type of map illustrated in Exhibit 82, but the relative densities are clearly indicated. In making such a map the value should be assigned to the dot that will best fit in between the

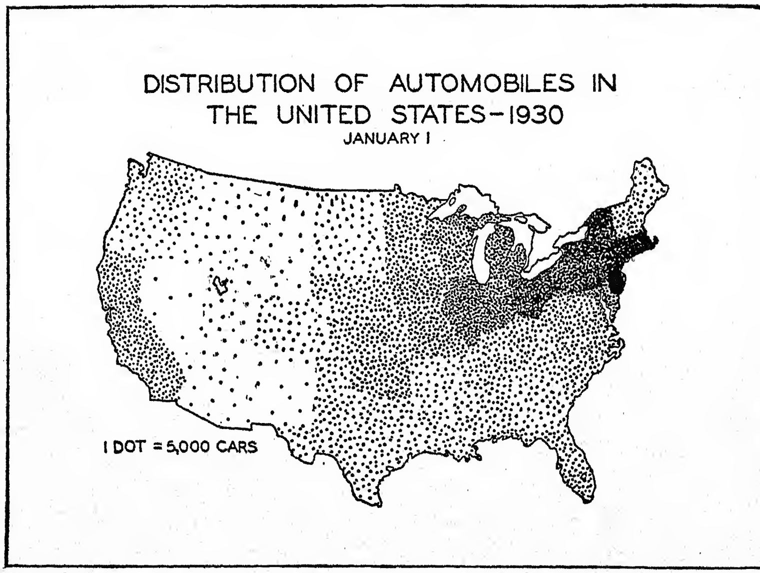

EXHIBIT 82.—The use of dots for the purpose of indicating density of distribution.

extremes—that is, the value should not be so small that there would not be room for the dots in concentrated areas (they may run together but there should not be more than one layer), and yet the unit should not be so large that important data in the sparse areas would be incorrectly represented or omitted. On a density map, the dots should be spread evenly over the unit of area and care should be taken to see that they are uniform in size. Since the areas of the states on the map are proportional to the actual areas, the dots are automatically distributed on a square-mile basis if they are spread evenly over each state. Density maps, like Exhibit 82, show the average distributions for each state, and do not show the concentration at large cities. For many purposes, it is necessary to use much smaller units of areas, such as counties, in a study of the

United States, or such as townships, towns, and city wards, in making local surveys.

Data Distributed Geographically Where Something Other than Area Is a Factor.—When a distribution is to be shown where something other than area is a factor, such as number of automobiles per thousand of population, the density dot map can not be used in the form described. One of the best forms for showing such data is the cross-hatched map

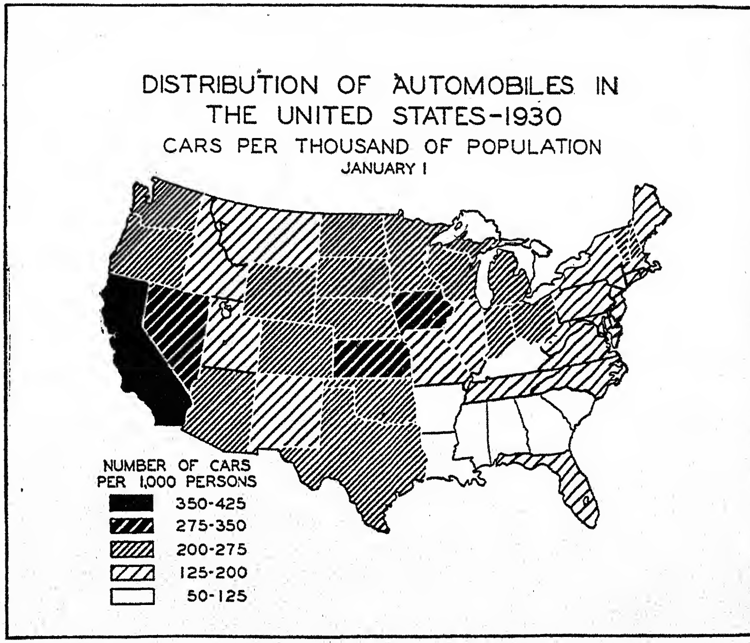

Exhibit 83.—A method of showing the geographical distribution of the relation between two factors. In this case, the relation of cars to population is shown by states.

illustrated in Exhibit 83, which presents the geographical distribution of cars in relation to population. The basic situation can be readily appreciated from such a presentation.[1]

Historical Distributions.—The fourth basis of classification is that in which *time of occurrence* determines the classes. In business statistics, distributions of data upon this basis are very common. Many business

[1] It will be noted that the shading on this map varies by 25 per cent from one class to another. That is, the bottom shade is 0 per cent black, the second 25, the third 50, the fourth 75, and the top is 100 per cent black. When only five shades are used, this simple method makes them easily distinguishable. Ideally, 100 per cent black would represent the saturation point. In this map it is used merely to represent the highest class.

facts that can be quantitatively stated are fully understood only when compared with conditions in other parts of the period under consideration. The number of automobiles produced in the United States during a certain month may have but little significance unless we know the production in other months.

Statistical distributions in which time of occurrence is the basis of classification are known as historical distributions. Thus, a historical distribution presents the total or aggregate cases occurring within a given period of time according to subperiods of time (Exhibit 84).

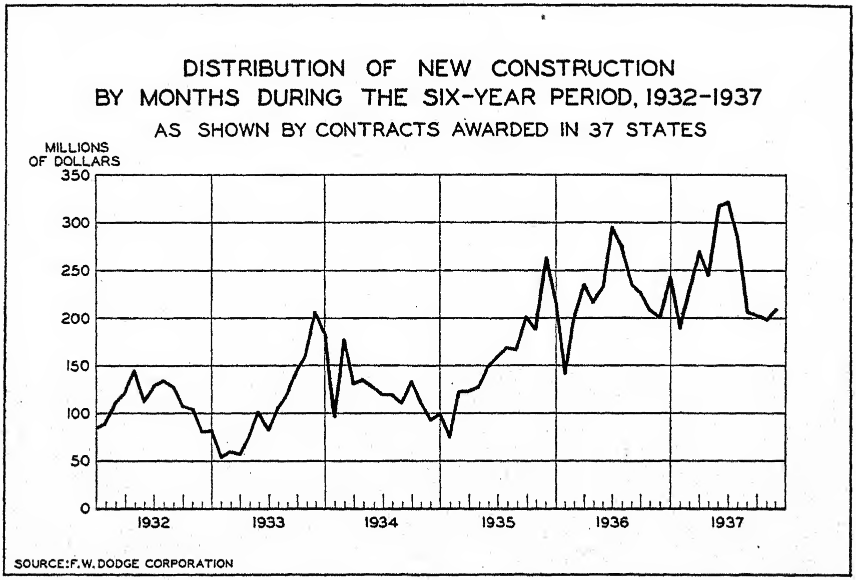

EXHIBIT 84.—A historical distribution of the total construction contracts reported for a given area during a certain period.

Some of the most common illustrations of historical distributions are found in production and marketing statistics. Thus, automobile production by months for the year 1938 would be a historical distribution; likewise, sales of automobiles by months for the year 1938 would be the same kind of a distribution.

All presentations of facts according to time of occurrence are not historical distributions, however. In the first place, to be *historical*, the data must be related to definite *dates*, such as certain months or years. A curve showing the increases in $1.00 at 5 per cent compound interest at 1, 2, 3, 4, and 5 years would not be historical, as it would not represent definite *dates*. In the second place, a distribution involves the subdividing of an aggregate upon some basis of classification. Many time-series

facts—that is, facts presented according to the time of occurrence—are not shown as a part of a total, and therefore are not historical distributions. For example, if the price of leather at various dates is presented in a table, this is a time-series presentation but not a time-series distribution. The individual prices, in this instance, are not parts of a total price; they are merely price observations at the particular dates. In later chapters of this book the subject of time series will be considered at some length. The present discussion refers to time-series presentations only as a basis of classifying the parts of an aggregate.

Questions and Problems

1. What are four common bases of classification?

2. What are the two principal types of distribution according to sizes?

3. Define "array," "variate value," and "variable."

4. When should the array be used?

5. Describe the frequency distribution.

6. Describe frequency distributions which show (*a*) the number of times each size occurs, and (*b*) the number of times sizes occur within size intervals.

7. What is the difference between "continuous" and "non-continuous" series?

8. Are wages continuous or non-continuous data? Wage rates? Earnings?

9. Define or explain: "range," "class interval," "class limits," "mid-point."

10. What are four important considerations in determining the class interval?

11. How does the range affect the size of the class interval?

12. Into how many classes should a range usually be divided in showing a frequency distribution?

13. Where should class limits fall? Class mid-points?

14. How are the frequencies indicated and how are the sizes of classes shown in plotting a frequency distribution curve? (See Exhibits 66, 68, and 71.)

15. What procedure should be followed in smoothing a frequency curve?

16. What are the fundamental types of frequency distributions?

17. Must the class intervals be uniform in size? May any be omitted within the range? What is an "open-end" table?

18. When should a classification by individual values be used? When should class intervals be used?

19. What is a "cumulative" frequency distribution?

20. What is a "percentage" frequency distribution? How do percentage distributions assist in comparing frequency distributions?

21. What is a "geographical" distribution?

22. Give a practical example of the use of a geographical distribution.

23. How are geographical distributions presented graphically?

24. What is a historical distribution?

25. Are all facts presented according to time of occurrence historical distributions?

26. What kinds of charts are used in presenting historical distributions?

27. Make a tally sheet, a frequency table, and a frequency graph (see Exhibit 68) of the wage data on pages 158 and 159. Use a class interval of $2.00. (Age and length of service data will be used in Problems 28 and 29.)

Analyze your graph and write a brief report of your analysis.

ALEXANDER MANUFACTURING COMPANY'S WEEKLY PAY ROLL
Employees' Ages and Length of Service—January 1, 1938

Name	Wage (dollars)	Age (years)	Service (years)	Name	Wage (dollars)	Age (years)	Service (years)
F. R. Adler	20.25	37	3.2	G. T. George	29.50	31	6.4
C. B. Adley	25.00	21	2.2	F. J. Geyer	22.50	35	.1
D. F. Allen	25.25	49	2.5	W. A. Gillis	27.00	39	4.3
W. S. Ambler	27.25	36	5.1	A. J. Gilmore	31.00	44	2.3
B. S. Anakin	25.50	37	4.3	O. E. Glass	22.25	26	4.1
W. G. Baird	26.00	34	2.6	A. P. Glenn	20.50	31	1.6
A. W. Baker	20.25	23	3.5	N. R. Goodall	22.00	46	.8
S. R. Bard	27.50	34	3.7	R. L. Gordon	23.75	43	1.1
H. S. Beard	25.00	31	5.2	A. J. Groat	24.75	37	1.7
J. C. Blick	22.25	41	1.4	A. E. Hadley	26.75	41	.3
E. H. Blum	33.75	45	12.9	R. C. Hait	30.50	36	1.9
M. S. Brandt	25.25	33	1.1	T. J. Hanna	23.00	37	1.7
E. E. Cary	24.75	28	2.5	L. B. Harnett	36.25	38	7.2
R. H. Cary	28.25	41	12.9	J. G. Harris	34.25	38	10.3
M. S. Chambers	25.00	31	3.6	G. C. Hart	27.25	67	3.9
D. E. Cheny	24.75	30	3.7	G. M. Harvey	27.50	24	1.6
T. V. Church	32.50	35	8.3	L. C. Henry	24.50	41	3.2
E. F. Clement	34.75	30	1.4	J. D. Heyer	34.75	34	8.5
A. L. Coffy	22.50	29	8.7	W. H. Hill	31.25	32	7.1
P. R. Cole	19.75	21	5.4	G. E. Horn	27.50	41	5.4
J. E. Collins	22.25	41	4.9	C. A. Hutchins	39.00	24	1.1
W. T. Comer	23.00	38	11.2	M. D. Hyman	30.00	28	1.4
G. W. Conn	20.25	41	4.3	E. L. Ide	41.50	35	14.6
R. L. Conte	35.00	37	11.3	O. D. Ingall	37.25	35	8.9
G. W. Coombes	23.50	45	3.6	A. A. Irvin	40.00	33	5.8
D. C. Coons	29.75	46	12.7	C. W. Irwin	20.00	79	7.1
J. W. Corbett	24.75	28	2.8	M. L. Jackson	25.25	57	6.0
B. A. Covey	27.25	43	3.2	E. B. Jamison	27.50	56	2.6
R. S. Dick	23.00	41	3.9	C. O. Jelm	26.50	31	4.1
H. W. Dickes	27.50	31	6.1	S. R. Jesse	28.25	45	13.3
C. C. Dillon	30.00	35	1.6	A. B. Johnson	22.50	25	7.2
F. S. Dolley	28.25	42	12.3	R. C. Johnston	32.50	51	14.6
W. E. Doran	29.75	40	3.1	M. H. Jolly	27.50	42	.4
D. D. Drake	22.50	45	1.2	H. P. Jones	21.50	52	.4
S. F. Duncan	18.50	50	.8	D. R. Jordan	26.50	38	4.2
C. W. Dunton	39.50	24	9.6	S. E. Kamp	30.75	36	6.8
H. A. Dyer	18.75	18	9.4	H. G. Kata	21.25	42	1.7
N. A. Dyke	18.75	22	2.9	I. W. Katz	24.50	34	3.2
T. G. Eades	23.50	20	1.4	V. D. Keating	34.25	43	6.3
W. J. Eden	22.25	38	5.1	M. D. Keller	22.50	42	13.0
B. W. Edgar	29.75	44	6.1	C. H. Kenney	22.50	23	5.2
S. E. Edwards	25.00	23	2.8	H. L. Kessel	24.75	50	2.6
J. A. Eleo	22.25	47	3.3	M. E. King	28.25	44	3.4
G. L. Elliott	22.75	41	1.7	A. M. Kirk	27.50	42	2.5
C. J. Elson	33.00	49	2.2	T. K. Knapp	31.50	43	8.9
A. T. Estes	29.25	31	3.8	G. R. Laine	18.00	17	4.1
E. L. Everett	34.25	51	11.9	C. E. Lance	22.00	29	1.3
C. L. Fair	34.75	48	2.6	W. R. Lange	22.50	34	2.8
D. W. Farmer	24.00	37	2.1	A. C. Latte	25.50	37	1.1
M. J. Faulk	21.25	25	3.7	R. W. Leas	24.50	53	1.2
B. P. Fenton	32.75	41	11.1	S. S. Leeds	27.00	52	13.5
W. J. Ferry	40.50	29	3.4	K. F. Leland	29.50	47	2.8
C. A. Fish	24.75	25	1.7	E. A. Levitt	22.50	46	1.5
N. J. Fleming	28.50	43	2.3	G. L. Lewis	37.50	32	8.1
L. W. Ford	23.00	25	.2	O. W. Lewis	26.00	24	11.5
D. L. Foster	32.50	43	4.7	F. A. Lindsay	24.75	32	6.4
M. S. Furby	27.50	44	2.4	F. W. Locker	23.00	40	3.8
W. L. French	22.75	49	.4	C. E. Long	22.25	28	2.6
W. E. Gear	26.75	46	.2	E. J. Louis	28.00	41	2.1

ALEXANDER MANUFACTURING COMPANY'S WEEKLY PAY ROLL.—(*Continued*)

Name	Wage (dollars)	Age (years)	Service (years)	Name	Wage (dollars)	Age (years)	Service (years)
C. D. Lucas	22.50	35	3.0	H. A. Racine	25.00	45	3.6
W. I. Lundy	23.00	38	6.3	L. L. Ralph	21.50	35	3.9
J. A. Lyon	27.50	27	2.6	J. L. Ramey	24.75	38	7.0
H. A. Mackey	28.25	42	1.2	E. C. Raper	23.50	47	4.5
L. R. Madsen	27.50	41	.5	C. I. Raymer	23.25	46	1.2
W. E. Main	22.50	40	.7	R. N. Redding	24.50	50	1.1
W. P. Manta	28.75	52	4.9	J. A. Reeve	25.25	47	1.4
W. M. Marion	28.50	57	.1	D. G. Rennie	30.50	41	.9
F. C. Marsh	21.00	62	.7	L. E. Richards	25.50	21	2.3
F. E. Martin	22.50	49	1.8	C. P. Ricks	26.75	24	4.5
G. A. Masters	23.25	53	1.9	L. D. Ripley	23.50	42	1.6
P. E. May	36.00	47	7.7	G. W. Robbins	24.50	24	2.4
C. H. Medde	22.50	47	4.3	F. P. Rolfe	27.50	30	1.5
A. I. Mellon	24.25	36	3.2	A. J. Romaine	22.50	29	7.2
J. S. Merlo	22.25	45	.7	P. M. Rose	21.25	26	.7
A. L. Miller	22.75	18	.1	W. F. Roth	21.50	36	.4
T. C. Miller	18.50	21	6.5	C. H. Roy	20.25	29	1.4
C. H. Mitchell	23.00	40	.2	M. S. Rudd	22.00	46	4.9
J. C. Moore	24.50	42	8.5	L. J. Russell	22.50	25	2.6
E. W. Morley	29.25	43	1.2	B. E. Ryder	22.00	19	3.5
W. L. Morrow	22.75	27	2.8	C. A. Saint	24.50	33	.3
W. R. Nash	26.50	51	1.5	W. P. Sage	25.75	29	1.7
R. G. Neale	24.75	43	3.2	G. E. Sammis	27.50	54	2.8
F. A. Nelson	26.50	26	1.1	D. B. Sands	22.00	37	1.0
F. J. Newman	24.25	32	.9	L. E. Sargent	23.50	34	2.7
N. V. Nicols	28.00	46	.1	R. M. Sax	27.50	23	1.2
V. D. Nissen	21.25	28	5.8	R. W. Schell	22.00	41	5.6
O. H. Norman	23.50	37	3.6	G. H. Shuler	28.50	45	.1
G. A. Nylin	23.00	31	1.5	I. M. Smith	25.75	27	1.1
C. H. Oakes	22.50	36	1.3	J. E. Smith	28.75	25	1.4
C. F. Oden	22.75	40	.4	P. L. Strong	23.50	31	.7
R. H. Oland	26.00	27	.8	D. D. Taylor	26.00	47	.9
A. D. Olds	24.50	23	1.3	J. G. Teagle	28.50	26	1.1
O. B. Olson	24.75	36	1.4	W. H. Terry	21.00	44	1.2
L. J. Oreck	28.00	21	1.6	C. J. Thom	27.75	41	4.3
P. H. Orkin	24.50	34	3.5	R. A. Thomas	22.00	29	1.4
W. E. Owens	25.75	44	8.7	L. M. Thorpe	24.25	35	1.2
J. C. Owings	22.50	48	4.2	E. R. Tilton	26.00	44	.1
A. E. Paine	22.50	27	1.9	W. H. Todd	20.50	39	1.8
D. S. Painter	24.00	31	1.4	C. M. Travis	24.00	45	1.5
G. A. Palmer	20.00	40	.3	F. B. Trotter	27.50	32	.8
R. S. Palmer	21.50	52	.5	E. P. Trout	21.50	41	1.7
M. E. Parker	21.75	56	.7	R. D. Tudor	28.25	36	2.1
W. S. Parks	25.50	30	1.6	S. M. Tuller	20.75	39	.4
J. H. Parsons	25.00	34	5.2	E. J. Ullery	28.50	36	.2
M. A. Patten	24.50	44	.1	S. J. Upton	21.00	40	1.8
G. P. Paxton	32.75	29	6.5	C. W. Urich	29.00	37	1.9
A. E. Peck	30.00	45	1.7	W. F. Valentine	29.00	32	.9
M. F. Pence	41.50	43	6.5	M. R. Vallens	22.00	34	2.4
F. R. Perkins	27.25	30	1.4	J. S. Vance	22.00	39	3.1
L. J. Peters	21.50	25	2.1	R. M. Varley	29.75	37	.1
T. R. Phelps	21.00	28	1.4	B. G. Varner	25.50	42	2.5
B. R. Phillips	28.75	35	.9	F. F. Vaughn	27.50	47	1.2
M. D. Pilson	27.25	40	2.3	P. T. Virgin	19.50	56	1.7
E. H. Porter	21.50	43	1.7	R. S. Waddell	21.75	32	.1
R. A. Priest	25.50	30	2.8	J. B. Wade	26.00	39	1.1
J. B. Pye	22.75	43	1.2	A. B. Walker	21.00	35	.9
B. E. Quick	27.50	35	2.6	M. S. Wallace	25.75	37	.7

28. Make a tally sheet, a frequency table, and a frequency graph (see Exhibit 68) of the age data given in the table in Problem 27. Use a four-year class interval.

Analyze your graph and write a brief report of your analysis.

29. Make a tally sheet, a frequency table, and a frequency graph (see Exhibit 68) of the length of service data given in the table in Problem 27.

Analyze your graph and write a brief report of your analysis.

30. The ratios of selling expenses to total expenses of 108 hardware stores are given below (ratios expressed in percentages):

31	52	43	17	57	44	32	33	42	33	24	43
17	25	11	22	27	19	55	25	26	28	25	22
47	9	32	29	18	45	33	24	37	31	33	22
5	31	26	27	29	22	36	42	26	47	32	39
31	22	36	25	43	19	21	36	24	26	34	28
27	18	36	44	57	42	24	34	27	36	39	37
53	47	33	39	31	27	29	26	38	33	44	25
14	63	16	29	38	37	31	29	34	25	19	22
23	46	17	42	29	24	50	31	39	21	22	24

a. Arrange the data in an array.

b. Make a tally sheet and a frequency table of the data. Use a class interval of 5 per cent.

c. Make a frequency graph (see Exhibit 68). Should the curve of this distribution be smoothed? If so, why?

31. The cost of radio advertising purchased by automobile manufacturers from the two major broadcasting systems in the United States in the years 1932–1936 is given below in thousands of dollars.

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1932	232	231	234	261	230	175	137	127	78	58	57	118
1933	176	171	215	126	121	115	128	234	209	261	273	289
1934	268	325	348	338	371	309	188	178	222	544	302	380
1935	408	363	426	342	312	275	232	200	244	721	400	487
1936	754	781	876	742	731	626	492	468	770	1,164	1,031	944

Source: Survey of Current Business.

Make a historical distribution of the cost of radio advertising to automobile manufacturers in the years 1932–1936 (see Exhibit 84).

32. (Geographical distribution.) Make a density map (see Exhibit 82) of the population data below:

POPULATION OF THE UNITED STATES—APRIL 1930
By States

Rank	State	Population	Rank	State	Population
1	New York	12,588,066	27	West Virginia	1,729,205
2	Pennsylvania	9,631,350	28	Maryland	1,631,526
3	Illinois	7,630,654	29	Connecticut	1,606,903
4	Ohio	6,646,697	30	Washington	1,563,396
5	Texas	5,824,715	31	Florida	1,468,211
6	California	5,677,251	32	Nebraska	1,377,963
7	Michigan	4,842,325	33	Colorado	1,035,791
8	Massachusetts	4,249,614	34	Oregon	953,786
9	New Jersey	4,041,334	35	Maine	797,423
10	Missouri	3,629,367	36	South Dakota	692,849
11	Indiana	3,238,503	37	Rhode Island	687,497
12	North Carolina	3,170,276	38	North Dakota	680,845
13	Wisconsin	2,939,006	39	Montana	537,606
14	Georgia	2,908,506	40	Utah	507,847
15	Alabama	2,646,248	41	District of Columbia	486,869
16	Tennessee	2,616,556	42	New Hampshire	465,293
17	Kentucky	2,614,589	43	Idaho	445,032
18	Minnesota	2,563,953	44	Arizona	435,573
19	Iowa	2,470,939	45	New Mexico	423,317
20	Virginia	2,421,851	46	Vermont	359,611
21	Oklahoma	2,396,040	47	Delaware	238,380
22	Louisiana	2,101,593	48	Wyoming	225,565
23	Mississippi	2,009,821	49	Nevada	91,058
24	Kansas	1,880,999			
25	Arkansas	1,854,482		Total continental	
26	South Carolina	1,738,765		United States	122,775,046

33. Make a density map of the data on automobile registrations given below (see Exhibit 82).

MOTOR VEHICLE REGISTRATIONS, 1937

State	Number of cars	State	Number of cars
Alabama	313,359	Nebraska	412,726
Arizona	129,210	Nevada	40,655
Arkansas	229,867	New Hampshire	125,939
California	2,484,653	New Jersey	994,497
Colorado	337,217	New Mexico	118,106
Connecticut	436,564	New York	2,561,703
Delaware	63,599	North Carolina	525,350
Florida	418,145	North Dakota	173,188
Georgia	441,847	Ohio	1,876,132
Idaho	142,110	Oklahoma	547,263
Illinois	1,768,946	Oregon	360,348
Indiana	956,016	Pennsylvania	1,984,821
Iowa	745,602	Rhode Island	167,586
Kansas	586,685	South Carolina	296,224
Kentucky	404,455	South Dakota	184,743
Louisiana	323,498	Tennessee	400,384
Maine	200,907	Texas	1,552,114
Maryland	387,410	Utah	126,692
Massachusetts	846,556	Vermont	88,958
Michigan	1,505,111	Virginia	440,713
Minnesota	822,598	Washington	535,483
Mississippi	226,286	West Virginia	290,837
Missouri	835,895	Wisconsin	854,374
Montana	173,892	Wyoming	81,837
		District of Columbia	184,119

Source: United States Bureau of Public Roads.

34. Present the data on per capita debt in each of the 48 states in the form of a geographical distribution by states (see Exhibit 83).

Per Capita Debt, State Governments, 1932

State	Dollars	State	Dollars	State	Dollars
Alabama	30.90	Maine	34.02	Ohio	1.15
Arizona	8.34	Maryland	18.99	Oklahoma	4.74
Arkansas	88.16	Massachusetts	14.69	Oregon	34.60
California	24.67	Michigan	12.21	Pennsylvania	7.81
Colorado	6.45	Minnesota	15.55	Rhode Island	24.15
Connecticut	.07	Mississippi	17.94	South Carolina	44.74
Delaware	8.63	Missouri	28.33	South Dakota	22.19
Florida	.26	Montana	17.33	Tennessee	35.55
Georgia	4.29	Nebraska	.67	Texas	1.75
Idaho	15.61	Nevada	14.89	Utah	11.08
Illinois	28.58	New Hampshire	13.96	Vermont	26.51
Indiana	1.45	New Jersey	15.05	Virginia	10.68
Iowa	6.66	New Mexico	26.53	Washington	5.22
Kansas	11.53	New York	36.14	West Virginia	49.20
Kentucky	6.16	North Carolina	50.91	Wisconsin	.40
Louisiana	39.41	North Dakota	7.32	Wyoming	24.42

Source: Statistical Abstract of the United States, 1936.

CHAPTER VIII

AVERAGES

If the general manager of a manufacturing company wished to compare the wage conditions in one branch factory with those in another branch factory, he might first compare the total pay roll of the one with that of the other. But would the total pay rolls indicate to him in which plant wages were higher? Would totals of the wages paid indicate anything whatever concerning the relative wage conditions in the two branches?

Again, the manager might have each actual pay roll made up in the form of an array of the wages paid at each branch. He would then have before him all of the details concerning the wages paid in the given period at both factories. But would these presentations permit easy comparison? Could he measure the conditions at each factory by looking at a list of the wages paid?

Even if the wages were grouped in two frequency distributions (Exhibit 85) showing the numbers of workers in each plant receiving wages between various wage limits, could the executive obtain quickly a picture of the relative wage conditions? In short, does a frequency distribution give a sufficiently concise picture for such purposes?

Methods of Measuring Data.—Neither the array nor the frequency distribution is a method of measuring data. They are methods of presenting data in orderly arrangement.

Whether data are in unorganized forms or in organized distributions such as those discussed in the preceding chapter, it is often necessary to obtain measurements which will give concise pictures or representations. There are several types of measurements, of which the most useful are:

1. Averages.
2. Quartiles, deciles, and percentiles.
3. Index numbers.
4. Measures of dispersion, or variation, and skewness.

In this chapter, the first two methods, *viz.*, (1) averages, and (2) quartiles, deciles, and percentiles, will be discussed. The following chapter (Chapter IX) will cover index numbers. In Chapter X, dispersion and skewness will be considered.

Nature and Types of Averages.—An average is a single value which is taken to represent a group of values. Such a representative value may

be obtained in several ways, for there are several types of averages. For a given case, one type may be more representative than another type, and it is therefore necessary to understand clearly the nature and purpose of each type.

Probably the most commonly used average is the *arithmetic average*, or *arithmetic mean*. This is an average obtained by adding the values of the cases and dividing by the number of cases.

Distribution of Employees in Plants 1 and 2 by Weekly Earnings
Week Ended September 10, 1938

Weekly earnings in dollars	Number of employees	
	Plant 1	Plant 2
1.00– 3.99	0	16
4.00– 6.99	4	23
7.00– 9.99	5	11
10.00–12.99	3	19
13.00–15.99	7	31
16.00–18.99	3	20
19.00–21.99	16	34
22.00–24.99	21	52
25.00–27.99	49	117
28.00–30.99	73	132
31.00–33.99	90	144
34.00–36.99	76	127
37.00–39.99	61	121
40.00–42.99	33	117
43.00–45.99	26	66
46.00–48.99	24	26
49.00–51.99	11	10
52.00–54.99	11	6
55.00–57.99	5	0
58.00–60.99	5	3
61.00–63.99	2	2
Total	525	1,077

Exhibit 85.—Frequency distributions of data taken from the pay rolls of two branches of a manufacturing company. (*Data adapted.*)

Another common average is the *mode*, or *modal average*. This is the value which occurs the greatest number of times, or, more exactly, the value about which the most cases occur; it is the most common value.

A third average is the *median*, which is entirely a position average. The median is the value of the "mid-case," or, more specifically, the value at the mid-point, when the data are arranged in order of size—that is, it is the value located where the number of larger values and the number of smaller values are the same.

A fourth average is the *geometric average*, or the *geometric mean*. This average is found by obtaining the product of the items and extracting the corresponding root of this product. The root to be obtained is the number of items—that is, if there are 500 items, the 500th root should be obtained. Such a root is easily found by the use of logarithms.

The four types of averages just mentioned will be discussed and illustrated in the pages which follow. Another average, which is seldom used in business statistics, and which, therefore, will not be considered in detail, is the *harmonic average*, or *harmonic mean*. This may be defined as the reciprocal of the arithmetic average of the reciprocals of the items. The reader may recall from elementary mathematics that the reciprocal of a number is a fraction of which the numerator is unity and the denominator is the number. Therefore, the harmonic mean is found by averaging arithmetically the fractions obtained by placing a numerator of 1 over each item. This resulting average of fractions[1] is the reciprocal of the harmonic mean.

Computation of the Arithmetic Average.—The method of computation of the arithmetic mean is not difficult when the exact values of all items are known, and it is even simpler where the total value of the items and the number of items are known. For instance, to return to the preceding example of a manufacturing company (Exhibit 85), if the weekly wages of the workers in Plant 1 are all available, the arithmetic average of the weekly wages can be obtained by adding together the individual wages and dividing by the number of wage earners. Or if only the total pay roll and the number of persons on the pay roll are known, the average wage can readily be found by dividing the latter into the former. Thus, if the total pay roll of Plant 1 for the week ended September 10, 1938, were $17,850 and the number of employees were 525, this average weekly wage would be $17,850 ÷ 525, or $34.

Obviously, for the purpose of computing the arithmetic average or mean, it does not matter in what order or form the exact values of the cases are used. The mean can be found just as quickly from items listed in a miscellaneous order as from items arranged in the form of an array, providing that they are arranged to facilitate addition. In obtaining the average from a frequency distribution, however, a slightly different procedure is necessary.

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[1] In this brief description, *common fractions* have been used to facilitate explanation. Ordinarily, calculations of the harmonic mean are made from a table of reciprocals which are stated decimally. The reciprocals in this form are found from the table, and are averaged arithmetically. The computed average reciprocal is then found in the table, and the absolute number corresponding to this is the harmonic mean. (See Table of Reciprocals, pp. 749–759.)

In Exhibits 65 and 66, pages 136 and 137, a frequency distribution of the sizes of men's shoes sold in 1937 by The Paris Store was presented. As in an array and in an unorganized list of items, each actual size of shoe sold is present in this type of frequency distribution, for there is no class interval. Yet, if the sizes listed were added together, the result would not be the sum of all sizes sold, for the number of sales of each size was greater than one. Therefore, before adding to find the sum of the sizes sold, we must multiply each size by the number of pairs sold, as in

Size (v)	Number of pairs (f)	Product (fv)
$4\frac{1}{2}$	5	$22\frac{1}{2}$
5	10	50
$5\frac{1}{2}$	20	110
6	25	150
$6\frac{1}{2}$	75	$487\frac{1}{2}$
7	150	1,050
$7\frac{1}{2}$	300	2,250
8	475	3,800
$8\frac{1}{2}$	410	3,485
9	375	3,375
$9\frac{1}{2}$	220	2,090
10	125	1,250
$10\frac{1}{2}$	75	$787\frac{1}{2}$
11	20	220
$11\frac{1}{2}$	15	$172\frac{1}{2}$
12	5	60
Total	2,305	19,360

19,360 ÷ 2,305 = 8.4, the arithmetic average size.

EXHIBIT 86.—Computation of the arithmetic average from a frequency table without class intervals.

Exhibit 86. Then, to find the arithmetic average of sizes sold, the sum of the products, 19,360, will be divided by the number of pairs sold, 2,305, giving 8.4. It will be noted that the sum of the products is the same sum as would have been obtained if each size had been listed the number of times indicated in the frequency (f) column and the resulting list of 2,305 individual sizes had been added together.

The method just described may be used to secure a very close approximation of the mean of data presented in the other type of frequency distribution—namely, where class intervals rather than exact class sizes are given. The only difference is that the mid-point of the class interval is used as the value of all cases within the class interval. In Exhibit 87, the computations of the average weekly wages earned for the week ended September 10, 1938, in Plants 1 and 2 are shown.

Sometimes it is advisable to obtain a *weighted arithmetic average.* The computation of the weighted mean differs from that of the simple mean in that the constituent items or values are first multiplied by certain weights. Then the total of the weighted products is divided by the sum of the weights, instead of by the number of items.

COMPUTATION OF ARITHMETIC AVERAGE OF WEEKLY EARNINGS (PLANTS 1 AND 2)
Week Ended September 10, 1938

Weekly earnings in dollars	Mid-point of class (m)	Plant 1		Plant 2	
		Frequency (f)	Product (mf)	Frequency (f)	Product (mf)
1.00– 3.99	2.50	0	0	16	40.00
4.00– 6.99	5.50	4	22.00	23	126.50
7.00– 9.99	8.50	5	42.50	11	93.50
10.00–12.99	11.50	3	34.50	19	218.50
13.00–15.99	14.50	7	101.50	31	449.50
16.00–18.99	17.50	3	52.50	20	350.00
19.00–21.99	20.50	16	328.00	34	697.00
22.00–24.99	23.50	21	493.50	52	1,222.00
25.00–27.99	26.50	49	1,298.50	117	3,100.50
28.00–30.99	29.50	73	2,153.50	132	3,894.00
31.00–33.99	32.50	90	2,925.00	144	4,680.00
34.00–36.99	35.50	76	2,698.00	127	4,508.50
37.00–39.99	38.50	61	2,348.50	121	4,658.50
40.00–42.99	41.50	33	1,369.50	117	4,855.50
43.00–45.99	44.50	26	1,157.00	66	2,937.00
46.00–48.99	47.50	24	1,140.00	26	1,235.00
49.00–51.99	50.50	11	555.50	10	505.00
52.00–54.99	53.50	11	588.50	6	321.00
55.00–57.99	56.50	5	282.50	0	0.00
58.00–60.99	59.50	5	297.50	3	178.50
61.00–63.99	62.50	2	125.00	2	125.00
Total		525	18,013.50	1,077	34,195.50

18,013.50 ÷ 525 = 34.31, the average wage in Plant 1.
34,195.50 ÷ 1077 = 31.75, the average wage in Plant 2.

EXHIBIT 87.—Computation of the arithmetic average from a frequency table with class intervals.

It may appear that the computation of the weighted mean is the same as that of the simple mean from data in a frequency distribution with class intervals, as the frequencies may appear to be weights applied to the mid-points of the classes. However, a weighted mean is one where the weights assigned to the constituent items are *estimated* or *arbitrarily determined,* and are not the actual number of occurrences or frequencies as found in a frequency distribution.

The weighted arithmetic average is particularly useful in studying prices of commodities where it is desired to weight each price according to the relative importance of each commodity. The use of weights in this and in other ways is illustrated in Chapter IX on Index Numbers.

Nature and Significance of the Arithmetic Mean.—It will be noted that the mean is affected by the value of every case, for all values (approximate values, in the case of frequency distributions with class intervals) are included in the sum of the values. However, undue weight may be given to extreme items. For example, in one factory, there may be a few highly paid skilled workers whose weekly earnings would cause the arithmetic mean to be somewhat higher than it would be without these earnings. Aside from these few skilled workers, wage conditions in that factory might be approximately the same as in another factory, and yet the arithmetic average of wages would be greater in the first factory because of the inclusion of a few extreme cases.

Furthermore, the mean very often is not the value of any observed case. In discrete (non-continuous) series, the mean may be an amount which does not actually exist. Thus, the average size of families in a certain city was found to be 3.46 persons; the average size of men's shoes sold by The Paris Store (see page 167) was determined to be 8.4. It is not to be considered that these characteristics are necessarily serious disadvantages. When averaging the prices obtained for a given product over a period of time, it might be essential to include all extremely high or low prices, because these affect earnings, and average selling prices are often compared with average earnings and average expenses per sale. Furthermore, even though the above stated average size of family does not actually exist, this average is useful in comparing cities from the standpoint of school requirements, the number of family dwellings required, etc.

The arithmetic average has a definite advantage in that it is commonly used and, therefore, is familiar to everyone. Furthermore, it is an amount which can be located with mathematical exactness, and it can be determined when only the total value of the cases and the number of cases are known.

It should be appreciated that the arithmetic average (as well as other averages) should be used only when the data are relatively homogeneous for the purpose at hand. The average wage discussed above, for instance, might be misleading if it included different kinds of labor, such as male and female, or men and boys. Suppose, for instance, that boys are 50 per cent as efficient as men. Then, if Factory A has 75 per cent boys averaging 60 per cent of men's wages, and Factory B has 25 per cent boys at 60 per cent of men's wages, Factory A will have lower average wages, but higher unit costs. From many points of view, such averages would be based on data that were not homogeneous, and would be misleading.

Arithmetic Average of Extreme Items.—For a rough approximation of the representative value or average of a series or group of data, the *mean of extremes* is useful. This is merely the arithmetic average of the highest and the lowest value. While ordinarily it is not sufficiently representative for careful work, it has the advantage of being easily computed. In instances where the computation of an arithmetic mean of all items would require so much work that it could not be used at all, and where it is known that the data are not subject to erratic fluctuations, the average of extremes serves a very useful purpose. For example, the average price of a certain stock on the New York Exchange on a given day should be obtained ordinarily by dividing the total number of shares sold into the total amount paid for the shares, but this is often so great a task that it cannot be done, especially if the study involves a large number of sales. If there are no unusual movements, the average of extremes may be accurate enough for the purpose at hand, and it may be much better to use this average in connection with a large number of stocks than to use the more refined methods with a small number of stocks. The average of extremes is often a practical average to use in analyzing the movements of the prices of commodities as well as the prices of stocks.

Computation of the Mode.—The mode has been defined as the value which occurs the greatest number of times, or, more exactly, the value about which the most cases occur. It is the point of greatest "density" or greatest frequency of occurrence in a frequency distribution; it is the most common value.

The modal average is a position average—that is, it is located at a certain position within an orderly grouping of the data. It can not be determined readily from unorganized data; it is not easy to locate within an array. For practical computation, grouping of the data in a frequency distribution is a prerequisite.

In a distribution where the cases are grouped by individual sizes rather than by classes of sizes, the mode ordinarily is that size which has the greatest frequency. Thus, in Exhibit 64, page 136, the modal average of men's shoes sold in 1937 is size 8, for there were more shoes of this size sold than of any other. Occasionally it happens that a certain size occurs more frequently than any other size, when there are only a relatively few frequencies of the adjacent sizes. If another size has occurred slightly less frequently, but this latter size has a greater number of adjacent items, it may be the point of greatest density. Then this latter item should be taken as the mode, rather than the size of greatest frequency. In some distributions, several points of considerable density may be found, and it may be difficult to determine at which point there is the greatest density of cases. In such distributions, it may not be advisable to compute or use the mode.

In frequency distributions having class intervals, the modal class is either the class having the most frequencies or, if this is not the place of greatest density, it is where the greatest density of cases is found. Sometimes several regroupings of the data, using class intervals of different sizes and placing the class limits at different points, are necessary to ascertain the modal class. Usually, however, in distributions where the modal class is not well defined it is not advisable to use the modal average. In obtaining the mode itself (within the modal class) it is usually satisfactory, when studying business data, to use the mid-point of the modal class. While it is true that the mid-point of the modal class can only be called the approximate mode, it is seldom necessary to use technical methods of locating the modal average in business statistics,[1] because mathematical exactness does not necessarily obtain a more representative mode.

Referring again to Exhibit 85, it will be seen that the class \$31.00 to \$33.99 is the "class of greatest density" both for Plant 1 and Plant 2, because more employees earned wages within this class than within any other, and also because the next greatest numbers of employees earned wages within the adjacent classes. Therefore, this class is the modal class, and the mid-point of this class, \$32.50, may be taken as the modal wage of employees in both factories.

Significance of the Mode.—In ordinary conversation, people often refer to an average which really is the mode. The "average" citizen is the modal citizen; by average income, we usually mean modal income;

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[1] A more scientific method of "interpolating for the mode" (although not often necessary in practical work) is as follows: The mode equals the lower limit of the modal class plus a fraction of the class interval, of which the numerator is the number of cases in the next higher class and the denominator is the sum of the number of cases in the next higher and the next lower classes. This may be expressed in formula form as:

$$M_0 = L + \left(\frac{fn}{fn + fm}\right)i$$

in which the symbols represent the items named above, namely, M_0 is the mode, L is the lower limit of the modal class, fn is the number of cases in the next higher class, fm is the number of cases in the next lower class, and i the class interval.

Referring to the modal wages in Plants 1 and 2 from the distribution of weekly wages in Exhibit 85, it is evident that the modal wage class for both plants is \$31.00 to \$33.99. Let us interpolate for the mode by applying the above formula. By substituting values in the above formula we find, for Plant 1, that the

$$\text{Mode} = 31.00 + \left(\frac{76}{76 + 73}\right)3 = 31.00 + \frac{228}{149} = 32.53$$

and for Plant 2 the

$$\text{Mode} = 31.00 + \left(\frac{127}{127 + 132}\right)3 = 31.00 + \frac{381}{259} = 32.47.$$

average efficiency is the most common or most frequently attained efficiency.

The mode is useful where extreme items should not affect the average. For example, if a banker wishes to know the average balance of depositors, he might wish to use the modal average rather than the arithmetic. The latter would be greatly influenced by a few balances of one million dollars or more, but the mode would not be affected by these extreme cases, and, therefore, would be a more representative average for many purposes. In fact, in obtaining the modal average, the size of extreme cases need not even be known.

The mode may not always be well defined or possible to locate properly. The "point of density" may be more than a point, and, therefore, the point taken as the mode may depend upon the judgment or desire of the person using the mode. In frequency distributions, it is essential that the class interval be properly chosen and that the class limits be correctly placed, or the modal point will not be accurately determined. Even where a representative mode is obtained, the product of the mode and the number of items will not give the total value of all cases, as does the product of the arithmetic average and the number of items (except in a perfectly normal distribution as explained on page 236).

Computation of the Median.—The median, like the mode, is also a "position" average. It is the value at the mid-point in an array. In order to determine the median, it is necessary to have the items arranged in order of size, as in an array or frequency distribution. The median can not readily be determined from unorganized data, except where the number of cases is sufficiently small to allow mental arrangement in the order of size.

When the data have been arranged in an array, it is only necessary to count the items and find the mid-item in order of size. Thus, in Exhibit 63, page 134, which presents an array of the sales of the Smith Motor Car Company, there are 21 items, and the median is the value of the 11th item (counting from either end of the array), namely $1,800.00. It will be noted that the median is so located that the number of items above it is the same as the number below it.[1]

If there is an even number of items, the median is halfway between the two mid-items. For instance, if the last sale listed in the array of the Smith Motor Car Company sales (Invoice 901 in the amount $1,450.00) were dropped from that array, there would be but 20 items (Exhibit 63).

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[1] The *position* of the median may be found by the formula $\frac{n+1}{2}$, in which n is the number of items. Thus, in Exhibit 63, there are 21 items, and $\frac{21+1}{2} = 11$. Therefore, the median is the 11th item.

The median would then be midway between the 10th and 11th items ($1,875.50 and $1,800.00 respectively), or $1,837.75. In other words, the midway point is the arithmetic average of the 10th and 11th items. In this case, there are also 10 items larger and 10 items smaller than the median ($1,837.75), but the median is not an actual item found in the array—it is midway between the two central items.

In a frequency distribution, where the items are grouped by *actual values* rather than by *class intervals of values*, the median can be found in a manner similar to that followed in the case of the array. It is only necessary to locate the class in which the mid-case falls, and this class value is the value of the median. In Exhibit 64, page 136, the median size of men's shoes sold by The Paris Store is the size of the 1,153d item in order of size, for there are 2,305 items included in the distribution. This item is found by counting from either end of the distribution by which it will be found to be among the 410 items of the size $8\frac{1}{2}$. The median size, therefore, is $8\frac{1}{2}$.

Location of the Median within a Class Interval.—Where classes are expressed in intervals of values, as in Exhibit 67, page 138, an additional problem arises. The class within which the median falls can be located in the manner described above, but which value within that class interval is the median? For example, in Exhibit 67, the median earning of employees in Plant 6 is within the class interval in which the 255th case falls, there being 509 cases in the distribution. This class is $24.00 to $25.99, in which 90 items or frequencies occur. But what value within that class range should be taken as the median?

Obviously, if the *actual* values of the 90 items were known, the median could be located exactly. It would only be necessary to count the arrayed items until the mid-item of the entire 509 wage earners (the 255th item) is found. But where the actual values of the 90 items are not known (they are not given by the frequency distribution and could be found only by reference to the data from which the frequency distribution was compiled), it is necessary to locate the median by somewhat arbitrary methods.

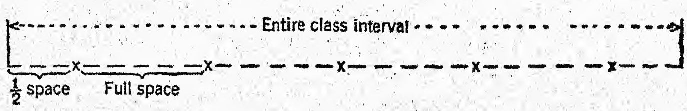
As a rough approximation, the mid-point of the class in which the median falls might be taken as the median, for it has been pointed out that the mid-point is usually considered to be representative of the class. However, this is not sufficiently accurate for some statistical work, particularly where it is evident that the median is more likely to be near one of the extremes of the class range.

A more exact method is to consider the median value as a *proportional* distance from the lower limit, or from the upper limit, of the class interval. Thus, in Exhibit 67 (page 138), if we count the number of wage earners receiving less than do the 90 employees in the median class ($24.00 to

$25.99), we will find the total to be 175. Since the median or mid-item is the 255th, it will be 80 items (*i.e.*, 255 minus 175) above the 175 items. In other words, if we count up from the smallest earning to the 255th earning, we will include the 175 items below $24.00 and 80 items in the class $24.00 to $25.99. As there are 90 wage earners in the class $24.00 to $25.99, we may locate the median as being $\frac{80}{90}$ of the entire interval ($2.00) from the lower limit, $24.00. Similarly, counting from the highest earning, there are 244 items larger than $25.99; therefore, we may locate the median as being $\frac{11}{90}$ of the interval below the upper limit.

A slight correction of these statements is necessary for mathematical exactness. The true proportions of the interval which we should take are 79.5/90 and 10.5/90, respectively, because the median must be considered to be $79\frac{1}{2}$ spaces distant from the lower limit rather than 80 spaces, and $10\frac{1}{2}$ spaces distant from the upper limit, in this example. We can prove this by noting that the median must be the same point, whether we find it by adding a fraction of the class interval to the lower limit, or by subtracting a fraction of the interval from the upper limit. This point would not be obtained by adding $\frac{80}{90}$ of the interval to the lower limit and subtracting $\frac{11}{90}$ from the upper limit, but it is obtained by using the fractions 79.5/90 and 10.5/90 respectively.[1] Thus, in this case the median is $24 + \frac{79.5}{90}(2)$, or 25.77; it is also $26 - \frac{10.5}{90}(2)$, or 25.77.[2]

[1] While the median is the 80th item *from the items in the next lower class* and the 11th item from those in the next higher class, it is only $79\frac{1}{2}$ spaces distant *from the lower limit of the class* and $10\frac{1}{2}$ spaces distant from the upper limit, if we assume that the items are evenly distributed throughout the class interval. The reason for this is that the first item should not be considered to be exactly at the lower limit nor the 90th item at the upper limit. If this latter condition were true, then the items in the next lower class would be distributed in the same manner and one of the cases in that class would fall at the upper limit of that class which is also the lower limit of the given class. Therefore, we must consider that the distribution of cases begins a certain distance from the limits, at a point which is one-half of the distance between any two cases. This is illustrated below by a graphic presentation of the positions of five items evenly distributed in a class interval.



Obviously, the third item from the lower limit is $2\frac{1}{2}$ spaces from that point, the second item is $1\frac{1}{2}$ spaces from the lower limit, etc.

[2] Reduced to rules of procedure, it may be said that when the actual values of the cases within the median class are not known, the median may be located within that class by either of the following:

1. Adding a proportion of the class interval to the lower limit of the median class—this proportion being a fraction having as the numerator one-half an item less than

Use of the Median.—The median average can be obtained even though the values of all items are not known. In fact, the only item which requires valuation is the mid-item, provided it is known that there are the same number of items larger and the same number smaller. For example, if a wholesale drug company wished to know the average annual sales of its customers, it could obtain the median, even though the sales data were not available for the smallest customers, nor the largest. It would only need to know whether the sales of the customers not reporting were larger or smaller than the median.

Sometimes it is difficult to measure qualities for which an average might be desired. The intelligence of the laborers in a factory cannot easily be measured in definite units, but it may not be difficult to arrange the laborers in order of intelligence and determine the laborer with the median average intelligence. In other words, the median may be determined even though it is not mathematically measured.

In locating the median no more weight is given to an extreme item than to an item near the median. Although it is not influenced by the size of extreme items, the effect is not likely to make it unrepresentative of the entire data.

If the items are few in number, however, the median is not likely to be representative. Neither the mode nor the median should ordinarily be used where the number of cases are few, because they are "position" averages. It requires a considerable number of cases to establish a reliable position for the mode or the median.

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the number of items located within the class which are needed to reach the median item of the entire distribution (counting up from the smallest case), and having as the denominator the total number of items within the median class. In formula form, this may be stated as:

$$\text{Median} = L + \frac{g}{f}i$$

where L is the lower limit of the class in which the median occurs, g is one-half of one item less than the number of items to be added to those below the class to reach the median item of the distribution, f is the total number of items within the class, and i is the class interval.

2. Subtracting a proportion of the class interval from the upper limit of the median class—this proportion being a fraction having as the numerator one-half an item less than the number of items located within the median class which are needed to reach the median item (counting down from the largest case), and having as the denominator the total number of items within the median class. In formula form this is:

$$\text{Median} = L' - \frac{g'}{f}i$$

where L' is the upper limit of the class in which the median occurs g' is one-half of one item less than the number of cases to be subtracted to get the median item, and f and i are the same as in the previous formula.

It is also to be noted that neither of these averages can be used to determine the total value of all the cases where the number of cases is known. The product of the number of cases and the median, or the mode, does not give the total of the data (except in a normal distribution as explained on page 236).

Finally, it should be appreciated that, although theoretically there should be the same number of cases larger and the same number smaller than the median, this may not be the case if an array contains groups of numbers of the same size. The median size of men's shoes sold by The Paris Store (see Exhibit 64, page 136) was stated to be $8\frac{1}{2}$, for this is the size of the 1,153d or mid-case. But there are 1,060 items smaller and 835 items larger, and the median might be taken as any one of the 410 items of the size $8\frac{1}{2}$. That is, the median might be taken as the 1,061st item or the 1,470th item, or any intermediate item, and yet the value of the median would still be $8\frac{1}{2}$, because this is the size of all 410 items. This indefinite quality of the median, it is to be observed, exists chiefly in discrete (non-continuous) series, and it is particularly evident in the above illustration because the data are particularly discrete.

Computation of the Geometric Average.—The geometric average (or geometric mean) is the nth root of the product of n factors. Thus, the geometric average of 10, 100, and 1,000 is the cube root of $(10 \times 100 \times 1{,}000)$ or $\sqrt[3]{1{,}000{,}000}$ which is 100. (Note that the arithmetic average is $[10 + 100 + 1{,}000] \div 3$ or $1{,}110 \div 3$ which is 370.) As there are usually more than three items to be averaged geometrically, there are usually more than three factors to be multiplied together and a higher root than the cube root to be extracted. Therefore, in computing this average, a knowledge of the more simple uses of logarithms is necessary.[1]

An easy rule to remember is that the geometric average is obtained by finding the arithmetic average of the logarithms of the items, which figure is the log of the geometric average. The procedure is just the same as in finding the arithmetic average of the data, except that the logarithms of the items are used instead of the items themselves. Therefore, to find the geometric average of several items in an unorganized presentation or in an array, add the logs of the items, divide by the number of items, and look up the antilog. To illustrate, the geometric average of 10, 100, and 1,000 may be found by adding the logs of these items, namely 1, 2, and 3, then dividing the sum (6) by 3 and looking up the antilog of the result (2), which is 100.

In a frequency distribution, the process of computing the geometric average with the use of logarithms is similar to that used in computing

[1] A brief explanation of logarithms and how to use them is given in Appendix VII, pp. 723–728.

the arithmetic average of the data. The computation is illustrated in Exhibit 88, in which it will be seen that the logarithm of each size (see second column) is multiplied by the corresponding number of items (third column). The resulting product (see fourth column) is the logarithm of a stated power of the given size. Thus, when the logarithm

THE PARIS STORE

Computation of the Geometric Average Size of Men's Shoes Sold in 1937

Size	Log of size	Number of pairs	Product of log and number of pairs
$4\frac{1}{2}$	.65321	5	3.26605
5	.69897	10	6.98970
$5\frac{1}{2}$	.74036	20	14.80720
6	.77815	25	19.45375
$6\frac{1}{2}$	.81291	75	60.96825
7	.84510	150	126.76500
$7\frac{1}{2}$	.87506	300	262.51800
8	.90309	475	428.96775
$8\frac{1}{2}$	.92942	410	381.06220
9	.95424	375	357.84000
$9\frac{1}{2}$	.97772	220	215.09840
10	1.00000	125	125.00000
$10\frac{1}{2}$	1.02119	75	76.58925
11	1.04139	20	20.82780
$11\frac{1}{2}$	1.06070	15	15.91050
12	1.07918	5	5.39590
Total		2305	2121.45975

Log of the geometric average = 2121.45975 ÷ 2305 = .92037.

Antilog of .92037 = 8.325, the geometric average.

Exhibit 88.—Computation of geometric average from frequency distribution of size values.

of size $4\frac{1}{2}$ is multiplied by 5, the result (3.26605) is the logarithm of the 5th power of $4\frac{1}{2}$.[1] The total of the logarithms of the 2,305 items is the sum of the fourth column, namely 2121.45975. By dividing this by 2,305, the logarithm of the 2,305th root of the product of 2,305 items is obtained. This is .92037, which is the logarithm of 8.325, the geometric average.

Significance and Use of the Geometric Average.—The geometric average is useful in averaging ratios, which express rates of change.

[1] Since the 5th power of $4\frac{1}{2}$ means that the number is multiplied by itself 4 times (that is, there are 5 factors of $4\frac{1}{2}$), the logarithm 3.26605 is the logarithm of the product of 5 items of size $4\frac{1}{2}$. Then the logarithm of size 5, namely .69897, multiplied by 10 gives the logarithm of the product of 10 factors having size 5 (*i.e.*, $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$).

These ratios are usually called "percentage relatives." For instance, if the price of sugar at one date is 5 cents and a year later it is 10 cents, the price at the first date may be taken as 100 and that at the later date will be 200. In other words, the second price is 200 per cent of the first. Now, if the price of bacon at the same dates is 80 cents and 40 cents, respectively, the percentage relatives are 100 and 50. What is the average price condition of these two commodities at the two dates? If we average the percentage relations arithmetically the results are as shown in the table below, and the average change is from 100 to 125.

	Relative price at first date	Relative price at second date
Sugar	100	200
Bacon	100	50
Total	200	250
Arithmetic mean	100	125

However, if we average geometrically, the average at the first and at the second date will be 100, and no change is found ($\sqrt{200 \times 50} = \sqrt{10{,}000} = 100$). The latter is in accordance with the facts, for the change in the price of sugar has been offset by a similar inverse change in the price of bacon, assuming that each change is to be given equal weight.

In studying index numbers in Chapter IX, further illustrations of the use of the geometric average as the best method of averaging percentage relatives will be presented. It is well to note that the geometric average is seldom used to average the observed values, as was done for purposes of illustration in Exhibit 88. Its use is necessary, however, in such problems as averaging percentage relatives or other expressions of rate of change.

Quartiles, Deciles, and Percentiles.—Sometimes it is desirable to obtain representative values other than the averages described in the preceding discussion. Thus, if salesmen are graded according to the dollars of sales attained, they may wish to know what amount of sales is necessary to place them above the first one-fourth of the salesmen, or above one-half of the salesmen, or above three-fourths of the salesmen. These dividing points, or rather the values at the points dividing the number of salesmen, are called the first, second, and third quartiles, respectively. They are found by dividing the number of items into four parts and finding the values of the items at the three resulting division points. Obviously, the items must first be arranged in order of size. Also, it will be noted that the second quartile and the median are identical.

The quartiles, deciles, etc., are always numbered from the smallest to the largest.

The first quartile in Exhibit 64, page 136, is the value of the item which is one-fourth of the total number of cases from the smallest case. Thus, it is approximately the 576th item. Counting up from the smallest item, we find this to be size $7\frac{1}{2}$. The second quartile, or median, is the value of the 1,153d item, which we previously found to be $8\frac{1}{2}$. The third quartile is approximately the 1,729th item, or size 9.

Deciles and percentiles are measurements similar to the quartiles, except that they are found by dividing the number of items into 10 and 100 parts, respectively. Thus, there are 9 deciles and 99 percentiles.

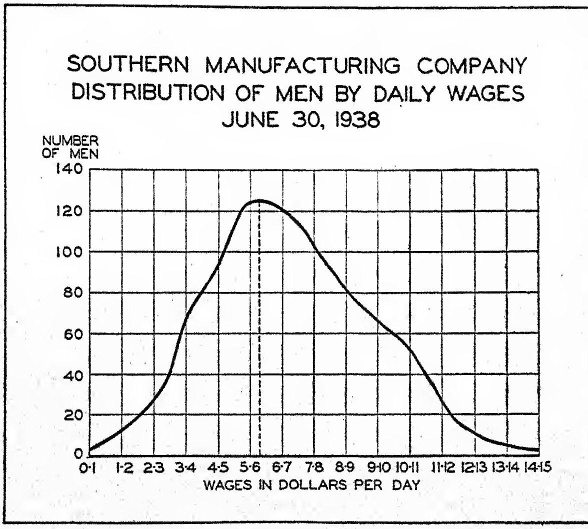

EXHIBIT 89.—In distributions approaching a bell-shaped curve, the mode is indicated by the highest point on the curve.

Determination of Averages, Quartiles, Etc., from Frequency Curves.— Two of the averages which have been considered can be determined from frequency curves. These are the mode, which can be found on an ordinary frequency curve, and the median, which can be determined on a cumulative frequency curve. The quartiles, deciles, and percentiles can also be located by means of the cumulative frequency curve.

In distributions which approach the bell-shaped curve,[1] the mode can easily be read from the graph, as it is determined by the highest point on the curve. That is, the mode is the value on the horizontal scale under the highest point on a frequency curve, providing, however, that the

[1] See Exhibit 72 for a graphic presentation of fundamental types of frequency curves.

adjacent portions of the curve are high enough to indicate that this point is the place of greatest density. In the curve of Exhibit 89, for instance, it is easy to see that the highest point is at the \$5.00 to \$6.00 class interval, or, in other words, that the most common wage is between \$5.00 and \$6.00. This curve also shows, however, that there are almost as many men receiving wages between \$6.00 and \$7.00. This fact should be taken into consideration when interpreting this curve. If a single figure or modal average is estimated from this curve, it is obvious that \$5.50 (half way between five and six) is probably not as close an approximation as \$5.75.[1]

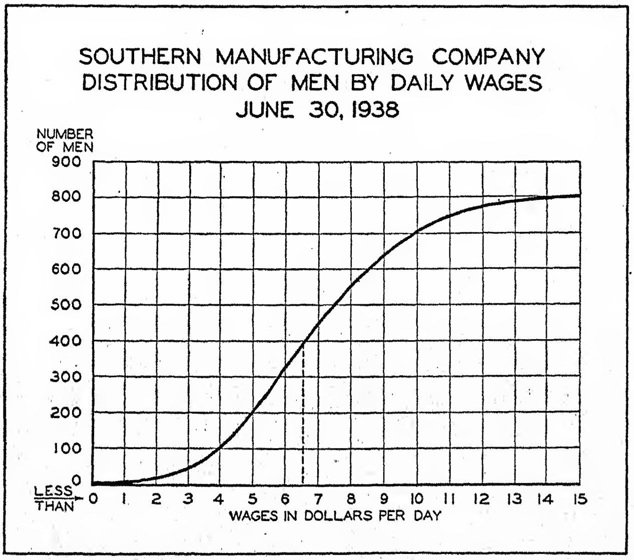

EXHIBIT 90.—Determining the median from a cumulative frequency curve. (The point is found where the curve crosses one-half the greatest ordinate value, and the median is read directly under this point on the horizontal scale.)

Since the cumulative frequency curve shows the cumulated frequencies as ordinates, the median is located by finding the abscissa value for the point at which the ordinate value on the curve is half way between zero and the largest ordinate value.[2] For instance, in Exhibit 90, the greatest ordinate value is approximately 800 men. Half way between zero and 800 is 400. Then, if we find the point where the curve crosses the 400 line and read the value in the horizontal scale directly below this point, we find that the median is between 6 and 7 or, in other words, approximately \$6.50.

[1] See footnote on page 171 for mathematical method of interpolating for the mode.

[2] As mentioned in Chap. VII, the ordinates are the frequencies, and the sizes are shown along the abscissa line.

If a "more than" curve were also plotted on the chart in Exhibit 90, it would cross the "less than" curve at the median. In other words, two cumulative curves of a frequency distribution will cross at the median.

Quartiles may be determined from a cumulative frequency curve in the same manner as the median, except that the highest ordinate value is divided into four parts instead of into two. In Exhibit 90, one-fourth of the highest ordinate value is 200 men. Thus, if we find the points where the curve crosses the 200, 400, and 600 lines and read directly below these points, we will have the first quartile ($5.00), the second quartile ($6.50), and the third quartile ($8.50), respectively. Deciles and percentiles can also be located by dividing the distance to the highest ordinate value into 10 or 100 parts, respectively. The abscissa values of the corresponding points on the curve are then read from the chart.

Because they are determined mathematically instead of according to their position in the data, the arithmetic and geometric averages are not ascertained by graphic methods.

Indicating Changes in Combinations of Statistical Series.—In this chapter, the methods of obtaining values which are condensed representations of data have been explained, and the use and significance of these representative values, or averages, have been considered. From this discussion, it will be realized that when the problem is one of analyzing changes in combinations of time or other series, something more than the ordinary use of averages is required. A device commonly used in such problems is known as the "index number," which is the subject of the following chapter.

Questions and Problems

1. Does the frequency distribution measure numerical data? What is its purpose?

2. Name four major types of measurements which may be applied to organized or unorganized data.

3. Describe the common types of averages. Explain the use of each.

4. Describe the procedure in obtaining an arithmetic average of data presented: (*a*) in unorganized form; (*b*) in an array; (*c*) in a frequency distribution not having class intervals; and (*d*) in a frequency distribution having class intervals.

5. What is the weighted arithmetic average? The average of extremes? How are they used?

6. Define the mode. How is it located? When should it be used?

7. Give the rule for interpolating within a class interval to locate the mode.

8. Compare the arithmetic average and the mode as to: (*a*) effect of extreme items in the computation; (*b*) existence of the average as an actual case; (*c*) ease of locating or determining; and (*d*) popular use.

9. Describe how the median is obtained from (*a*) an array; (*b*) a frequency distribution not having class intervals; and (*c*) a frequency distribution having class intervals.

10. Is the median easier to locate than the mode? Is it more definite? Is it always a representative average? When is it particularly not likely to be representative?

11. Explain how the geometric average is computed.

12. When should the geometric average be used in preference to other averages?

13. What are quartiles, deciles, and percentiles? How are they used?

14. Which average can be obtained by reading from a frequency curve? Which by reading from a cumulative frequency curve? How are these readings made?

15. Can quartiles be located from a graphic presentation of a frequency distribution? From what type of graphic presentation?

16. A study was made of the amounts spent for lunches by university students which revealed the following distribution of meal checks in a given week:

Amount of meal check in cents	*Number of customers*
2.5– 7.5	6
7.5–12.5	101
12.5–17.5	327
17.5–22.5	489
22.5–27.5	577
27.5–32.5	556
32.5–37.5	583
37.5–42.5	621
42.5–47.5	280
47.5–52.5	126
52.5–57.5	63
57.5–62.5	17
62.5–67.5	4
Total	3,750

From the above series compute the average meal check, by the following methods:

a. Mode (estimate and compute the mode and note the difference).
b. Median (compute).
c. Arithmetic mean.
d. Geometric mean.

17. What is the average wage paid by the Alexander Manufacturing Company according to data presented in Problem 27, pages 158–159?

Compute the following:

a. Simple arithmetic average. (Total pay roll is $6,047.50.)
b. Average of extremes. (Arithmetic average of highest and lowest wage items.)
c. Mode. (Use the frequency table of wage data made for Problem 27, page 157, and find the mid-point of the modal class.)
d. Median. (Use the frequency table of wage data made for Problem 27, page 157, and the actual data on pages 158–159 to determine the actual median—not the computed median.)

Write a brief report on the advantages and disadvantages of the preceding averages.

18. What is the average age of workers employed by the Alexander Manufacturing Company according to data presented in Problem 27, pages 158–159?

Compute the following:

a. Simple arithmetic average. (Total of ages is 8,760 years.)
b. Average of extremes. (Arithmetic average of highest and lowest wage items.)
c. Mode. (Use the frequency table of age data made for Problem 28, page 160 and find the mid-point of the modal class.)
d. Median. (Use the frequency table of age data made for Problem 28, page 160, and the actual data on pages 158 and 159 to determine the actual median—not the computed median.)

Write a brief report on the advantages and disadvantages of the preceding averages.

19. What is the average length of service of workers employed by the Alexander Manufacturing Company according to data presented in Problem 27, pages 158–159?

Compute the following:

a. Simple arithmetic average. (Total of years of service is 808.7.)
b. Average of extremes. (Arithmetic average of longest and shortest length of service items.)
c. Mode. (Use the frequency table of length of service data made for Problem 29, page 160, and find the mid-point of the modal class.)
d. Median. (Use the frequency table of length of service data made for Problem 29, page 160, and the actual data on pages 158–159 and find the actual median, not the computed median.)

Write a brief report on the advantages and disadvantages of the preceding averages.

20. From the following table of prices of Class AAA bonds on April 1, 1937, compute the average price of such bonds. Compute:

a. Arithmetic mean.
b. Median.
c. Geometric mean.

PRICES OF 30 CLASS AAA BONDS
Week Ending April 2, 1937

Name of company	*Price*	*Name of company*	*Price*
Alleghany Valley Ry	$107\frac{3}{8}$	Erie and Pittsburgh RR	$103\frac{1}{2}$
American Tel. and Tel	$106\frac{1}{8}$	Hackensack Water Co	106
Atchison, Topeka, & Santa Fe Ry	$108\frac{3}{4}$	Hudson County Gas Co	$118\frac{1}{4}$
Atlantic Refining Company	$100\frac{7}{8}$	Illinois Central RR	$110\frac{1}{8}$
Brooklyn Edison Company	$98\frac{7}{8}$	Illinois Steel Co	107
Canadian Pacific Ry	$111\frac{1}{2}$	Inland Steel Co	$101\frac{1}{2}$
Chesapeake and Ohio Ry	$106\frac{3}{8}$	Kansas City Terminal Ry	$106\frac{1}{8}$
Chicago Union Station Co	105	Liggett & Myers Tobacco Co	121
Cincinnati Gas and Electric Co	98	Little Miami RR	109
Cleveland Electric Illumin. Co	107	(P.) Lorillard Co	$117\frac{1}{2}$
Columbus and Toledo RR	107	Michigan Central RR	$105\frac{1}{2}$
Consolidated Edison Co. (N.Y.)	$103\frac{3}{4}$	Montana Central Ry	$101\frac{1}{2}$
Consolidated Gas Co. (N.Y.)	$106\frac{1}{8}$	National Steel Corp	$103\frac{1}{4}$
Detroit Edison Co	$112\frac{1}{4}$	Ontario Power Co	$112\frac{3}{8}$
Duquesne Light Co	$102\frac{1}{2}$	Richmond Terminal Ry	$103\frac{1}{2}$

Source: Commercial and Financial Chronicle.

CHAPTER IX

INDEX NUMBERS

How much have general business conditions changed since a year ago and since five years ago on the average? How much has the general price level risen as compared with 1933? How has industrial production changed during the past ten years? Questions like these are common among those interested in business affairs, and a valuable aid in answering them is the special statistical device known as the index number.

About the middle of the eighteenth century, an Italian named Carli wished to know the effect of the discovery of America upon the level of prices in Italy. To measure the price change which had taken place, he obtained a representative price of each of three commodities in the year 1500 and in the year 1750. The three commodities chosen were grain, oil, and wine, which were considered to be representative of all Italian commodities. These prices were then expressed as percentage relatives; that is, the prices in 1500 were each taken as 100 and the prices in 1750 were expressed relatively to their corresponding prices for the year 1500. Carli then averaged the three percentage relatives of the year 1750 arithmetically, and the resulting relative was his *index* of the change in prices between the years 1500 and 1750.

Since the time of the introduction of Carli's simple index number, the use of index numbers has increased until now we have indexes (some of them based upon hundreds of items) of wholesale and retail prices, general business conditions, financial situations, prices of stocks and bonds, commodity market growth, and other changes in business activities. The use and value of both general and special index numbers have increased very rapidly with the growing interest in statistics.

Illustration of the Purpose of Index Numbers.—In Exhibit 91, the prices of 24 commodities from 1910 to 1937 are charted. In each curve the relative changes in a particular commodity price can be compared. Also, since all of the curves are presented upon the same logarithmic scale, it is possible to compare the changes in each commodity price with the changes in all of the other commodity prices during the years shown. But will such a detailed comparison of 24 curves indicate the change in the group as a whole? Can an accurate interpretation of the movement of the entire group of commodity prices be obtained from a study of these individual curves or from a study of each of the series of data which

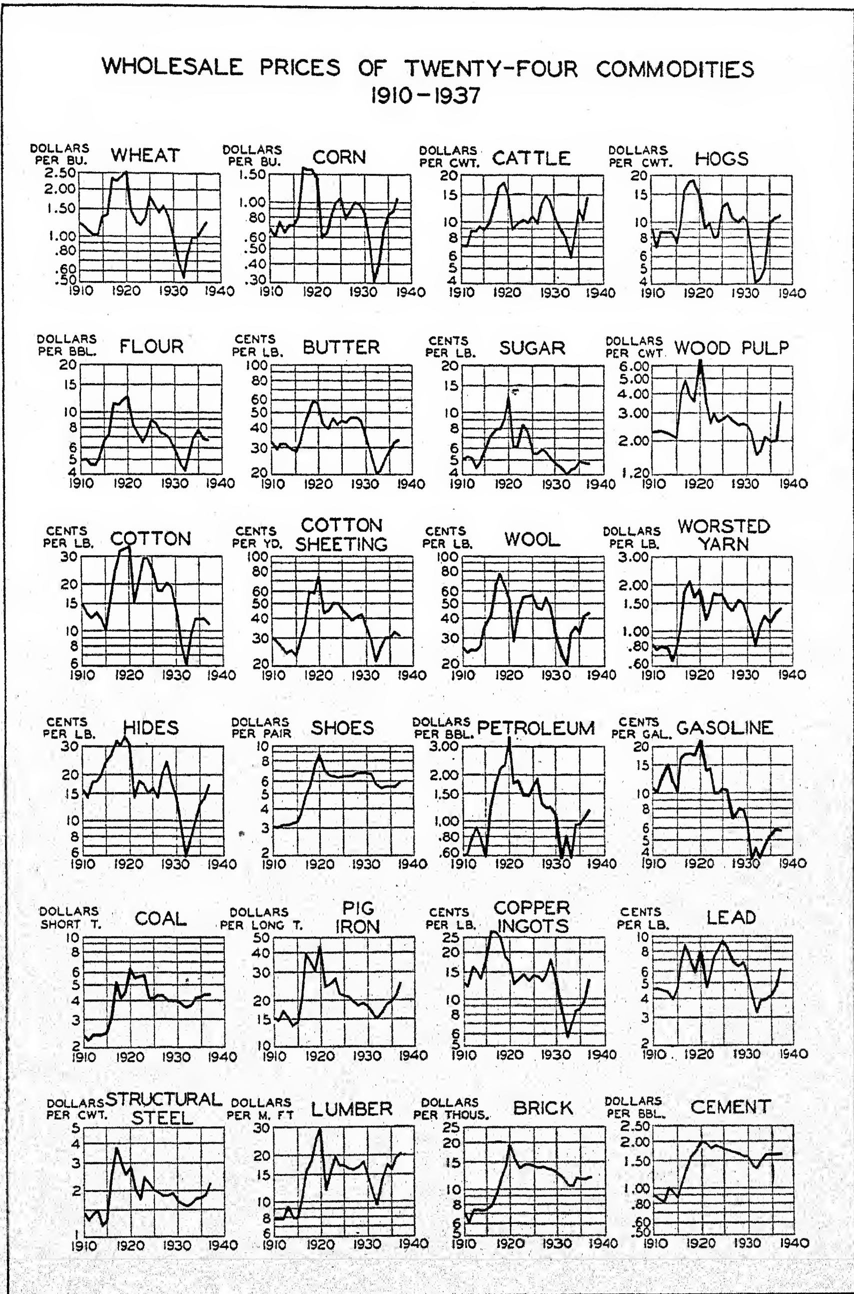

EXHIBIT 91.—In order to show what changes have taken place in the group as a whole, the above curves may be combined as an "index number."

they represent? It will be noted that the units differ greatly and that the changes vary greatly in direction and in relative magnitude. Can a picture of the composite change in the group as a whole be obtained?

Clearly, a device which would summarize the movement of the whole group and which, at the same time, would be sensitive to the movement and to the influence of each individual element of the group in its proper relation to the whole, would be of great practical usefulness. To do this is the purpose of an index number. An index number is not only capable of summarizing masses of data, but, as will be noted later, it is capable of being so constructed that it will be more sensitive to a certain element or group of elements than to others, which procedure may be desirable for special purposes.

Instead of representing the difficulty of obtaining an accurate conception of the movement in the prices of 24 commodities as a whole, Exhibit 91 might quite as readily have represented the movement of the production of 24 flour mills, the revenues of 24 different railroads, the deposits of 24 banks, the meals served by 24 restaurants, the automobiles produced by 24 different manufacturers, the number of persons employed in 24 factories, the sales of 24 retail stores, or other similar data. It suggests the need for some device by which the movements of complex groups can be portrayed accurately, and it illustrates the difficulty of obtaining an accurate picture of complex groups without the use of some summarizing device such as an index number. Without this device, no definite knowledge of the changes in a complex group of variables can be obtained. Are prices higher today than last year; are wages higher; are building costs higher; is industrial activity greater? Knowledge of every individual price, wage, building cost, or production change will not answer these questions. To obtain a definite picture, the individual price changes, wage changes, etc., must be summarized in a composite expression. This is both the nature and the value of an index number, as applied to problems of measuring business activity. It is *an "indicator" of the composite variation in a large number of elements.* In many instances, the variations in a number of individual series are not indicative, and it is only when they are properly merged that they are definitely significant.

Present-day Use of Index Numbers in Business.—As a statistical device, the chief purpose of present-day index numbers in the business world is to measure business conditions, either generally as to a wide range of business activities, or specifically as to one or a few phases of business activity. Carli, in the eighteenth century, had a particular purpose in inventing an index number to measure the change in the general price level, although he used but three commodity prices to represent the entire price field. The price level is one of the most important subjects

of index-number studies, and we have today many published index numbers which indicate the changes in price conditions in the wholesale field and in the retail field.[1] Other index numbers are published regularly to indicate changes in the volume of trade, in stock and in bond prices, in production of commodities, in the cost of living, and in many other business data. Some of these indexes have a general purpose, such as the measurement of the cost of living throughout the United States or the volume of trade in the country as a whole. Other indexes are prepared, although not always published, for special purposes, such as the cost of living among the factory workers of a certain factory, or the volume of trade in one class of commodities within a small area.

Importance of Proper Construction.—If properly constructed, an index number is of great value; if not properly constructed, it is a dangerous device. It will be readily recognized that the problem of measuring changes in the general level of prices, for example, is not an easy one. Thousands of commodities are dealt in daily. For some commodities, representative prices are easily obtained; for others they are not. While some prices are advancing from day to day, others are declining. Also, in the general price level, some commodities are of far greater importance than others. These and many other obstacles must be overcome in obtaining one figure—an index number—to represent the change in the entire price level from one period to another.

Steps in the Selection and Construction of an Index Number.—Although the conditions under which index numbers are constructed are not uniform, in general the steps are as follows:

1. *Definition of the purpose of the index number.* Until the purpose for which it is to be used is clearly and exactly stated, decisions can not be made in the steps that follow.
2. *Determination of the number and the kinds of units in which the data are to be collected.* Specifically, if we are seeking an index of wholesale prices, how many commodity prices are to be included, and which of the thousands of items that might be chosen shall be used to represent all wholesale prices? Also, will price quotations be for dozens, tons, pounds, bushels, or some other unit of measure?
3. *Determination of the relative importance of the items selected and the assignment of weights to these items.* Obviously sugar is more important than nutmegs, but to what degree? This problem of weighting will be considered at length later in this chapter.
4. *Decision as to the sources from which the data are to be collected and the methods of collection.* Cotton prices and wheat prices are quoted on several exchanges and in local markets. Other items, such as machinery, furniture, or clothing, are not regularly sold in exchanges or auctions, and a definite price may be more difficult to obtain. Care must be taken that the quotations are representative. It frequently happens that the prices quoted do not represent important volumes of transactions, because large purchases may be made on the basis of long-time contracts, and the market prices

[1] See the discussion of published index numbers at the end of this chapter.

may apply only to a very small volume of goods. In all cases, however, definite decisions must be made as to what sources of quotations shall be used.

5. *Collection of the data.* In collecting data, it is essential that the decisions made in the four preceding steps be constantly kept in mind. For example, in collecting prices for a given month, the question will arise as to whether the opening price on the first day of the month, the average price of the day, the closing price of the day, one of these prices on the fifteenth day, an average of daily opening prices for the month, etc., is to be obtained. This is a problem of the kind of unit, and it should be determined in connection with Step 2.

6. *Choice of the general method of calculating the index number itself from the data collected.* In the pages that follow, a number of the methods that are commonly used in calculating business index numbers will be described and illustrated.

7. *Selection of the basis of reference, or "base period," if one is used.* As described previously, it is usual to base index numbers upon the value for a given period, which is taken as 100. Problems in selecting this base period will be considered later.

8. *Selection of the form for final expression and completion of the arithmetical computations.* Usually, the final form for expression of the index number is a percentage relative—that is, on the basis of 100. Sometimes, however, a sum total of the items at a given date is used as the final form. Various procedures will be illustrated in certain methods of computing index numbers, which are described below.

Methods of Calculating Index Numbers.—There are many ways of calculating index numbers, but only a few of the more important will be described here. They are as follows:

A. Aggregates.
  1. Unweighted totals of actual prices (or other data).
  2. Weighted totals of actual prices (or other data).

B. Averages of Percentage Relatives.
  1. Unweighted averages.
    a. Arithmetic mean.
    b. Median.
    c. Geometric mean.
  2. Weighted averages.
    a. Arithmetic mean.
    b. Geometric mean.

C. Fisher's "Ideal" Index Number.

Each of the methods will be described by giving the steps in the calculation, and by showing an illustration of the calculation. In order to facilitate comparison of the different methods and their results, the same original data (the prices of 11 basic commodities for the year 1926 and the month of January 1938) have been used. In the illustrations the methods are applied to prices, but they may also be applied to general business conditions, industrial production, trade, factory employment, retail sales, and many other types of data.[1]

[1] For additional information on index numbers, see F. C. Mills, "Statistical Methods," 2nd. ed., Chaps. VI and IX, Henry Holt and Company, New York, 1938; and *Bulletin* 284, United States Bureau of Labor Statistics (see footnote p. 203).

Unweighted Aggregative Index Number.—This index number is sometimes called a "simple" aggregative index number. The steps in its calculation are:

1. Express the items in terms of some common basis of measurement (such as dollars in the case of prices).
2. Find the total of the values of the items for both the base period and the period for which the index is being computed.
3. Divide the total for the period for which the index is to be found by the total for the base period and point off as a percentage or multiply by 100 (to express the result as a percentage relative). The result is the unweighted aggregative index number.

Exhibit 92 shows the computation of an index number of the prices of 11 basic commodities in January 1938 as compared with the year 1926, by the method of unweighted aggregates.

Commodities*	Units of quotation	1926 prices	January 1938 prices
Wheat	Bushel	$ 1.542	$ 0.978
Hogs	100 pounds	13.115	8.605
Cotton	Pound	.170	.084
Wool	Pound	.457	.318
Sugar	Pound	.043	.032
Hides	Pound	.140	.141
Coal	Net ton	4.314	4.441
Petroleum	Barrel	1.884	1.160
Pig iron	Gross ton	20.616	25.890
Lumber	1,000 feet	45.109	43.740
Rubber	Pound	.487	.148
Total		$87.877	$85.537

Divide total of January 1938 prices by total of 1926 prices and point off as a percentage or multiply by 100. Thus: ($85.537 ÷ $87.877)100 = 97.34.

* The following bulletins of the United States Bureau of Labor Statistics were the sources of prices and weights used in this chapter in the illustrations of the computation of index numbers: *Bulletin* 453, "Revised Index Numbers of Wholesale Prices, 1923 to July 1927" (for average prices for 1926 and weights); January 1938 *Bulletin*, "Average Wholesale Prices and Index Numbers of Individual Commodities" (for average prices for January 1938).

The price quotations are for the following classes of commodities: wheat, cash—number 2, red winter, Chicago; hogs, Chicago—good to choice, light butchers; cotton, middling—Galveston; wool, Boston—domestic—Ohio, grease basis—medium grades; sugar, New York—raw, 96 degrees centrifugal; hides, Chicago—packers'—heavy native steers; bituminous coal (composite price)—mine run; petroleum, crude, wells—Kansas-Oklahoma; pig iron—foundry, number 2, northern, Pittsburgh; lumber—pine, yellow, flooring, mill; and rubber, crude, New York—plantation, ribbed, smoked sheets.

The weight assigned to each commodity quotation in the illustrations presented later in this chapter is the total weight assigned to all quotations of the commodity by the Bureau of Labor Statistics. These were determined by that bureau from data for the years 1923 to 1925.

EXHIBIT 92.—Calculation of unweighted aggregative index number.

From this calculation the unweighted aggregative index number for January 1938 is found to be 97.34, which indicates a net decrease of less than 3 per cent for the period from 1926 to January 1938.

The chief advantage of this index number is that it is simple to compute. It is merely the percentage that the total of prices for a given period is of the total of prices for a base period.

A very important disadvantage of this index number is that it is easily influenced in such a manner as to give biased results when some of the elements used in its construction are quoted in units which differ from the others by large amounts. Thus, if one changes the method of quoting pig iron in the above illustration to a price per pound, the index number is 88.69 instead of 97.34; if the cotton price is changed to a price per 500-pound bale, the index is changed from 97.34 to 73.80; if the quotation for hides is changed from a price per pound to a price per 100 pounds, the index number becomes 97.80 instead of 97.34; and if the quotation for rubber is changed to a unit of 100 pounds, the index number is 73.62. Again, so far as an effect upon the final index found is concerned, a change of 25 per cent in the price of lumber has more than 1,000 times as much influence upon the index as a change of 25 per cent in the price of sugar, because the magnitude of the former is more than 1,000 times the latter. That is to say, the influence of each item upon the resulting index number depends upon its size in relation to the sum of all the items. In Exhibit 92, the sum of all the items consists chiefly of the prices of lumber, pig iron, and hogs; and the items of sugar, cotton, and hides have very little influence upon the total. Clearly, unless it is desired to give more weight to certain items than to other items, the unit of quotation should be such that the size of the price per unit is as nearly uniform as possible (at least in the base year). If the unit of quotation is not uniform. the items will be unequally weighted.

Computation of Weighted Aggregative Index Number.—The steps involved in the computation of this index number are as follows:

1. Multiply each item, both in the base period and in the period for which the index number is being computed, by the weight used for each respective item. (Thus, if a price index is being computed, the weight by which each price is multiplied is usually the quantity produced or marketed. Problems involved in the determination of the proper weights for index numbers of prices are discussed later in this chapter.)
2. Find the total of the results in Step 1 for both the base period and the period for which the index is being computed.
3. Divide the total for the period for which the index is being computed by the total for the base period and point off as a percentage or multiply by 100. The result is the index number (expressed as a percentage of the figure for the base period).

Exhibit 93 shows the computation of a weighted aggregative index number for the prices of the 11 basic commodities for January 1938 as compared with the year 1926.

As indicated by Step 3, this index number is usually expressed in the form of the percentage relation of totals, although it may be expressed merely as the totals themselves (Step 2).[1] In general, it is one of the best index numbers for practical use, since it weights each item in proportion to both its price and quantity.

Commodities	Units of quotation	1926 prices (in dollars)	January 1938 prices (in dollars)	Weights* (in thousands)	Total values at 1926 prices (in thousands of dollars)	Total values at January 1938 prices (in thousands of dollars)
Wheat.......	Bushel	1.542	.978	628,711	969,472	614,879
Hogs.........	100 pounds	13.115	8.605	123,305	1,617,145	1,061,040
Cotton.......	Pound	.170	.084	6,629,267	1,126,975	556,858
Wool........	Pound	.457	.318	426,910	195,098	135,757
Sugar........	Pound	.043	.032	10,360,669	445,509	331,541
Hides........	Pound	.140	.141	1,330,230	186,232	187,562
Coal.........	Net ton	4.314	4.441	496,244	2,140,797	2,203,820
Petroleum....	Barrel	1.884	1.160	811,265	1,528,423	941,067
Pig iron......	Gross ton	20.616	25.890	8,594	177,174	222,499
Lumber......	1,000 feet	45.109	43.740	30,822	1,390,350	1,348,154
Rubber......	Pound	.487	.148	740,725	360,733	109,627
Total......	....	....	....	....	10,137,908	7,712,804

Dividing the total value in January 1938 by the total value in 1926 (7,712,804 ÷ 10,137,908) and pointing off as a percentage or multiplying by 100, one finds the index number for January 1938 to be 76.08.

* See footnote in Exhibit 92 for source of weights used.

EXHIBIT 93.—Calculation of weighted aggregative index number.

Percentage Relatives.—The second general method of computing index numbers is that of averaging percentage relatives (see group *B* on page 188). Before describing the various forms of index numbers constructed under this general method, it is essential that we become thoroughly familiar with the term "relatives," which is more accurately described as "percentage relatives." This term is used to indicate the percentage ratios of the items to a certain base. If wheat were 80 cents a bushel in 1935 and 90 cents in 1938, the percentage relative for wheat in 1938 on a 1935 base would be said to be 112.5.[2] The percentage relative

[1] Two of the best known indexes of wholesale prices in the United States (Bradstreet's and Dun's) were not expressed as percentage relatives, but as totals in dollars and cents. (These indexes were discontinued in 1937.)

[2] Simple percentage relatives are sometimes referred to as "index numbers" in certain publications. In this book, the term "index number" is not applied to a "per-

PIG IRON PRODUCTION IN THE UNITED STATES, 1925–1937

Years	Production (in thousands of tons)	Percentage relatives 1931 = 100 (m)	Percentage relatives 1933 = 100 (n)
1925	36,408	199	276
1926	39,072	214	296
1927	36,228	198	274
1928	37,836	207	286
1929	42,288	231	320
1930	31,404	172	238
1931	18,276	100	138
1932	8,688	48	66
1933	13,212	72	100
1934	15,912	87	120
1935	21,012	115	159
1936	30,624	168	232
1937	36,612	200	277

EXHIBIT 94A.—Actual data and percentage relatives with two different bases.

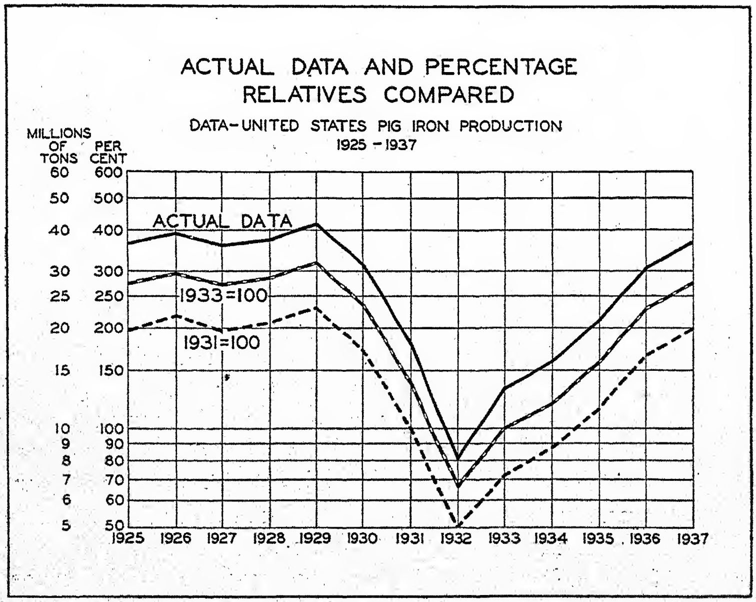

EXHIBIT 94B.—Comparison of relative fluctuations of actual data and percentage relatives with different bases.

centage relative," unless it is also an index number in the sense that it is an indicator of the composite variation in a large number of elements, as previously explained.

for any other commodity would be obtained in the same manner. Similarly, if one were considering the cost of living in the different states of the United States, one might speak of the percentage relative of the cost of living in Texas as compared with the average cost in all the states.

The table in Exhibit 94*A* shows the percentage relatives of the production of pig iron in the United States with two different years chosen as the bases of reference. In the first column of percentage relatives (marked *m*) the production of each year is stated as a percentage of the production of 1931; that is, the amount of production in each year has been divided by the amount of production in 1931 and the quotient has been pointed off as a percentage or multiplied by 100. In the second column of percentage relatives (marked *n*) the basis of reference is 1933.

Commodities	Units of quotation	1926 prices	1926 percentage relatives	January 1938 prices	January 1938 percentage relatives
Wheat..............	Bushel	$ 1.542	100.0	$ 0.978	63.4
Hogs...............	100 pounds	13.115	100.0	8.605	65.6
Cotton.............	Pound	.170	100.0	.084	49.4
Wool...............	Pound	.457	100.0	.318	69.6
Sugar..............	Pound	.043	100.0	.032	74.4
Hides..............	Pound	.140	100.0	.141	100.7
Coal...............	Net ton	4.314	100.0	4.441	102.9
Petroleum..........	Barrel	1.884	100.0	1.160	61.6
Pig iron...........	Gross ton	20.616	100.0	25.890	125.6
Lumber.............	1,000 feet	45.109	100.0	43.740	97.0
Rubber.............	Pound	.487	100.0	.148	30.4

EXHIBIT 95.—Calculation of percentage relatives of eleven basic commodity prices for January 1938, using 1926 as 100 per cent.

It is customary, when stating the basis of reference of percentage relatives, to state it: "1931 = 100" or "1933 = 100," as illustrated at the tops of the last two columns of the table in Exhibit 94*A*.

Exhibit 94*B* shows on a semi-logarithmic chart both of the series of percentage relatives, and the actual production of pig iron. The percentage relatives, it will be noted, form two curves which are identical in shape with the curve of actual production. (The reduction of widely differing series to percentage relatives, for purposes of ready comparison, has many uses other than as index numbers.)

In Exhibit 95, a second table of simple percentage relatives is given for the prices of 11 basic commodities in January 1938 as compared with the year 1926.

It will be noted that the percentage relatives for all of the commodities are 100 per cent in 1926. This is because each of the prices for 1926 was

used as the basis of reference. It follows that any kind of average of the 1926 percentages is 100 per cent. In most index number calculations, it is not necessary to insert the 100 per cent column, since it is always the same and can be assumed. In the illustrations given in the following examples, it has been omitted.

Unweighted Arithmetic Mean of Relatives Index Number.—The steps in the calculation of this index number are:

1. Express each item in the period for which the index number is being calculated as a percentage (or "percentage relative") of the same item in the base period. (That is, divide the value of each item in the period for which the index is calculated by its value in the base period and point off as a percentage or multiply by 100.)
2. Add the percentages obtained in Step 1.
3. Divide by the number of items, *i.e.*, the number of percentages computed in Step 1. The result is the index number.

Exhibit 96 shows the computation of an unweighted arithmetic mean of relatives index number of the prices of 11 basic commodities for January 1938 as compared with the year 1926.

Commodities	Units of quotation	1926 prices	January 1938 prices	January 1938 percentage relatives
Wheat	Bushel	$ 1.542	$ 0.978	63.4
Hogs	100 pounds	13.115	8.605	65.6
Cotton	Pound	.170	.084	49.4
Wool	Pound	.457	.318	69.6
Sugar	Pound	.043	.032	74.4
Hides	Pound	.140	.141	100.7
Coal	Net ton	4.314	4.441	102.9
Petroleum	Barrel	1.884	1.160	61.6
Pig iron	Gross ton	20.616	25.890	125.6
Lumber	1,000 feet	45.109	43.740	97.0
Rubber	Pound	.487	.148	30.4
Total	....	....	....	840.6

Divide the total of the January 1938 price relatives by the number of items (840.6 ÷ 11) = 76.4.

EXHIBIT 96.—Calculation of unweighted arithmetic mean of relatives index number.

From the calculation, we find the index number for January 1938 to be 76.4. This value (76.4 for January 1938) shows the average percentage change from 1926, taking 1926 (the base year) as 100 per cent. The conclusion from this calculation would be that prices, on the average, declined by approximately 24 per cent during the interval from 1926 to January 1938.

The strength of this index number lies in its relative ease of calculation and in the ease with which it may be understood.

The peculiar shortcomings of this index number are that it may be unduly influenced by items of small importance and that it responds more readily to an upward movement than to a downward movement. The first shortcoming may be illustrated by imagining that we had included in our calculation some element of little importance, such as the price of carpet tacks, which, let us suppose, had risen from 3 cents a box to 15 cents a box, an increase of 400 per cent, giving a percentage relative of 500 for January 1938. Adding 500 per cent to 840.6 per cent and dividing by 12 instead of 11 gives an index number of 111.7 which would indicate that prices rose nearly 12 per cent, whereas from the viewpoint of the average person they fell nearly 24 per cent. The second shortcoming is a bias which is always found in the arithmetic average of relatives. The bias arises from the fact that the upward change in the price of any commodity may be indefinite, whereas the downward movement is limited to 100 per cent. Thus, if the price of wheat moves upward from 99 cents to $2.99, the upward change would be greater than 100 per cent. However, it cannot move more than 100 per cent in the downward direction (see also pages 177 and 178), for, if a price decreases 100 per cent of itself, it becomes zero.

Unweighted Median of Relatives Index Number.—The steps in the calculation of this index number are:

1. Express each item in the period for which the index number is being calculated as a percentage (or "percentage relative") of the same item for the base period.
2. Form an array of the percentages obtained in Step 1.
3. Find the median percentage, which will be the index number.

Exhibit 97 shows the computation of an unweighted median of relatives index number of the prices of 11 basic commodities[1] for January 1938 as compared with the year 1926. From this index number (69.6) we would draw the conclusion that prices declined over 30 per cent for the period 1926 to January 1938.

The median is not influenced by unusual items which show a large change to any greater extent than by items of small change. It also possesses the very desirable characteristic of being easily calculated.[2]

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[1] If there is an even number of items instead of an odd number as in Exhibit 97, it is customary to find the median by taking a point midway between the two mid-values.

[2] The median possesses another characteristic which should be mentioned. It is not ordinarily susceptible to weighting, and never is unless the weights may be expressed in whole numbers. Whenever the median is weighted, however, it becomes a somewhat artificial type, unless many elements are being used in computing the index number.

However, it is often desirable that the index number be computed in such a way that the influences of extreme variations are recognized. In such cases, the method of the median of relatives is not to be recommended. Also, it is not a trustworthy method in cases where only a small

Commodities	Units of quotation	1926 prices	January 1938 prices	January 1938 percentage relatives
Wheat	Bushel	$ 1.542	$ 0.978	63.4
Hogs	100 pounds	13.115	8.605	65.6
Cotton	Pound	.170	.084	49.4
Wool	Pound	.457	.318	69.6
Sugar	Pound	.043	.032	74.4
Hides	Pound	.140	.141	100.7
Coal	Net ton	4.314	4.441	102.9
Petroleum	Barrel	1.884	1.160	61.6
Pig iron	Gross ton	20.616	25.890	125.6
Lumber	1,000 feet	45.109	43.740	97.0
Rubber	Pound	.487	.148	30.4

The percentages array as follows: 30.4, 49.4, 61.6, 63.4, 65.6, 69.6, 74.4, 97.0, 100.7, 102.9, 125.6. Five items are larger than 69.6 and five are smaller than 69.6. The median item is, therefore, 69.6, the index number sought.

EXHIBIT 97.—Calculation of unweighted median of relatives index number.

number of items are included; probably it should not be used unless there are 50 or more items.

Unweighted Geometric Mean of Relatives Index Number.—The steps in the calculation of this index number are:

1. Express each item of the period for which the index number is being calculated as a percentage (or "percentage relative") of the same item in the base period.
2. Find the logarithm of each percentage found in Step 1.
3. Add the logarithms found in Step 2.
4. Divide by the number of logarithms. That is, find the arithmetic mean of the logarithms.
5. Find the antilogarithm of the quotient in Step 4. This is the index number.

Exhibit 98 shows the computation of an unweighted geometric mean of relatives index number of the prices of 11 basic commodities for January 1938, as compared with the year 1926. From the calculation, the geometric mean is found to be 71.6, which is the index number sought, and from which one would conclude that the prices of the 11 basic commodities declined over 28 per cent from 1926 to January 1938.

The geometric mean, while somewhat more complicated than the indexes computed thus far, has the important advantage of possessing no bias and being affected as easily by a rise as by a fall in prices. How-

ever, it is influenced to a relatively greater extent by small changes than by large changes, the effect of changes in the various items being in proportion to the logarithms of the percentage changes, and not in proportion to the absolute percentage changes, as is the case in the arithmetic mean of relatives. The geometric mean has a decided disadvantage in that it is somewhat difficult to explain to persons not versed in mathematics.

Commodities	Units of quotation	1926 prices	January 1938 prices	January 1938 percentage relatives	Logarithms of relatives
Wheat.........	Bushel	$ 1.542	$ 0.978	63.4	1.80209
Hogs..........	100 pounds	13.115	8.605	65.6	1.81690
Cotton........	Pound	.170	.084	49.4	1.69373
Wool..........	Pound	.457	.318	69.6	1.84261
Sugar.........	Pound	.043	.032	74.4	1.87157
Hides.........	Pound	.140	.141	100.7	2.00303
Coal..........	Net ton	4.314	4.441	102.9	2.01242
Petroleum.....	Barrel	1.884	1.160	61.6	1.78958
Pig iron.......	Gross ton	20.616	25.890	125.6	2.09899
Lumber........	1,000 feet	45.109	43.740	97.0	1.98677
Rubber........	Pound	.487	.148	30.4	1.48287
Total........	....	....	....	.....	20.40056

Divide total of logarithms by number of items (20.40056 ÷ 11) = 1.85460.

Find antilogarithm of quotient (1.85460) which is 71.6, the index number sought.

EXHIBIT 98.—Calculation of unweighted geometric mean of relatives index number.

Computation of Weighted Arithmetic Mean of Relatives Index Number.—The steps in the computation of the weighted arithmetic mean of relatives index number are as follows:

1. Express each item of the period for which the index number is being calculated as a percentage (or "percentage relative") of the same item in the base period.
2. Multiply the percentages, as found in Step 1 for each item, by the weight which has been assigned to that item. (The type of weight to be used is discussed in subsequent paragraphs.)
3. Add the results obtained from the several multiplications carried out in Step 2.
4. Divide the sum obtained in Step 3 by the sum of the weights used. The result is the index number.

Exhibit 99 shows the computation of a weighted arithmetic mean of relatives index number of the prices of 11 basic commodities for January 1938, as compared with the year 1926, which is 76.0. From this calculation, we would draw the conclusion that prices declined 24 per cent during the interval from 1926 to January 1938.

The weighted arithmetic mean of relatives corrects some of the difficulties mentioned in the unweighted arithmetic mean of relatives index. The degree to which it is representative of the various elements depends upon the weights assigned to the elements.

Commodities	Units of quotation	1926 prices	January 1938 prices	Weights assigned	January 1938 percentage relatives	Weights times relatives
Wheat..........	Bushel	$ 1.542	$ 0.978	9.6	63.4	608.64
Hogs............	100 pounds	13.115	8.605	16.0	65.6	1,049.60
Cotton..........	Pound	.170	.084	11.1	49.4	548.34
Wool...........	Pound	.457	.318	1.9	69.6	132.24
Sugar...........	Pound	.043	.032	4.4	74.4	327.36
Hides...........	Pound	.140	.141	1.8	100.7	181.26
Coal............	Net ton	4.314	4.441	21.1	102.9	2,171.19
Petroleum.......	Barrel	1.884	1.160	15.1	61.6	930.16
Pig iron.........	Gross ton	20.616	25.890	1.7	125.6	213.52
Lumber.........	1,000 feet	45.109	43.740	13.7	97.0	1,328.90
Rubber.........	Pound	.487	.148	3.6	30.4	109.44
Total.........	....	....	....	100.0	.....	7,600.65

Dividing 7,600.65 by 100.0 we obtain 76.0* which is the index number.

* The weighted arithmetic mean of relatives and the weighted aggregative indexes are identical if the weights assigned to the former are proportional to the total values of the items in the base year.

Exhibit 99.—Calculation of weighted arithmetic mean of relatives index number.

Computation of Weighted Geometric Mean of Relatives Index Number.—The weighted geometric mean of relatives index number is computed in the same manner as the unweighted geometric mean of relatives index number, which is described on pages 196 and 197, except that weights are introduced by applying them to the logarithms of the relatives. The steps are:

1. Express each item of the period for which the index number is being calculated as a percentage relative of the same item in the base period.
2. Find the logarithm of each percentage relative found in Step 1.
3. Multiply the logarithms by the weights assigned.
4. Add the results obtained in Step 3.
5. Divide the total obtained in Step 4 by the sum of the weights.
6. Find the antilogarithm of the quotient obtained in Step 5. This is the weighted geometric average (or mean) of relatives index number.

In Exhibit 100, the computation of the weighted geometric mean of relatives index number is shown for the prices of 11 basic commodities in January 1938 as compared with the year 1926. Computed by this method, the index number is 72.7. From a mathematical standpoint, the

weighted geometric mean of relatives is an excellent method for computing index numbers. Because of the labor of computation it is not often used where many items are included in the index.

Commodities	Units of quotation	1926 prices	January 1938 prices	January 1938 percentage relatives	Logarithms of relatives	Weights assigned	Weights times logarithms of relatives
Wheat....	Bushel	$ 1.542	$ 0.978	63.4	1.80209	9.6	17.300064
Hogs......	100 pounds	13.115	8.605	65.6	1.81690	16.0	29.070400
Cotton....	Pound	.170	.084	49.4	1.69373	11.1	18.800403
Wool......	Pound	.457	.318	69.6	1.84261	1.9	3.500959
Sugar......	Pound	.043	.032	74.4	1.87157	4.4	8.234908
Hides......	Pound	.140	.141	100.7	2.00303	1.8	3.605454
Coal.......	Net ton	4.314	4.441	102.9	2.01242	21.1	42.462062
Petroleum.	Barrel	1.884	1.160	61.6	1.78958	15.1	27.022658
Pig iron...	Gross ton	20.616	25.890	125.6	2.09899	1.7	3.568283
Lumber....	1,000 feet	45.109	43.740	97.0	1.98677	13.7	27.218749
Rubber....	Pound	.487	.148	30.4	1.48287	3.6	5.338332
Total....	....	......	....	....	....	100.0	186.122272

Dividing 186.122272 by 100.0 we obtain 1.86122272.

Find antilogarithm of quotient (1.86122272), which is 72.7, the index number sought.

EXHIBIT 100.—Calculation of weighted geometric mean of relatives index number.

Fisher's "Ideal" Index Number.—This index number is really the geometric mean of two index numbers. The steps in its calculation are as follows:

1. Compute a weighted aggregative index number for the period for which the "ideal" index is to be calculated, using as weights the quantities of the base year.
2. Compute a weighted aggregative index number for the period for which the "ideal" index is to be calculated, using as weights the quantities of the period for which the index is to be calculated.
3. Find the geometric mean of the two index numbers found in Steps 1 and 2. This is the "ideal" index number.

The "ideal" index number is a compromise of two systems of weights. It possesses the very desirable characteristic, when used in computing a price index, of making it possible to take into consideration changes in the demand for, or consumption of, commodities. However, it is often impractical for ordinary business problems, not only because of the time and effort required in its computation, but also because it is necessary to obtain new weights constantly. In many cases it is particularly difficult to obtain the data for new weights based upon recent periods.

Nature and Purpose of Weighting Index Numbers.—Whether an index number is obtained by one of the methods of averaging relatives, or whether it is the aggregate of actual items, it is usually advisable to apply weights to the individual elements. In almost any collection of numerical data covering a group of complex phenomena, it will be found that the items are of varying importance. If weights are not scientifically assigned to the items, the importance given to each item in the determination of the index number will either be a matter of chance occurrence or will be determined by irrelevant factors. It is to be observed that it is not possible to construct any index number without weighting the component items in some manner; even where weights are not recognized, they exist. Therefore, an intelligent analysis of the weighting problem is desirable in constructing any index number, even though it may be found in some instances that the inherent weights obtained in the so-called "unweighted" index are satisfactory for the problem at hand.

In considering the problem of weighting index numbers, one must not be confused by the mass of detail and lose sight of the main objective which is to apply some device or method whereby each commodity shall exercise an influence upon the final results proportionate to its relative importance. Obviously, the best data with which to determine the relative importance of each commodity in a general wholesale price index, for instance, would be the quantities sold at wholesale but, as these data are not available for many commodities, production figures are commonly used as the best available substitute. If the problem is one of measuring the cost of living and it is necessary to weight retail prices, the corresponding sales transactions are ordinarily too far removed from production, both in time and place, and, consequently, production figures will not furnish satisfactory weights. The proportionate expenditures upon the different articles as indicated by representative family budget data, however, will make appropriate weights for application to retail prices of consumers' goods.

Methods of Weighting Index Numbers.—There are three types of weights which may be used in index-number computations, *but each type is applicable only under certain general methods of computation.* These types or systems of weights are (1) physical quantities, (2) monetary values, and (3) arbitrary magnitudes. The first type, physical quantities, should be used only when the method of aggregates is the method of calculating the index number. The second, weights based on monetary values, should be used only in weighting relatives. The third, arbitrary magnitudes, is an unscientific method and ordinarily should be used only in the crudest forms of index numbers. However, in using an arbitrary weight, it is essential to recognize that the arbitrary weights which might

be applied in calculating an index number by averaging relatives cannot be correctly applied in calculating an aggregative index number.

Physical quantities are properly used as weights when applied to actual prices or other values. Thus, it is proper to weight each price by the quantity produced, or marketed, or consumed, etc., in obtaining an index by the method of aggregates. This requires no adjustment or change in the weights; the quantity produced, or marketed, etc., is applied directly to the price of the article. An illustration of the application of these weights is given in Exhibit 93.

To obtain correct monetary value weights for index numbers which use weighted relatives, one must obtain the relative monetary value of each item, commodity, or article used in the index-number computation. In other words, the weight to be applied to each computed relative is determined by ascertaining the monetary importance of each type of commodity or article. This process is described in detail in the following paragraph, and applications of the weights obtained are given in Exhibits 99 and 100.

Computation of Monetary Value Weights.—The steps in the computation of weights according to this system are as follows:

1. Multiply the price of each item by the amount of the item in the period selected as the base for the weights.
2. Sum the results of Step 1.
3. Express each item (money value) as a percentage of the total money value.[1]

Exhibit 101 illustrates the method used in calculating the weights used in the computation of the weighted arithmetic and the weighted geometric mean of relatives index numbers (see Exhibits 99 and 100).

The percentage, which the money value of each element is of the total money value, is shown in the last column, and is the weight determined for each of the elements in the group shown. If one of the elements should be omitted or dropped from the calculation, or if another element were added, all the weights should be recalculated. Essentially, both quantity and value weights represent proportions of a total—the former standing for the proportion of the total number of units, the latter indicating the proportion of the total worth or value which each of the elements composes of the total money value.

The Base Period for Weights.—Just as index-number calculations are related to a base period, so also are weights usually selected as of some fixed period. In many instances weights are selected which represent conditions in the base period of the index number, as it is often found that when conditions appear best for the index base period, they also are

[1] The easiest way to do this is to multiply each item by the reciprocal of the total and point off two places (or multiply by 100).

best for the weights. If, however, either the physical quantities or monetary values in some other period appear to present more logical material for weighting, these data should be used. Thus, an index number based on the year 1925, for example, may have physical quantity or monetary value weights taken from data of the year 1923.

Commodities	Units of quotation	1926 prices (in dollars)	Weights* (in thousands)	Total values at 1926 prices (in thousands of dollars)	Percentages of total value
Wheat............	Bushel	1.542	628,711	969,472	9.6
Hogs.............	100 pounds	13.115	123,305	1,617,145	16.0
Cotton...........	Pound	.170	6,629,267	1,126,975	11.1
Wool.............	Pound	.457	426,910	195,098	1.9
Sugar............	Pound	.043	10,360,669	445,509	4.4
Hides............	Pound	.140	1,330,230	186,232	1.8
Coal.............	Net ton	4.314	496,244	2,140,797	21.1
Petroleum........	Barrel	1.884	811,265	1,528,423	15.1
Pig iron.........	Gross ton	20.616	8,594	177,174	1.7
Lumber...........	1,000 feet	45.109	30,822	1,390,350	13.7
Rubber...........	Pound	.487	740,725	360,733	3.6
Total..........	....	....	....	10,137,908	100.0

* See footnote in Exhibit 92 for source of weights used.

Exhibit 101.—Calculation of weights based on monetary values.

Since the relative importance of one commodity in a group varies over a period in relation to the group as a whole, it follows that the weights should be revised from time to time. In making these revisions, the purpose of the index must be very definitely kept in mind. If it is to show price fluctuations only, care must be taken to see that an index change is not due to a change in weighting. For instance, if we were computing an annual index of wholesale prices, weighted on the basis of production data, and if we were setting up a new set of weights every year, we would not have an index of prices alone, but rather an index of total values of production. Consequently, if the aim is to show price changes alone as far as possible, revisions in weights must be made by periods much longer than those for which the prices are reported (such as once in a decade for the above index), and the weights may be adjusted between the dates for which they are taken so that the intervening change will be a gradual one corresponding to a straight line or a smoothed curve.

In considering the problem of prices and price indexes, however, it should be appreciated that "since the importance of price fluctuations

depends largely upon the accompanying changes in the quantity of goods bought, there is use for index numbers that do not attempt to measure the price factor in isolation. By changing weights each year it is possible to make these constantly occurring changes in quantities bought influence the price index, and therefore to secure results better fitted for certain uses than the results of an unambiguous measure of fluctuations in prices."[1] Again it should be emphasized that a precise understanding of the purpose of an index is imperative before work is begun on its construction.

If a new base is selected for the index number, it is often advisable to change the weights to the new base at the same time; similarly, if a new set of weights is introduced, it is often advisable to change the base period. When this practice can be followed, it is a valuable means of calling attention to the fact that the new index is not strictly comparable to the old index number.

The General Problem of Weighting—Common Errors and Difficulties. The problems of weighting are often confusing, even to those who are trained in the use of figures and who might be expected to have no difficulty in correctly solving them. The student should distinguish accurately (1) between a "complete" arithmetic average and a weighted arithmetic average, and (2) between quantity weights and value weights. The importance of these distinctions may be emphasized by pointing out the fact that business men, even those who are adept at figures, often reach wrong conclusions because they do not understand clearly these factors in weighting. For instance, the N.R.A. Lumber Code provided that the minimum price of each product be based upon the weighted average of the cost per unit. In this connection, there was much confusion in the use of the term "weighted average." The drafters of the code probably intended to provide that a "complete" average be used. But in the administration of the code, an average, weighted on the basis of value, was used, which resulted in inaccurate figures. In this case complete data were available, and if an average weighted on the basis of quantity had been used, the result would have been the same as the "complete" average, and would not have been inaccurate, but the use of value weights exaggerated the effect of the price influence because of the fact that this influence was already included in the value figures.

The above problem of weighting may be explained by referring to the table in Exhibit 102, which illustrates a price-index problem similar to the preceding code cost problem. In this problem the question is: What is the weighted average price of Product Z? Column A in the table shows

[1] *Bulletin* 284, "Index Numbers of Wholesale Prices in the United States and Foreign Countries," p. 66, United States Bureau of Labor Statistics, Washington, D. C., 1921.

Company	*A* Quantity of production (in cu. ft.)	*B* Value of production (in dollars)	*C* Price per cu. ft. $(B \div A)$ (in dollars)	*D* Quantity per dollar $(A \div B)$ (in cu. ft.)	*E* Quantity weights (percentage each item in *A* is of total of *A*)	*F* Value weights (percentage each item in *B* is of total of *B*)	*G* Price per cu. ft. weighted on basis of quantities $(C \times E)$ (in dollars)	*H* Price per cu. ft. weighted on basis of values $(C \times F)$ (in dollars)	*I* Quantity produced per dollar weighted on basis of quantities $(D \times E)$ (in cu. ft.)	*J* Quantity produced per dollar weighted on basis of values $(D \times F)$ (in cu. ft.)
1	800	400	.50	2.0	55.9	24.3	.2795	.1215	1.118	.486
2	10	5	.50	2.0	.7	.3	.0035	.0015	.014	.006
3	600	1,200	2.00	.5	42.0	73.0	.8400	1.4600	.210	.365
4	20	40	2.00	.5	1.4	2.4	.0280	.0480	.007	.012
	1,430	1,645			100.0	100.0	1.1510	1.6310	1.349	.869

Totals $B \div A$ = \$1.15 per cu. ft.
Totals $A \div B$ = .869 cu. ft. per dollar.

EXHIBIT 102.—Calculation of the average price of Product *Z*.

the quantities produced by the different concerns in a particular industry, and column B shows the value of these products at the prices given in column C. If we divide the total of column B by the total of column A, the resulting figure ($1.15) is the true or "complete" average based on the total value of the "complete number" of units, and no weighting is necessary. If, however, our problem is to obtain a representative average of the unit price items in column C, and if the data in column A are not available, we would then have to weight the items in column C with the best available substitutes for the items in column A, which might be the quantities produced during some previous period.

For purposes of illustration, two sets of weights are shown in the table in Exhibit 102 (see columns E and F). In order that the "complete" average, quantity-weighted average, and value-weighted average may be easy to compare, they are all based on the same period, namely, that to which the production and prices apply. Column E is a percentage distribution of the quantities in column A, and column F is a percentage distribution of the value figures in column B. The price items of column C, weighted by the quantity weights of column E, are presented in column G. The total of this column (G) gives an average ($1.15) which is, of course, the same as the average obtained by dividing the total of the value of production (column B) by the total of the quantity produced (column A), and it is actually the true or "complete" average rather than a weighted average, strictly speaking. However, the fact that these figures are the same shows that quantities rather than values should be used in computing a similar weighted average. Column H shows the effect of weighting the items in column C by the value weights of column F and it is seen that the average obtained ($1.63) is much higher than the correct figure ($1.15).

The figure obtained by using value weights ($1.63) is incorrect because of the double influence of prices. The price influence appears both in the prices themselves and in the value of production. Suppose that we carry our illustration still further and assume that, instead of our question being: What is the average price per cubic foot? it is: What is the average number of cubic feet produced per dollar? Our problem is now one of cubic feet per dollar instead of dollars per cubic foot. In such an instance, if the quantity-per-dollar data in column D were weighted on the basis of the percentage distribution of the quantity units in column E, as shown in column I, the influence of the number of units would appear twice and the total (1.349) would not be the correct answer. However, if we weight the quantity items in column D by the value weights in column F, we will arrive at the correct answer (.869) which is the same as the figure obtained by dividing the total of column A by the total of column B.

In connection with the data of Exhibit 102, it should be emphasized that the complete data were used to illustrate methods of weighting for the

specific purpose of facilitating ease of comparison, and it should be clearly understood that, under the usual circumstances, weights are used only when complete data are not available. When complete data are used the "complete" average should be computed as illustrated in Exhibit 102 by dividing the total of column *B* by the total of column *A* to obtain the average price per cubic foot (or by dividing the total of column *A* by the total of column *B* to obtain average cubic feet produced per dollar).

In using the table in Exhibit 102 for purposes of illustration, the quantity weights were shown in column *E* in terms of percentages to aid in making comparisons. This method of stating the weights facilitates computation for the reason that the process of "dividing the total by the sum of the weights" involves simply dividing by 1 (100 per cent mathematically stated). It should be noted, however, that this procedure can be followed only when the quantities are stated in terms of a common unit (cubic feet in this instance). When the prices to be averaged are stated in terms of unlike units, they should be weighted directly by quantity data expressed in corresponding units as illustrated by the weighted aggregative index number calculation in Exhibit 93. If, however, relative prices are being used, as in the weighted arithmetic mean of relatives index number in Exhibit 99, "the weights should be reduced to a common denominator. As multipliers, of course, weights may be regarded as merely abstract numbers; but in studying the weights themselves, it is necessary to have some common standard by which the relative importance assigned to various commodities can be accurately compared. The only common denominator for all commodities that is significant for economic ends and capable of quantitative expression is money value. But it is ill advised to weight by money values when actual prices are being used, for the common denominator is already present in the quotations themselves. These price quotations are best multiplied by the physical quantities of the goods produced, exchanged, or consumed, as the case may be."[1]

Misapplication of Index Numbers.—No index number may be used for every purpose. In fact, every index number has one best use and this use is the particular use for which it was originally and, it is assumed, properly designed. Ordinarily, any use of the index number other than that for which it was especially designed is almost certain to introduce error into the interpretations based upon it.

The art of applying an index number, even for the specific purpose for which it was calculated, is acquired only after careful study and

[1] *Bulletin* 284, "Index Numbers of Wholesale Prices in the United States and Foreign Countries," p. 65, United States Bureau of Labor Statistics, Washington, D. C., 1921.

considerable experience. The beginner would do well always to check his new calculations or his applications of old index numbers with someone well trained in statistical interpretation. An index of production never would be confused with and used as a substitute for an index of demand; yet how frequently one finds a wholesale price index used where only a retail price index should be used.

In almost every practical problem, the decision must be made between the construction of an index number for the purpose at hand and the use of one already constructed, which might prove more or less unreliable.

Published Index Numbers.—A few of the more important published index numbers which are of value to the business man will be introduced at this point.

One of the best known general-purpose index numbers of wholesale prices in the United States is that of the United States Bureau of Labor Statistics. Beginning with January 1938, the index has included 813 commodities or price quotations weighted according to the importance of each commodity.[1] It is expressed as a percentage of 1926. The index numbers are published monthly in a pamphlet on wholesale prices and in the *Monthly Labor Review*. They are also published in a press release. Figures are available by years since 1801 and by months since 1890. The monthly movements of this index over the period from 1926 to 1937, inclusive, are shown in Exhibit 103.

Fisher's weekly index number of wholesale prices is published as relative prices. The relatives are derived from weighted aggregates of actual prices.

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[1] The weighting factors used in computing the present series of index numbers back to 1913 are as follows: For the year 1913 the mean of 1909 and 1914 data, taken from Reports of the Census Bureau and the Department of Agriculture; for 1914 to 1919, inclusive, the mean of 1914 and 1919 data; for 1919 to 1921, inclusive, the mean of 1919 and 1921 data; for 1921 to 1923, inclusive, the mean of 1921 and 1923 data; for 1923 to 1927, inclusive, the mean of 1923 and 1925 data; for the period since January 1930, the mean of 1925 and 1927 data. Since 1934 no changes in weights have been made, because the weights which would normally have been introduced in 1936 were those relating to the depression years and they appeared to be unrepresentative. At the present time the quantities used as "quantity weights" are averages for 1929 and 1931, except for farm products, for which the quantities used are averages for 1929, 1930 and 1931. In the case of farm products, data for all years of the period are averaged to form the weights. In certain cases where census or other reliable data were lacking, estimates based on the best information available have been used. An explanation of the methods used in computing this index is given on pp. 2–5 of United States Bureau of Labor Statistics *Bulletin* 493. See also Jesse M. Cutts and Samuel J. Dennis, "Revised Method of Calculation of the Wholesale Price Index of the United States Bureau of Labor Statistics," *Journal of the American Statistical Association*, December 1937.

Dun and Bradstreet, Inc., compile a weekly index of wholesale food prices, which represents the sum total of the wholesale prices per pound of 31 commodities in general use. This organization also compiles a daily weighted price index based on 30 basic commodities.

An index of retail food prices is compiled by the United States Bureau of Labor Statistics. This index covers 84 food items, and is currently computed by applying the per cent change of a weighted list of prices of

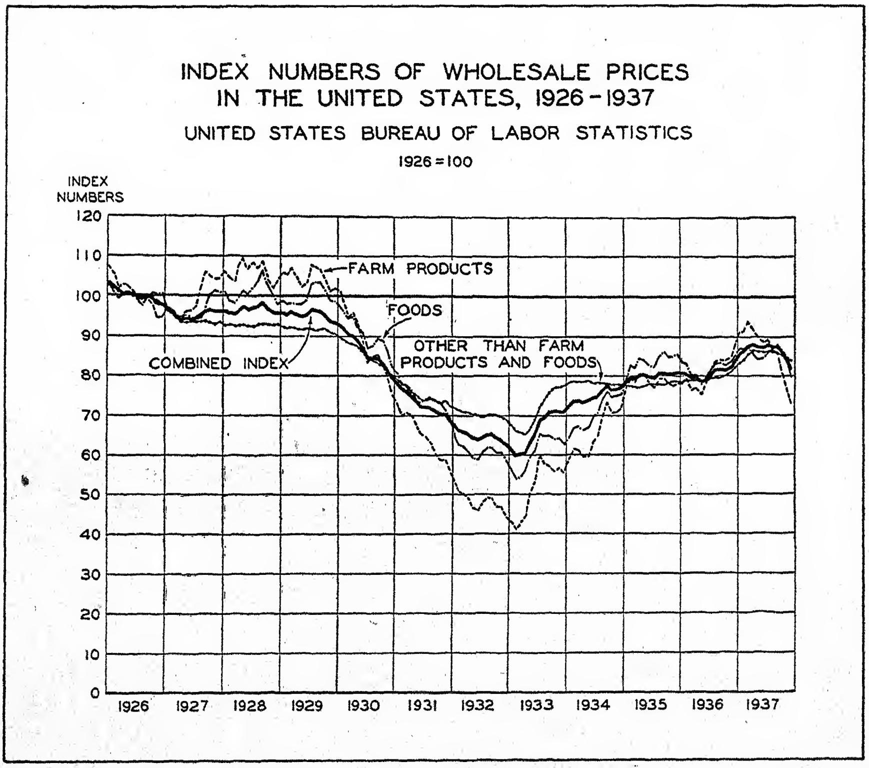

EXHIBIT 103.—United States Bureau of Labor Statistics general index of wholesale prices, with breakdown by types of commodities.

identical foods, between two successive periods, to the index for the earlier of the two periods to obtain the current index for each of the 51 cities covered.

Fairchild's textile and house furnishings index compiled by *Fairchild Publications* represents prices collected as of the first of each month on 105 items from 36 retail trade organizations, including independent and chain department stores, mail-order organizations, and specialty stores, located in 23 principal cities broadly distributed over the country. It is heavily weighted with textiles.

A world price index is compiled by the United States Bureau of Foreign and Domestic Commerce from prices of important commodities quoted in representative world markets.

The *Engineering News-Record* index of construction costs is designed to indicate the cost situation in the construction industry. It is of the aggregate type, weighted according to the relative importance of the four components: steel, cement, lumber, and labor.

The National Industrial Conference Board compiles a cost of living index designed to show the trend of living costs of wage earners' families. The index is based on a comprehensive list of retail prices and rents. It is an arithmetic average of weighted relatives using the 1923 average as a base.

The Federal Reserve Board index of industrial production is a general indicator of changes in the volume of the country's industrial output. The index is made up of two components: manufactures and minerals, covering industries which represent directly and indirectly about 80 per cent of total industrial production. In constructing the index, the monthly physical volume figures are reduced to daily averages, adjusted for seasonal variation where necessary, using adjustment factors derived by the ratio-to-moving-average method,[1] and weighted according to value added by manufacture for the index of manufactures, and by average value of product for the minerals index. The aggregate of the weighted, seasonally adjusted, daily averages for a given month is related to the identical aggregate for the base years 1923–1925.

The *Printers' Ink* magazine advertising index covers approximately 80 per cent of the total linage in all magazines. After making various adjustments to obtain homogeneity, the data are corrected for seasonal variation by using the ratio-to-moving-average method, and reduced to relatives on a 1928–1932 base. The resulting relatives are weighted according to the estimated amount spent in the base period and averaged geometrically.

The Federal Reserve Board index of department store sales is based on the monthly dollar sales of about 425 department stores located mostly in the larger cities in various parts of the country. The index is based on daily average dollar sales of an identical group of stores, related to the 1923–1925 daily average, and adjusted for seasonal variation by the ratio-to-moving-average method. Allowances are made for changes from month to month in the number of Saturdays and Sundays and for the six holidays most commonly observed. An allowance is also made for the changing date of Easter.

[1] See pp. 320–322 for a discussion of a ratio-to-moving-average method of adjusting for seasonal variation.

Indexes of employment and wages are compiled by the United States Bureau of Labor Statistics by industries and by industrial areas.

The Standard Statistics Company, Inc., compiles an index of industrial corporation profits.

Moody's Investor's Service compiles an index of dividend payments based on six hundred stocks.

Dow, Jones and Company compiles an index of bond prices from the yields of the average prices of the bonds for each day of the month, the average yields for the ten bonds in each class being capitalized at 4 per cent to give the combined index.

The Standard Statistics Company, Inc., compiles an index of bond prices and an index of bond yields. The corporate bond price index is based on the quotations of 45 high-grade corporate bonds converted to the equivalent price for a $4\frac{1}{2}$ per cent bond having 30 years to date of maturity. The index of bond yields is an arithmetic average of the yield to maturity of the same 45 bonds that are included in the index of bond prices.

Indexes of stock prices are compiled by Dow, Jones and Company for 30 industrials, 20 public utilities, and 20 railroads; by *The New York Times* for 25 industrials, 25 railroads, and total; and by the Standard Statistics Company, Inc., for 347 industrials, 40 public utilities, 32 railroads, and total.

Questions and Problems

1. What is an index number?

2. What kinds of situations make necessary the use of index numbers?

3. How are index numbers used in present-day business?

4. What is the difference between a general-purpose index number and a specific-purpose index number?

5. What are the dangers if an index number is not propertly constructed?

6. What are the usual steps in the selection and construction of an index number?

7. List the more important methods of constructing index numbers.

8. What are the steps in calculating an "unweighted aggregative index number"? What are the advantages and disadvantages of this index number?

9. List the steps in the calculation of a "weighted aggregative index number." What are the shortcomings of this index number?

10. What are percentage relatives and for what are they used? When plotted on an arithmetic scale, does a curve showing percentage relatives differ from a curve of the original data? When plotted on a semi-logarithmic chart, do the two curves differ? Why?

11. What are the steps in calculating an "unweighted arithmetic mean of relatives index number"? Discuss the advantages and disadvantages of this index number.

12. Explain the steps in calculating an "unweighted median of relatives index number." What are its strong and weak points?

13. What are the steps in the calculation of an "unweighted geometric mean of relatives index number"? Discuss its desirable and undesirable characteristics.

14. List the steps in computing a "weighted arithmetic mean of relatives index number." How does this index number compare with the unweighted arithmetic mean of relatives index?

15. Give the steps in calculating a "weighted geometric mean of relatives index number." What are its strong and weak points?

16. What are the steps in computing "Fisher's 'ideal' index number"? What are its advantages and disadvantages?

17. Explain the nature and purpose of weighting index numbers.

18. Discuss the methods of weighting index numbers. Can the same weight be applied to relatives that is used in weighting the items entering into an aggregative index? Explain.

19. Describe the computation of monetary-value weights.

20. How is the base period for weights usually selected?

21. When is it proper to use quantity weights and when value weights in constructing index numbers of prices? Can actual prices be weighted by value weights? Explain.

22. Discuss the misapplication of index numbers.

23. Name and describe briefly some of the more important published index numbers which are valuable to the business man.

24. (Percentage relatives.) Reduce the data in Problem 24, page 128, (both stores) to percentage relatives of January 1936 (January 1936 = 100.) Use a slide rule in making your computations.[1]

Present graphically on both arithmetic and logarithmic scales and analyze.

25. From the following table reduce the data on the volume of lumber production to percentage relatives:

a. Use January 1935 = 100.

b. Use June 1937 = 100.

PRODUCTION OF LUMBER IN THE UNITED STATES, 1935–1938
(In millions of board feet)

Month	1935	1936	1937	1938
January	1,189	1,614	1,571	1,249
February	1,238	1,469	1,596	1,275
March	1,355	1,718	2,177	1,698
April	1,499	2,005	2,233	1,473
May	1,495	2,052	2,398	1,550
June	1,478	2,052	2,500	1,696
July	1,798	2,218	2,352	1,606
August	2,007	2,171	2,342	
September	1,937	2,151	2,297	
October	2,156	2,352	1,969	
November	1,779	1,816	1,671	
December	1,608	1,827	1,452	

Source: Survey of Current Business.

[1] A very convenient slide-rule setting for computing percentage relatives is as follows (see Appendix II, p. 655):

C scale	Base number (24)	Other number (36)
D scale	100	Percentage relative (150)

26. The wholesale prices of the commodities used in Exhibits 92, 93, 95, 96, 97, 98, 99, 100, and 101 for the last five months of 1936 were as follows:

Commodity	Unit of quotation	August	September	October	November	December
Wheat	Bushel	1.142	1.157	1.186	1.200	1.340
Hogs	Cwt.	11.345	10.544	9.850	9.435	10.131
Cotton	Pound	.121	.122	.120	.119	.124
Wool	Pound	.389	.385	.391	.428	.484
Sugar	Pound	.037	.036	.034	.036	.038
Hides	Pound	.131	.147	.149	.153	.156
Coal	Net ton	4.217	4.229	4.224	4.228	4.233
Petroleum	Barrel	1.040	1.040	1.040	1.040	1.040
Pig iron	Gross ton	21.390	21.390	21.390	21.640	22.390
Lumber	M feet	36.613	37.290	38.308	38.590	39.282
Rubber	Pound	.163	.165	.166	.179	.202

Source: Wholesale Prices published by the United States Bureau of Labor Statistics. See details in footnote to Exhibit 92, p. 189.

Using the base, weights, etc. given in the exhibits listed above, compute index numbers for each of the last five months of 1936 by the following methods:

a. Unweighted aggregative (see Exhibit 92).
b. Weighted aggregative (see Exhibit 93).
c. Unweighted arithmetic mean of relatives (see Exhibit 96).
d. Unweighted median of relatives (see Exhibit 97).
e. Unweighted geometric mean of relatives (see Exhibit 98).
f. Weighted arithmetic mean of relatives (see Exhibit 99).
g. Weighted geometric mean of relatives (see Exhibit 100).

Summarize the seven sets of index numbers in a single table and discuss the strong and weak points of each.[1]

27. Compile price data for the commodities listed in Problem 26 for five months from the latest available issues of *Wholesale Prices* published by the United States Bureau of Labor Statistics, and compute seven indexes according to the instructions in Problem 26.

28. It is the purpose of this problem to illustrate the more important methods of computing index numbers by means of a simple set of figures. Using 1913 as a base, construct index numbers of the following commodity prices:

Commodity	Average price, 1913	Average price, March 1938
Wheat, per bushel	$0.990	$0.920
Coal, bituminous, per ton	3.000	9.564
Cotton, per pound	.128	.089
Wool, per pound	.570	.680
Leather, sole, per pound	.449	.305

[1] To save time in computation it may be desirable to assign the different months to different sections of the class, and then gather the data together for the final analysis.

(A great many more commodities would have to be included, of course, in the computation of a representative price index. The present problem is designed merely to illustrate the method employed.)

a. Construct index numbers for the year 1913 and for the month of March, 1938, in the form of *unweighted aggregates of actual prices.* Express these as relatives, with 1913 as the base.

b. Construct index numbers for the same periods in the form of *weighted aggregates of actual prices.* Prices are to be weighted by quantities marketed in 1919, which follow:

Commodity	*Quantity marketed in* 1919
Wheat	807,816,000 bushels
Coal, bituminous	400,200,000 tons
Cotton	5,710,382,000 pounds
Wool	367,382,000 pounds
Leather, sole	296,326,000 pounds

(The figures for quantities marketed relate to all grades of the given commodities, not merely to the grades for which prices are given above. Prices of the grades named are to be taken to represent all grades of the various commodities, and are to be weighted by the total quantities marketed.)

Express these weighted aggregates as relatives, with 1913 as the base.

c. Construct index numbers for the same periods in the form of *aggregates of prices per pound.* (The quotation for coal, as given above, is by long tons. Assume 60 pounds of wheat to the bushel in reducing prices to the per pound basis.) Express these as relatives, with 1913 as the base.

d. Construct index numbers for the same periods in the form of *medians of relative prices.*

e. Construct index numbers for the same periods in the form of *simple arithmetic averages of relative prices,* with 1913 as the base.

f. Construct index numbers for the same periods in the form of *weighted arithmetic averages of relative prices,* weights to be based upon the total values of the various commodities marketed in 1919. The following table, showing the percentage distribution of these values, gives the weights to be employed:

Wheat	35
Coal, bituminous	26
Cotton	29
Wool	7
Leather, sole	3
Total	100

The report should include tables showing the method employed in computing each index number.

Summarize in a single table the six sets of relatives secured.[1]

29. The wholesale prices of certain representative food commodities and their respective weights were as follows:

[1] Problem by courtesy of F. C. Mills.

Commodity	Average prices		Unit	Commodity weights,* (in millions)
	Year 1913	March 1938		
Wheat, No. 1, northern spring, Minneapolis	$0.874	$1.190	Bushel	683
Cotton, upland, middling, United York	.128	.089	Pound	5,410
Sugar, granulated	.043	.046	Pound	13,683
Hogs, heavy	8.365	9.130	Cwt.	76
Eggs, firsts, fresh	.226	.210	Dozen	927

* These weights are those used by the United States Bureau of Labor Statistics in computing index numbers of wholesale prices in the United States for 1913 and 1914 by groups of commodities. See Bureau of Labor Statistics *Bulletin* 181, "Wholesale Prices 1890 to 1914."

Using the year 1913 as a base, compute an index number of wholesale prices for March 1938, by each of the following methods:

a. Unweighted aggregate.
b. Weighted aggregate.
c. Unweighted arithmetic mean of relatives.
d. Unweighted median of relatives.
e. Unweighted geometric mean of relatives.
f. Weighted arithmetic mean of relatives.
g. Weighted geometric mean of relatives.

30. The volume of production in six major industries for the months of January 1933 and January 1938 was as follows (data hypothetical):

Industry	Unit of quotation	Production January 1933 (in thousands)	Production January 1938 (in thousands)
Automobile	Unit	121	368
Cotton	Bale	442	664
Electric power	Kilowatt-hour	6,842,000	10,078,000
Lumber	Board foot	1,321,000	1,742,000
Petroleum	Barrel	64,342	96,421
Steel	Long ton	1,024	4,642

Compute an index of industrial production for January 1938, using January 1933 as a base period:

a. By the method of unweighted arithmetic mean of relatives.
b. By the method of unweighted geometric mean of relatives.

CHAPTER X

DISPERSION AND SKEWNESS

Although it is very useful to have, for instance, an average of the earnings of workers in a factory, this one value may not be sufficient to indicate the real wage conditions. An average does not indicate the spread or range over which the data are scattered; it gives only a single amount, which is considered to be representative of the whole data. The same average wage might be found in the pay rolls of two factories, and yet all the wages in one plant might approximate the computed average, while in the other plant the wages might be distributed between two widely separated extremes. For example, the average of 10 items of $30 each is $30 (whatever method is used in obtaining an average). Yet 10 other items distributed between $1 and $100 might be found to average $30 by one or more of the methods of computing averages. That is, in both factories there might be the same arithmetic average, or the same median, or the same mode, or the same geometric average. Although this is an extreme illustration, it will be recognized that the same limitations in using averages exist where distributions are more nearly alike.

Measures of Dispersion.—In order to determine whether the average is really representative or to what extent the data vary from the averages, and in order to measure the spread of the data irrespective of the relation of the items to the computed averages, it is necessary to compute *measures of dispersion*. There are four common measures of dispersion: the *range*, the *quartile deviation*, the *average deviation*, and the *standard deviation*.

These measures can be stated in two ways. One method of statement shows the *absolute amount of deviation*, while the other presents the *relative amount of deviation*. We shall first consider the methods of determining the absolute amount of deviation.

The Range.—If one desires to know only the extent of the extreme variations, the range may be used as a measure of the dispersion. If either of the extreme items is exceptional (that is, if they are far distant from the other items), the range indicates nothing concerning the usual or normal spread of the items. Therefore, the range is usually most useful when it is known that the extreme items are not exceptional in nature, and it is often satisfactory to state a range excluding specified extreme items.

Stock prices and interest rates are often stated in terms of their ranges over a period of time (Exhibits 104 and 105). If the movements have not

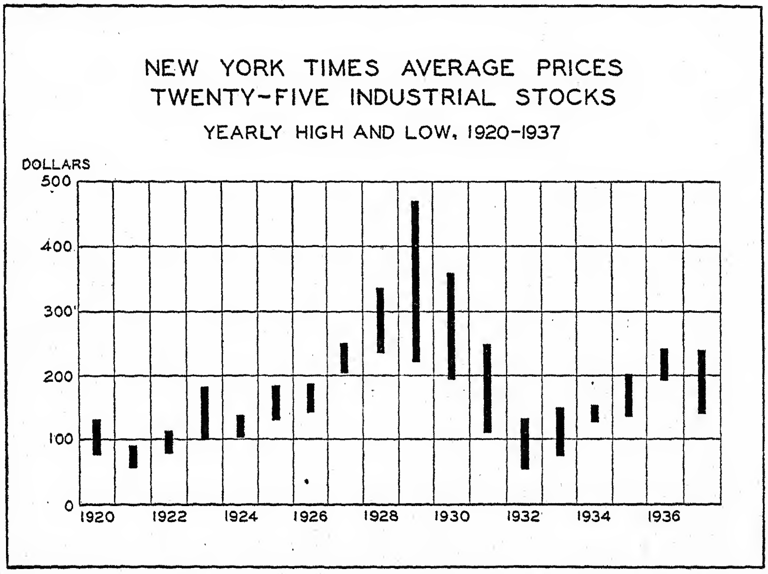

EXHIBIT 104.—The "range" of stock prices for each year from 1920 to 1937.

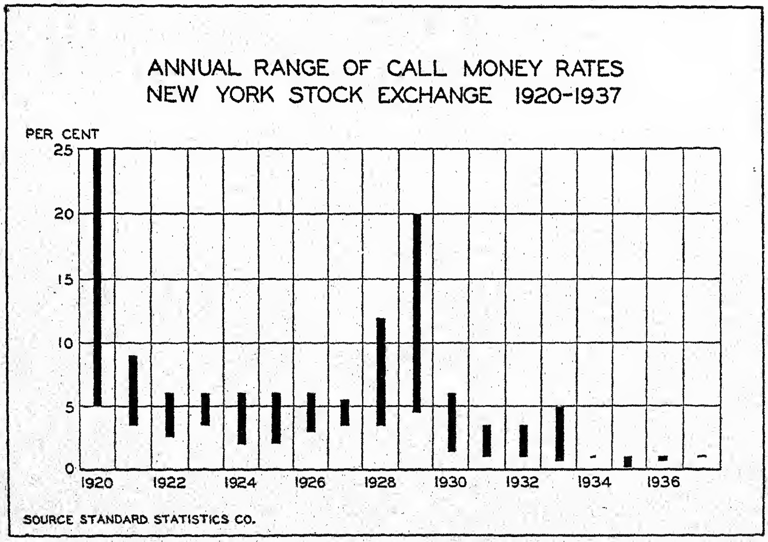

EXHIBIT 105.—The "range" of call-money rates for each year from 1920 to 1937.

been unusual, a quoted range may measure the ordinary dispersion, but if unusual movements of a temporary nature have occurred, the range

shows the effect of the temporary and unusual conditions (see the year 1929 in Exhibit 104 and the years 1920, 1929, 1934, and 1937 in Exhibit 105). Thus, the amount of dispersion shown by the stock-price range indicates market conditions, and comparisons of ranges in two different series have different meanings under different circumstances. Such a comparison at one time may show the effect of an unusual market condition on different stocks; whereas in a normal market, it would indicate differences in stock fluctuations under ordinary trading conditions.

Quartile Deviation.—The quartile deviation measures the spread of the *central* items only. It is not affected by the size of the extreme items, as is the range.

To obtain the quartile deviation, the value of the first quartile is subtracted from the value of the third quartile and this difference is divided by two. The formula for this procedure is $(Q_3 - Q_1) \div 2$. The quartile deviation is often called the *semi-interquartile range,* for it is one-half the difference between the values[1] of the third and the first quartile.

To illustrate the process of determining the quartile deviation, we may again refer to Exhibit 64, page 136, in which the third quartile was found to be 9 (the value of the 1729th item) and the first quartile was found to be $7\frac{1}{2}$ (the value of the 576th item). The quartile deviation is $(9 - 7\frac{1}{2}) \div 2$, or $\frac{3}{4}$ of 1 size. This shows that one-half of the items are included in the interquartile range, and we now know that one-half of the men's shoes sold by The Paris Store did not vary from the quartiles in size by more than three-fourths of a size. (Due to the discrete character of the data, this variation cannot be more than a half-size, for the sizes 8 and $8\frac{1}{2}$ are the only sizes between the quartiles.) In other words, there was a small amount of variation in the sizes of the central half of the items in the distribution.

Average Deviation.—The two measures of deviation just described are rough measures of the spread or scatter of items. One depends upon the extreme items only; the other upon the central items only. In many analyses of business data, however, it is necessary to have an exact measurement of the *average or usual deviation of all items from a central point.* This is obtained either by the average deviation (also called the mean deviation) or by the standard deviation.

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[1] Confusion sometimes arises as to the meaning of Q_3 and Q_1. It should be noted that these terms do not mean the *rank* of these quartiles, but they refer to the *value* of the quartiles. Obviously, one-half the difference in rank is the number of items which these quartiles are distant from the median item, but one-half the difference in value will not be the amount of difference between the median value and the quartile values, unless the median value happens to be equidistant from the values of Q_3 and Q_1.

The average deviation is merely the arithmetic average of the deviations from an average of the data. That is, first an average of the data is obtained (ordinarily the median, the mode, or the arithmetic average), then the deviation of each value in the entire data from the average is computed, and finally the arithmetic average of these deviations (disregarding plus and minus signs) is computed. Although deviations from any of the averages mentioned above may be used, it is theoretically best to use the median, because the arithmetic sum of the deviations from this average is less than from any other point. Unless another average is specified average deviation will be measured from the median in this book.

	Smith		Jones	
	Production (in pounds)	Deviations from median (sign omitted)	Production (in pounds)	Deviations from median (sign omitted)
Monday	550	100	670	20
Tuesday	775	125	600	50
Wednesday	720	70	650	0
Thursday	650	0	660	10
Friday	630	20	640	10
Total deviation	...	315	...	90

Average deviation of Smith's production = 315 ÷ 5 = 63.
Average deviation of Jones' production = 90 ÷ 5 = 18.

EXHIBIT 106.—Computation of average deviation.

Computation of Average Deviation.—When the exact size of each item in the data is known, the computation of average deviation merely involves the subtraction of the median from *each size* (to obtain the deviation of each size from the median size), the summation of the resulting deviations (disregarding signs), and a division of this total by the number of items.

For example, if a foreman in a factory wishes to determine which of two employees is the more "steady" worker, he could find this from the average deviation of their output. If employee Smith produced 550 pounds on Monday, 775 on Tuesday, 720 on Wednesday, 650 on Thursday, and 630 on Friday, his median output of the five days would be 650. Likewise, if employee Jones produced 670 pounds on Monday, 600 on Tuesday, 650 on Wednesday, 660 on Thursday, and 640 on Friday, his median output would be 650. The average deviations of output of Smith and Jones, measured from the median production of each, are determined

in Exhibit 106. Obviously, Jones is much the steadier of the two, because his output fluctuated on the average only 18 pounds whereas the output of Smith fluctuated on the average 63 pounds.

In formula form, the method of computation is:

$$\text{Average deviation (A.D.)} = \frac{\Sigma d}{n}$$

the Greek letter Σ (sigma) meaning "sum of," d being used for "deviation," and n for "number of items." Thus, in Exhibit 106, Σd in Smith's production is 315 and Σd in Jones' production is 90. In both cases the value of n is 5.

The procedure illustrated (Exhibit 106) is all that is required where the data have not been arranged in a distribution, or where they are arranged in an array. Where the data are arranged in a frequency distribution, the procedure is essentially the same.[1] However, in the field of business statistics, the average deviation (or standard deviation) is seldom computed from a frequency distribution. Practically always, in business problems, the computation of average or standard deviation is made from data that have not been arranged in a frequency distribution.

Standard Deviation.—The standard deviation, like the average deviation, is also a measurement of the variation or spread of all the values throughout the entire distribution or series. It is obtained by squaring the deviations from the arithmetic average of the data, then finding the arithmetic average or mean of the squared deviations, and extracting the square root of this mean. It is frequently called the *root-mean-square* deviation, since it is the square root of the mean of the squared deviations.

In measuring the variation or spread throughout a distribution, the average from which deviations are measured in computing standard deviation is the arithmetic average, because the standard deviation is smallest when computed from this average.[2]

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[1] It is only necessary to determine the amount by which the value of each class in the frequency distribution deviates from the median (or mode, or arithmetic average), and then multiply the class deviation by the number of items in the class. When the resulting products are added, the sum of the deviations is obtained. This sum divided by the number of items gives the average deviation.

[2] In time-series analyses (see Chaps. XIII–XVI), standard deviations (and also average deviations) are computed from a "normal" which changes over the period under consideration. That is, the standard deviation in such instances is the square root of the mean of the squared deviations from this normal. Since the standard deviation about a line of average relationship is a measure of the accuracy of estimates, it may be termed the "standard error of estimate." In studies where it is necessary to distinguish between these computations as made from the arithmetic average and from a trend line, the term "standard deviation" is understood to refer to the root-

Computation of Standard Deviation.—If the variation in output of Smith and Jones in the preceding illustration of the two factory employees were measured by the method of the standard deviation, instead of the average deviation, the deviations would be measured from the arithmetic average, and the procedure would be as shown in Exhibit 107. In this computation, the deviations from the arithmetic average are each squared, the squared deviations are added (giving 29,800 and 2,920, respectively), and each total is divided by 5 to obtain the arithmetic

	Smith			Jones		
	Production (in pounds)	Deviations from mean	Deviations squared	Production (in pounds)	Deviations from mean	Deviations squared
Monday.........	550	−115	13,225	670	+26	676
Tuesday.........	775	+110	12,100	600	−44	1,936
Wednesday......	720	+ 55	3,025	650	+ 6	36
Thursday........	650	− 15	225	660	+16	256
Friday..........	630	− 35	1,225	640	− 4	16
Total.........	...	.....	29,800	...	....	2,920

Arithmetic average of Smith's production = 665.
Arithmetic average of Jones's production = 644.
Standard deviation in Smith's output is the square root of (29,800 ÷ 5), or

$$\sqrt{5{,}960} = 77.20.$$

Standard deviation in Jones's output is the square root of (2,920 ÷ 5), or

$$\sqrt{584} = 24.17.$$

Exhibit 107.—Computation of standard deviation.

averages of the squared deviations. The square root of each respective mean is the standard deviation of output of each employee.

In formula form the method of computing standard deviation may be stated as

$$\text{Standard Deviation (S.D. or } \sigma) = \sqrt{\frac{\Sigma d^2}{n}}$$

In this formula, Σd^2 is the sum of the squared deviations from the arithmetic average and n is the number of items in the entire group of

mean-square deviation about the arithmetic mean, while the root-mean-square deviation from the normal trend line is referred to as the "standard error of estimate." (For further information see Frederick C. Mills, "Statistical Methods," pp. 330–334, Henry Holt & Company, Inc., New York, 1938.)

data. Thus, in the example of Exhibit 107, Σd^2 for Smith is 29,800, and for Jones, Σd^2 is 2,920. In both cases n has a value of 5.

By comparing the A.D. with the S.D. in the preceding Smith and Jones production problem, it will be noted that the standard deviations are larger than the average deviations. This is a characteristic relation, for squaring the deviations gives more weight to the large deviations than is the case where the deviations are added without squaring, as in computing average deviation.

Coefficients of Variation.—For purposes of comparison, the *absolute amount* of a measurement is not always so valuable as an expression of the *relative amount.* Thus, we might find the average deviation in the annual incomes of residents in one part of a city to be \$500, whereas in another district of the city the average deviation is \$50. This would not prove that there was relatively greater variation in the incomes in the first district than in the second, for if the average income were \$10,000 in the first district and \$1,000 in the second district, the relative degree of variation would be the same. In other words, a variation of \$500 where the usual income is \$10,000 would be the same *percentage of variation* as a variation of \$50 where the usual income is \$1,000.

Percentages of variation are known as *coefficients of dispersion* or *coefficients of variation.* They state the *degree of variation.* There are three common coefficients of variation, and they are obtained by expressing the amount of variation as the percentage which it is of a central value.

The coefficient most commonly used is obtained by expressing the *standard deviation as a percentage of the average from which it was measured:* namely, the arithmetic average. Thus, in the above example the coefficient of dispersion in Smith's production is found by this method to be 77.20 ÷ 665, or 11.6 per cent. For Jones, the coefficient is 24.17 ÷ 644, or 3.8 per cent.

In the second method of measuring relative variation, *the average deviation is expressed as a percentage of the median,* or whatever average has been used in measuring the deviations. In the example of the production of Smith, the coefficient is obtained by dividing the average deviation (63) by the median (650). This gives a coefficient of 9.7 per cent. For Jones the coefficient is 18 ÷ 650, or 2.8 per cent.

In the third method, the quartile deviation is stated as a percentage of one-half the sum of the first and third quartiles. This is stated below in the usual symbols for the quartiles:

$$\text{Coefficient of variation} = \frac{(Q_3 - Q_1) \div 2 \times 100}{(Q_3 + Q_1) \div 2}$$

which is reduced for convenience to the formula

$$(Q_3 - Q_1) \div (Q_3 + Q_1) \times 100$$

The coefficient of variation in sizes of men's shoes sold by The Paris Store (Exhibit 64, page 136) based on the quartiles as previously determined is, therefore, $(9 - 7\frac{1}{2}) \div (9 + 7\frac{1}{2}) \times 100$, or 9.09 (per cent).

Value of Average and Standard Deviation Measurements.—Especially in the study of business time-series data, the use of either average or standard deviation is indispensable as a tool. In later chapters we shall study business data in the form of time series, and in the course of this study the importance of measuring deviations in the series considered will be evident. At the present point, however, it is desired to call attention to two particular uses of average and standard deviations, which will be illustrated further in the pages that follow. These uses are: (1) in comparing time series having widely different amplitudes of fluctuation; and (2) in combining several series of business data into one composite series called a "business index." In order to use measures of dispersion for these purposes, however, it is first necessary to reduce the deviations to units of their standard or average deviation.

Units of Standard or Average Deviation.—Referring again to the variations in production of Smith and Jones (Exhibit 107), it will be seen that on Monday Smith's production was 115 pounds below his arithmetic average for the week, and on Tuesday his production was 110 pounds above the average. Are these large fluctuations for Smith? And when Jones is 26 pounds over the mean of his week on Monday, and 44 pounds under on Tuesday, are these large variations for him?

These questions can be answered by reducing the variations to units of their respective standard deviations (or average deviations, if desired). If 77.20 pounds is the standard deviation for Smith, then a deviation of 115 pounds is $(115 \div 77.20)$ standard deviations, or 1.49 standard deviations. Similarly, 110 pounds is a variation of 1.42 standard deviations. For Jones, a deviation of 26 pounds is, in terms of standard deviation units, a deviation of $26 \div 24.17$, or 1.07 standard deviations, since *his* standard deviation is 24.17. Likewise, 44 pounds is a deviation of 1.82 standard deviations. Thus we see that on Tuesday Smith's variation of 110 pounds was not as great for him as Jones' variation of 44 pounds was for him.

Units of average deviation are obtained in the same manner, namely, by dividing the average deviation into the individual deviations, these individual deviations usually being from the median.

The reduction of series to terms of their respective standard (or average) deviations places data having widely different amplitudes of fluctuation on a comparable basis. One series, say bank clearings, may have deviations from the average amounting to millions of dollars,

whereas another series, such as interest-rate percentages, may have deviations from the average that never exceed five or six. When the deviations of such series are reduced to units of their own respective standard deviations, the fluctuations in value become readily comparable, because they are stated in terms of a "normal" for each series. For example, a deviation of $10,000,000 in bank clearings may be found to represent the same number of "normal" or standard deviations as a deviation of 1 per cent in interest rates.

In studying business cycles, time series are often expressed as percentage deviations from a changing base or growth trend. When two series are being compared or combined, for instance, it is important to have them on such a basis that the fluctuations are of similar significance. Suppose that we are comparing cycles of bank clearings and cycles of pig-iron production for a certain period, and that bank clearings tend to fluctuate within a range of ± 20 per cent and that pig-iron production tends to fluctuate within a range of ± 45 per cent. Obviously, a 10 per cent fluctuation in pig-iron production is less than one-half as significant as a 10 per cent fluctuation in bank clearings. The two series can be placed on a comparable basis from this point of view by stating them in terms of their respective average or standard deviations. This use of average and standard deviation will be explained in greater detail on pages 339 to 341, in the chapter on business cycles.[1]

In order to illustrate further the procedure of reducing series to terms of their standard deviations, let us consider Exhibits 108 and 109. In the table of Exhibit 108, columns *A* and *E* represent sales (in dollars) of tires and automobiles in an assumed case. Columns *B* and *F* represent deviations from the average of each series. In the manner previously described, it is found from columns *C* and *G* that the standard deviations are .7 and 5.5 for sales of tires and sales of automobiles, respectively (see note at bottom of Exhibit 108). It is then a simple matter to reduce these series to terms of their standard deviations. To do this, the deviations from the average are simply divided by the respective standard deviations. For example, the first item in column *B* (−1) is divided by .7, and the resulting figure (−1.4) means that January sales of $1,000 were 1.4 standard deviations under the average. The two series are stated in terms of their standard deviations in columns *D* and *H*, Exhibit 108. It will be seen from columns *A* and *E*, Exhibit 108, and from Part *A*, Exhibit 109, that the automobile series fluctuates much more widely than the tire series. However, from a point of view such as that of analyzing business conditions from the movements of these curves, a small movement in the tire curve is as significant as a large movement in

[1] See also second footnote on p. 219 relative to standard error of estimate.

the automobile curve. That this is true is shown by Part B, Exhibit 109, in which the two series are presented in terms of their standard deviations. It is obvious that if the two series in Part A, Exhibit 109, were averaged together, the resulting curve would be dominated by the automobile

	Sales of tires				Sales of automobiles			
	A Sales in thousands of dollars	*B* Deviations	*C* Deviations squared	*D* Sales in terms of S.D.	*E* Sales in thousands of dollars	*F* Deviations	*G* Deviations squared	*H* Sales in terms of S.D.
January........	1	−1	1	−1.4	2	− 6	36	−1.1
February.......	2	0	0	0	10	+ 2	4	+ .4
March.........	1	−1	1	−1.4	2	− 6	36	−1.1
April...........	3	+1	1	+1.4	15	+ 7	49	+1.3
May...........	2	0	0	0	4	− 4	16	− .7
June...........	3	+1	1	+1.4	15	+ 7	49	+1.3
July...........	2	0	0	0	4	− 4	16	− .7
August.........	2	0	0	0	4	− 4	16	− .7
September......	3	+1	1	+1.4	18	+10	100	+1.8
October........	2	0	0	0	10	+ 2	4	+ .4
November......	1	−1	1	−1.4	2	− 6	36	−1.1
December......	2	0	0	0	10	+ 2	4	+ .4
	12/24		12/6		12/96		12/366	
Averages.....	2	...	.5	......	8	....	30.5	

$\sqrt{.5} = .7$
S.D. or σ = 0.7*

$\sqrt{30.5} = 5.5$
S.D. or σ = 5.5*

* Since sales are in thousands of dollars, three places having been dropped to simplify computation, .7 really represents $700 and 5.5 really represents $5,500. That is, S.D. of tires is actually $700 and S.D. of automobiles, $5,500. However, since the three places were dropped in columns A and E, the entire computation was carried through on this basis, using .7 and 5.5 in computing columns D and H.

EXHIBIT 108.—Reduction of two time series to terms of their standard deviations.

fluctuations. In Part B, however, the two series are on a comparable basis, and if they are averaged together, the resulting curve will be influenced approximately as much by the fluctuations in tire sales as by the fluctuations in automobile sales.[1]

[1] The effects of expressing certain series in terms of their standard deviations may be further appreciated by referring to Chart I, Exhibit 60, p. 119.

Skewness.—The measures of variation which we have considered are used in determining either the absolute or the relative amount of variation in the data. They do not, however, measure skewness. Skewness is the lack of symmetry. When a frequency distribution is plotted on a chart, skewness is present if the items tend to be dispersed more on one side of the mean than on the other.

Since measures of skewness are used only occasionally in business statistics, this discussion on the subject will be confined to the "bell-shaped" curve and distributions that are similar to it. That is, U-shaped curves and J-shaped curves (see Exhibit 72, page 144) will not be included.

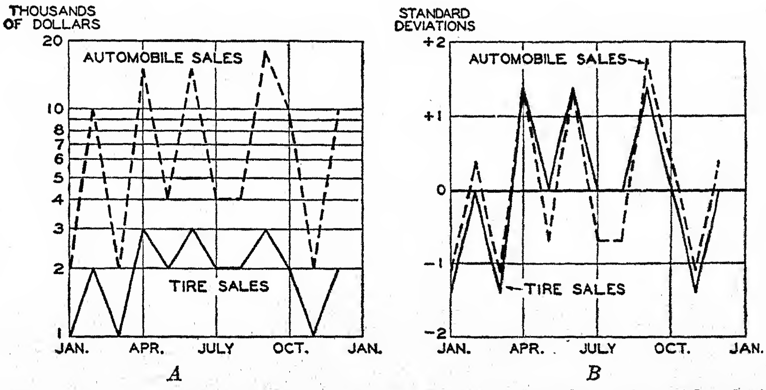

EXHIBIT 109.—A series with relatively large fluctuations and a series with relatively small fluctuations are presented in Part *A*. These series are presented in terms of their standard deviations in Part *B*. (Tabular data are given in Exhibit 108.)

In measuring the extent to which the data are skewed to one side of the central or most "dense" part of the distribution, the relative positions of the averages are useful. In a perfectly symmetrical distribution (such as Part *A*, Exhibit 72, page 144), the mode, median, and arithmetic average coincide. Asymmetrical distributions, however, show variation in the values of these averages. In moderately asymmetrical data (Part *B*, Exhibit 72), the median usually is approximately one-third of the distance (along a scale of values) from the mean to the mode. It is usual that the greater frequencies in economic data occur toward the side of smaller values (positive skewness), and therefore the mode is usually less than the median and the arithmetic average in these data.

The most common method of measuring skewness is

$$\text{Skewness} = \frac{\text{arithmetic mean} - \text{mode}}{\text{standard deviation}}$$

The result will be positive if the mode is smaller than the arithmetic mean, and negative if the mode is larger than the mean (see Exhibit 110).

The difficulty which sometimes exists in locating a true mode has resulted in the suggestion of a formula based on the median, namely:

$$\text{Skewness} = \frac{3(\text{arithmetic mean} - \text{median})}{\text{standard deviation}}$$

This formula may be used instead of the first formula for the reason that the difference between the arithmetic mean and the mode tends to be approximately three times the difference between the arithmetic mean and the median.

Whether or not skewness is measured by mathematical methods, its presence can be observed from the distribution if it exists to any marked extent. It is important to determine the existence and the direction of skewness, for two distributions of data might have very similar averages and the same amount of dispersion, and yet be very different, because

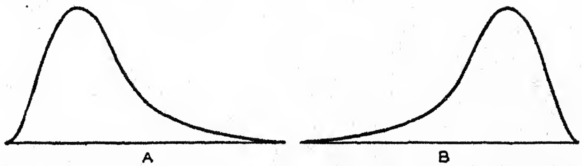

EXHIBIT 110.—Frequency curves with skewness. *A. Positively skewed* (mode smaller than mean—long tail of curve extends to the right). *B. Negatively skewed* (mode larger than mean—long tail extends to the left).

skewness existed on opposite sides of the central tendencies in the two distributions.

Questions and Problems

1. What is meant by "dispersion"?
2. Name four measures of dispersion.
3. What are two ways of expressing each measure of dispersion? Explain.
4. Does the range indicate normal dispersion?
5. How is the quartile deviation determined?
6. Does quartile deviation describe the dispersion within the entire distribution?
7. Describe the average deviation method of measuring dispersion.
8. How is a standard deviation computed?
9. Describe three coefficients of dispersion.
10. Explain how measures of dispersion are useful in comparing different series of data.
11. How are deviations from the average reduced to units of standard deviation?
12. What is skewness? How may it be measured?
13. An automobile manufacturer operating plants in Detroit and Los Angeles finds a wage distribution of the employees in his two plants to be as shown in the table at the top of page 227.

From the data shown in the table at the top of page 227 determine which of the plants has the higher wages and in which plant the wages fluctuate more, absolutely and relatively. For your comparisons use the average deviation (compute

the median), the standard deviation, and any other measurements you may find necessary. Write a brief report based on your comparison.

Daily wage	Number of employees Detroit plant	Number of employees Los Angeles plant
\$ 1.50–\$ 2.49	0	7
2.50– 3.49	59	102
3.50– 4.49	259	111
4.50– 5.49	282	360
5.50– 6.49	1,300	159
6.50– 7.49	384	33
7.50– 8.49	122	13
8.50– 9.49	39	11
9.50– 10.49	49	0
10.50– 11.49	0	0
11.50– 12.49	6	4
Total	2,500	800

14. Sales of the James Locomotive Company and of the Diamond Variety Stores Inc. for the years 1924–1937 were (in millions of dollars):

Year	James Locomotive Co.	Diamond Variety Stores, Inc.
1924	1.9	56
1925	2.3	62
1926	2.2	69
1927	2.4	76
1928	2.9	78
1929	2.1	86
1930	1.6	74
1931	1.1	69
1932	.9	30
1933	.1	38
1934	.2	52
1935	1.1	60
1936	2.2	72
1937	2.8	88
Total	23.8	910

Compare the relative fluctuations in sales of the two companies by two methods of computation.

15. The data on retail sales and bank debits in Crown City for the years 1936 and 1937 (in millions of dollars) are shown in the table on page 228.

a. Reduce the two 24-month series to units of their respective standard deviations and make a graph in units of standard deviation (see Exhibit 109*B*).

b. Write a brief report of business conditions in Crown City.

Crown City Business Data

Month	Retail sales		Bank debits	
	1936	1937	1936	1937
January	17	21	36	67
February	18	21	39	69
March	24	26	41	72
April	22	25	44	77
May	17	21	40	66
June	14	18	47	71
July	13	17	49	64
August	15	23	53	62
September	19	24	55	69
October	21	25	61	71
November	24	27	64	74
December	30	34	79	82

16. Compare the relative fluctuations of the two 24-month series on retail sales and bank debits given in Problem 15, using the average deviation in your computations. Comment briefly on your results.

CHAPTER XI

THE NORMAL CURVE, PROBABLE ERROR, AND THE PROBLEM OF SAMPLING

The business man must continually deal with occurrences which have not been the subject of direction or control but which "just happen." In practical considerations, this category is a broad one and includes many kinds of events which might be controlled but which are allowed to occur without any attempts at direction. Typical problems may be indicated by the following questions confronting the executives of a shoe manufacturing company: How many of each size of shoe should we make? Can we cut down the accident rate in our plant? Have we too much sickness? Can we reduce the number of fires? Are our inspectors missing too many defective parts?

The Importance of Distinguishing Chance Phenomena.—In the field of business, some occurrences are due to chance, some to directed causes, and some to a combination of chance and directed causes. Inasmuch as problems of chance occurrences ordinarily require entirely different solutions from those of directed causes, any method that will aid in distinguishing them is obviously of much value.

Chance happenings often can be segregated because they occur in accordance with certain laws which make it possible to determine their distribution. Any variations from the expected distribution may be indicative of outside causes. Once this distribution is known, undesirable events can often be provided against on a systematic basis such as that underlying the principles of insurance. The results not due to chance, once the causes are known, can often be controlled or eliminated.

Even though it may sometimes be possible to remove or control the causes of undesirable happenings, it is often preferable to make a systematic provision for compensating for them if they tend to follow the laws of chance. For instance, in a large, widespread community, it is possible to determine closely the chances of fire losses for the community as a whole, and it is considered more practicable to provide fire insurance than to eliminate all possibility of fire.

If one is to be able skillfully to consider chance phenomena, in an effort to take advantage of favorable opportunities and avoid pitfalls or make proper provision therefor, he must study the causes that give frequency curves certain shapes, and in this effort he will profit by

considering some of the fundamentals of the theory of "probability." It is not properly within the scope of the present work to consider the subject of probability at any great length, but a brief discussion of the subject will be given for the purpose of introducing the reader to the possibilities of this type of analysis. It will be seen that some understanding and appreciation of the theory of probability is of great practical value in analyzing business problems, and that it is important for the business student to realize this fact. Even if he does not have, or does not wish to acquire, the background necessary for making the more elaborate probability analyses, he should at least appreciate, in a general way, the possibilities of such an approach so that he will not overlook the distinctions between distributions due to controlled causes and those due to pure chance.[1]

The Normal Curve.—In Chapter VII on statistical distributions, the symmetrical bell-shaped frequency curve, known as the "normal curve," was briefly described. This curve is also known as the "normal probability curve," the "normal curve of error," and the "Gauss-Laplace curve." It is known as the normal probability curve because it represents the theoretical probabilities of certain chance phenomena (see later explanation of Exhibits 112 and 113). It is known as the "normal curve of error" because of the use of the curve to describe a distribution of errors made in the taking of measurements. When repeated measurements are taken of a certain astronomical distance, for instance, they will not all be the same but will have a marked central tendency from which they will vary above and below according to a symmetrical pattern. It is called the "Gauss-Laplace curve" after the mathematicians Gauss and Laplace to whom is credited much of the earlier work on the analysis of the curve and the theory of probability. The so-called "Gauss-Laplace law," however, is believed to have been first discovered by DeMoivre in 1732. The curve is known as the normal frequency curve, not only because of the tendencies just mentioned, but also because natural as well as chance phenomena tend to be grouped or distributed according to the probability curve. For instance, if the leaves on a tree are classified by lengths into a frequency table, a curve plotted therefrom will approximate the symmetrical bell-shaped curve shown in Exhibit 113.

The normal curve is but one of a number of types of curves which may be used to describe frequency distributions, but since it probably is the most significant single type, an understanding of its characteristics is of great help in making statistical analyses where the problems of

[1] For a more complete discussion, see Frederick C. Mills, "Statistical Methods," 2d ed., Chap. XIII, "Elementary Probabilities and the Normal Curve of Error," and Chap. XIV, "Statistical Induction and the Problem of Sampling," Henry Holt & Company, Inc., New York, 1938.

frequency distributions are involved. It follows that a limited introduction to the elementary principles of probability may properly be based upon a consideration of this curve, and, accordingly, this procedure is followed in the present discussion.

Probability.—The term "probability," as used in statistics, refers to the probability of occurrence of an event, or the expected relative frequency of an event in an infinite number of observations or trials. This expected ratio may be based upon a priori knowledge of the conditions determining the probability, as in simple games of chance, or upon actual experience data, as is usual in economic and social phenomena.[1] The probability ratio is a fraction between the limits zero (impossibility) and unity (certainty). Within the practical limits of human knowledge and belief, the probability that the sun will rise in the east tomorrow is certainty or 1; the probability that the sun will set in the east is impossibility or 0; and the probability that it will be cloudy in the east is somewhere in between.

The law of chance as applied to the tossing of a coin is generally understood. That is, assuming a haphazard toss and a perfect coin, the chance is one out of two that a head will turn up. Or there is one chance that a head will show to one chance that a tail will show.

Next let us suppose that two coins are thrown. What are the chances of heads appearing and what are the chances of tails appearing? Both coins may show heads, both tails, or either a head with the other a tail. Letting H indicate head and T indicate tail and letting (*a*) and (*b*) identify the respective coins, the possible combinations may be listed as follows:

(*a*)		(*b*)	(*a*)		(*b*)	(*a*)		(*b*)
H	and	H	H	and	T	T	and	T
				or				
			T	and	H			

Thus there is possible one throw of both heads (HH), two of a head and a tail (HT), and one of both tails (TT). This may be expressed:

$$H^2 + 2HT + T^2$$

where the exponents or powers to which H and T are raised represent the number of H's and the number of T's and, thus, the number of heads, the number of tails, and the number of heads and tails. The resemblance

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[1] See Harry Jerome, "Statistical Method," p. 165, Harper & Brothers, New York, 1924, and Arne Fisher, "The Mathematical Theory of Probabilities," pp. 82–85, The Macmillan Company, New York, 1915.

of this result to that obtained when a binomial is squared becomes evident.[1]

Developing the coin-throwing illustration still farther, let us consider the combinations possible if we throw four coins designated (*a*), (*b*), (*c*), and (*d*) so that the individual coins can be identified. The possible combinations are as follows:

4 heads 0 tails	3 heads 1 tail	2 heads 2 tails	1 head 3 tails	0 heads 4 tails
(*a*)(*b*)(*c*)(*d*)	(*a*)(*b*)(*c*)(*d*)	(*a*)(*b*)(*c*)(*d*)	(*a*)(*b*)(*c*)(*d*)	(*a*)(*b*)(*c*)(*d*)
H H H H	H H H T	H H T T	H T T T	T T T T
	H H T H	T H H T	T H T T	
	H T H H	T T H H	T T H T	
	T H H H	H T T H	T T T H	
		H T H T		
		T H T H		

If we consider 4 heads in the above illustration as "heads to the fourth power," which means the product of 4 H's, etc., the combinations may be expressed as follows:

$$H^4 + 4H^3T + 6H^2T^2 + 4HT^3 + T^4,$$

where H^4 is the probability of exactly 4 heads in a single trial, $4H^3T$ is the probability of exactly 3 heads and 1 tail in a single trial, etc. From the above it is apparent that there seems to be a regular law which governs the combination of a given number of heads and tails. In fact, the results obtained from the above throw of four coins are the terms of the binomial expansion resulting from raising the binomial (H + T) to the fourth power. If ten coins had been tossed at one time, the binomial would be raised to the tenth power and so on.

If ten coins are tossed, we might list the possible combinations as we did in the above example of four coins. A much more simple procedure, however, is to determine the coefficients (the number of times each combination occurs) by the use of the binomial triangle.

The Binomial Triangle.—The binomial triangle (Pascal's arithmetical triangle) is an ingenious labor-saving device for determining the coefficients of a binomial expansion. To set up this triangle, first list the powers in a table stub as far as the power to which the given factor is to be raised, see Exhibit 111. Then, in the first column, list as many 1's as there are powers (10 in the illustration). Next, in the adjoining

[1] The student will recall from elementary algebra the rule for squaring the binomial $a + b$, *viz: the square of the sum of two numbers equals the square of the first, plus twice the product of the first by the second, plus the square of the second.*

column on the top line enter a 1. Then, to complete the table, obtain each coefficient in the successive rows by adding the coefficient located just above it to the coefficient immediately to the left of this above coefficient. When the end of each row is reached and there is no term above, enter a 1 at the end of the row. Thus, in Exhibit 111, after the column of 1's has been set up, the top row is completed by placing another 1 in the next column. For instance, in the second row the second item (2) is obtained by adding the 1 above to the 1 to the left of the 1 above, and the row is completed by setting down a 1 at the end. For the second item in the third power the coefficient (3) is obtained by adding 1 and 2 together; and that for the fourth term in the tenth power (120) is 84 + 36.

Referring again to our example of 10 coins, we can read from the bottom row of the table in Exhibit 111 the coefficients of the expansion of the

BINOMIAL TRIANGLE

Power	Coefficients										
1st	1	1									
2d	1	2	1								
3d	1	3	3	1							
4th	1	4	6	4	1						
5th	1	5	10	10	5	1					
6th	1	6	15	20	15	6	1				
7th	1	7	21	35	35	21	7	1			
8th	1	8	28	56	70	56	28	8	1		
9th	1	9	36	84	126	126	84	36	9	1	
10th	1	10	45	120	210	252	210	120	45	10	1

EXHIBIT 111.—The binomial triangle (Pascal's arithmetical triangle)—a device for determining coefficients of the terms in the expansion of a binomial to a given power.

binomial (H + T) to the tenth power. The total of the coefficients is 1,024. Thus, if 10 coins are thrown, the chances are 1 out of 1,024 that all will be heads, 10 out of 1,024 that 9 will be heads, 45 out of 1,024 that 8 will be heads, and so on. All of the probabilities added together give 1,024/1,024 or unity, which is the symbol for certainty, as previously mentioned.

Comparison of Actual and Theoretical Distributions.—In the table of Exhibit 112 the middle column shows the theoretical distribution of throws of 10 coins 1,024 times and the last column shows the actual distribution in a certain experiment in which the coins were actually thrown 1,024 times. These data are shown graphically in Exhibit 113 on which the smoothed curve represents the theoretical distribution and the heavy black dots connected by light lines represent the actual results of the experimental throw. (It should be appreciated that a smoothed curve is not, strictly speaking, an exact representation of the binomial coeffi-

cients for the tenth power, as there are no intermediate values between the coefficients, and the smoothed curve implies a continuous series. However, as the power becomes larger, the coefficients approach more closely the smoothed normal probability curve, and, therefore, the smoothed curve may be used to indicate the relationships between the binominal distribution and the probability curve.)[1]

The Law of Normal Frequency.—The preceding discussion illustrates how chance phenomena tend to be grouped according to the probability curve. Previous reference has also been made to the tendency of natural phenomena to be grouped according to the probability curve. The reason

COMPARISON OF
THEORETICAL AND ACTUAL DISTRIBUTIONS OF THROWS
OF TEN COINS 1,024 TIMES

Number of heads up in a throw	Frequency of throws	
	Theoretical number	Actual number
0	1	2
1	10	11
2	45	41
3	120	129
4	210	223
5	252	248
6	210	194
7	120	124
8	45	42
9	10	8
10	1	1

EXHIBIT 112.—Theoretical probabilities compared with actual occurrences in an experimental throw. (Shown graphically in Exhibit 113.)

for this similarity is that the variations in natural phenomena, as, for instance, in the lengths of leaves, are, like the results of coin throwing, due to chance.

A chance phenomenon, strictly speaking, is not due to abstract happenings without cause, but is the result of an intricate multiplicity of causes which are largely uncontrollable and often unknown, at least with respect to the result being considered. When a coin is tossed, for instance, a great number of small factors, which influence or control the result with varying degrees of effect, come into play, such as the original position, a turn of the hand, air currents, and resilience of the floor or

[1] See G. U. Yule and M. G. Kendall, "An Introduction to the Theory of Statistics," 11th ed., Chap. X, Charles Griffin & Company, London, 1937.

table top. Under the conditions being considered, however, the combinations of factors are very finely balanced and there is an equal tendency or a 50–50 chance that the variation may be in either direction. Likewise the variations in the lengths of leaves are due to many causes which "happen" to affect them with the result that the lengths are distributed according to the laws of chance.

The tendency of natural phenomena, such as the lengths of leaves, heights of men, or sizes of hands, when classified into a frequency table and plotted as a graph, to approximate a symmetrical bell-shaped curve

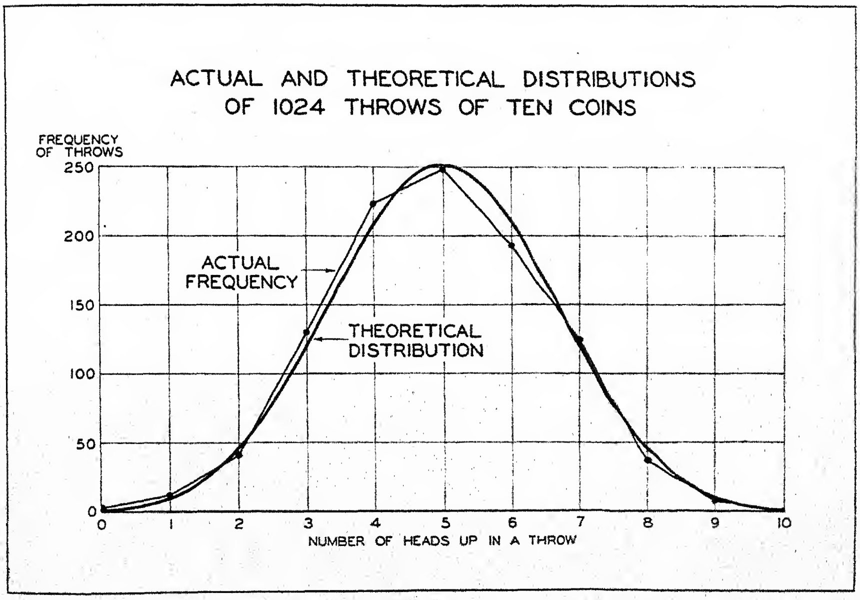

EXHIBIT 113.—Smoothed curve of theoretical distribution compared with actual occurrences in an experimental throw. (Data from Exhibit 112.)

similar to the probability curve (Exhibit 113) is known as the "law of normal frequency." In more precise terms the law is as follows: *Variate values of a natural phenomenon tend to be symmetrically distributed about the mode in proportions determined by the law of chance distributions.*[1]

So far the theory of probability or normal distribution has been stated in terms implying the dominance of the law of equal chance, that is, that there must be an equal tendency or a 50–50 chance that the variation may be in either direction. When, however, one or more causes tend to be dominant, a skew distribution results. Such a condition is common in business and economic phenomena, but often such skewness

[1] JEROME, HARRY, "Statistical Method," p. 170, Harper & Brothers, New York, 1924.

is not so great as to prevent the application of the law of normal probability with reasonably accurate and valuable results. Extreme care must be taken, however, if there is a marked departure from a normal curve. The importance of appreciating this point when applying statistical formulas cannot be overemphasized for the reason that most of them are based upon the assumption of a symmetrical or only moderately skew distribution.

Properties of the Normal Curve.—The heavy curve in Exhibit 113 represents a normal probability curve smoothed from the theoretical distribution of tossing ten coins. The shape of the curve would be different for different exponents of the expression of the binomial. That is, the lower the exponent the more "peaked" is the curve; and, conversely, the higher the exponent the flatter is the curve. However, all of these curves are alike in that they are symmetrical about a maximum and they shade off equally in both directions in a regular and systematic manner. These curves have certain mathematical characteristics of which the following are among the most important:

1. The arithmetic mean, the median, and the mode coincide.
2. The standard deviation cuts the curve at the points of inflection (the points where the curvature reverses).
3. Within a distance of one standard deviation above and below the mean, 68 per cent of the instances will occur.
4. The average deviation is approximately $\frac{4}{5}$ (or .7979) of the standard deviation.
5. The semi-inter-quartile range, $\frac{Q_3 - Q_1}{2}$, is equal to the probable error, that is, a distance above and below the mean within which one-half of the instances fall.
6. The semi-inter-quartile range, $\frac{Q_3 - Q_1}{2}$, when added to the lower quartile or when subtracted from the upper quartile is equal to the mean, the median, and the mode, and is equal to approximately $\frac{2}{3}$ of the standard deviation.[1]
7. Within a range of approximately $\frac{2}{3}$ (or .6745) of the standard deviation, measured plus and minus from the mean, one-half of all the instances occur. (This measure is the "probable error" which added to and subtracted from the mean determines the range within which the chances are even that an item selected at random will fall.)
8. As all of the instances are included between the curve and the horizontal axis, the area between the curve and the horizontal axis represents the total number of items. (A clear appreciation of this point is important in connection with the discussion of probable error on pages 237 and 239.)

Referring to Item 1 above, it is obvious from either the table or the graph of theoretical frequencies (Exhibits 112 and 113) that the mode is 5. It is likewise obvious that the mid-item of an array would be in the group of 252 fives and that, therefore, the median is 5. Since half

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[1] See Horace Secrist, "An Introduction to Statistical Methods," pp. 368–369, The Macmillan Company, New York, 1925.

of the total of the values is above and half is below the mid-point, it is also obvious that the arithmetic mean is 5. (This figure may be checked, if desired, by multiplying each number of heads by the corresponding theoretical frequency, obtaining the total of these products, 5,120, and dividing by the number of throws, 1,024.)

Probable Error.—It follows from the preceding discussion that the tendencies and characteristics of the normal distribution will provide a basis for determining and expressing the degree of precision of averages and coefficients insofar as they are conditioned by chance elements. In this connection the measure of dispersion known as the "probable error" (P.E.) will now be considered. "Error" is used in the sense of deviation, and "probable" is used in the sense of equally probable.

The term "error" must not be confused with "mistakes" or "blunders" due to carelessness or incompetency in reading figures, making computations, or transcribing data. Mistakes can be avoided by exercise of care and use of checking methods. But error cannot be avoided for the reason that measurements cannot be absolutely exact. Measurements repeated with the greatest care will not yield the same results, but the most probable value may be obtained from a number of observations by averaging the results, providing the errors of the separate observations are accidental, or due to chance, and tend to balance each other. Error is the difference between the observed value and the most probable value. And error is a deviation—not a mistake.

Various averages and coefficients are calculated from samples. One may measure the precision of these averages and coefficients, insofar as it is conditioned by elements of chance, by determining the amount or size of the probable error. If the deviations from the average are arrayed in order of size, irrespective of whether they are above or below the average, the central deviation (median) is the "probable error," or it is the deviation within which it is just as likely as not (that is, the chances are even) that any given case chosen at random will fall.

An average computed from the limited number of observations in a sample is not likely to be the same as would be obtained from another sample of the same size or if all of the items in the universe were included. Similarly, differences are likely to occur in the standard deviation and other statistical measures. That is, these statistical measures are themselves variables. As a practical matter it is not feasible to describe a distribution with greater precision than the facts warrant. Therefore, it is necessary to know how the statistical measures obtained from limited samples are distributed. If successive samples of the same kind are chosen at random, and averages or other statistical measures are computed for each sample, the distribution of the different computations will follow laws of probability, some according to the bell-shaped curve or

normal curve of error which has been described, and others according to laws quite different from the normal curve of error. The amount of unreliability may be stated in terms of this *probable error,* which is .6745 times the standard deviation. (It is to be noted that P.E. in cases other than the normal distribution is .6745 times the standard deviation, but in these cases it is a range which does not necessarily contain 50 per cent of the instances.)

In further explanation of the probable error concept, let us suppose that we have a large number of averages that have been computed from different samples selected at random from a large group of items. As indicated previously, these averages will differ somewhat from one another, although they have been taken from the same universe, and if they are arranged in a simple frequency table and plotted as a curve, the result will approximate a bell-shaped normal curve. Different normal frequency curves vary according to the degree to which the data concentrate about the mean. Some are relatively "peaked" while others are relatively flat. If the averages computed from the samples as mentioned above tend to vary but little from the mean, the curve will be relatively high and narrow, which indicates only a slight dispersion. If the averages tend to vary widely from the mean, the curve will be relatively wide and flat, indicating great dispersion.[1] In both cases, however, the curves have the characteristics of the normal curve.

Regardless of such differences in the normal frequency curves as those mentioned above, if we measure from the central ordinate a distance to the left equal to the standard deviation of the series, and if we measure a like distance to the right, and erect ordinates at these points, they will cut the curves at the points of inflection. And the area under the curve between these two ordinates will be in all cases approximately 68 per cent of the total area under the curve and, consequently, will represent approximately 68 per cent of the total frequencies. That is, 68 per cent of the total deviations are within a range described as "plus one" and "minus one" standard deviation from the central ordinate, which, in a normal distribution, represents the mean, as well as the median and the mode.[2] Similarly, if we measure approximately two-thirds (.6745) of the standard deviation in both directions from the mean and erect perpendiculars at these points from the base line to the curve, the area enclosed will be an even half of the total under the curve and hence will represent one-half of the total frequencies, or the area within which there is a 50–50 chance that an item selected at random

[1] It is understood, of course, that the curves are drawn to the same or comparable scales.

[2] Similar percentages can be determined for any fraction or multiple of the standard deviation.

will fall. In Exhibit 114 this procedure is applied to a curve of the theoretical frequencies of coin throws tabulated in Exhibit 112. Ordinate *A* is approximately two-thirds of one standard deviation to the left of the mean (5), and ordinate *B* is the same distance to the right. The shaded portion represents one-half of the area under the curve, and is the area within which one-half of the items fall. The difference between the mean and the value at ordinate *A* or ordinate *B* represents the "probable error" of the distribution. The standard deviation (S.D. or σ) in this case is 1.58 and the probable error (P.E.) is 1.07. That is, there is a 50–50 chance that an item selected purely at random will fall

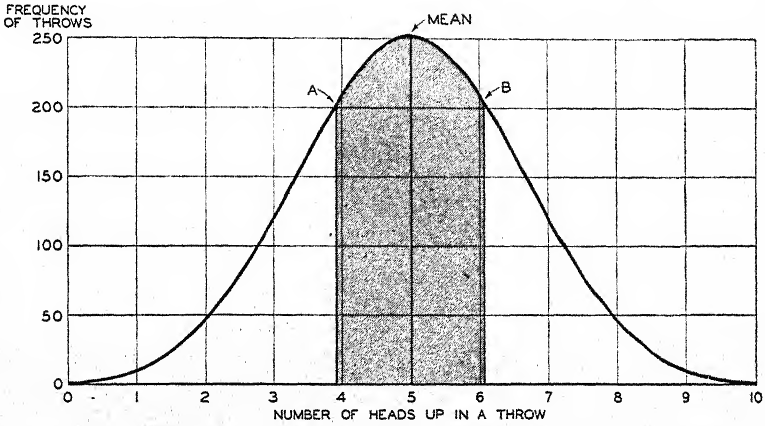


EXHIBIT 114.—Ordinate *A* is approximately two-thirds (.6745) of one standard deviation to the left of the mean (5), and ordinate *B* is the same distance to the right. The shaded portion represents one-half of the area under the curve, and is the area within which one-half of the items fall. The range of approximately two-thirds (.6745) of one standard deviation to the left and to the right of the mean is known as the probable error of the distribution. (In this case P.E. = ± 1.07; see Exhibit 115.) (Exhibit 114 is based upon the theoretical distribution tabulated in Exhibit 112.)

within 5.00 ± 1.07 or between 3.93 and 6.07. The method of computing S.D. and P.E. of this distribution is illustrated in Exhibit 115. (Again it should be appreciated that for purposes of simple illustration the coin series, which really is a discrete series, has been treated as a continuous series.)

Referring again to the curves of averages, mentioned on pages 237 and 238, if the dispersion in a group is small (that is, if the items are concentrated closely about the mean item and the curve is relatively high and narrow) the probable error will be small. And, conversely, if the dispersion is great (that is, if the items are spread out and the curve is relatively low and flat) the probable error will be large.

Probable error formulas have been developed for each of the statistical averages and coefficients that may be calculated from a sample. In

each case the probable error is the measure, in the sense here used, of the reliability of the given average or coefficient, in terms of the probable distribution of a large number of estimates, computed from similar samples, about the true value. A detailed or extended consideration of probable error is beyond the scope of this study, but it is important

Number of heads (class) (*a*)	Frequency of throws (*b*)	Deviations from mean (5) (*c*)	Deviations squared (*d*)	Product of frequency times deviations squared ($b \times d$)
0	1	−5	25	25
1	10	−4	16	160
2	45	−3	9	405
3	120	−2	4	480
4	210	−1	1	210
5	252	0	0	0
6	210	+1	1	210
7	120	+2	4	480
8	45	+3	9	405
9	10	+4	16	160
10	1	+5	25	25
Total..........	1,024	...	..	2,560

Substituting the proper figures from the above table in the formula for the standard deviation, we have:

$$\text{S.D.} = \sqrt{\frac{2560}{1024}} = \sqrt{2.5} = 1.58$$

$$\text{P.E.} = .6745 \times \text{S.D., or } .6745 \times 1.58, \text{ or } 1.07$$

EXHIBIT 115.—Calculation of the standard deviation and probable error of the theoretical frequencies presented in Exhibits 112 and 113. The above computation of S.D. is made by the usual process, but it will be noted that it differs from the illustration on page 220 as the above table is a frequency table and the squared deviations are multiplied by the corresponding frequencies.

to consider a few of the more common types. Some of the more important probable error formulas are listed at the top of the next page.[1]

[1] For the purposes of the present discussion, the well-known formula for S.D. used in Exhibit 115 is considered satisfactory, but it should be appreciated that while this formula is satisfactory for large samples, it is not adequate if the sample is small. Small samples are not so generally representative and corrections are necessary. In a case of a small sample, the best estimate of the standard deviation of the universe, σ, is given by

$$\sigma^2 = \frac{n}{n-1}S^2$$

where S is the standard deviation as found from the sample and n is the number of items in the sample.

1. P.E. of any distribution = 0.6745 S.D.
2. P.E. of the mean = $0.6745 \frac{\text{S.D.}}{\sqrt{N}}$
3. P.E. of the S.D. = $0.6745 \frac{\text{S.D.}}{\sqrt{2N}}$
4. P.E. of the coefficient of correlation (r) = $0.6745 \frac{(1 - r^2)}{\sqrt{N}}$

The explanation of the probable error of a distribution was included in the preceding discussion of Exhibits 114 and 115.

The probable error of the mean is used to determine the reliability of an arithmetic average computed from a given sample. It depends upon the value of the standard deviation and the number of items in the sample. In applying the above formula, let us assume that we are trying to determine the average weekly wage of 100,000 factory workers in a certain state on the basis of a sample of 1,000. Suppose that the average as determined from the sample is $20.00 per week. How much should the average obtained from this sample be expected to differ from the average of the entire 100,000 workers? Applying the above formula for $\text{P.E.}_{\text{mean}}$ (assuming the standard deviation of the data in the sample to be $2.00), we have the following:

$$\text{P.E.}_{\text{mean}} = .6745 \frac{\$2.00}{\sqrt{1{,}000}}$$
$$= \$.043$$

The average found in the above investigation, therefore, would be stated as $20.00 ± .043. This means that 50 per cent of the samples have a mean which has a value lying between the true value minus $.043 and the true value plus $.043. In other words, it is just as likely as not that the true value differs from $20.00 by at most $.043.

The probable error of the standard deviation is computed in a similar manner. Substituting the above weekly wage data in the formula we have:

$$\text{P.E.}_{\sigma} = .6745 \frac{\$2.00}{\sqrt{2 \cdot 1000}}$$
$$= .03,$$

which gives us a measure of reliability of our "measuring stick." It is interpreted in a manner analogous to P.E. of the mean. The probable error of the coefficient of correlation will be explained in the next chapter in connection with the coefficient of correlation.

Standard Error.—Another method of checking reliability is by means of the standard error. The standard deviation of a distribution of means

computed from samples is termed the *standard error of the mean* (σ_M). (Without the subscript σ refers to the standard deviation.) The standard deviation of a distribution of any other statistical measures computed from samples is the standard error of that particular measure. The *probable error* is equal to .6745 times the *standard error*.

The formula for the standard error of the mean (the standard deviation of the distribution of the means of samples) is:

$$\sigma_M = \frac{\sigma}{\sqrt{N}}$$

It will be noted that the computation is like that for P.E. previously described except that the factor .6745 is not used.

Significance of Probable Error and Standard Error.—The significance of the measures of reliability should be clearly understood. When we use a mean, standard deviation, or coefficient of correlation, which has been computed from a sample, we assume that it is applicable to a larger group or to the universe within certain limits. And the measures of reliability enable us to determine the limits within which there is a fifty-fifty chance that the results computed from the sample would not vary from the true value for the universe.

It should be clearly understood that probable error and standard error can be used to measure only error resulting from variations in random samples of the same universe. It should also be understood that in economic and business statistics the conditions of purely random sampling are rarely fulfilled. The probable error and the standard error can give no indication of variations in successive samples due to causes other than those arising from the process of simple sampling. Variations due to directed or controlled sampling, bias, or mistakes, cannot be checked by these methods of determining reliability.

The limitations of the above measures of error are so great when applied to business or economic data that they are not often directly applied. It is nearly always more satisfactory to make actual statistical tests of reliability or stability, by breaking up a given sample into significant parts and then comparing these parts and studying them for reasonableness in connection with tests based on outside data. The chief value of the study of probability, and measures of error, in business and economic statistics is the training that it gives in the analysis of samples and distributions. Study and practice in using or in attempting to apply the mathematical methods previously described in this chapter will enable one to appreciate much more clearly the problem of making other kinds of checks to test the reliability of a sample to see if it is satisfactory. In other words, while these mathematical methods are seldom applied

directly in business or economic problems, *indirect* applications are very commonly made and are of great value as background material to the practical statistician.

Questions and Problems

1. Why is it important to distinguish between occurrences due to chance and those due to directed causes?

2. What is the "normal curve of error"?

3. What is meant by "probability"?

4. Explain the law of chance, using coin tossing as an illustration.

5. How is Pascal's arithmetical triangle constructed and how is it used?

6. What is the "law of normal frequency"?

7. What are some of the important mathematical characteristics of the normal curve?

8. What is meant by "probable error"?

9. What is meant by "standard error"?

10. Discuss the significance of probable error and standard error.

11. Construct a binomial triangle (see Exhibit 111) so that you can determine coefficients to the twelfth power.

12. Toss 10 coins 1,024 times and compare results with the data in Exhibits 112 and 113.

13. Compute the probable error of the distribution of the data obtained in Problem 12 (see Exhibit 115). Present graphically as in Exhibit 114.

14. Suppose that you are trying to determine the average weekly wage of 50,000 factory workers in a certain industrial area on the basis of a sample of 1,000. Suppose also that the average determined from the sample is \$18.00 per week and that the standard deviation is \$1.60. What is the probable error of the mean?

15. What is the probable error of the standard deviation in Problem 14?

16. What is the standard error of the mean in Problem 14?

CHAPTER XII

ANALYSIS OF ASSOCIATED CHARACTERISTICS—CORRELATION

Business men are constantly studying a great number of relationships, such as the influence of population on the markets for their products, the degree to which advertising may be depended upon to increase sales, the relation of the price of cotton to the size of the crop, the variation of the production of workers with the length of their experience, or the relation between bank profits and the average balance maintained by depositors. Such analyses are practically always correlation analyses. As a matter of fact, business is a continual process of association or correlation, and the successful business man is an expert at correlation analysis, though it is not always in statistical form.

Correlation Defined.—If a department store manager finds that he sells many more umbrellas on rainy days than on bright days, he knows that there is "correlation" between the sales of umbrellas and rainy weather. However, he knows that as long as there is a possibility of rain at all, he will sell a few umbrellas even though the days are clear. He also knows that if there are twice as many rainy or cloudy days in one month as in another, he will probably not sell exactly twice as many umbrellas in the rainier month. But there will be some correspondence between sales of umbrellas and the number of rainy or cloudy days in the month, and this correspondence is spoken of as "correlation."

When one speaks of *correlation* between variables, one refers to the causal relationship between them. The changes in one variable may be the direct cause or effect of those in another, the changes in certain variables may be the effects of a common cause, some of the causes and effects may be mutually reactive, several independent causes may be operating, and so on, but there must be some kind of a causal connection if there is to be correlation. If one should find, for instance, that the variations in canoe production in Samoa corresponded for a period very closely with wool prices in Boston, it would not indicate correlation between the two series, unless it could be shown that there was a causal connection between them. In analyzing his problems, the business man appreciates that there are causal relationships between such series as population and markets, sales and advertising, price and production, or skill and experience. Just what is cause and what is effect may be diffi-

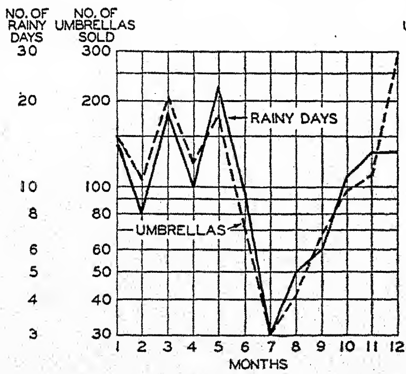
cult or impossible to determine, but the degree of correlation that results may be measured and analyzed.

Methods of Comparing Variables.—As time series occupy such a dominant position in business statistics, and as they are so often used as illustrations in the present chapter, it is necessary at the outset to understand clearly the two commonly used graphic methods of presenting time-series correlation. These and other methods will be considered again, in greater detail, later in the chapter, but a brief explanation of the

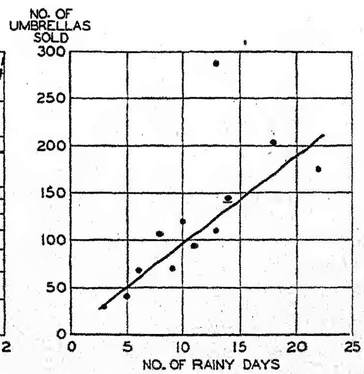
NUMBER OF RAINY DAYS AND UMBRELLAS SOLD

MONTH	NO. OF RAINY DAYS	NO. OF UMBRELLAS SOLD	MONTH	NO. OF RAINY DAYS	NO. OF UMBRELLAS SOLD
JANUARY	14	147	JULY	3	30
FEBRUARY	8	108	AUGUST	5	41
MARCH	18	203	SEPTEMBER	6	68
APRIL	10	120	OCTOBER	11	96
MAY	22	175	NOVEMBER	13	110
JUNE	9	70	DECEMBER	13	289

PART A.–TABLE OF DATA



PART B.–HISTORICAL CURVES



PART C.–SCATTER DIAGRAM

EXHIBIT 116.—Two common graphic methods of comparing variables in correlation analysis.

two kinds of correlation graphs will be given at this point in order that they may be used at once in explaining the different types of correlation.

One method that the above department store manager could use in studying the correlation between rainy days and umbrella sales is to plot the data as two historical curves as shown in Part *B*, Exhibit 116. (The numerical data of the two series are presented in Part *A*.) The movements of the two curves correspond very closely (which indicates a high degree of correlation) until the month of December is reached, when there is a wide divergence, obviously due to the effect of Christmas sales. As this method is the same as that previously used in presenting

time series in this book, it need not be discussed further at this point.

The other graphic method of showing correlation is known as the "scatter diagram." By this method, the correlation between rainy days and umbrella sales may be shown as in Part *C*, Exhibit 116, where the number of rainy days is plotted on the x-axis and the number of umbrellas sold is plotted on the y-axis. That is, to plot the data for January, find 14 on the rainy-day scale and plot this value opposite 147 on the umbrella scale (see Part *C*, Exhibit 116, in which this particular point is underlined). When the corresponding values of two series are paired in this manner, the relation between the two is indicated. The reader of Part *C*, Exhibit 116, will recognize rainy days as the independent causal factor, and the effect is shown in the dependent fluctuations of umbrella sales. In this example, it is clearly indicated that, as the number of rainy days per month increases, the sales of umbrellas tend to increase. (The trend line through the points helps to indicate this tendency.) However, one marked exception to this tendency appears on the chart, in that one dot (near the top of the graph) is far out of line, because of Christmas sales, as mentioned previously.[1]

Types of Correlation.—Correlation is described or classified in several different ways. Three of the most important are: (1) *direct* and *inverse;* (2) *linear* and *non-linear* (*curvilinear*); and (3) *simple, partial,* and *multiple.* These will now be discussed briefly.

1. The classification of correlation as *direct* or *inverse* is based upon *direction of change.* If, as one variable increases (or decreases), the other changes in the same direction, the relation, correlation, or association between the two statistical series is said to be *direct.* Thus, employment increases when general business activity increases, and decreases when business declines (see Part *A*, Exhibit 117). If, as one of the variables increases, the other decreases—that is, if the changes in the two variables are in opposite directions—the correlation between two statistical series is said to be *inverse.* To illustrate, business failures decrease in frequency as business improves, and increase as business declines (see Part *B*, Exhibit 117).

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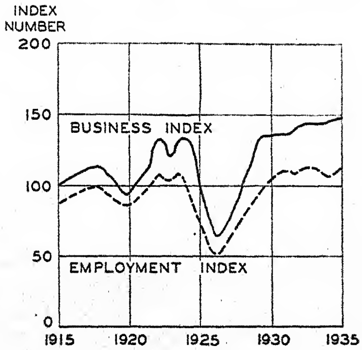
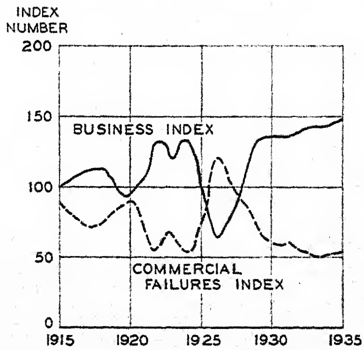
[1] The two methods discussed above are sometimes designated as *secondary* and *primary* correlation methods, respectively. The first method (illustrated in Part *B*, Exhibit 116) is a secondary correlation method of determining the association or relationship between two variables. Their relationship is determined by first relating each variable to a common third variable (time, in this case). In the second method (illustrated in Part *C*, Exhibit 116) the variables are directly paired, and the relationship is determined irrespective of any relationship which may exist between each and a common third factor. Thus, in secondary correlation, the relationship of the two variables is determined through the relationship of each variable to a third common causal factor, while in primary correlation the direct causal relationship between the variables is measured.

2. Whether correlation is *linear* or *non-linear* (*curvilinear*) depends upon the *constancy of ratio of change.* If the amount of change in one variable tends to bear a constant ratio to the amount of change in the other variable, then the correlation is said to be *linear.* Thus one finds, ordinarily, that if income is doubled, wealth also is doubled; or if wealth is doubled, income is doubled (see Part *A*, Exhibit 118). This ratio of increase tends to remain constant irrespective of the amount of wealth or the amount of income. If the amount of change in one variable does not bear a constant ratio to the amount of change in the other variable, then the correlation is said to be *non-linear,* or *curvilinear.* Thus, if one should double the amount of rainfall, the amount of wheat per acre is not necessarily doubled (see Part *B*, Exhibit 118). In most elementary work, the assumption is made that the scatter diagram tends to form a



A.—Direct correlation.



B.—Inverse correlation.

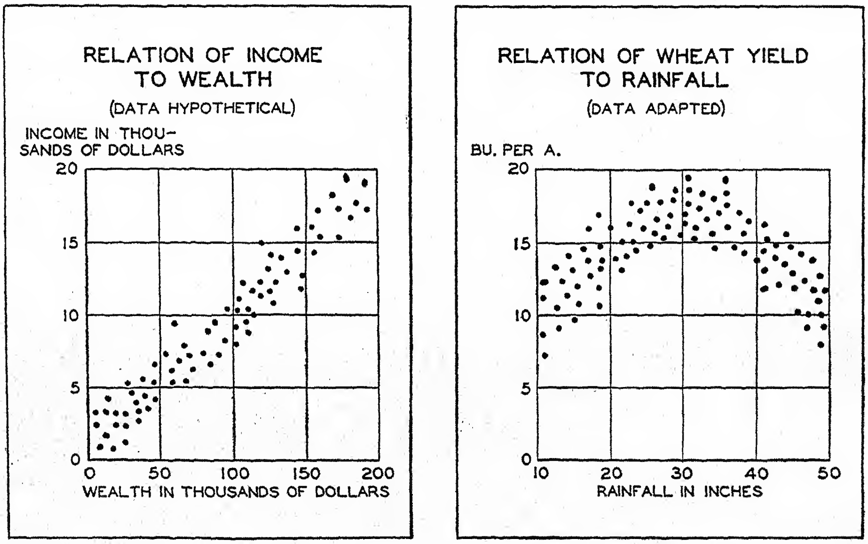
EXHIBIT 117.—Direct and inverse correlation illustrated on a secondary basis. (*Hypothetical data.*)

straight line; that is, that the correlation is linear. The methods for measuring non-linear correlation are much more complicated than those for measuring linear relationship.

It should be pointed out that there are very few cases in which the relation between the associated series is linear throughout the entire possible range of the series. Almost all statistical series have a limiting value below which they do not go or above which they do not rise, and as the paired values of the series fall nearer and nearer to the limiting value, the relation between the correlated series becomes non-linear. Thus, in Part *A*, Exhibit 118, showing the correlation between wealth and income, there are some people who have incomes, but who are entirely without wealth in the ordinary sense of the term. As a consequence a "line of best fit" would tend to have a greater slope near the lower left corner. For practical purposes in business statistics a good rule is: Unless there is a fairly well-defined curve relation between the paired values, within their ordinary and expected range, the correlation, if any, should be

presumed to be linear. However, if there is positive evidence that the relation is not linear, it is not correct to apply linear-correlation methods.

3. The distinction between *simple, multiple,* and *partial* correlation is based upon the *number of series correlated.* When but two attributes or characteristics are considered, the problem is one of *simple correlation.* If more than two series are considered, the problem is one of *multiple* or *partial correlation.* If the correlation is considered as if it included more than two variables, it is said to be *multiple correlation.* If we recognize more than two variables, but consider only two variables to be influencing each other, the problem is one of *partial correlation.*



A.—Linear correlation. *B.*—Curvilinear correlation.

EXHIBIT 118.—Simple examples of linear and non-linear (curvilinear) correlation.

To illustrate the problem of *multiple correlation* we might consider the association between the yield of wheat per acre and both the amount of rainfall and the average daily temperature. Or, the earnings of factory workers might be considered in relation to the amount of output, the amount of waste, and the length of experience with the company. In such problems the association is considered as a "combined" correlation, not as the relationship between individual series.

The problem of *partial correlation* is that of determining the association or correlation between two variables when certain other influencing variables are held constant, or when corrections and allowances are made for these other influencing variables. Thus, in the wheat problem mentioned above, if we limit our correlation analysis of yield and rainfall to periods when a certain average daily temperature existed, or if we treat

the problem mathematically in such a way that changes in temperature are allowed for, the problem becomes one of partial correlation.

In this discussion, only *simple correlation* will be considered.

Associated characteristics or correlation will now be studied in three major parts: (1) those processes having to do with the logical analysis of correlation; (2) those processes by which the amount or degree of correlation between data is approximated by graphic and tabular methods; and (3) those processes by which more precise mathematical measurements are made of the degree of correlation.

LOGICAL ANALYSIS OF ASSOCIATED CHARACTERISTICS

In logical analysis of associated characteristics in business statistics, one seeks to explain the correspondence between the series by means of some principle or theory of business. Or, if such principles or theories do not adequately explain the correspondence, one seeks to formulate such hypotheses as will give a consistent explanation of the co-relationship.

As was pointed out previously, simple comparison sometimes suggests that fundamental relationships may exist between the variables, but an understanding or appreciation of the degree and nature of these relationships requires more than simple comparison; it requires a *correlation* analysis. The correlation analyses that are constantly being made by men in all types of business are ordinarily simply reasoned out with but little use of graphic or mathematical methods. The previously mentioned department store manager, for instance, knows in a general way about what the relation is between rainy days and umbrella sales without referring to charts or figures. Graphic and mathematical methods, however, will greatly improve the business man's knowledge of the correlations in his business, but they still must be accompanied by good judgment. These more accurate methods are becoming more and more necessary as competition increases. In all business correlation studies, logical analysis of the associated characteristics, which seeks to explain why certain relationships exist, is a fundamental requirement if the results are to be reliable and usable.

Simple Cause and Effect Analysis.—Let us first imagine that we have two ideal statistical series, of which one is the only cause and the other the only effect. Let us further imagine that the cause operates independently; that is, that it is in no way influenced by the effect. Correlation analysis in such a problem would be simple, but such ideal relations do not exist. However, the *principal* cause may account for nearly all of the observed effects, as in the umbrella rainy-day correlation previously discussed. The logical analysis then consists of setting up an

hypothesis, theory, or principle by which the relation of simple cause and effect is demonstrated between the two series.

When the correspondence between unemployment and building activity, for instance, is analyzed, a somewhat more complicated situation is found. Thus, it is quite certain that the amount of unemployment is affected by factors other than building activity, such as the introduction of machinery, importation of cheap labor, and increase in efficiency of laborers already employed. Moreover, it is certain that building activity causes changes in a large number of series other than unemployment.

Association Involving Common Causes.—In the last illustration above, the associated series (unemployment and building activity) contained but one of the causes (building activity), and it was pointed out that many causes may be operating to give a certain measured effect.

In certain problems of correlation, it is definitely known that neither of the two series is a cause, but that both are effects of a common cause or causes. In such problems, an additional complication enters, because it is necessary to include in the logical analysis not only the common cause, but also the causes which affect one series but which do not affect the other. Thus, suppose the problem involves the association between unemployment conditions and bank reserves. There would be some factors which would affect bank reserves but which would not affect unemployment at all, or which affect unemployment in a decidedly different manner than bank reserves. Likewise, there would be factors which would affect unemployment but which would not affect bank reserves at all, or which would affect bank reserves very differently than employment. Importation of cheap labor might have little or no effect upon bank reserves, but it might have a decided effect upon unemployment. Bank laws requiring a definite ratio of reserves to deposits would not have much bearing upon unemployment.

It frequently happens that a common cause will affect one of the series before it affects the other. For instance, a certain manufacturer of heating equipment has found that the general conditions in the building industry correlate very closely with his volume of business. But, he also has found that his business increases about three months after building permits increase, and decreases about the same length of time after building permits decrease. Such relations between two time variables make it possible to *forecast* the movements of the series moving latest in point of time.

Mutually Reactive Series.—Finally, we come to the most common situation in the analysis of associated characteristics in business, namely, that in which some of the causes and effects are mutually reactive, in which there are some common causes and some common effects, and in

which there are some independent causes and some independent effects. Of the causes in general, some are known, while some are unknown. Of the effects, some are known, while some are unknown. Some of the causes and some of the effects are not measurable, or, at least, not reducible to such form that they can be analyzed statistically. Exhibit 119 illustrates the situation diagrammatically.

It may appear to the reader that such a problem as that indicated in Exhibit 119 is impossible to solve. Of course a perfect solution is impossible. But it is representative of the problems that the business man is called upon continually to solve as best he can. In his logical analysis, the business man sorts out the principal causes and effects,

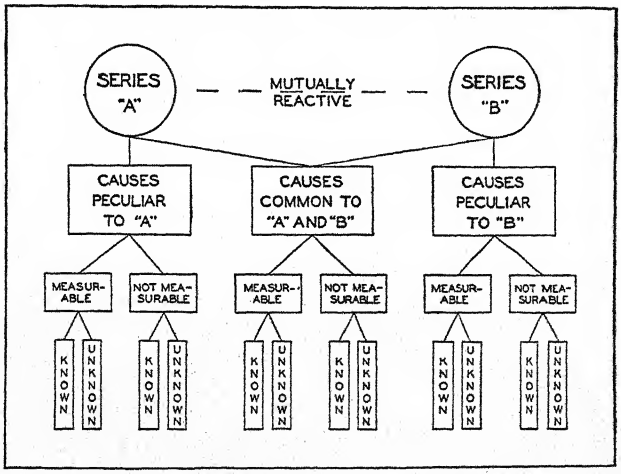


Exhibit 119.—Illustration of a common problem in the analysis of correlation in business.

and studies their relationships in a practical way. The more he appreciates the theory of correlation, the better will be his practical analyses of such business problems.

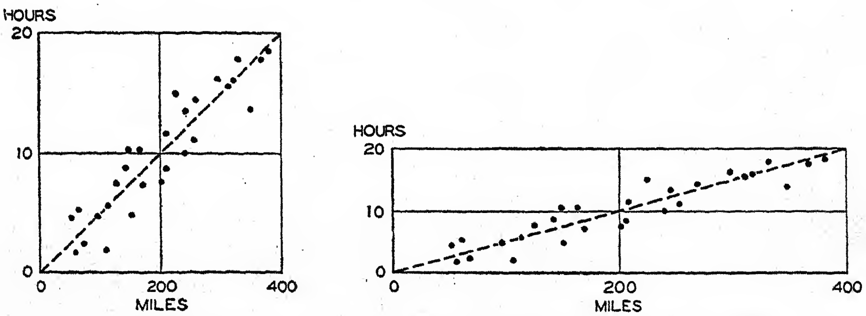
Importance of Logical Analysis of Association.—Logical analysis of association or correlation is of great value as a supplementary check upon the mechanicalness of mathematical methods (which will be described later). It possesses a distinct advantage over mathematical analysis in that while a small number of cases may destroy the reliability of some phases of mathematical analysis, such paucity of statistical information does not operate to the same extent to destroy the usefulness of logical analysis. In logical analysis of association, it is possible to distinguish between causes and effects of major and minor importance, and to discriminate between primary and secondary, and between direct and indirect causation. This characteristic of logical analysis makes it peculiarly useful where multiple and partial correlations are involved, a

field in which our present-day mathematical methods are none too well developed and understood.

Logical analysis should always be employed: (1) to demonstrate the likelihood of correlation between statistical series; and (2) to explain in a consistent manner the association which appears to exist.

GRAPHIC AND TABULAR ANALYSIS OF CORRELATION

Graphic methods are very commonly used in business correlation problems. On the whole, carefully handled and skillfully interpreted graphs have certain advantages over mathematical methods of determining correlation in the usual business problems. The elements of judgment and special knowledge of conditions can be more easily introduced in studying correlation graphically. Mathematical correlation is often much too rigid for the data at hand.



A.—Good proportions. *B*.—Distorted proportions.

EXHIBIT 120.—Relations of scales for correlation charts.

The Scatter Diagram.—Scatter diagrams, such as presented in Exhibits 116 and 118 earlier in this chapter, represent a widely used graphic form for showing the correlation between two statistical series. In this type of diagram, the two series are *paired*, and the paired values are then plotted in the same manner as the points were plotted on the graphs discussed in Chapters V and VI.

In preparing a scatter diagram, the horizontal and vertical scales should be so chosen that adequate space is provided for the range of the data of both series. In Exhibit 120, Part *A* illustrates the use of approximately equal scales, while Part *B* illustrates the use of different ranges. The dashed line or "line of best fit" through the dots in Part *A*, Exhibit 120, approximates an angle of 45 degrees and the dots are distributed along a narrow band which follows this line.[1] In Part *B* the same data

[1] Such "lines of best fit" may be mathematically computed by the method of least squares. In Exhibit 120, however, the lines were drawn in free hand. This method of "fitting the line by inspection" is sufficiently accurate for most practical

are scattered along a narrower band and a lesser slope. The narrower band and lesser slope result because the scale on the x-axis is relatively large in relation to the scale on the y-axis.

A number of methods are available by which it is easy to make a straight line of best fit (if there is one) approximate a 45-degree angle. Probably one of the simplest is that by which the ratio between the scales is made inversely proportional to the ratio of the ranges of the data. That is, if the range of the x data is 400 building permits and the range of the y data is $5,000,000, then the scale for x would be to the scale for y in the ratio of 5,000,000:400, or 12,500:1. A convenient approximation is usually satisfactory, which in the preceding case might be 10,000:1, so that the same linear distance would represent $10,000 for y as represented 1 building permit for x. If the data include items of unusual size, it may be better to make the x- and y-scales inversely proportional to the differences between the upper and lower quartiles. Thus, if the difference between the upper and lower quartiles on the x-axis is $20,000,000 of bank debits, and that on the y-axis is $25,000,000 of bank clearings, then the x-scale to the y-scale would be (inversely) 25,000,000:20,000,000, or 5:4. That is, the same distance would indicate $4,000,000 on the x-axis as would indicate $5,000,000 on the y-axis.

Scatter Diagrams May Indicate Both the Degree and the Type of Association.—The extent to which the paired data points tend to fall in a narrow band gives an indication of the degree of association between the two statistical series. In the charts in Exhibit 121, the first two (A and B) show a high degree of correlation because the points fall within a narrow band, and the last one (C) shows low or no correlation because the points are so widely scattered.

The direction of the slope of the line of best fit (if the data lie fairly close to it) gives an indication of the type of association. If the correlation is *direct*, the data will be distributed in a narrow band running from the lower left-hand corner to the upper right-hand corner of the diagram (see Part A, Exhibit 121). If the correlation is *inverse*, the data will be distributed in a narrow band running from the upper left-hand corner to the lower right-hand corner of the diagram (see Part B, Exhibit 121).

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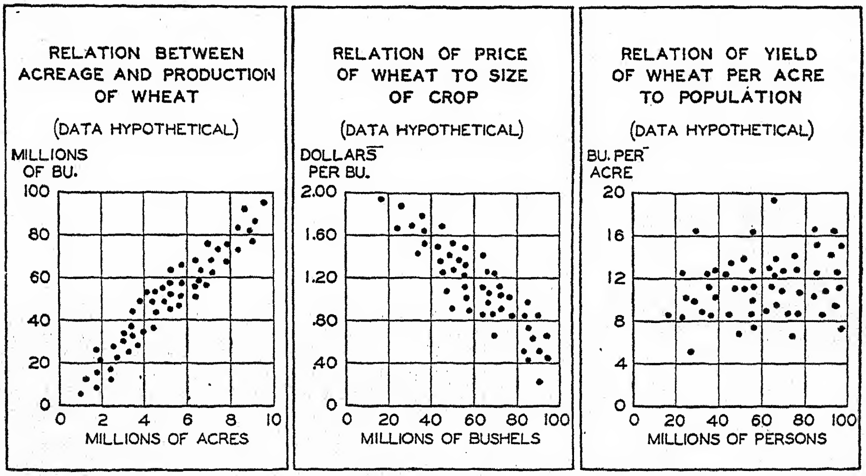
business problems, and it facilitates a judicious weighting of the observations. There are two ordinary lines of best fit or "regression lines," one of y on x, and one of x on y. The dashed line in Exhibit 120 lies halfway between them. The ordinary least squares line of best fit (see pages 297 and 298 for computation method) when y (hours) is the dependent variable is the one which minimizes the sum of the squares of the vertical y (hour) deviations. Note that in Exhibit 126, page 259, the lines are generalized to read x on y as well as y on x. To give the best estimates of y from x, the lines should be so drawn as to minimize the sums of the squares of the vertical deviations. (See also footnote on page 259 for regression line formula.)

The use of the scatter diagram as a means of estimating the degree of correlation possesses the very desirable advantage that it is no more influenced by items of extreme size than by items of usual or average size. Some of the statistical methods of a mathematical nature, however, are decidedly influenced by extreme items. In fact, an item of extreme variation may have such influence as to give a mathematical expression of low association when there is really high association present and *vice versa*.

The Correlation Table as a Means of Estimating Association.—If, instead of recording the positions of the paired x and y values by means of a dot, the number of values falling within certain scale limits



A.—High direct correlation. *B.*—High inverse correlation. *C.*—Low or no correlation.
EXHIBIT 121.—Scatter diagrams indicating type and degree of correlation.

were counted and recorded, we would have what is known as a *correlation table.* The choice of scales for the construction of this table is exactly the same as for the construction of the scatter diagram. Three correlation tables are presented in Exhibit 122. Part *A* shows a high degree of direct correlation; Part *B* shows a high degree of inverse correlation; and Part *C* shows very low or no correlation.

The interpretation of the correlation table is the same as the interpretation of the scatter diagram. It has an advantage to those who are numerically minded, in that it gives the number of cases within the selected class intervals. It has a disadvantage, as compared to the scatter diagram, in that it does not indicate accurately the position of the actual points except as falling between certain limits. In fact, it is necessary to assume that all the points indicated by the frequencies fall at the intersection of the means of the class limits.

Correlation of Historical Curves.—If two historical curves are compared and it appears that one of the series tends to move in a definite

RELATION BETWEEN TOTAL ACREAGE AND TOTAL PRODUCTION OF WHEAT
(Data hypothetical)

Production (in millions of bushels)	Acreage (in millions of acres)				
	0 to 2	2 to 4	4 to 6	6 to 8	8 to 10
80 to 100	..	..	..	..	5
60 to 80	..	..	2	8	2
40 to 60	..	2	12	4	..
20 to 40	2	9	1	..	..
0 to 20	4	2	..	..	..

A.—High direct correlation.

RELATION OF PRICE OF WHEAT TO SIZE OF CROP
(Data hypothetical)

Price (in dollars per bushel)	Crop (in millions of bushels)				
	0 to 20	20 to 40	40 to 60	60 to 80	80 to 100
1.60 to 2.00	1	5	1	..	..
1.20 to 1.60	..	2	10	3	..
.80 to 1.20	..	..	5	10	3
.40 to .80	..	..	..	1	7
0 to .40	..	..	..	..	1

B.—High inverse correlation.

RELATION OF YIELD OF WHEAT PER ACRE TO POPULATION
(Data hypothetical)

Yield (in bushels per acre)	Population (in millions of persons)				
	0 to 20	20 to 40	40 to 60	60 to 80	80 to 100
16 to 20	..	1	1	1	2
12 to 16	..	3	4	6	5
8 to 12	1	7	6	7	5
4 to 8	..	1	2	1	1
0 to 4	..	..	..	..	..

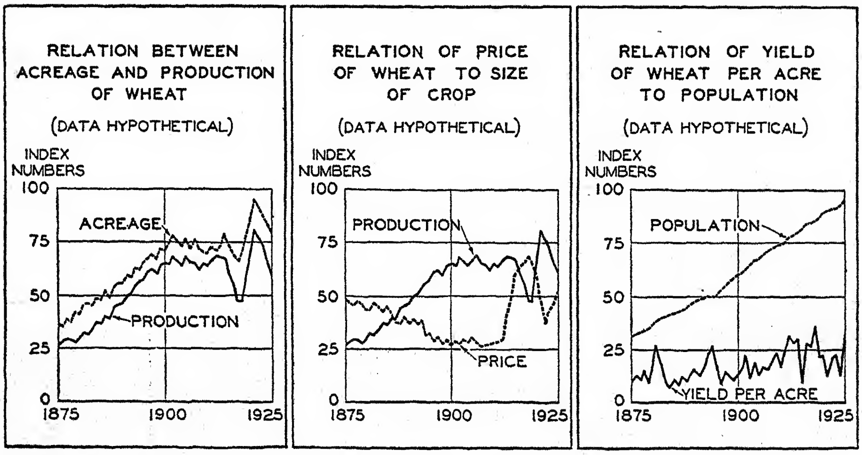
C.—Low or no correlation.

EXHIBIT 122.—Illustrations of simple correlation tables. (Data same as shown on scatter diagrams in Exhibit 121.)

relation to the other, then the two curves are said to show correlation. Such correlation studies of time series represent the most common form

of correlation in business. The charts shown in Exhibit 123 indicate how such curves appear when they are plotted as ordinary time curves. Part *A* shows high direct correlation, Part *B* shows a high degree of inverse correlation, and Part *C* shows very low or no correlation. Exhibit 124 indicates a high degree of correlation in the major movements of building activity in seven sections of the United States.

In making correlation analyses of historical curves, great care should be taken that instances of close correspondence, which do not represent actual correlation, be recognized. Spurious correlation is often found between two time series, and frequently it can be made very high by



A.—High direct correlation. *B.*—High inverse correlation. *C.*—Low or no correlation.

EXHIBIT 123.—Historical curves indicating type and degree of correlation.

wrongly introducing the element of time lag, particularly when the series are short.[1]

Certain causes take effect on some series much more slowly than on others. In such instances it is necessary to correct for lag before attempting to determine the degree of correlation. Another important correction that often must be made is that for price changes. These corrections will now be discussed.[2]

Graphic Determination of Lag.—There are two commonly used graphic methods of determining the amount of lag, when it exists, between

[1] See M. C. Rorty, "Statistics and the Scientific Method," Presidential Address, Ninety-second Annual Meeting, American Statistical Association, December 31, 1930, published in the *Journal of the American Statistical Association*, pp. 1–10, March 1931.

[2] Correction of time series for trend and seasonal variation in correlation analysis will be discussed in Chap. XVI (pp. 341 and 342) after statistical methods of determining and eliminating long-time trend and seasonal variation have been explained.

two correlated time series. One is by means of the scatter diagram; and one is by means of ordinary historical curves.

To determine the amount of lag by the scatter diagram, it is necessary to construct scatter diagrams with the data paired with different lag. Thus, one might construct one diagram with no lag at all; that is, with the Januarys paired together, Februarys paired together, etc. Another might be constructed with January of one series paired with February of the other, etc., which would be one month lag. Still other scatter diagrams might be constructed with two, three, four, etc., months lag.

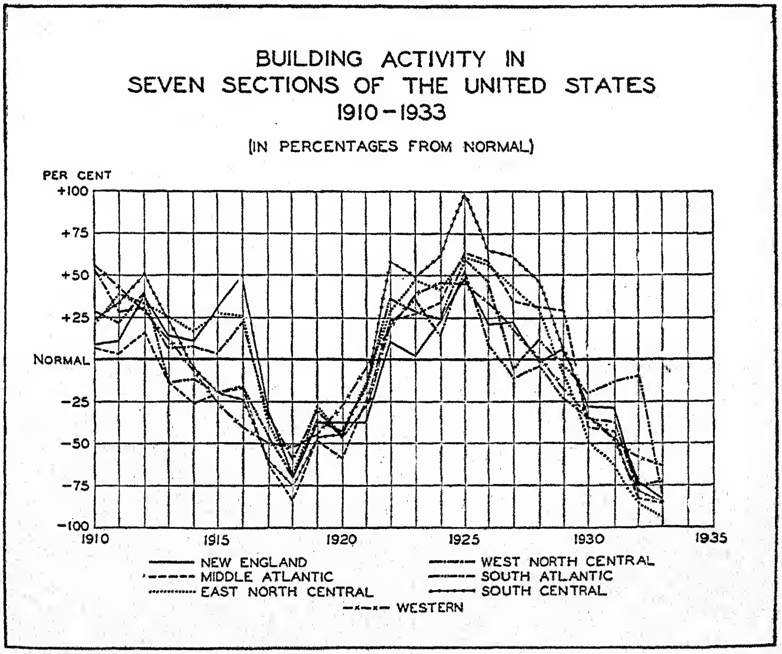
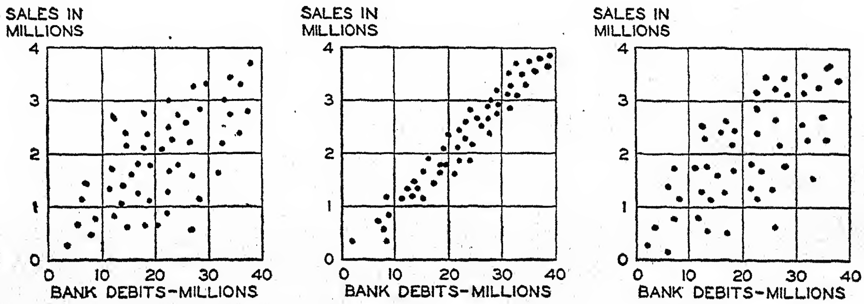


EXHIBIT 124.—Correlation of a group of time series.

The scatter diagram which most closely groups the data along a straight line indicates the lag. Thus, in the three charts in Exhibit 125, the data for one and three months lag are not so closely concentrated as the data for two months lag. Consequently, it is said that the series has a two months lag. With this method it is never possible to determine the exact amount of the lag closer than the time interval of the data. Thus, with monthly data, one month is the closest possible determination.

To determine the amount of lag by means of ordinary historical curves, it is necessary first, to make the proper corrections as if for purposes of correlation only, and second, to plot the two curves (to the same horizontal scale) on two separate sheets of paper (translucent paper is preferred).

With the two curves on separate sheets of paper, they are held before a light sufficiently strong to make both curves appear fairly clear. Next, the curves are moved back and forth horizontally until the best fit is obtained and the amount of displacement on the x-scale is noted. The amount of this displacement is the amount of the lag. Thus, assuming that speculation moves ahead of general business, the monthly data representing the two series are plotted, and then the curves are shifted until they come closest to coinciding with each other. The amount of displacement is the lag. This method gives a somewhat more accurate estimate of lag than does the scatter diagram, since the lag can be estimated to the fraction of a month. Although neither method can be depended upon for precise measurement, especially where the degree of correlation is not high, both are accurate enough for most practical business purposes.



A.—One month lag. *B.*—Two months lag. *C.*—Three months lag.

EXHIBIT 125.—Determining lag with scatter diagrams.

Correcting for Price Changes in Correlating Time Series.—If two series are not affected in the same way by *prices* or *changing monetary values*, a correction must be made before correlating them or an incorrect idea of the correlation may be gained. Thus, suppose that the value of pig iron is correlated with the tonnage of steel over a period of years, or that the volume of bank clearings is correlated with the number of men employed. If the series containing the price elements are not corrected for changes in the value of the dollar, the apparent degree of correlation may be very inaccurate. The customary method of correcting such series is to divide through by the proper index numbers of prices. Care should be exercised, however, to use the *proper* index.

Estimating from a Scatter Diagram.—If a scatter diagram is constructed for the purpose of estimating the degree of association between two statistical series, it is possible to make a rough estimation of one of the variables from a known value of the other, by a purely empirical process. The first step after constructing the scatter diagram is to draw,

as accurately as possible, a line of best fit. Part *A*, Exhibit 126, illustrates such a scatter diagram with a straight line of best fit drawn in free hand.[1]

To make an estimate from Part *A*, Exhibit 126, suppose that the amount of bank debits in a given week is $30,000,000, and we desire to estimate the corresponding bank clearings. A line is drawn from $30,-000,000 on the *x*-scale (bank-debit scale) parallel to the *y*-axis until it

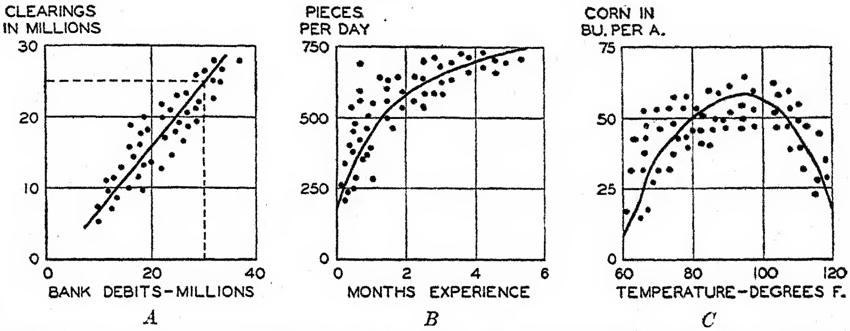


EXHIBIT 126.—In Part *A*, estimation is easy throughout the range. In Parts *B* and *C*, it is very difficult in portions of the curves. (*Data hypothetical.*)

intersects the line of best fit. Then a line is drawn horizontally from this point of intersection to the *y*-scale (bank clearings). The desired

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[1] In order to estimate the magnitude of one variable from a known value of the other, the line of best fit may be determined mathematically by the computation of the "line of regression." This line is the line of typical relationship of values of one variable (y) upon values of the other variable (x). By computing the line of regression, the probable or typical value of the one variable (y) can be read as the ordinate corresponding to the abscissa which represents the given value of the other variable (x).

The slope of the line of regression (when it refers to the relationship of two variables only and when the line of relationship is a straight line) may be computed by determining the regression coefficient (b) of the one variable upon the other (y upon x) by the formula

$$b = r\frac{\sigma_y}{\sigma_x}$$

This formula gives the slope of the line of regression, and the procedure for the determination of the points on the line is similar to the computation of the ordinates of trend in the method of least squares after the trend increment has been determined. The regression coefficient is applied to the average of the y-values in the same manner as the trend increment is applied in the computation of the line of trend.

It is also possible to determine by formula a line of "multiple" regression, which can be used to estimate an unknown variable that is dependent upon two or more known variables. This line shows the characteristic relationship between the two or more causal factors or variables, and the one result or dependent variable. For example, it might show the typical relationship between rainfall, sunshine, and temperature on the one hand, and the size of the cotton crop on the other.

estimate of the bank clearings is thus found to be $25,000,000. The value for any other estimate would be found in the same way.

It is possible to estimate one of the variables from the other in cases where the data do not approximate a straight line, although the accuracy of such an estimate is not always so satisfactory as where the relation is linear.[1] Thus, in the last two examples given in Exhibit 126, it is difficult to estimate the y-variable from certain values of the x-variable. In Chart B, the number of pieces change so rapidly during the first month that it is difficult to make an estimate during the period. In Chart C,

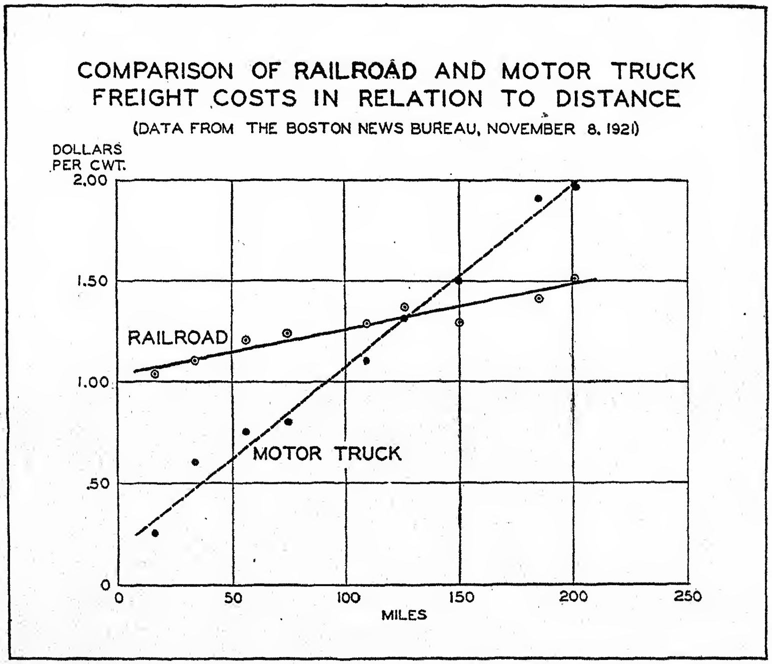

EXHIBIT 127.—Graphic comparison of correlations.

yield is difficult to estimate between 60 and 70 degrees, and between 110 and 120 degrees, because it (the yield) changes so rapidly. If, in this chart (Exhibit 126, Part C), one should wish to estimate the values of x from y (which is the reverse of the customary process as previously mentioned) he would find two values for x, each of which, so far as the chart is concerned, is equally true. In both of the charts the dependent variable has been plotted along the y-axis.[2]

[1] It is important that the "causal" or independent variable, if it is known, be plotted along the x-axis. This has come to be the custom in making scatter diagrams. Estimation is not always feasible from the dependent or "effect" variable.

[2] See footnote on p. 252.

Comparison of Correlations.—A practical example of graphic comparison of correlations is presented in Exhibit 127, which shows the relation between cost and distance for both railroad and motor-truck freight. By referring to this chart, one sees that according to the sample there was a tendency for truck transportation to be cheaper than railroad freight transportation up to about 125 miles, but for greater distances railroad freight costs were the lower.

MATHEMATICAL MEASUREMENT OF CORRELATION

The most common mathematical expression of the degree of correlation is known as the *coefficient of correlation.* It is customary to use the letter r as the designation of this coefficient. The common coefficients of correlation fluctuate between two numerical limits, $+1$ and -1. The limiting values, $+1$ and -1, represent perfect direct and perfect inverse correlation, respectively. Zero represents complete absence of correlation; a value of r between zero and $+1$ indicates a degree of direct correlation, while a value between zero and -1 indicates a degree of inverse correlation.

Many methods have been developed to give a mathematical expression of the degree of association or correlation between two or more series of quantitative data. Only one of the most commonly used of these methods (the Pearsonian) will be discussed in this text.

The Pearsonian Coefficient of Correlation.—If two statistical series, when plotted on a scatter diagram, *tend to approximate a straight line,* then the most widely used mathematical method of measuring the degree of association or correlation between the series is what is known as the *Pearsonian coefficient of correlation.*

The formula for computing Pearsonian r is:

$$r = \frac{\Sigma xy}{N\sigma_X\sigma_Y},$$

in which x and y are the deviations from the means while σ_X and σ_Y are the standard deviations of the X- and Y-series. N has the usual meaning.[1]

[1] A large number of formulas or transformations of this same formula have been proposed for expressing the calculation of the Pearsonian r, but the one given is the most common. Since

$$\sigma_X = \sqrt{\frac{\Sigma x^2}{N}} \text{ and } \sigma_Y = \sqrt{\frac{\Sigma y^2}{N}},$$

the formula given above may be expressed

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2}\sqrt{\Sigma y^2}} \text{ or } \frac{\Sigma xy}{\sqrt{\Sigma x^2 \cdot \Sigma y^2}}.$$

If the data are expressed in terms of their standard deviations, r becomes the

The steps in the calculation of r by this method may be summarized as follows:

1. Set up the data in two columns, which we shall call X and Y (see Exhibit 128).
2. Find the arithmetic mean of each column.
3. Find the differences of the individual items from the mean by subtracting the mean from the items, and enter these differences, with the proper sign, in two new columns headed x and y.
4. Square the individual differences set up in each of the columns x and y and enter in two additional columns headed x^2 and y^2.
5. Multiply each pair of differences together (the items in columns x and y found in Step 3), and enter in column xy. Then find the total, taking the signs into consideration. (This total, 1,156 in Exhibit 128, is the numerator of the formula.)
6. Divide the total of the x^2 column (496) by the number of items (11) and extract the square root (6.7) to get the standard deviation of X. Then repeat the process with the y^2 column to get the standard deviation of Y (19.1).
7. Multiply together the number of items (11), the standard deviation of X (6.7), and the standard deviation of Y (19.1). (This is the denominator of the formula.)
8. Divide the total obtained in Step 5 (1,156) by the product obtained in Step 7 (1,407.6). The answer (.82) is the coefficient of correlation.

Probable Error of the Coefficient of Correlation.—Coefficients of correlation are ordinarily computed from samples. The above coefficient, for instance, was computed from a sample time period of 11 years (Exhibit 128). When a sample is used as the basis for statistical analysis, it is important from an interpretative point of view to inquire into the degree of reliability which may be present in the results. As pointed out in the preceding chapter, the reliability insofar as it depends on the conditions of random sampling can be determined by computing the *probable error*.

The probable error of the coefficient of correlation is computed by the formula:

$$\text{P.E.}_r = .6745\frac{1 - r^2}{\sqrt{n}},$$

in which r is the coefficient of correlation and n is the number of pairs of items.

Carrying out the computation of the probable error, assuming a coefficient of correlation of .90 computed from a sample of 25 pairs of items we have:

$$\text{P.E.}_r = .6745\frac{1 - .90^2}{\sqrt{25}} \text{ or } .03.$$

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average product of such deviations, or

$$r = \frac{\Sigma x'y'}{N},$$

x' and y' being the deviations of the respective variables from their means reduced to terms of their standard deviations.

Any advanced textbook on statistical methods will give other formulas, and other ways of expressing the above formulas.

According to the foregoing computation, therefore, there is a 50:50 chance that the value of r is located within $\pm .03$ from the true correlation coefficient for the universe. A complete statement of this coefficient of correlation (with its probable error), then, would be $.90 \pm .03$.

Year	Number of building permits (thousands) X	Value of building permits (millions of dollars) Y	Deviations from mean of numbers x	Deviations from mean of values y	Deviations from mean of numbers squared x^2	Deviations from mean of values squared y^2	Deviations multiplied together xy
1927	21	41	−10	−23	100	529	+230
1928	18	34	−13	−30	169	900	+390
1929	23	38	− 8	−26	64	676	+208
1930	34	67	+ 3	+ 3	9	9	+ 9
1931	36	68	+ 5	+ 4	25	16	+ 20
1932	38	84	+ 7	+20	49	400	+140
1933	38	76	+ 7	+12	49	144	+ 84
1934	36	72	+ 5	+ 8	25	64	+ 40
1935	32	99	+ 1	+35	1	1,225	+ 35
1936	33	67	+ 2	+ 3	4	9	+ 6
1937	32	58	+ 1	− 6	1	36	− 6
Total	341	704			496	4,008	1,156
Average	31	64					

$$\sigma_X = \sqrt{\frac{496}{11}} \text{ or } 6.7 \qquad \sigma_Y = \sqrt{\frac{4{,}008}{11}} \text{ or } 19.1$$

Then, since $\Sigma xy = 1{,}156$, $N = 11$, $\sigma_X = 6.7$, and $\sigma_Y = 19.1$, substituting in the formula,

$$r = \frac{\Sigma xy}{N\sigma_X\sigma_Y}$$

we have

$$r = \frac{1{,}156}{11 \cdot 6.7 \cdot 19.1} = .82$$

EXHIBIT 128.—Calculation of the Pearsonian coefficient of correlation.

The measures of probable error can be properly used, however, only when the three following conditions exist: (1) The whole of the data must approximate a normal frequency curve (bell-shaped curve). (2) The statistical measure for which the probable error is computed must have been calculated from a sample. (3) The sample must have been selected in an unbiased manner, and the individual items must be independent. Since these conditions are seldom even approximated in business data, especially in time series, and since the significance, if any, of probable

error computed from time series is not well understood,[1] the degree of reliability of a statistical result or coefficient in most business data must be determined largely, if not entirely, on the basis of exterior tests of reasonableness which are often of a non-statistical character.[2]

Interpretation of the Coefficient of Correlation.—General rules for interpreting the size of the coefficient of correlation are more or less arbitrary, but are of considerable aid to the beginning analyst. For general work the following are suggested (it being understood that the coefficient to be reliable must be at least 6 $P.E._r$ in all instances and that the results of other tests are satisfactory):

1. If the coefficient is greater than .95, there is a high degree of correlation between the variables and one of them may be quite accurately estimated from a known value of the other.
2. If the coefficient is greater than .75, but less than .85, there is a decided amount of association between the two series, and one of the variables may be estimated fairly well from a known value of the other.
3. If the coefficient is greater than .40, but less than .60, there may be a fair degree of association between the two series, but any estimate of one variable from a known value of the other would ordinarily be of but little practical value.
4. If the coefficient is less than .35, there is but little association present between the two series, and one of the series cannot be used as the basis for estimating the value of the other.

These general rules, of course, are arbitrary, and for this reason gaps were left between the general zones.

In interpreting the coefficient of correlation, with regard to the extent to which the coefficient is subject to the variations of a chance character, the usual statistical practice requires that the size of the coefficient shall be at least six times as large as the probable error.

There are two conditions frequently found in correlation problems which may more or less destroy the usefulness of the Pearsonian coefficient mentioned above. They are: (1) the data may differ materially from a straight line distribution on the scatter diagram; and (2) items of extreme deviation from the main data may unduly influence the coefficient. If a Pearsonian coefficient of correlation is computed for a series such as that shown in Part *B*, Exhibit 118, a very low coefficient will be obtained, whereas the correlation is, in fact, very high.

Multiple and Partial Correlation.—Mathematical methods for computing multiple and partial correlation and other devices used in estimating correlation are sometimes valuable to the professional statistician.

[1] See Warren M. Persons, "Some Fundamental Concepts of Statistics," *Journal of the American Statistical Association*, pp. 4–8, March 1924, Presidential Address, Eighty-fifth Annual Meeting of the American Statistical Association.

[2] See Edmund E. Day, "Statistical Analysis," pp. 378–381, The Macmillan Company, New York, 1925.

Those who wish to pursue such study further will find the subjects discussed in the more mathematical or advanced texts on technical methods.[1]

Special Corrections in the Mathematical Correlation of Time Series.—Before time series are correlated mathematically, it is often necessary to make several special adjustments or corrections. Four of the more important types are: (1) trend; (2) seasonal variation; (3) price changes; and (4) lag. The customary method of allowing for *trends* is to reduce the series to terms of deviations from their trends. The type of trend, however, should be carefully selected, and only that trend should be used which is the most representative. Differences in types of trends in the different series (straight lines, curves, etc.) do not have an important bearing on the calculation of the coefficient of correlation. As in the case of the trend, any satisfactory method of calculating the *seasonal variation* is acceptable for correlation purposes. These corrections will be discussed again in Chapter XVI, after trends and seasonal variation have been studied in some detail. The customary way of correcting the *price changes* (or changing monetary values) is to divide by the *proper* index number of prices. When the effects of a common cause do not occur at the same time, the correlation should be determined after allowing for the proper *lag*. The determination of lag may be facilitated by the use of the coefficient of correlation. Since the corrections for price and lag have been considered previously in the graphic correlation of time series, it will not be necessary to consider them in detail at this point.

Determination of Lag by Coefficient of Correlation.—It has been shown how an approximation of the amount of lag between two statistical series may be made by means of the scatter diagram, and, also, how a somewhat closer estimate may be made by horizontal adjustment of the curves when plotted on separate sheets of paper and held up to the light. A more precise measurement of the amount of lag may be made through the use of the coefficient of correlation. The method is simple and is applied as follows:

[1] Excellent discussions of mathematical correlation will be found in F. C. Mills, "Statistical Methods," Henry Holt & Company, New York, 1924 and 1938; E. E. Day, "Statistical Analysis," The Macmillan Company, New York, 1925; R. E. Chaddock, "Principles and Methods of Statistics," Houghton Mifflin Company, New York, 1925; W. L. Crum and A. C. Patton, "Economic Statistics," McGraw-Hill Book Company, Inc., New York, 1925; G. U. Yule, "An Introduction to the Theory of Statistics," Charles Griffin and Company, Ltd., London, 1924; Mordecai Ezekiel, "Methods of Correlation Analysis," John Wiley & Sons, Inc., New York, 1930; George R. Davies and Walter F. Crowder, "Methods of Statistical Analysis," John Wiley & Sons, Inc., New York, 1933; Frederick E. Croxton and Dudley J. Cowden, "Practical Business Statistics," Prentice-Hall, Inc., New York, 1934; and George R. Davies and Dale Yoder, "Business Statistics," John Wiley & Sons, Inc., New York, 1937.

1. Calculate coefficients of correlation with different pairings of the data. Thus, for the first calculation the months would be paired without any lag; a second calculation with one month lag; a third with two months lag, etc. The number of coefficients necessary may usually be determined from an estimate of the lag by one of the methods suggested above.

2. Plot the coefficients obtained from Step 1 on arithmetic coordinate paper and draw a smooth curve through them (see Exhibit 129). (For very precise work the curved line is fitted mathematically.)

3. Determine the point on the smoothed curve which is highest. (See "Z" on the curve in Exhibit 129.) This indicates the amount of lag; that is, when this amount of lag is allowed for, the correlation will be greatest.[1]

Limitations Must Be Appreciated in the Analysis of Associated or Correlated Characteristics.—In the practical application of these methods

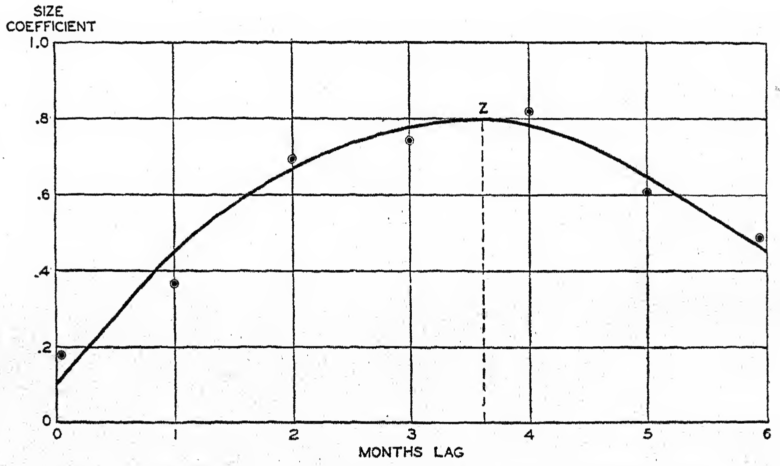

EXHIBIT 129.—Graphic determination of the amount of lag from a smoothed curve fitted to coefficients of correlation.

for analyzing associated or correlated characteristics, it is exceedingly important that due care be taken to observe their limitations.[2] While

[1] For very accurate work one would take the first derivative of the equation of the fitted curve, equate it to zero and solve for x.

[2] "The dangers that lie in undue reliance upon coefficients of correlation, the harmonic analysis, and other forms of rigid mathematical treatment of complex statistical data can hardly be overestimated. I am inclined, in fact, to believe it should be a matter of fundamental principle for all statistical conclusions to be reached primarily through logical and experimental processes, supplemented by a very simple numerical and graphical analysis. The more elaborate mathematical treatment should then be applied, if necessary, as a check and verification of the results that have been arrived at by these other means." From M. C. Rorty, "Statistics and the Scientific Method," Presidential Address, Ninety-second Annual Meeting, American Statistical Association, December 31, 1930, published in the *Journal of the American Statistical Association*, pp. 1–10, March 1931.

ordinary static series, or geographic series not involving a consideration of time, may ordinarily be analyzed satisfactorily, the methods herein described must be applied with care in purely time series. Constant changes in business make it dangerous to go back very far in time to prove anything about today. Quantitative methods do not possess that discriminatory power which enables them to judge between good and bad figures, between applicable and non-applicable facts, or between cause and effect. Accordingly, they often give peculiar results; in every case they should be checked by other methods not subject to the same type of error. It is not final to check one statistical method by another; both are quantitative and both are subject to similar limitations. In no part of statistical analysis is this caution more important than in the analysis of associated characteristics or correlation.

Questions and Problems

1. What is meant by correlation?

2. State two methods of comparing variables, and explain how each method can be presented graphically.

3. When is correlation *direct?* When is it *inverse?*

4. Describe *linear* correlation. When is correlation *curvilinear?* Which is the more common, and which is usually assumed in elementary statistical work?

5. How many series of variables can be measured as to their correlation?

6. What is "simple" correlation? What is "multiple" correlation? What is "partial" correlation?

7. What are three processes or ways in which correlation may be studied?

8. In what way are variables studied in logical analysis of associated characteristics?

9. Explain how causes may act on the variables studied, naming at least four ways.

10. For what two uses should logical analysis always be employed?

11. How are charts and tables used in studying correlation? What is a scatter diagram? How is it made? What does it show?

12. In fitting a "line of best fit" to data on a scatter diagram, what direction or angle should the line take in relation to the base line? How is the presence or absence of correlation determined after the line of best fit has been drawn? What relationship or ratio should exist between the values on the two scales in order to obtain the proper angle for the line of best fit?

13. Describe the construction of the correlation table. Does its purpose differ from that of the scatter diagram?

14. How is the direction and degree of correlation determined from correlation charts upon which historical data have been plotted?

15. In correlating historical (time-series) data, what influences may operate to indicate spurious correlation?

16. How can the approximate period of lag of one series in relation to another be determined?

17. When should corrections be made for price influences before correlating time series? How may they be made?

18. Describe how and when the estimation of the value of one variable can be made from a known value of another variable by use of the scatter diagram.

19. What is meant by the "coefficient of correlation"?

20. State the steps in the computation of the Pearsonian coefficient of correlation.

21. What is meant by "probable error" of the coefficient of correlation? How is this probable error computed?

22. Before undertaking the mathematical measurement of correlation, what four important adjustments or corrections does one often find necessary?

23. Describe the method of determining the amount of lag by the use of coefficients of correlation.

24. What are some of the limitations of quantitative methods of measuring or analyzing associated or correlated characteristics?

25. Determine by means of a scatter diagram (see Exhibit 121) the type and degree of correlation between the bank debits of Blank City and the sales of ABC Department Store (data in thousands of dollars):

Month	1936		1937	
	Bank debits	Sales	Bank debits	Sales
January	28	1.8	35	2.3
February	23	1.6	31	2.1
March	32	2.2	41	2.8
April	40	3.1	62	4.2
May	43	2.8	53	3.6
June	31	2.0	41	3.0
July	25	1.6	32	2.2
August	29	1.9	41	2.8
September	44	2.9	50	3.4
October	51	3.4	62	4.2
November	47	3.2	61	4.1
December	64	4.3	96	5.9

26. Present the data given in Problem 25 in the form of a correlation table (see Exhibit 122).

27. The height and weight of 10 men picked at random from a crowd were found to be as follows:

Man	Height (in inches)	Weight (in pounds)
1	68	140
2	69	180
3	65	165
4	70	190
5	71	170
6	73	195
7	67	145
8	66	140
9	62	155
10	69	180

By the Pearsonian coefficient of correlation determine the relationship between the height and weight of men in the particular crowd.

28. Plot the following series on arithmetic and logarithmic charts, analyze, and write a report on the "correlation" between the movements of the series:

VALUE OF ANNUAL PRODUCTION
(In millions of dollars)

	Wheat	Cotton	Bituminous coal	Automobiles
1920	1,540	1,069	2,130	1,809
1921	843	676	1,200	1,092
1922	818	1,116	1,275	1,562
1923	703	1,454	1,515	2,275
1924	1,050	1,561	1,063	2,041
1925	961	1,577	1,060	2,545
1926	1,013	1,121	1,183	2,746
1927	1,042	1,308	1,030	2,266
1928	912	1,302	934	2,704
1929	853	1,245	953	2,981
1930	595	659	795	1,721
1931	366	484	589	1,154
1932	289	424	407	651
1933	410	664	446	795
1934	446	596	628	1,204
1935	521	590	658	1,789
1936*	643	764	768	2,092

* Preliminary.

Source: Statistical Abstract of the United States.

CHAPTER XIII

TIME SERIES IN BUSINESS

One of the chief problems in modern business is that of estimating what the future changes in business conditions will be. This makes it necessary to analyze data over a period of time. If the data are merely descriptive of a situation at a certain time, the methods described in the preceding discussions of frequency distributions, averages, and dispersions may be all that are necessary in making an analysis. But, if the data represent changes that are taking place over a period of time, it is necessary to use special methods which will describe change or progress as well as describe a static situation. Data representing change over a period of time are known as time series, and it is the object of the next few chapters to describe the specialized methods that are necessary in time-series analysis.

Importance of Time-series Analysis.—The importance of time-series analysis can be best appreciated by considering a few examples of the difficulties encountered when proper analyses have *not* been made of business conditions. These examples are taken from several different periods during the past fifty years.

The early 1890's, for the most part, was a period of great prosperity in the United States. The National Cordage Company, for instance, considered its business so good that early in 1893 it declared a stock dividend of 100 per cent. Three months later its stock had fallen from \$147.00 per share to the equivalent of \$14.00 per share on the old basis, and it went into the hands of receivers. In the early 1890's the Reading Railroad overexpanded its lines into new territory, engaged in a bitter fight with a competing capital group, and ventured beyond its depth in undertaking the sale of coal. This was followed by strained financial conditions, the collapse of a coal combination, and the receivership of the Reading Railroad. In 1892 railroad earnings were considered, on the whole, to be favorable, but during the second half of 1893 such important lines as Erie, Lake Shore, Northern Pacific, Union Pacific, and Santa Fe passed into the hands of receivers. Why do such changes take place?

Business was again prosperous in the early 1900's and, excepting a brief interruption in 1903, continued to boom into 1906. But 1907 witnessed the failures of many industrial concerns, including the Pope Manufacturing Company, which had recently switched from making

bicycles to making automobiles. Later in the year, the Knickerbocker Trust Company, with $1,200,000 in capital and $48,387,000 in deposits, closed its doors. This action was followed by a large number of failures among banks and trust companies. Why are times of general prosperity followed by periods in which so many concerns fail or find great difficulty in remaining in business?

Additional illustrations are furnished by the experiences of the Goodyear Tire and Rubber Company, Ford Motor Company, General Motors Corporation, and the silk industry during the general depression of 1921, and by the stock market slump of 1929 and the depressions of the 1930's.

The Goodyear Tire and Rubber Company was incorporated in 1898 and became the largest manufacturer of rubber tires in the world. From the time of its incorporation until the middle of 1920, the company's operations were extremely successful. In 1919 the company had a surplus of more than $30,000,000. This was changed to a deficit of more than $15,000,000 by the end of 1920, and the president was ousted by the company's creditors from control of the mammoth organization which he had built up. What is the explanation? The immediate explanation is that the Goodyear Company had recklessly expanded its fixed investment at high costs, and that it had a large merchandise inventory which had been accumulated at high prices. This made it necessary to write off many millions when prices dropped. But why did this company and many others expand fixed investment and accumulate high-priced inventories, and why were prices high, and why did they fall?

Though more cars were sold in 1920 than in any previous year, all of Ford's factories closed at the end of December and remained closed for several weeks. Ford owed the government $55,000,000 in taxes. Notes for nearly $30,000,000 originally issued to buy out his minority partners would be due in a few weeks. He had only $23,000,000 in cash and treasury bonds. Unsold cars were piled high in the factory and were choking salesrooms throughout the country. The public was not buying automobiles. Why were Ford's factories and salesrooms piled high with unsold cars, and why did the public stop buying cars of any kind?[1]

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[1] Henry Ford has always been opposed to extensive borrowing from banks, and desires to have complete ownership and control of his organization. Due to his buying out his minority stockholders in 1919 and his unusual expansion policies, however, he was in desperate need, late in 1920, of $75,000,000. It is reported that one New York banker went so far as to pay Mr. Ford a special visit in order to suggest to him who the treasurer of his refinanced organization should be. In this case, however, Mr. Ford was able to shift the burden to his dealers. He shipped automobiles to 17,000 willing and unwilling dealers who had to pay if they were to hold their agencies. In this way he converted a frozen inventory into $69,000,000 cash by April. (Data from *Boston News Bureau*, July 14, 1921.)

In 1920 and 1921 the General Motors Corporation underwent severe financial losses. "The reduction of the surplus materials purchased at high prices, and of inventory and other commitments made prior to December 1920, resulted in a total liquidation loss of $84,869,893."[1] In 1921 a new management was placed in control of the organization. Again the question arises—why did this company, as well as many others, overexpand their inventories, and why were prices abnormally high?

In the silk industry wholesale prices increased very rapidly from February 1919 to February 1920. Wholesalers and retailers talked of the great demand for silk and the supposed great shortage of raw goods in the country. Those in the silk industry "honestly believed that there was a lack of raw silk, and that prices would continue to rise. The extent of the accumulation of speculative stocks was not realized in the trade. It was not until insurance companies discovered that the amount of silk which they had collectively insured was beyond their insurance limits that any one understood that there was really an excess supply of silk in storage. Reacting to this news, all prices of silk dropped at once, and manufacturers, wholesalers, and retailers found themselves with huge inventories to liquidate. Manufacturers and wholesalers suffered from a general cancellation of orders, which still further added to their difficulties."[2] Why is it possible for such a situation to develop?

The stock-market boom which reached its height in 1929 was the greatest boom of its kind in history. In approximately six years, the Dow Jones daily averages of prices of 30 industrial stocks rose from a low of $85.80 (October 29, 1923) to a high of $386.10 (September 3, 1929). The enormous profits made from the increase in prices led to an unprecedented demand for stocks. Great volumes of marginal purchases were made by those who had never bought stocks before. Prices were paid for stocks on which the dividends could not reasonably be large enough to net the "investor" as much as 3 per cent upon his "investment." (Good bonds could be bought to yield 4 and 5 per cent.) Then in less than two and one-half months (from September 3 to November 13) the average dropped from a high of $386.10 to $195.35 and thousands of speculators were ruined. Why did people believe that the upward tendency in stock prices would continue indefinitely? Could they have determined the actual situation accurately? After the abrupt decline from September 3 to November 13, 1929, a less steep downward tendency continued to July 11, 1932, when the Dow Jones averages reached a low

[1] Annual Report to the Stockholders, 1922, p. 4.

[2] "Business Cycles and Unemployment." By a Committee of the President's Conference on Unemployment, published by United States Department of Commerce, Washington, and McGraw-Hill Book Company, Inc., New York, 1923.

point of $40.56. Then there was an irregular rise to $195.59 on March 10, 1937. This was followed by a rather rapid erratic decline reaching $98.00 on March 31, 1938.

The chief reasons that the foregoing difficulties arose are lack of information and wrong information regarding the true state of developments over the periods involved. By using skillful statistical analyses, many firms are now not only escaping the effects of these difficulties, but are actually taking advantage of such situations. The General Motors Corporation, for instance, whose difficulties in 1920 and 1921 are mentioned on the preceding page, has since developed an extensive system of basing its policies on careful technical analyses of business conditions, and it came through the depression of the early 1930's with a net profit for every year.[1] Many investors took advantage of the stock-market rises and falls just mentioned, selling in 1929 when prices were high, buying in 1932 and 1933 after the decline, and selling again early in 1937.

Use of Statistics in Analyzing Business Changes.—In analyzing the changes and developments in business from time to time, the business man has three direct questions that time-series methods must attempt to answer: First, what factors caused business in the past to develop as it did? Second, what causal factors are operating now? Third, what significance has the present combination of causal factors in relation to the future?

The primary purposes of time-series analysis are indicated by the three questions above. Through logical analysis or by direct observation, we may arrive at the conclusion that certain elements in past situations have been responsible for the developments which occurred, but it is only by statistical methods that we can measure quantitatively the real importance of the various factors. If current operating causes can be recognized and evaluated in the light of their significance in the past, it is then possible to estimate the future on a more systematic basis.

Obviously, prediction of the future is the aim of every business executive. To buy on a rising market, and to sell before a decline; to stock heavily enough for a good year, and to buy carefully for a poor one; to build at low prices; to borrow at low money rates; to extend credit when safe, and to refuse it at other times; to expand or reduce the personnel at the right time; to advertise when the business is available; and to expand when growth is justified and permanent—these mean prosperity and profit. To guess wrongly means loss or failure. Simple statistics cannot answer all these questions, to be sure; but proper analysis of available data, combined with sound interpretation and good business judgment, will provide an extremely valuable guide.

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[1] See Twenty-ninth Annual Report of General Motors Corporation, year ended December 31, 1937.

Multiplicity of Causes Affecting Business Development.—The practical business man is interested in the almost infinite number of factors which may affect his enterprise. The political situation has its bearing through the possibilities of regulation, taxation, tariff changes, war, development policies, and the like; natural physical circumstances, such as weather and crop conditions, are undeniably important; intellectual progress, with its new inventions and discoveries, must be considered; while popular psychology, manifested through willingness or unwillingness to do business with faith in the outcome, exercises a marked effect. From this maze of influences, which are often contradictory, the business man must derive his conclusions as to the probable course of business during the coming months or years.

It is apparent that the fluctuations of business activity and prosperity are not mathematical totals which vary on a regular schedule but are the net resultants of many factors, some of which are tending in one way and some in another. "Bearish" sentiment based upon large inventories and declining prices may be offset by the stimulating effects of favorable credit conditions, improving foreign markets, and steady consumption of goods. Since the beginning of the present century, interest has greatly increased in efforts to measure and analyze these factors.

Causal Elements Do Not Operate Uniformly.—Every type of business is responsive in some degree to general business conditions. However, not all industries will react in the same manner to the same stimuli; some will even run counter to the general trend. The invention of a new electric storage battery, which would make electric automobiles much superior to the gasoline type, would boom certain parts of the automobile industry, but tend to depress the petroleum industry. There are all kinds of variations and degrees of responsiveness to a given situation; and it is imperative that the relationship of general business conditions to every type of business venture be analyzed for that type specifically. A given business may prosper in response to causes that may be harmful to other businesses.

Of equal significance is the fact that no two communities are composed of exactly the same economic constituents, and hence may not always be expected to flourish or slump simultaneously. In 1925, Florida was booming, while Iowa was depressed. Obviously, these communities were reacting to the economic forces that dominated them at the time, with unmistakably diverse results. Undoubtedly the two areas felt many influences in common, but the intensity of the conflicting factors and their relative importance varied considerably, as the outcome indicates.

What is true of different industries and of different geographical areas is even more significant as regards individual enterprises. In addi-

tion to these external economic factors there are the influences of the various internal conditions of the individual business.

Business Characterized by Tendency to Increase though Growth Is Irregular.—It is apparent to every student of business that most lines of commercial and industrial activity show gradually increasing volumes as the years go by—a fact associated with population increases, widening markets, higher standards of living, new inventions and processes, and many other developments. Not all of the causes of general economic growth are operating at once or in the same direction, but their great number results in a net tendency toward maintained progress, which varies but little in its rate-of-growth tendencies for years at a time.

Though the tendency to grow seems to prevail unceasingly, there are also times, as has been pointed out, when business is better or worse than at other times. In 1908, 1914, 1921, 1932, and 1938, conditions were depressed; many concerns became bankrupt, production slumped, men were out of work, and the standard of living was temporarily lowered. The opposite condition characterized 1907 (early part), 1910 (early part), 1916, 1920, 1923, 1925, 1926, 1929 (early part) and to a certain extent the first half of 1937, which were times of relatively vigorous business activity. These frequent periods of boom and depression are probably more important to the business man than the tendency to grow over a long period of time, because their effects upon his enterprise are more sudden and severe. The factors causing these fluctuations are probably as numerous as those responsible for general growth, and appear in various combinations from time to time, so that the problem of anticipating these irregularities in business development is difficult.

Four Distinct Movements in Time Series.—Although different industries and different communities do not always show the same business tendencies at the same time, their fluctuations have characteristics similar enough to warrant the use of fairly generalized methods of statistical approach. In most business time series, it is possible to distinguish four well-defined movements. These are:

1. Basic or long-time trend.
2. Seasonal variation.
3. Business cycles.
4. Erratic fluctuations.

These are of primary importance in statistical work, because they constitute a basis for both explanation of the past and prediction of the future. The major tendency of each type of movement is due largely to the group of causal factors which creates the movement. Isolation of the results (according to the four movements listed above) is the first step toward explaining the effects and importance of the causal factors.

Basic or Long-time Trend.—Although most economic time series fluctuate noticeably from month to month, and even from year to year, inspection of their movements over a period of several years commonly discloses a definite and regular basic tendency toward increase or decrease. For example, the expansions of successful business enterprises ordinarily have smooth and regular basic trends, which go on despite temporary vicissitudes in business activity. Ordinarily in a community there will be a gradual growth in population. Or there may be a gradual increase in industrial output due to improved methods. Or there may be a gradual increase in the production of automobiles and a gradual decrease in the production of carriages. *Basic* or *long-time trend*[1] may, therefore, be defined as the underlying tendency to grow or decline over a period of years; it is the movement the series would have taken had no seasonal, cyclical, or erratic factors existed.

Exhibit 130 illustrates the nature of basic trends. Were it not for the disturbing factors previously mentioned (seasonal variation, business cycles, and erratic influences), the sales data of the American Mercantile Company would have followed the straight trend line (Part *A*), which indicates the prevailing tendency of growth. The trend in this series is an "arithmetic straight line," indicating that the yearly increase tends to be a constant amount.

All basic trends are not of the same nature. Sometimes the predominating tendency will be a constant amount of growth (which would be shown by a straight line on an arithmetic scale); sometimes it will be a constant percentage increase (which is shown by a straight line on a semi-logarithmic chart); sometimes the characteristic tendency is best represented by the "logistic" or "S" curve; and sometimes it is a combination of types or a special irregular curve. Examples of these trends are presented in Chapter XIV.

Properly recognizing and accurately measuring basic trends is one of the most important problems in time-series analysis. Trend values are used as the base from which the other three movements are measured, and inaccuracy may vitiate the entire work. Fortunately, the causal elements controlling trend growth are relatively stable; trends do not commonly change their nature quickly and without warning. It is, therefore, usually reasonable to assume that a representative trend, which has characterized the data for a past period, is prevailing at present, and that it may be projected into the future for a year or so. It should be appreciated, however, that such projections should always be supported by a cross-section analysis of the current economic factors, as serious errors have been made in making these projections, especially

[1] The term "secular trend" is also commonly used.

in the case of the straight geometric increase or compound interest curve.

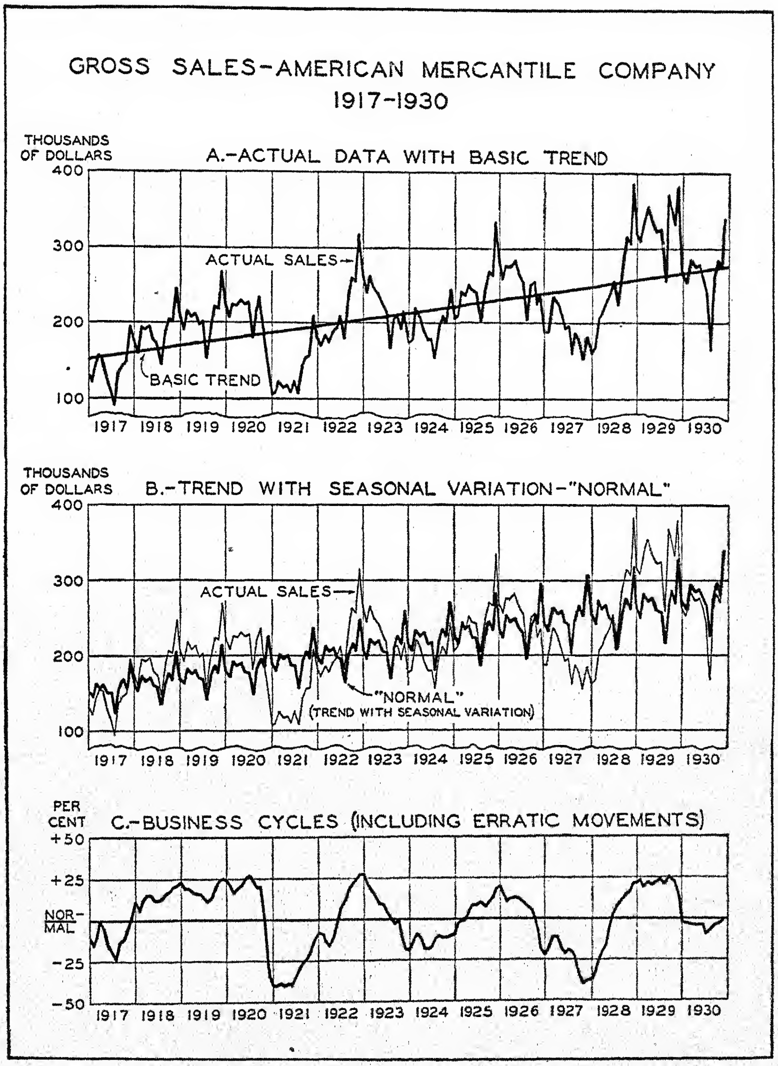

EXHIBIT 130.—Basic trend, seasonal variation, and business cycles (including erratic movements). (*Data adapted.*)

Seasonal Variation.—Examination of Exhibit 130 will reveal a second well-defined movement in the series, with its Christmas peaks and summer slumps. That is, for instance, it can be seen from Part *A*, Exhibit 130, that on the average December is high and August is low. A mathe-

matical determination of this average tendency of the different months of the year to vary from a straight-line trend is shown by the heavy curve in Part *B* (Exhibit 130). Other series will show various types of these short-range fluctuations, each of which is repeated each year. These fluctuations are known as *seasonal variations*, taking their name from the fact that they are reflections of temperature influences, popular trading habits, and other factors which operate in similar fashion in the same season of each year. Sales of ice are heavy in the summer, and decline in importance in the winter; retail clothing stores may be extremely busy during the weeks preceding Christmas and Easter, and may have slack periods at certain other times; and so on. This seasonal movement may vary in the same series in its quantitative extent; thus, a year in which the total volume of activity is large may show a much greater difference between busy times and slack times than would a year when little business is done.

It is customary to measure seasonal variation as a percentage of the trend rather than in absolute quantities, a method which conforms reasonably closely to the need. The *seasonal index* for any month (week, quarter, etc.) may be defined as the ratio of the normally expected month (excluding the business cycle and erratic movements) to the corresponding trend value; thus, a January seasonal of 110 means that a normal January is 10 per cent greater in volume than the trend[1] at that point would indicate.

The seasonal index is merely a set of percentages, indicating that the various months (or weeks, quarters, etc.) of the year are, respectively, greater or less than the corresponding trend points in a normal year. They are usually determined by measures of actual occurrence over a certain period, under the apparently well-justified assumption that any well-defined seasonal tendencies are relatively stable over periods of years. If an average January for the period 1930–1939 approximated 110 per cent of its trend ordinate, it is likely that the same figure would be usable in 1940.

The Meaning of "Normal" in Business Statistics.—Business is often said to be "about normal," or above or below "normal." When so used the term "normal" is generally recognized to mean a level of activity which is characteristic of (1) the degree of business progress attained at the time, and (2) the season of the year. In other words, if there were no changes except those due to basic trend and seasonal variation, the situation would always be "normal" (see heavy curve of Part *B*, Exhibit 130). If, for instance, crops fail, if monetary problems cause a panic, or

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[1] In a series in which the trend change is negligible, or from which trend has been removed, the seasonal index will indicate also the relative importance of the months.

if there is an overproduction of commodities, the normal course of events is interrupted, and business activity departs from normal levels.

Economic society has never been so well ordered that business activity could long proceed at a normal pace. Even factors ordinarily extremely steady by nature may, in certain combinations, be responsible for booms or depressions. Added to those, there are many factors whose influences are habitually indicated as causes of cyclical and erratic movements, which may be roughly defined as departures from normal.

The Business Cycle.—Because of the persistent tendency for business to prosper, decline, stagnate, recover, and prosper again, in apparently never-ending sequences, the third characteristic movement in economic time series has been called the *business cycle*. Unlike the seasonal movement, the business cycle does not recur on a regular time schedule, but moves sporadically in response to causes which develop intermittently out of complex combinations of economic and other circumstances.

When the business of a country or of a community is above or below normal, the excess or deficiency is usually attributed to the business cycle. Measuring the business cycle, therefore, becomes a process of contrasting actual occurrences with a normal estimate arrived at by combining the calculated trend and seasonal movements. The measurement of the variations from normal may be made in terms of actual quantities (dollars, tons, or whatever the unit of the series may be), or it may be made in such terms as percentage deviations, which is generally the more satisfactory method as it places the measure of cyclical tendencies on a comparable base throughout the entire period under analysis.

It will be noted in Part *B*, Exhibit 130, that actual sales fluctuate above and below normal every few years. The percentages by which these sales rise above or fall below normal are shown in Part *C*. These fluctuations are the business cycles (including erratic movements) expressed in "percentage deviations from normal."

Erratic Movements.—The last of the four types of time-series fluctuations, the *erratic movements*, are exceedingly difficult to disassociate quantitatively from the business cycle. Their causes are such irregular and unpredictable things as droughts, floods, fires, pestilence, fads, and fashions, which operate as spurs or deterrents upon the progress of the cycle. The severity of the depression of the early 1930's in the United States, for instance, was partly due to the fact that an unusual combination of adverse erratic factors were operating at the same time that business was in the depression phase of the cycle. Among these were the unprecedented drought of 1930, and revolutionary disturbances in China, India, and South America. Other types of unusual combinations of business influences often play a large part in the progress of business conditions. A simultaneous decision, for instance, of a number of large

corporations to inaugurate policies of expansion might start a boom, whereas a reverse policy might start a decline. Separation of erratic movements from the cycle is generally limited to a process of pointing out certain outstanding deviations from normal as the results of specific causes. For practical purposes, actual separation of the cycle and erratic movements is seldom attempted, analysis being more easily based upon the unadjusted deviations from normal, which include both movements.

It is worth noting that the importance of these erratic movements increases as the range of study narrows. On a series representing nation-wide activity, a Mississippi flood or a San Francisco fire may be of minor importance, but to Louisiana or California, respectively, they are catastrophic events.

Estimation of the Future—Forecasting.—If trend movements have been established definitely in the past, and no reason for radical change is apparent, there is ample justification for projecting the same trend line into the future for a year or two. A trend value for every week, month, or quarter in the year to be forecasted is thus easily determined. If the seasonal percentages are believed to be representative, each projected trend month (week, quarter, etc.) of the year may be adjusted for the known seasonal variation to attain "normal" values for all items being forecasted. The normal values may then be adjusted to allow for any changes implied in the forecast of the business cycle, when its expected movement can be derived from a study of prevailing conditions and operating causal elements. While forecasts thus made are generalizations, they are useful as guides in planning for future business. The subject of business forecasting will be discussed at greater length in Chapter XVII.

The principal value of statistical methods in analyzing time series is not in forecasting definite movements, but in determining the degree to which business is above or below normal. The skater on the lake can tell when he is in danger if he knows the thickness of the ice, but he cannot tell when it will break. If the business man has statistical information so that he knows when "he is skating on thin ice," he will not be liable to be caught in the difficulties experienced by the business concerns and investors mentioned at the beginning of this chapter (pages 270–273).

Statistically, one's position in relation to normal may be determined as follows:

1. Determine the basic trend.
2. Measure the seasonal variation and compute a set of indexes.
3. Determine the normal. (This is the trend combined with the seasonal variation.)
4. State the item for each month (or other period) as a percentage of the corresponding normal figure.

The foregoing procedure, however, requires a careful determination of trend and seasonal variation. The determination of basic trend is the subject of Chapter XIV, and the measurement of seasonal variation is treated in Chapter XV. Chapter XVI explains how the relation to normal may be stated and analyzed as cyclical and erratic movements. Further application of the methods described is then made in Chapter XVII on Business Forecasting. Before proceeding to study these methods, however, it is advisable to consider briefly the preliminary

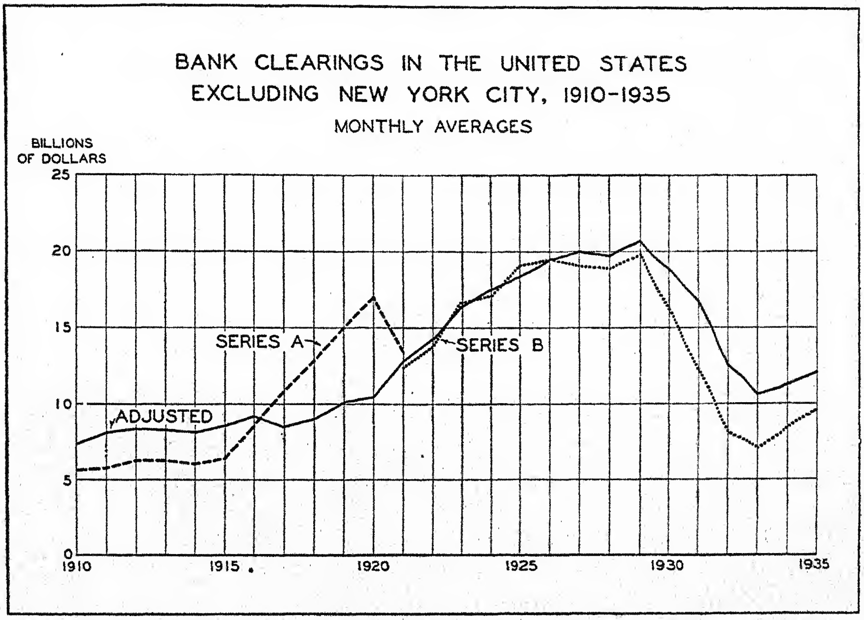

EXHIBIT 131.—The upward tendency of this series from 1910 to 1920 and the downward tendency from 1929 to 1935 are less pronounced when adjusted for differences in price levels. (The "adjusted" series has been placed on a 1926 dollar basis by dividing through by the Bureau of Labor Statistics All Commodity Wholesale Price Index Number (1926 = 100), and an adjustment has been made in the 1910–1921 data so that the series can be spliced at 1921.)

work necessary in preparing data and the importance of studying technical methods.

Preliminary Preparation of Data.—In all cases, business time series should be carefully examined for homogeneity before attempting to measure such movements as trend or seasonal variation. Sometimes the data can be made more homogeneous by applying certain corrections. In other instances, corrections cannot be made directly, but allowances for the lack of homogeneity must be made in the analysis and interpretation.

Preliminary preparation of the data is often necessary because of price inflation, and because of uneven time intervals, before fitting trends,

computing seasonal variations, and measuring business cycles. Price inflation (or deflation) is usually corrected by dividing through by the proper price index number. The effect of adjusting a series for this factor is illustrated in Exhibit 131. Unequal time intervals, such as months (which are not only unequal in length, but which are still less uniform because of holidays, etc.) may be made more nearly equal by reducing them to an average daily basis (see Exhibit 132). Unequal time intervals will be discussed at greater length in connection with seasonal variation in Chapter XV.

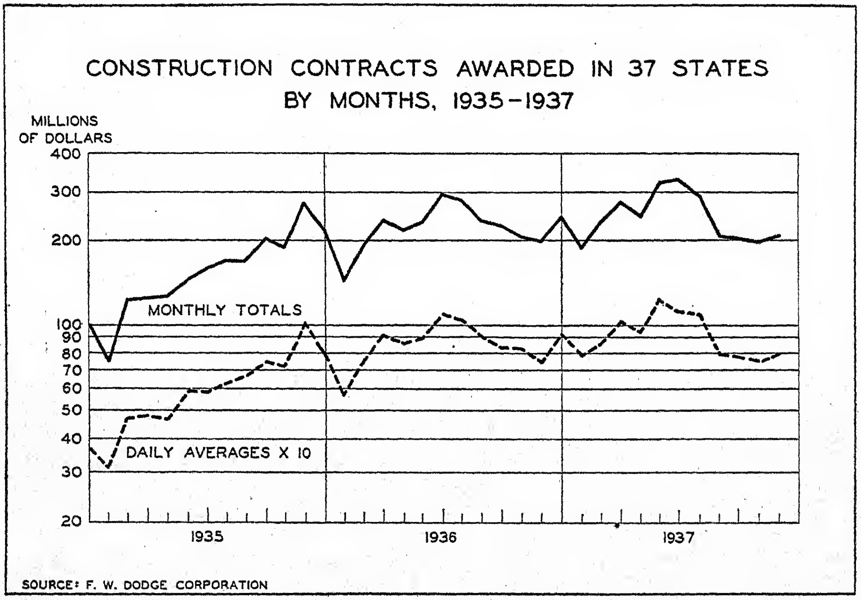

EXHIBIT 132.—Effects of unequal lengths of working months adjusted by average per working day. While the major swings are practically the same, there are marked differences in the monthly changes as shown by the two curves.

Importance of Studying Technical Time-series Methods.—Although the methods just mentioned are technical, and require considerable skill, experience, and energy, they are widely used in practical business research. If a business man is to make the best use of the information furnished by the research department in his own firm or by outside statistical organizations, it is highly desirable that he understand, to a certain extent at least, the methods that have been applied. In other words, correct use of the results often depends upon a correct understanding of the methods of obtaining the results.

If an executive does not use mathematically computed trends, or seasonal or cyclical variations in his own business, these factors, neverthe-

less, must be recognized and approximated. Even the manager of a very small enterprise must recognize the effects of these elements on his business, although he may not measure them in an exact mathematical manner. Actual computation of these factors is not so necessary as an understanding of them and a realization that they exist. In practical work, the trained business or economic analyst considers probably hundreds of series without computed trends or seasonal indexes to every one for which he has such computations. Understanding the nature and characteristics of trend and seasonal effects, he can often make the necessary analyses of the cycles of a series from unadjusted data. In fact, the greatest value of a study of technical methods of time-series analysis is the training that it gives in making allowances for such factors as long-time trend and seasonal variation without actually making the computations.

In the three following chapters, the methods introduced in this chapter will be developed from the point of view of not only enabling the student of business conditions to understand and to apply the methods in time-series analysis, but also helping him, when analyzing specific business problems, to make the proper allowances for trend, and seasonal, cyclical, and erratic variations where the available data (on production, sales, financial conditions, etc.) have not been adjusted for these factors.

Questions and Problems

1. What questions does the business man desire to have answered by time-series analyses? How can such analyses assist in the adoption of successful policies? Give specific examples of difficulties encountered when correct analyses have not been made of business conditions.

2. Are changes in business conditions regular in time of occurrence?

3. What causes irregular variations in business?

4. Is the business man affected by few or many economic factors? Is his situation simple or complex? Explain fully.

5. Do the conditions in all industries always coincide? In all countries? In all sections of the same country?

6. Name four well-defined movements common to most time-series data in the business world.

7. Define or describe long-time or basic trend.

8. What type of increase or decrease does an arithmetic straight line indicate? Are all trends of this type?

9. Define or describe seasonal variation. How is it usually expressed in statistical work?

10. What is meant by "normal" business? What factors, mentioned previously, are included in the "normal" condition?

11. Describe business cycles. How may they be measured?

12. What is meant by erratic movements? Can they be measured accurately? How are they usually measured?

13. How can the measurement of the underlying factors or movements assist in forecasting future conditions?

14. What are some of the corrections to make business data homogeneous which often should precede measurement of the underlying movements?

15. Why is it important to understand technical time-series methods when analyzing business conditions?

16. Using the tables given below, eliminate the effects of price changes from the sales of Sears Roebuck and Co. for the years 1932–1936 by dividing each month's sales by the corresponding price index. Make a graph of the original sales and the sales adjusted for price fluctuations. (Use semi-logarithmic paper.) Compare the two curves and write a brief analysis.

SALES OF SEARS ROEBUCK AND CO., 1932–1936
(In millions of dollars)

Year	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1932	19.9	20.6	19.9	23.6	24.5	25.0	20.3	20.8	24.5	25.6	24.7	30.5
1933	16.9	16.1	16.3	19.8	22.7	22.8	20.0	24.7	26.6	30.5	31.3	36.9
1934	22.0	20.6	25.3	25.2	30.1	27.1	21.5	25.2	29.9	34.4	33.7	41.9
1935	23.8	23.7	32.0	34.1	35.2	35.1	29.6	29.6	34.3	44.0	40.9	51.3
1936	27.7	27.6	36.1	39.0	44.9	46.4	39.6	38.9	47.6	58.1	49.0	66.4

Source: Survey of Current Business.

FAIRCHILD INDEX OF RETAIL PRICES, 1932–1936
(January 1, 1931 = 100)

Year	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1932	81.3	80.1	79.2	78.2	76.8	75.1	74.0	73.6	73.6	73.3	72.6	71.8
1933	71.1	69.9	69.7	69.4	70.4	72.3	76.1	82.5	86.0	87.1	88.0	88.0
1934	88.5	89.5	89.6	89.4	88.6	88.2	87.9	87.7	87.7	87.4	87.4	87.2
1935	86.8	86.6	86.3	86.3	86.1	85.7	85.2	85.7	86.6	87.6	88.0	88.2
1936	88.3	88.3	88.1	88.1	88.1	87.9	88.1	88.5	89.3	90.0	90.8	91.7

Compiled and published by Fairchild Publications, Inc.

17. Using the index of construction costs given on page 285, adjust the following data on building permits in Temple City for price fluctuations.

BUILDING PERMITS IN TEMPLE CITY, 1932–1937
(In thousands of dollars)

Year	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1932	72.1	68.7	83.6	106.5	64.0	79.2	62.4	67.1	50.2	36.2	37.6	40.1
1933	40.2	37.6	29.1	42.6	62.7	89.3	91.2	78.6	68.4	72.4	86.4	87.8
1934	92.5	86.7	138.6	142.0	151.6	134.4	162.6	172.8	191.6	160.2	158.4	120 0
1935	139.7	160.6	279.1	197.5	163.6	142.7	146.7	162.4	206.3	219.4	180.0	186.1
1936	194.6	241.7	293.4	271.6	203.4	186.9	193.6	207.8	242.0	200.1	202.4	200 4
1937	196.2	236.8	284.7	242.1	201.3	175.2	186.4	200.7	261.2	243.1	186.4	175 0

Index of Construction Costs, 1932–1937
(1930 = 100)

Year	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1932	79.0	78.0	77.4	77.1	74.0	73.0	72.0	71.8	71.4	71.1	70.0	69.0
1933	69.2	69.4	67.8	67.6	68.1	68.8	71.0	74.0	75.1	75.4	76.0	77.0
1934	77.0	78.4	78.8	79.0	80.1	80.4	81.2	81.6	81.9	82.3	82.6	82.8
1935	82.1	83.6	83.4	82.8	82.4	82.1	81.6	81.4	81.0	82.1	82.4	83.0
1936	83.6	82.8	82.6	83.4	83.6	84.0	84.4	85.0	86.1	87.1	87.6	86.9
1937	88.2	88.6	89.1	90.4	91.3	90.4	89.8	87.6	86.4	84.2	85.1	85.3

Present graphically the original and the adjusted building permits data and comment briefly. (Use semi-logarithmic paper.)

CHAPTER XIV

TIME SERIES—BASIC TRENDS

Basic or long-time trend may be defined as the tendency, prevailing over a period of years, for an activity to grow or to diminish. It is stable and fundamental; it is the record the series would have made had there been no seasonal, cyclical, or erratic factors.

Importance of Trend Measurement.—Accurate trend measurement is important not only in analysis, but also in many types of forecasting. In the absence of evidence to the contrary, it is not unreasonable to assume that the present tendencies toward growth are fairly well indicated by past records, and that factors operating in the recent past will continue their influences. Forecasting, by simply assuming that the present trend will continue, is thus not impossible in certain instances; but obviously the trend must be an accurate and representative one, for any error of direction will cumulate rapidly, thereby reducing the value of the forecast.

Nature of Trends.—As was pointed out in a previous discussion, the development of a business series at the present time is a resultant of many factors which seldom operate on different industries, communities, or enterprises in the same way. Naturally, it is not to be expected that there would be a striking degree of similarity between the trends of unrelated series. Differences in extent of growth, time of growth, nature of growth, and the length of periods during which the same growth tendencies prevail, all provide intricacies with which the statistician must deal, and there are few series whose trends do not present some peculiar aspects of their own.

There is, of course, no limit to the number of kinds of trends which may exist; nevertheless, the problem is simplified somewhat if we assume that irregularity is not likely to characterize trend behavior. While trends do change, there are but few series whose long-time development may not be described by smooth and regular curves or lines. Though few time series are controlled by exactly the same economic factors, and few trends are exactly the same, it is possible to distinguish four fairly common types of trends. The first is the simple arithmetic progression—a regular increase or decrease by the same *amount* each year, which may be shown graphically as an arithmetic straight line (see Exhibit 133). The second is the simple geometric progression—a regular increase or decrease by the same *percentage* each year, which may be shown as a

straight line on a semilogarithmic chart, and is, therefore, called a "logarithmic straight line" or "compound interest trend" (see Exhibit 134, upper graph). The third type of trend is the "S-curve" which is characteristic of the growth of many industries; that is, small gains at first, then relatively large gains, and then small gains again when the industry reaches maturity and the saturation point is approached in the

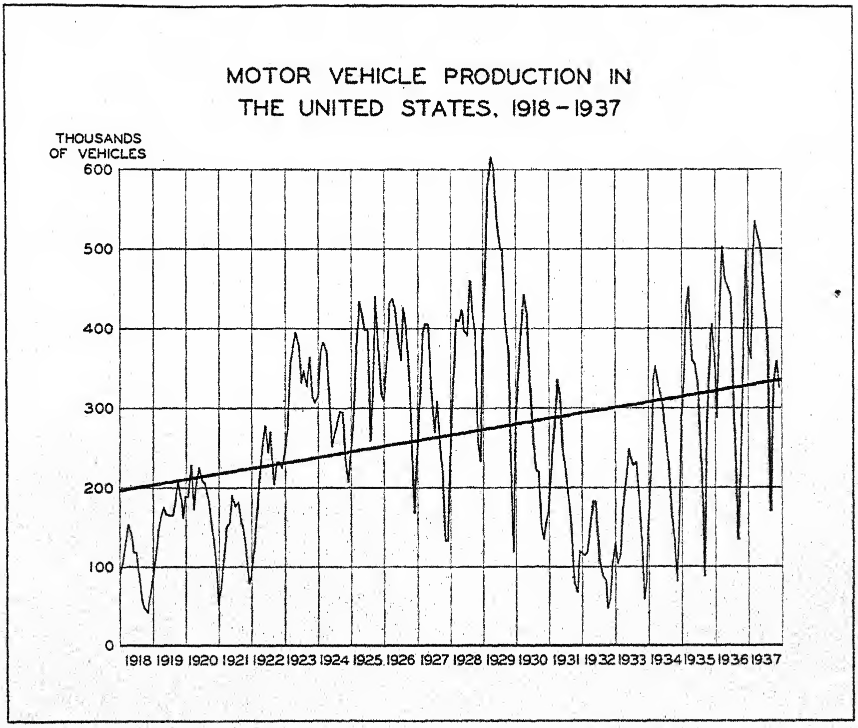

Exhibit 133.—An arithmetic trend. (Tendency to increase on the average by the same amount each year.) (The computation of this particular trend line is illustrated in Exhibit 168 on page 350.)

market. (An illustration of an S-curve is presented in Exhibit 145, page 303.) The fourth type of trend is an irregular smoothed curve which usually has to be fitted largely by inspection or estimates. Other types of computed curves are sometimes used, but it is usually difficult to justify their use on a logical basis, especially if the period is short.

Occasionally, any of the types of trends just discussed may change in direction abruptly, as illustrated in Exhibit 135. In these "broken trends" the curve may even change from one type to another, as is the case in this exhibit.

Determination of the Proper Trend Formula.—In fitting a trend, the first essential is to chart the data, as this is usually the best way to reveal

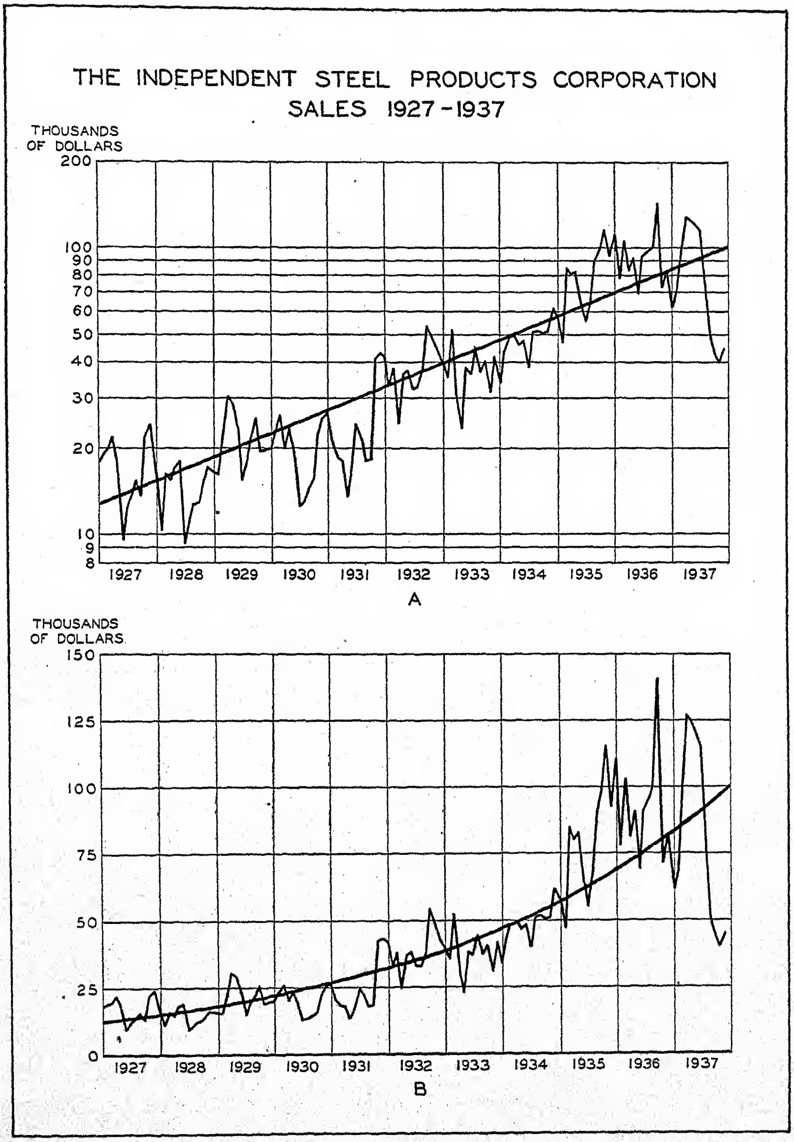

EXHIBIT 134.—A compound-interest trend or constant rate of increase. (Tendency to increase by the same percentage each year). *A.* Trend line with a constant rate of growth on a semi-logarithmic chart. *B.* Same trend on an arithmetic chart. (*Data adapted.*)

the movements. Both arithmetic and semi-logarithmic charts should be employed, as the true nature of the trend may not always be revealed on

either one alone. It is not usually necessary to plot all of the data in the series. If monthly data are used, it may be sufficient to plot the average month of each year, as the use of one point per year gives practically the same trend line. Close study of the charted data will ordinarily indicate what type of curve should be used in fitting a trend line, and the approximate period over which that trend should be fitted. It is usually a good plan to sketch the computed trend line upon the charts of the actual data, as a check upon the judgment of the worker and upon his mathematics as well. If the computed trend fits the data when plotted and meets the test of rational criticism, it is probably satisfactory.

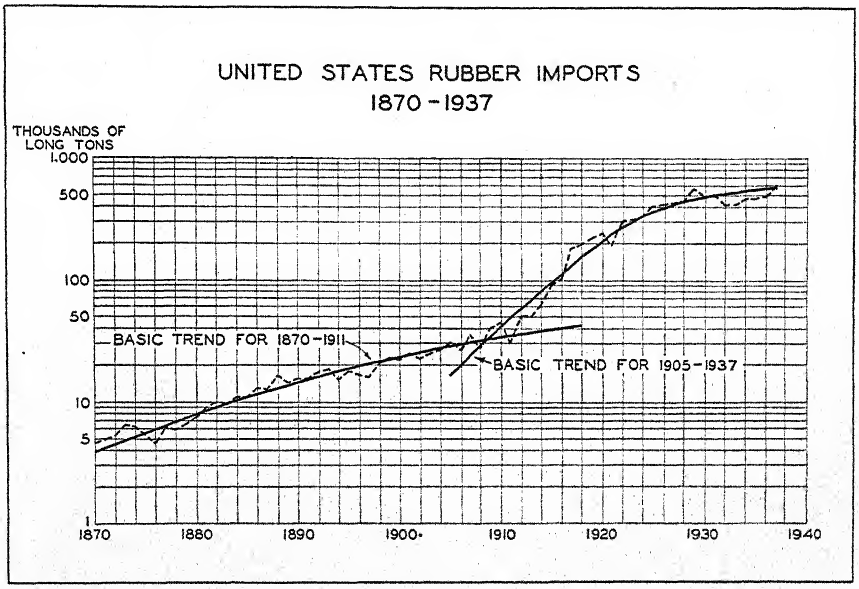

EXHIBIT 135.—An abrupt change in slope and type of trend.

When charting trends, the scale should be selected with care. A relatively erratic series might move in almost a straight line if the horizontal and vertical scales do not maintain the proper degree of proportionality. A horizontal movement of about 1.5 times the vertical in actual distance, is a proportion which usually gives good results in series covering from 10 to 20 years.

Proper Period Essential in Trend Measurement.—It is always possible to obtain close-fitting so-called trends, by fitting lines or curves over an unrepresentative short period of time. It must be remembered that we are obtaining these trends by fitting lines through data which are greatly influenced by the business cycle. Care must be taken that the cycle does not tilt the trend line, a contingency which is very likely to

happen if the period of fit is short, or if it begins in a depression and ends in a boom, or *vice versa.* This difficulty is illustrated by the steeper trend line in Exhibit 136. Any trend period of less than 10 years should be regarded with suspicion, even though circumstances may force its use. It is very essential to remember, also, that trend fitting does not mean simply curve fitting; the trend describes economic rather than mathematical developments. Often the statistician's knowledge that a boom or depression took place in a given year will guide him in properly

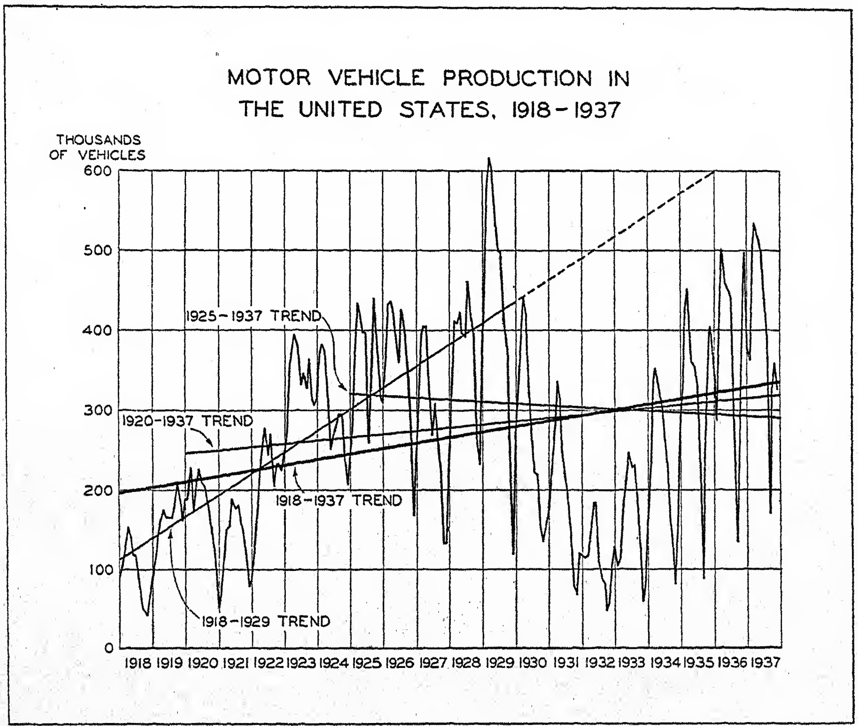

EXHIBIT 136.—Trends based upon different periods. This chart indicates the importance of basing the trend upon the proper period.

placing his trend, when the period is too short or the data are too imperfect to indicate a proper trend line.

Unreliable results, very similar to those obtained by using short periods of time for trend fitting, are sometimes obtained by the use of mathematical formulas which are entirely too rigid for the purpose at hand. Use of a formula *involving powers higher than the second* is justified only when the statistician has a set of data which clearly conform to the characteristics of that mathematical curve *over a period of many years.* A peculiar temporary cyclical twist does not justify the assumption that the trend requires a very sensitive device for its measurement.

When there is a question as to what period is the most representative of the current trend, a process of experimentation is often valuable. By comparing trend lines fitted to the same series by the use of different groups of data, *i.e.*: 1919–1930, inclusive; 1919–1937, inclusive; 1920–1930, inclusive; 1920–1937, inclusive; 1921–1932, inclusive; etc., one may determine roughly whether or not the trend is changing, and what set of data provides the most reasonable trend.

Elimination of Extreme Items.—Occasionally a series will be found which has one or two observations decidedly out of proportion to the

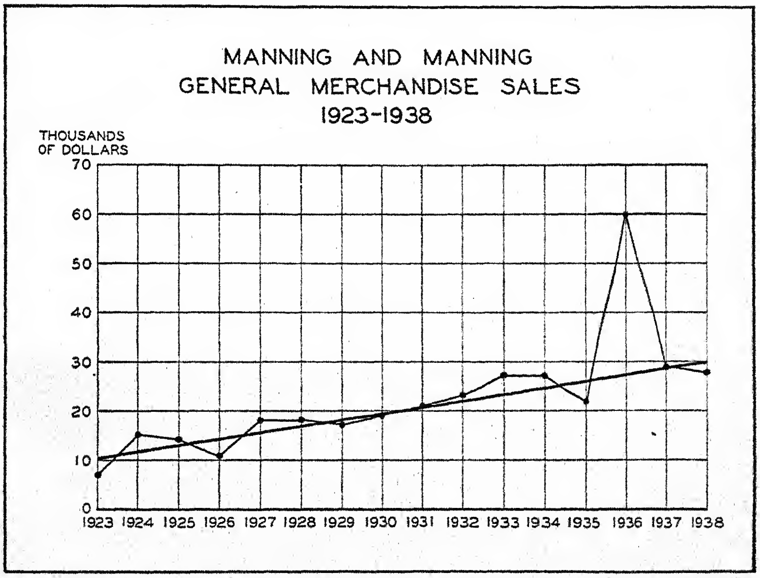

EXHIBIT 137.—The year 1936 was very abnormal because of the activity caused by the construction of a railway tunnel and bridge near the city. Therefore, it was omitted when determining basic trend. (*Data adapted.*)

others. Exhibit 137 shows such a series. The year 1936 in this exhibit appears to be freakish, and its inclusion in a trend calculation would be unwise; and the trend as shown, which fits the normal periods of the data, has been computed without any reference to the peak of 1936. Naturally, it is necessary to recognize such erratic disturbances when they occur, and to avoid confusion of non-recurring extremes with the cyclical disturbances which are recurring. In another instance, a railroad was built through a wheat-growing valley which was already served by two railroads. While the new railroad was being built, business activity in the valley was greatly accelerated, because of the presence of construction workers, etc.; but that era has passed, and, since the new railroad apparently did not influence the later development of the valley, it is not

desirable that the years of construction activity be allowed to affect the results at all when fitting trends to data on the valley's business. The valley is affected by the same types of business and crop conditions that affected it before the railroad building. It is sometimes prosperous and sometimes depressed. In the long run these conditions are offsetting, and a trend fitted to the data over a properly selected period will not be unrepresentative. When trends are being determined for areas influenced by such activity as the construction of the Boulder Dam, the Grand Coulee Dam, the Passamaquoddy power development, or the Florida Ship Canal, the data must always be carefully analyzed to find out whether the activity is merely of temporary effect, whether it caused a definite change in the slope of the trend, or whether it caused a vertical displacement in the trend with or without a change in the slope.

Relative Importance of Later Years of Trend Period.—If the trend of the data is to be used for forecasting purposes or as a basis for analysis of present circumstances, it is especially important that the trend items for the recent years be very carefully computed. Whereas slight errors in earlier years, where comparisons both backward and forward are possible, may not be important, in the later years they may be serious. Especially in forecasting is this true; for errors in arbitrarily extended trends are cumulative, and such cumulated errors as those in a production or sales budget may cause a program to be attempted which would result in a serious loss.

It will be recalled that many economic factors are concerned in the growth of a business, an industry, or a commercial area. Some of these factors operate continuously, some intermittently, and some drop out at the same time that new ones make their appearance. Such shifting of causal circumstances may or may not cause a change in a trend; but when a change in a trend occurs, it is, of course, due to changes in the underlying causes. If we wish to establish a trend line which can be extended into the future, it may be desirable to give more weight to present tendencies than to those of the past, since the present causal elements will unquestionably be more effective in the near future than those of the more distant past periods. Just how this should be done is an open question. Deliberate weighting of recent observations, selection of sensitive trend formulas, use of short periods for trend fitting—each has its peculiar advantages and disadvantages. Since the best course of action must always be determined upon a basis of good judgment in the individual case, it is impossible to make a general rule on procedure.

Available Methods of Trend Fitting.—It is possible to use many formulas of varying nature and varying degree of mathematical flexibility in the measurement of trends. Among the wide selection available are formulas whose nature will enable them to meet even the most peculiar

needs, though in some cases the computations become too laborious to be justified. There is always a point beyond which additional mathematical accuracy is not warranted, due to the time and cost necessarily entailed. Moreover, in many cases, extreme mathematical exactness does not add greatly to the accuracy of the final estimate, due to the probable human error in selecting the formula and in determining the period of fit.

Probably the most essential thing to know in trend fitting is the general form that the growth curve must assume. It is frequently possible to determine that a growth curve must have certain characteristics. If the periods are relatively short, this growth is often described by a straight line on either arithmetic paper or semi-logarithmic paper. Frequently, conditions develop in such a manner that a trend changes abruptly or a new trend is established. Sometimes the growth characteristics are described by a logistic or S-curve. When the logical form has been determined, a rough fitting of the data is next in order to determine the adjustments that should be made. Then usually the adjusted data may be fitted mathematically.

For the reasons just outlined, it seems essential for the business statistician to be equipped with methods which are simple, economical, and reasonable, but not too exacting. The processes described in the following sections will meet most ordinary needs.

Free-hand Curves as Trends.—One common trend-fitting device, which is exceedingly flexible and simple, is nothing more than an ordinary free-hand curve, sketched in arbitrarily. This device in the hands of an experienced statistician is of unquestionable value, but for the inexperienced it is unsafe.

As a matter of fact, after one understands the mathematical methods of fitting trends, he will ordinarily use them only occasionally. Free-hand fitting, tempered with mathematical knowledge and experience, often will be the most practical procedure as it facilitates a judicious weighting of the observations and a judicious consideration of other outside factors. It cannot be too strongly emphasized that a rough fitting along logical lines based on good general judgment is much better than a refined fitting along illogical lines. A good general procedure in determining free-hand trends for practical purposes is to draw three free-hand curves, two of which represent the extreme variations in either direction, and the third the compromise or most probable fit. The two extreme curves help to indicate the probable accuracy of the central curve. The free-hand curve trend will be discussed in greater detail (pages 307 to 309) after consideration has been given to a number of mathematical methods.

Moving Averages as Trends.—The moving average is another simple and flexible process of trend measurement which is quite accurate under

certain conditions. This method establishes a trend by means of a series of averages covering overlapping periods of the data, as illustrated in Exhibit 138. The moving-average item for 1933 is the simple arithmetic average of the items for 1932, 1933, and 1934; that for 1934 is the average of the data for 1933, 1934, and 1935; and so on. This process of successively averaging three years' data, and of establishing each average as the moving-average value for the central year in the group, is carried through the entire series. For a five-item, seven-item or other moving average, the same procedure is followed; the average obtained each time being considered representative of the middle period of the group.

The choice of a 5-year, 7-year, 9-year, or other moving average is determined by the length of period necessary to eliminate the effects of

COMPUTATION OF THE MOVING AVERAGE
(Three-item period)

Year	Original data	Three-item total	Moving average
1932	13		
1933	15	46	15.3
1934	18	57	19.0
1935	24	64	21.3
1936	22	72	24.0
1937	26	78	26.0
1938	30		

EXHIBIT 138.—Example of the calculation of a three-year moving average of annual data.

the business cycle and erratic fluctuations. A good trend must be free from such movements, and if there is any definite periodicity to the cycle, it is well to have the moving average cover one cycle period. Ordinarily the necessary periods will range between three and ten years for general business series but even longer periods are required for certain industries. Exhibit 139 shows a series of monthly data and moving-average trends based on 5 years or 60 months and 10 years or 120 months.

In the preceding discussion, the moving averages of odd numbers of years were representative of the middle years. If the moving average covers an even number of years (or other time periods), each average will still be representative of the mid-point of the period covered, but this mid-point will fall half way between the two middle items. In the case of a four-year moving average, for instance, each average represents a point half way between the second and third years covered. In such a case, a second moving average may be used, if desired, to "recenter" the averages. That is, if the first moving average gives averages center-

ing half way between the years, a further *two-item* moving average will recenter the data exactly on the years.

As a trend-measuring device, the moving average is subject to several criticisms. First, it is sensitive to freakish movements in the data which would scarcely affect a mathematical curve; second, it cannot be brought up to date, as the averages are representative of the mid-point of each period; and third, in an upward trend it is subject to an upward bias when the trend is increasing by increasing amounts, and to a downward bias when the trend is increasing by decreasing amounts. In the case of a downward trend, the bias operates in the reverse of these directions.

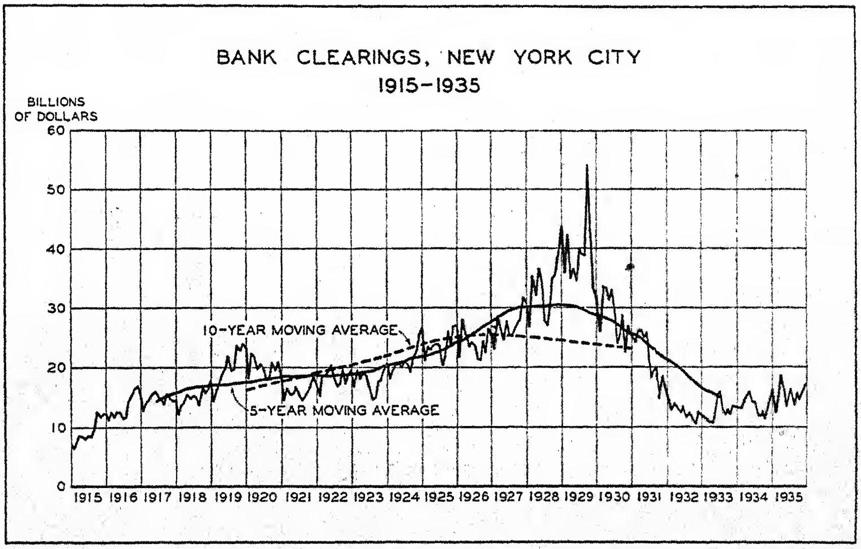

EXHIBIT 139.—Use of 5-year and 10-year moving averages of monthly items in determining trend.

However, the moving average is valuable in approximating trends in a period of transition, when the mathematical lines or curves may be inadequate, and it often serves as a basis for testing other types of trends, even though the data are not such as to justify its use otherwise.

Method of Semi-averages.—The method of semi-averages is another simple method of locating a line of trend. Two points are enough to establish a straight line. These two points may be established by dividing the trend period into halves and plotting the average of the observations falling in each half as representative of that period. Each of these semi-averages is plotted at the center of the half of the time period represented (see Exhibit 140). By drawing a line between the points thus determined, the trend is ascertained.

This method is flexible, for it is permissible to select representative periods to determine the two points. Unrepresentative years or groups of years may be ignored. For example, in Exhibit 140, it is apparent that the year 1934 is freakish and should not be considered in computing a trend. In determining the trend shown in Exhibit 140 (using the method of semi-averages), point *A* was established by averaging the years 1930–1933, inclusive, and point *B* was established by averaging the years 1935–1938, inclusive.

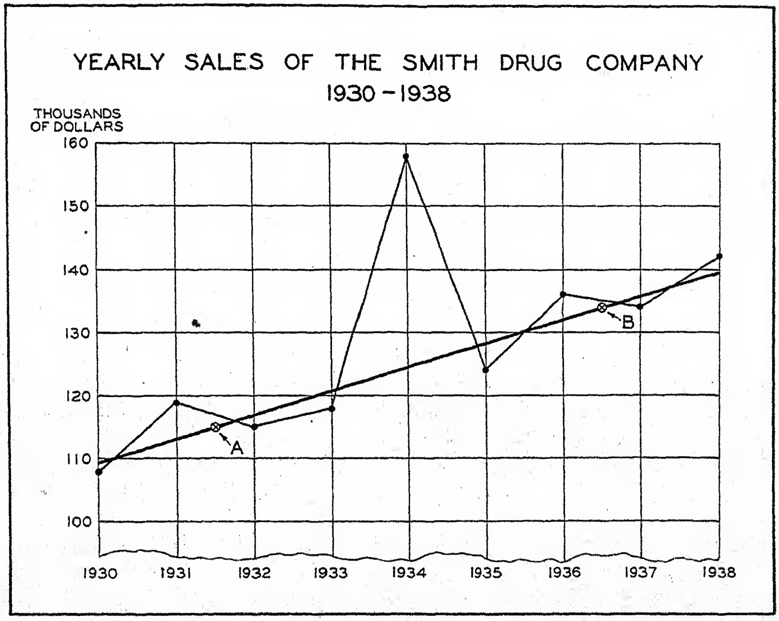

EXHIBIT 140.—Fitting a trend by method of semi-averages. (*Data adapted.*)

The yearly and monthly trend growth may be determined by dividing the difference between the values of point *A* and point *B* (Exhibit 140) by the number of months or years intervening. This method produces satisfactory results, although there is danger that the business cycle may affect the averages determining the two points, if the trend period is short.

Sometimes it will be desirable to determine a semi-average point by the use of a non-continuous period of years, or from a group of observations not occurring at regular intervals. For example, in Exhibit 140, it might be desired to represent the period 1930–1933, inclusive, by an average of the three years 1930, 1932, and 1933. In such a case, the semi-average point *A* should be centered nearer to the two points for 1932 and

1933 than to that for 1930. To be more specific, it should be located at about March 1, 1932.[1]

The use of the method of semi-averages need not be confined to arithmetic trends. If, when the data are plotted on a semilogarithmic chart, the desirability of a straight-line trend is indicated, the method of semi-averages may be applied to the logarithms of the actual data. The trend items for each year and month may then be obtained in terms of logarithms, and converted into an actual trend series by looking up their antilogarithms. In other words, in a compound interest trend, the semi-average points are located at the geometric means instead of at the arithmetic means.

Method of Least Squares.—If a straight line fitted to the data will serve as a satisfactory trend, perhaps the most accurate method of fitting it is that of least squares. The method of least squares is a mathematical device which places a line through a series of plotted points in such a way that the sum of the squares of the deviations of the actual points above and below the trend line is at a minimum. This is as near as it is conveniently possible to get to a line of least deviations, and is, therefore, accepted as the "line of best fit."

Mathematical methods of obtaining "lines of least squares" are available for both straight-line and curved trends. The determination of such a line of best fit, unless it is a very complex curve, is not a difficult task. The determination of a *straight line* by the method of least squares for data by years is as follows:[2]

1. Find the arithmetic average of the data (see column Y in the table of Exhibit 141) and find the mid-year (mid-point in time).
2. List opposite each item the number of years which it is distant from the middle year. Give those before the mid-year the sign of minus ($-$), and those after the mid-year the sign of plus ($+$). See column x in the table of Exhibit 141. These are commonly called the "time deviations" and are usually designated by x.
3. Find the squares of the time deviations found in Step 2. See column x^2 in Exhibit 141.
4. Multiply each time deviation found in Step 2 (column x) by the corresponding item in column Y. See column xY in Exhibit 141.
5. Divide the sum of the squares found in column x^2 into the algebraic sum of the multiplications in column xY. The result is the slope of the line of least squares.

From the calculation we find the slope of the line of least squares to be 5, which indicates the amount of increase in the trend of total sales from

[1] The date from which the sum of the time deviations (in months) equals zero, is the proper location for the semi-average point.

[2] Only the least-squares method applicable to straight lines will be discussed in detail. The formula, however, for a second-degree parabola is given in the footnote on p. 306.

year to year. The formula for the calculation of the slope of the line of least squares is

$$\Sigma xY \div \Sigma x^2$$

in which the terms ΣxY and Σx^2 stand for the totals of the fifth and the fourth columns, respectively, in Exhibit 141.

The slope of the line of least squares gives what is called the annual increment. It is the amount, "on the average," by which the data increase from year to year. If the data had been shown for every month of the year, then the slope would have been the amount of increase, on the average, from month to month.

Year X	Sales (thousands of dollars) Y	Time deviations of each year from middle year x	Squares of x-deviations x^2	xY	Trend ordinates (thousands of dollars)
1934	45	−2	4	− 90	50
1935	56	−1	1	− 56	55
1936	78	0	0	0	60
1937	46	+1	1	+ 46	65
1938	75	+2	4	+150	70
Total	5)300		10	+ 50	
Average	60				

$$\frac{\text{Sum of } xY}{\text{Sum of } x^2} = \frac{50}{10} = 5.$$

EXHIBIT 141.—Simple illustration of the determination of a line of best fit by the method of least squares.

The middle-point value on an arithmetic trend line is the arithmetic average of the data. In the problem of Exhibit 141, then, the trend-point value for 1936 would be 60. To determine the trend value for any given year, it is only necessary to add to or subtract from the middle-point value the proper number of *annual increments*. Thus, to determine the trend value for the year 1934, we would subtract *two* annual increments (2×5, or 10) from the annual average (60), which would give 50 as the trend value for that year (see last column of Exhibit 141). Similarly, by adding four annual increments to 1936, we would find the future trend value for 1940 to be $60 + (4 \times 5)$, or 80. The sales data and trend line of Exhibit 141 are presented graphically in Part A, Exhibit 143.

Computation of Monthly Trend Ordinates (by Method of Least Squares).—Most time-series studies are made on a basis of monthly data, but it has been previously pointed out that the fitting of trends to twelve

or even four points per year is a tedious task. For this reason the trends are ordinarily fitted to the *average month* (or *average quarter*) of each year.

Obviously, by using the *average* month of the year the data are reduced to $\frac{1}{12}$ the size of *total* annual sales data. Therefore, the resulting trend increment (when the trend is determined on the basis of an average month of each year instead of the total of the year) is $\frac{1}{12}$ as large as that of total annual sales. But this is not the monthly trend increment; it is also an annual increment, although it is the annual increment in average monthly sales instead of in total annual sales. In order to reduce this *annual* increment in average monthly sales to a *monthly* increment it is necessary to divide again by 12. As a result, the *monthly increment of sales is* $\frac{1}{144}$ *the size of the annual increment of total annual sales.* Similarly, a quarterly increment is $\frac{1}{16}$ as large as an annual increment in total sales.

For example, if average monthly data had been used in Exhibit 141, the sales column would show amounts $\frac{1}{12}$ as large as the sales given, and the average would be 5 instead of 60. This is the trend ordinate for 1936 on the new basis, and the trend increment will be found to be $\frac{1}{12}$ as large as that on the former basis, or $\frac{5}{12}$ of one thousand dollars of sales. To get the trend ordinates for the other years it is necessary only to add or subtract the increment cumulatively, as described in the illustration.

To reduce the preceding annual trend increment of average monthly sales ($\frac{5}{12}$) to a monthly increment, we again divide by 12, giving $\frac{5}{144}$ as the monthly increment. However, in obtaining the trend ordinates of the 12 months of each year a new problem arises. The trend ordinate for the average month of the year 1936 we have found to be 5 thousand dollars. This average month is what month of the year? Is it at the beginning, the middle, or the end of the year? The answer is that the average of the year is centered at the middle of the year. This will include one-half of June and one-half of July, or the period from June 16 to July 15, and the trend ordinate for this period of 1936 is 5 thousand dollars. To find the trend points for June and July, one-half a monthly increment is subtracted from the middle-point value to obtain the June trend point and one-half an increment is added to obtain the July point. The ordinates of the other months are found by adding or subtracting cumulatively a full month's trend increment.

The method of obtaining monthly trend ordinates, which has just been described, may not obtain results exactly identical with those obtained by fitting the line of least squares directly to the monthly sales data. But it is commonly used because the mathematical computation of trend is much more laborious when monthly sales data are employed. The difference in results is ordinarily negligible for practical purposes,

except where the monthly fluctuations are wide and erratic throughout the period.

The Compound-interest Curve.—Many series of business data tend to gain, on the average, by a constant percentage each year, instead of a constant amount. Such a tendency is best represented by the compound-interest curve, or geometric progression. An example of such a curve is presented in Exhibit 134, page 288.

Different rates of growth provide curves of varying shapes, but the compound-interest trend is readily recognized when plotted on a semilogarithmic chart because on such a chart it becomes a straight line.

Year X	Sales (thousands of dollars) Y	Logs of actual data (sales) log Y	Deviations of each year from middle year		x (log Y)	Logs of trend ordinates	Trend ordinates (thousands of dollars)
			x	x^2			
1934	62	1.7924	−2	4	−3.5848	1.8581	72
1935	180	2.2553	−1	1	−2.2553	2.0335	108
1936	115	2.0607	0	0	0	2.2090	162
1937	190	2.2787	+1	1	+2.2787	2.3845	242
1938	455	2.6580	+2	4	+5.3160	2.5600	363
Total		5)11.0451		10	1.7546		
Average*		2.2090					

$$\text{Logarithm of annual increment is } \frac{1.7546}{10} = .17546.$$

* Arithmetic average of the logarithms, or logarithm of the geometric mean.

EXHIBIT 142.—Simple illustration of the computation of a compound-interest trend.

Because of this characteristic, the compound-interest curve is sometimes called a "logarithmic straight line."

The compound-interest curve is fitted by the method of least squares, substituting logarithms for the actual data (see Exhibit 142). The entire process, even including the determination of monthly or quarterly trend ordinates, is carried through in terms of logarithms. The antilogarithms of the resulting trend logarithms are then obtained and used as trend ordinates. Exhibit 142 (next to last column) illustrates how the logarithm of the annual increment is successively added to or subtracted from the arithmetic average of the logarithms of the sales data. This average logarithm is the logarithm of the geometric mean, which is the trend ordinate for the middle year. The *actual* trend ordinates are the antilogarithms of the resulting logarithms (see last column of Exhibit

142). The data and trend of Exhibit 142 are presented graphically in Part *B*, Exhibit 143.

A close approximation to the results obtained with least squares on logarithms may be attained by using the method of semi-averages on logarithms. In fact, any device usable with the actual data is also suitable for use on the logarithms, giving the compound-interest characteristics a degree of flexibility which makes them much more useful than they would otherwise be. Of course, the semi-logarithmic chart should be used as a guide for such manipulations, as it reflects tendencies toward geometric progression much more clearly than does the arithmetic chart.

The S-curve Trend.—The tendency of the growth of an industry, especially if it is expressed on a per capita basis, to have small gains at first, then relatively large gains, and then small gains again as it approaches

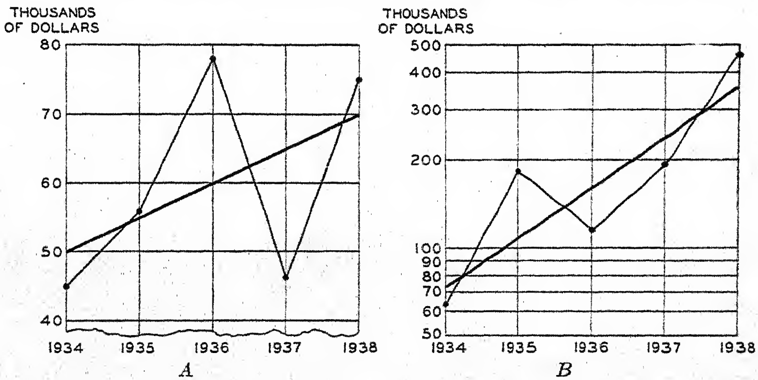

EXHIBIT 143.—*A*. Arithmetic trend line computed in Exhibit 141. *B*. Compound-interest trend line computed in Exhibit 142.

the market saturation point, is generally appreciated. Such series as automobile registrations and telephone and electric-light installations over a period of time have shown these growth characteristics in a manner which is very similar to the characteristics of a cumulative normal frequency distribution. Many other series of business data show tendencies of growth that vary in such a manner that they show characteristics of cumulative frequency curves other than those of the normal or bell-shaped curve when cumulated. Trend curves showing these tendencies toward an S form are known as "S-curves," and although there are theoretical limitations upon their use in the analyses of many time series, they have been found to be of practical value in analyzing trends which show a persistent tendency toward an S form.

As an introduction to the study of trends of this sort, a brief description will be given of the use of the S-curve, which, when plotted, corresponds in appearance to a cumulative frequency curve made from a

bell-shaped or symmetrical distribution. It is to be emphasized that this particular S-curve can be used in time series analyses only when it fits the data presented. The use of other S-curves of this type in analyses of time series can be found in the more extended discussions of this subject.[1]

The use of the concept of a cumulative normal frequency distribution in analyzing the trend of a time series is greatly facilitated by plotting the curve on cumulative probability paper[2] using a simple arithmetic

TOTAL AUTOMOBILE REGISTRATIONS IN THE UNITED STATES PER 100 PERSONS
(December 31, 1905 to 1937)

Year	Number of registrations per 100 persons	Number multiplied by 4.3	Year	Number of registrations per 100 persons	Number multiplied by 4.3
1905	.09	.39	1922	11.15	47.94
1906	.13	.56	1923	13.60	58.48
1907	.16	.69	1924	15.77	67.81
1908	.22	.95	1925	17.50	75.25
1909	.35	1.50	1926	18.97	81.57
1910	.50	2.15	1927	19.87	85.44
1911	.65	2.79	1928	20.65	88.79
1912	.97	4.17	1929	21.98	94.51
1913	1.33	5.72	1930	21.57	92.75
1914	1.75	7.52	1931	20.80	89.44
1915	2.48	10.66	1932	19.30	82.99
1916	3.50	15.05	1933	18.98	81.61
1917	4.99	21.46	1934	19.71	84.75
1918	5.98	25.71	1935	20.57	88.45
1919	7.28	31.30	1936	21.97	94.47
1920	8.72	37.50	1937	23.05	99.11
1921	9.75	41.92			

EXHIBIT 144.—Table of data to accompany Exhibits 145 and 146.

scale to represent time. The use of probability paper enables one to apply probability laws to business problems without laborious mathematical treatment. Exhibit 145 shows total automobile registrations per 100 inhabitants in the United States from 1905 to 1937, and how the

[1] KUZNETS, SIMON S., "Secular Movements in Production and Prices," Houghton Mifflin Company, Boston, 1930; PEARL, RAYMOND, "The Logistic Curve and the Prediction of the Population of the Chicago Region," *Journal of the American Statistical Association*, Vol. 23, p. 361, December 1928; YULE, G. UDNY, "Laws of Population Growth," *Journal of Royal Statistical Association*, Vol. 88, pp. 1–58, January 1925; PERSONS, WARREN M., "Forecasting Business Cycles," Chap. XII, John Wiley & Sons, Inc., New York, 1931; HAY, WILLIAM WREN "'Estimation of Population Growth," *Roads and Streets*, Vol. 81, No. 6, pp. 45–47, June 1938.

[2] The probability paper recommended for this purpose is that designed by Allen Hazen and sold by the Codex Book Company, Inc., Norwood, Mass. The paper is often referred to as "Hazen's Cumulative Probability Paper."

trend corresponds to the familiar S-shaped cumulation of a normal frequency curve. Exhibit 146 shows how this curve trend forms a straight line when plotted on cumulative probability paper to the proper scale.

The probability paper illustrated in Exhibit 146 is so designed that cumulative normal frequency curves can be plotted as straight lines. This fact makes it possible to construct a trend with a much smaller number of observations than would otherwise be required and, what is

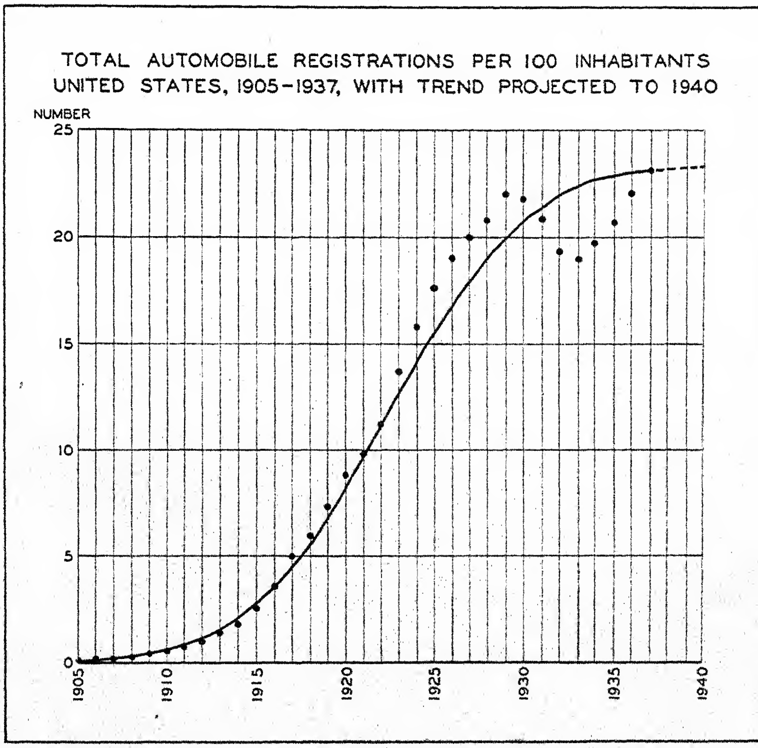

EXHIBIT 145.—An S-curve trend plotted on arithmetic paper.

more important, it also permits the projection of the trend into the future with the likelihood that such projection will be in accordance with the actual development. "Although the theory governing the probability curve has been well understood for a long time, its application has been limited almost entirely to technical problems, owing to the difficulty of plotting it. The designing of probability paper has opened up through its simplicity a much wider field, and unquestionably there are many problems that may be solved by it that have as yet not been worked out. The law of probability is applicable to a wide variety of fields in experimental and statistical work, testing, sampling, insurance, in fact to

all sorts of observations which are not susceptible to an exact mathematical analysis, but which may be solved approximately through their dependence upon the laws of chance."[1]

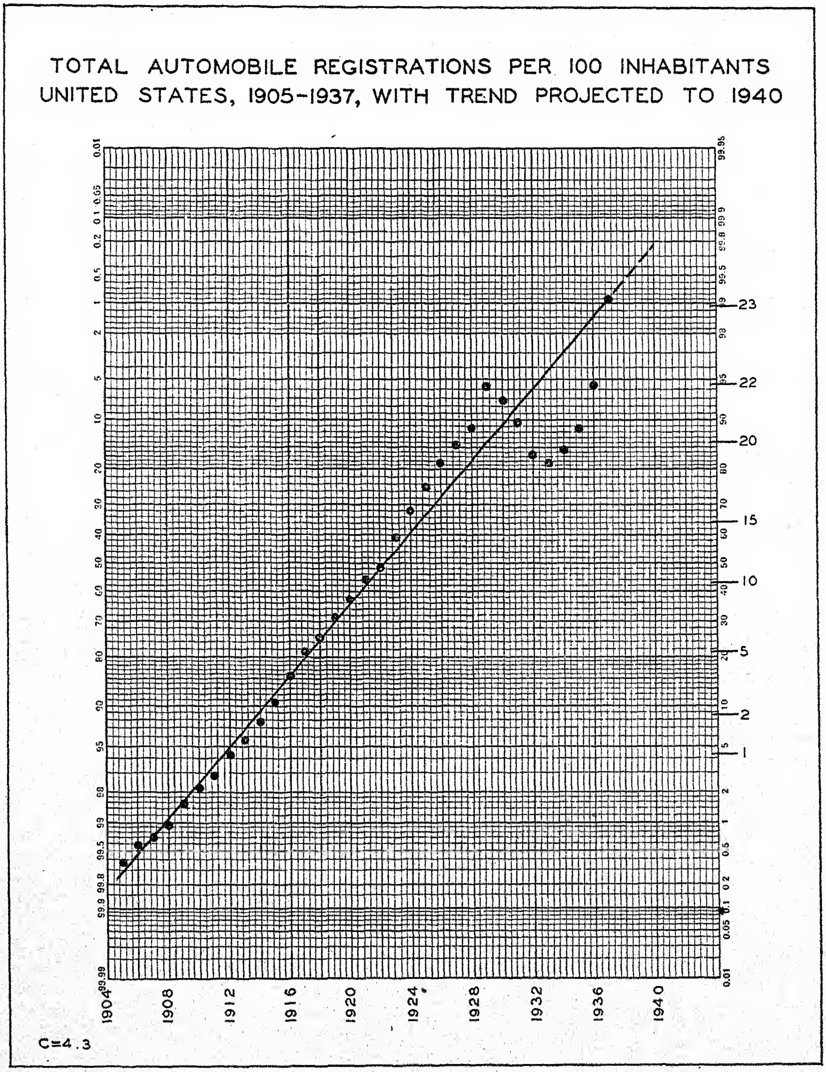

EXHIBIT 146.—An S-curve trend plotted on probability paper. (*Adapted from American Founders Corporation.*)

The scales of small figures on the sides of the graph in Exhibit 146 are in percentages. These figures appear on the paper as it comes from

[1] HASKELL, ALLAN C., "Graphic Charts in Business," p. 149, Codex Book Company, Inc., Norwood, Mass., 1928.

the manufacturer. The limits of a cumulative series are no items and all items—that is, 0 and 100 per cent of the total of the series. The probability scale laid out in percentages will never reach either 0 or 100 per cent in either direction, for the true normal curve is asymptotic to (*i.e.*, approaching but never reaching) the horizontal axis, and its 100 per cent parallel. Therefore, we can never plot 0 and 100 per cent on this paper. We can only approach these limits by extending the scale into small fractions toward each limit.[1]

In order to plot an S-curve as a straight line, it is necessary to multiply (or divide) the original data by a constant in order to adjust the sizes of the data to correspond to the scale of the probability paper. The size of this constant is determined by experiment. If we multiply by a constant that is too large, the line will curve upward, and if we multiply by a constant that is too small, the line will curve downward.

To illustrate the use of probability paper in greater detail, let us consider the data on total auto registrations per 100 inhabitants presented in Exhibit 144. Suppose that our horizontal scale of years is identical with that in Exhibit 146.[2] Then as a trial, let us take three points, one from near the beginning of the series, one from near the middle, and one from near the end—namely, the figures for 1907, 1922, and 1937 (see Exhibit 144). Now the problem is to see if we can multiply these data by a constant that will enable us to plot them (according to the scale of small figures on the right, Exhibit 146) so that they will lie in a straight line, running diagonally from the lower left to the upper right. If 3 is used as a constant, the curve turns downward. If 5 is used, the curve turns upward. If 4 is used, the points lie more nearly in a straight line. Further experiment shows that 4.3 is a satisfactory constant.[3] The original data are then multiplied by 4.3 as shown in the last column of the table of Exhibit 144 and plotted according to the detailed scale of percentages on the right of the graph (Exhibit 146). This trend has been projected to 1940. For convenience in reading, a scale in terms of registrations can be set up as shown by the large figures

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[1] This is not an important difficulty, however, as "unless we deal with large series, containing over ten thousand items and so grouped that the terminal groups have only one item each, we shall not need to plot points less than .01 per cent or greater than 99.99 per cent." See Karl G. Karsten, "Charts and Graphs," p. 460, Prentice-Hall, Inc., New York, 1923.

[2] The probability paper referred to in this illustration, namely as illustrated in Exhibit 146, can be used only for the type of S-curve to which this discussion is limited.

[3] It should be appreciated that we cannot determine our constant on the basis of the ratio of the average of the data to the average of the percentage scale, because at the outset we do not know what part of the complete range our data represent. Therefore, the best fit is determined experimentally as described.

on the right of the graph. In this case the units 1, 2, 5, 10, 15, 20, 22, and 23 were selected as convenient in relation to the spacing. By multiplying these by 4.3 and finding the positions on the percentage scale, we have the proper locations for the corresponding scale figures.

Having determined the trend in the above manner, it is sufficiently accurate for most purposes to read the values of this trend line from the graph, but they can be mathematically computed if greater accuracy is desired. In either case, the corresponding data can then be expressed in percentage deviations from the trend, and these can be converted to standard or average deviations.

While the fitting of growth curves to straight lines on this special plotting paper may have but little significance if only a fraction of the growth is shown, the significance is great when a close fit is obtained with respect to a growth curve that has approached to within 20 per cent of its saturation point.[1] Of course, an S-curve trend, like all other trends, should not be used unless supported by a strong logical justification.[2]

One of the greatest difficulties in using the S-curve as above described is to make the proper selection of the three points on which to base the straight line. Here careful judgment must be exercised if the line is to serve as a normal above and below which the cycles fluctuate. If the values fluctuate as much as they do toward the end of the period in Exhibit 146, it is particularly important to apply external checks to see if the position is reasonable. Probably the greatest importance of the S-curve in time-series trend fitting is not in its direct use, which will be only occasional, but rather in its indirect value in the training and help that it gives the statistician in fitting representative trends of other types. A keen appreciation of the long-time tendency of a mature industry to flatten out as it approaches a saturation point in the market is

[1] See M. C. Rorty, "Statistics and the Scientific Method," Presidential Address, Ninety-second Annual Meeting, American Statistical Association, December 31, 1930, published in the *Journal of the American Statistical Association*, March 1931, pp. 1–10.

[2] Further information on frequency curves and probability paper is given in Allan C. Haskell, "Graphic Charts in Business," pp. 143–151 and 340–351, Codex Book Company, Inc., Norwood, Mass., 1928; and Karl G. Karsten, "Charts and Graphs," pp. 450–471, Prentice-Hall, Inc., New York, 1923.

Occasionally it is advisable to measure a curving trend (not of the S type) whose amounts of change are increasing or decreasing, or whose percentage rate of change is gradually changing (see Exhibit 176, p. 374). For most practical purposes a second-degree curve of the potential series (second-degree parabola) is satisfactory. The general formula for the trend points is, $y = a + bx + cx^2$, in which y represents the trend value of y or the trend ordinates. The constants a, b, and c are found by solving three normal equations in three unknowns. For further explanation of the mathematical process, see Frederick C. Mills, "Statistical Methods," pp. 253–279, Henry Holt & Company, New York, 1938.

imperative to the business statistician, and a knowledge of S-curve trend fitting will be of much help to him in making the necessary allowances and adjustments, when, as will usually be the case, he is attempting to fit a straight line, a free-hand curve, or other simple type of trend.

Mathematical Methods of Trend Measurement Not "Fool-proof."—As will be inferred from the description of types of trends, accurate trends cannot always be fitted mathematically. Since the approach is generally that of fitting a line or curve to the charted original data, the particular mathematical device used is the one which the statistician believes will best enable him to measure the movements apparent to his eye. However, mathematical methods of fitting trend are not "fool-proof"; in fact, they are the source of some of the most serious errors that are made in statistical work. They should *never* be used unless rigidly controlled by a separate logical analysis. Trend fitting depends upon the judgment of the statistician, and a free-hand sketch is often more practical than a refined mathematical formula.

Effective use of mathematical methods in trend measurements is possible only when the statistician is reasonably familiar with the inherent characteristics of each type of trend line. While the fitted curves tend to follow the observed points to which they are fitted, the curve formulas control the characteristics of the trend lines to such an extent that the use of the wrong one necessarily means a poor fit. For this reason, as indicated previously, it is necessary that one appreciate mathematical methods of trend fitting if he is to be able to fit a free-hand trend skillfully.

Trend Fitting Not Merely Curve Fitting—Importance of Free-hand Curves.—In fitting trends it must be thoroughly appreciated that the objective is to determine the fundamental tendency over a considerable number of years. Suppose that in January 1938 we are fitting a trend curve to steel-ingot production for the period from 1900 to 1937, inclusive. The position of the trend at the end of the period should be representative, not only of the data of the past period, but also for at least a few years into the future. It should be located at the level above and below which past and future cycles will fluctuate. Obviously, a mathematically fitted line, for instance, even though it be the most representative line that could be fitted to the curve, may not represent the fundamental growth tendencies of the series. In Exhibit 147 a line (or curve) mathematically fitted to the period from 1920 to 1930 would indicate a downward tendency into the future. Economic analyses of the conditions surrounding steel-ingot activity, however, indicate that such a trend would not be truly representative of the long-time fundamental tendencies of the series.

A trend-fitting problem like that of Exhibit 147 requires a combination of methods together with a thorough understanding of the nature of

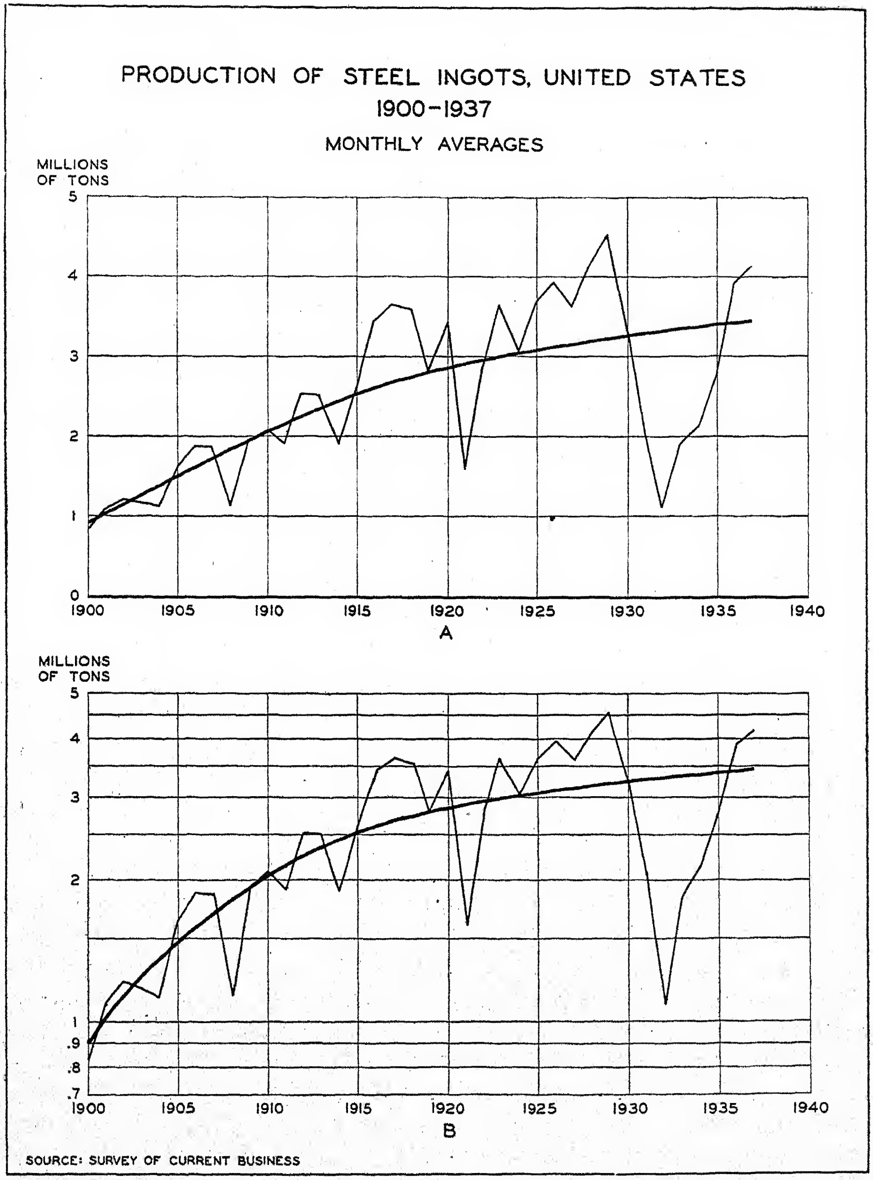

EXHIBIT 147.—A free-hand curve trend fitted by inspection but based partly upon a moving average for the early part of the period and a number of computed trends for the later part of the period.

long-time tendencies and good judgment on the part of the statistician. For the most part, the trend shown in Exhibit 147 is a free-hand curve based, in turn, upon a 10-year moving average for the early part of the period, and upon a number of efforts to compute a trend for the later part of the period. At the end of the period an arbitrary adjustment is included, based upon a general knowledge of economic growth tendencies and a careful consideration of the influence of a major depression on the usual methods of curve fitting. The resulting free-hand curve stands the test of rational analysis when plotted on both arithmetic and logarithmic paper. Such trends must be carefully watched when making current analyses and forecasts, and checks must be made with outside data to determine whenever possible whether the trend level or slope is too high or too low to be reasonable.

Year	Data of Exhibit 141			Data of Exhibit 142		
	A Sales in thousands of dollars	*B* Trend ordinates	*C* Series in percentage deviations from trend	*D* Sales in thousands of dollars	*E* Trend ordinates	*F* Series in percentage deviations from trend
1934	45	50	−10	62	72	−14
1935	56	55	+ 2	180	108	+67
1936	78	60	+30	115	162	−29
1937	46	65	−29	190	242	−21
1938	75	70	+ 7	455	363	+25

EXHIBIT 148.—Simple examples of eliminating trend.

Elimination of Trend.—Usually the best method, and in many instances the only practical method of eliminating the trend influence, is to express the actual data as percentages of the corresponding trend ordinates—that is, to divide each trend ordinate into the corresponding actual sales or other items. When the resulting percentages of trend are plotted, the points will appear above or below the horizontal 100 per cent line, and the relative variations from the trend can easily be measured by noting the distances (on the percentage scale) of each point from the horizontal base. If desired, 100 per cent may be subtracted from each percentage of trend item, giving positive and negative percentage deviations from trend. Exhibit 148 illustrates the elimination of the trends computed in Exhibits 141 and 142 (see columns *B* and *E*) from the original sales data (columns *A* and *D*) with the adjusted series expressed in terms of percentage deviations from trend (columns *C* and *F*).

It is also possible to determine variations from trend by subtracting each trend ordinate from the corresponding actual datum. But the

result is in size units, such as dollars or pounds, rather than in percentage terms. For further statistical use, it is usually desirable to state the variation from trend in percentage terms, and these percentages are, in turn, often converted to terms of percentage deviations, average deviations, or standard deviations.

Questions and Problems

1. What is basic or long-time trend?

2. Describe three common types of trends. What is meant by "broken trends"?

3. What would you do first to determine the proper trend formula?

4. In charting data for trend purposes, what is ordinarily a proper relationship for the horizontal and vertical scales?

5. Discuss the problem of determining the proper time period for trend measurement.

6. Is it ever advisable to disregard any items in a series for trend measurement purposes? Explain.

7. If weight is to be given to any portion of the period for which the trend is to be measured, to which portion should it be given? Why?

8. Name four methods of determining a line of trend.

9. What are the advantages and disadvantages of the free-hand curve trend?

10. Describe the method of moving averages as used in trend measurement. What factors determine the period of the average? What are the disadvantages of this method of trend computation?

11. Explain the method of semi-averages for trend measurement. What are some of the advantages of this method?

12. State the steps in the computation of a straight-line arithmetic trend by the method of least squares.

13. What is meant by "trend increment"? By "trend ordinate"?

14. Is the monthly trend increment one-twelfth the annual trend increment? Explain.

15. Why are trends often fitted to the average month of the year by the method of least squares instead of to annual data?

16. How is the method of least squares used in fitting a compound-interest curve? Describe the process.

17. In connection with what types of growth should the S-curve trend be used?

18. Explain the use of probability paper in determining S-curve trends.

19. Why should the S-curve trend (as well as other trends) be supported by a strong logical justification?

Sales of Jones Wheel Co., 1922–1937

1922	$59,000	1930	$99,000
1923	42,000	1931	84,000
1924	47,000	1932	61,000
1925	58,000	1933	32,000
1926	67,000	1934	40,000
1927	89,000	1935	52,000
1928	78,000	1936	68,000
1929	92,000	1937	84,000

Note: This table is part of Problem 22.

20. Explain "mathematical methods of trend fitting are not fool-proof."

21. Describe two methods of eliminating the effect of trend from a time series. How are "percentage deviations from trend" obtained?

22. From the annual sales of Jones Wheel Co. for the years 1922–1937, as shown in the table on page 310, compute the trend ordinates for these calendar years by the moving-average method. Use a 4-year moving average.

23. The Morton Lumber Company had average quarterly sales (in millions of dollars) as follows:

Year	*Average quarterly sales*
1930	2.1
1931	2.0
1932	1.6
1933	1.7
1934	2.2
1935	4.3
1936	6.7
1937	7.6

a. By the method of least squares find the quarterly trend ordinates for each of the eight years.

b. What is the annual trend ordinate for 1937, and what is the annual increment of the annual trend?

c. What are the quarterly trend ordinates for the four quarters in 1937?

24. Sales of the Pacific Book Company by quarters for the years 1932–1937 were (in thousands of dollars):

Year	1st quarter	2d quarter	3d quarter	4th quarter
1932	34	46	40	60
1933	37	23	41	19
1934	17	24	28	31
1935	35	45	37	43
1936	29	44	38	49
1937	40	47	38	55

By the method of least squares what is the annual increment of the annual trend? What is the quarterly trend ordinate for the third quarter of 1937, and how does this figure compare with the actual sales of that quarter?

25. Electric power sales and revenues in the United States, 1928–1936, were as shown in the table at the top of page 312.

a. What is the monthly trend ordinate for September 1936 of the sales of power? Use the method of least squares.

b. What is the monthly trend increment per month for the revenues from power sales? Use the method of least squares. What would be the monthly trend ordinates for revenues in 1937?

c. Fit a free-hand trend to the data taking into consideration possible and probable future growth. Show graphically.

Year	Sales (in billions of kilowatt-hours)	Revenue (in billions of dollars)
1928	67.0	1.78
1929	75.3	1.94
1930	74.9	1.99
1931	71.9	1.98
1932	63.8	1.83
1933	66.0	1.77
1934	70.8	1.84
1935	77.6	1.92
1936	90.0	2.07

Source: The Edison Electric Institute.

26. By the method of semi-averages fit a straight-line trend to the following Dow Jones averages of railroad-stock prices:

a. For the years 1899–1936.

b. For the years 1918–1936.

Year	Average price	Year	Average price	Year	Average price
1899	$ 79.76	1912	$119.64	1925	$102.96
1900	83.99	1913	109.30	1926	112.87
1901	105.26	1914	98.42	1927	132.06
1902	120.55	1915	98.07	1928	142.65
1903	105.04	1916	105.70	1929	158.59
1904	105.39	1917	88.26	1930	124.80
1905	124.03	1918	85.06	1931	72.50
1906	129.33	1919	82.38	1932	27.27
1907	106.68	1920	76.60	1933	39.98
1908	103.05	1921	71.54	1934	43.08
1909	124.18	1922	83.71	1935	34.58
1910	117.75	1923	83.71	1936	50.28
1911	116.83	1924	89.74		

Source: The Wall Street Journal, Pacific Coast Edition, March 1937, *Review and Supplement.* The prices quoted are averages of the high and low for the respective years.

In both *a* and *b* chart the data together with the trend line (see Exhibit 140).

27. Fit a 3-year moving average trend to the data given in Problem 26.

28. Fit a trend by the method of least squares to the data given in Problem 26.

29. Fit a free-hand trend to the data of Problem 26.

CHAPTER XV

TIME SERIES—SEASONAL VARIATION

The various weather and climatic conditions, which characterize different portions of each year, have a direct bearing upon trade and production and, consequently, are very important in determining slack and busy seasons. Nearly every type of business activity is susceptible to seasonal influence, to a greater or a lesser degree, and the seasonal effects are present whether the year be a prosperous one or a dull one. Seasonal variation is, therefore, to be accepted as a normal phenomenon, recurring every year, and one which should be accounted for in attempts to study the causes of past developments or to forecast coming events.

Nature of the Seasonal Movement.—The seasonal movement in the form of recurring peaks and valleys of each year can easily be observed in Part *A*, Exhibit 149, which shows the volume of sales of a machinery firm for 21 years, from 1918 to 1938. It is also apparent that the seasonal movement is much more pronounced in the more recent years when sales have been larger. However, when we place the same data on the logarithmic scale (Part *B*, Exhibit 149), the seasonal fluctuations appear to be relatively about the same throughout the entire series. This test illustrates two important characteristics of most seasonal movements: (1) that their extent tends to be a certain proportional deviation from the average business at the time, rather than a definite amount; and (2) that this proportional deviation ordinarily does not change rapidly.[1]

In Chapter XIII, a normal year in any series was described as one in which only the trend and regularly recurring seasonal factors were operating, and in which there were no effects due to cyclical and erratic factors. Assuming that such a year is found, and the monthly trend values are known, the measurement of the seasonal movement is a simple task. If we divide the actual monthly values by the corresponding trend values, the several quotients represent the seasonal variation, *stated as percentages of the trend.*

To illustrate, we may find that, on account of seasonal acceleration, January of our assumed normal year is 110 per cent of the trend for

[1] These conclusions are not based upon universal characteristics, but rather upon the tendencies in the majority of cases. Not infrequently seasonal variation may change. In such cases, methods for measuring changing seasonal variations should be used, if accurate measurement is necessary (see pp. 324 and 325).

January; or that due to seasonal dullness, August of our assumed normal year is 80 per cent of the August trend. These percentages are the "seasonal indexes" for January and August, respectively. Such a

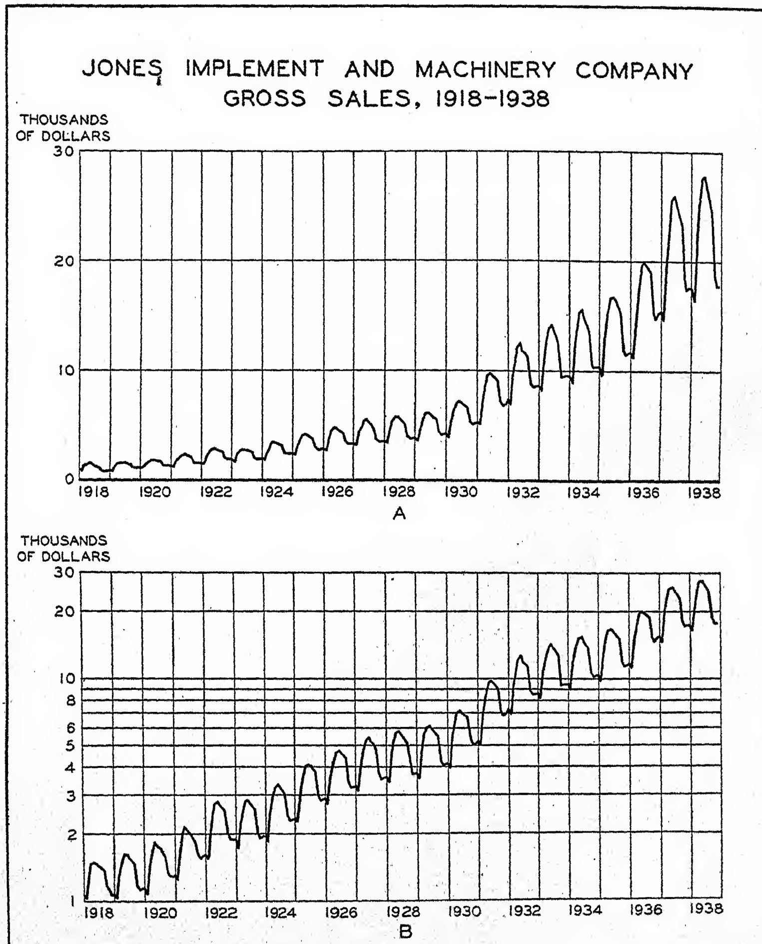

EXHIBIT 149.—A series with distinct seasonal fluctuations presented on arithmetic and semi-logarithmic graphs. It is evident from the unadjusted data that the relative seasonal variations are about the same over the period (lower graph or Part *B*). (*Data adapted.*)

seasonal index indicates the relationship of any "normal" month to the computed trend for that month. But the completely normal or "ideal" year (which has only trend and seasonal factors) is practically never found in business data, because of cyclical and erratic influences. Over a period of years, however, the cyclical and erratic effects are likely to be offsetting, and if *averages* are taken of the Januarys, of the Februarys,

of the Marchs, etc., one should obtain a normal year. This may be done by averaging the ratios to trend for each January (over a period of years), for each February, etc., to obtain a normal ratio to trend for each month.

Preliminary Considerations.—Before attempting to measure seasonal variation, certain preliminary decisions must be made. It is, of course, necessary to decide whether weekly, quarterly, or monthly indexes are required, which will be determined by the nature of the problem and the type of data available.

It is possible to make the computation of seasonal indexes more accurate by some methods if trend influences are eliminated before the seasonal computations are made. This may be done by computing the trend values and dividing them out of the series, as was explained in the preceding chapter.

The problem of varying lengths of months, which was discussed previously in connection with Exhibit 132 on page 282, is important in determining seasonal variation. It is sometimes necessary to make special allowances for varying lengths of months from one year to the next because of differences in the numbers of Sundays, holidays, etc. Adjustments may sometimes be made in the seasonal percentages, or in the original data for each month, by dividing by the number of working days in the month to arrive at an average daily figure. Then the analysis may proceed in the usual manner, using the daily averages instead of the monthly totals.

The fact that the same months in different years do not always contain the same number of business days, owing to changes in the calendar, is significant when handling the statistics of certain lines of business, where the amount of activity varies directly with the number of business days. In other types of business there is little difference between a Sunday or holiday and an ordinary work day. Between these two extremes are various degrees of difference.

The ordinary seasonal percentage may often be made more accurate by adjusting it to allow for the number of working days in the month. For example: July has 31 days, of which 1 is a holiday and $4\frac{3}{7}$ (the average number) are Sundays, thus leaving 25.57 working days. When there are 5 Sundays in July the number of working days is 25, or 2.2 per cent less than the average, and the seasonal index should thus be decreased that much. Conversely, when July has only 4 Sundays there are 26 working days, or 1.7 per cent more than the average; the normal seasonal index should therefore be increased by 1.7 per cent. This sort of scheme is logical only when the volume of activity is known to be a function of the number of work days, and when the 6 work days of the week are equal in importance.

No arbitrary rule can be laid down which will determine whether or not the average daily basis is better or worse than the monthly total basis for any particular series. In most cases, it is evident that corrections for differences in additional days at the ends of the months are sound, but whether or not adjustments should be made for Sundays and holidays is a more difficult question. It is obvious, for instance, that if a restaurant closes on a holiday, the day's business is lost, while if a clothing store closes on a holiday, its sales will be made up largely on the immediately following and perhaps preceding days. The fluctuations in a series like bank clearings, however, are due to an intricate combination of many kinds of activities, and it is very difficult to determine the real effect of holidays. Average daily figures based on so-called working days, therefore, should not be used blindly, but only when a careful analysis indicates that they are more representative than the totals.

Period of Seasonal Calculation.—Just as in the case of basic trends, the seasonal-index computations must be made from data which are homogeneous and representative. A seasonal index of short-time money rates covering the period 1900 to 1938 would be fallacious, for the data are not homogeneous. Because of the inauguration of the Federal Reserve System and recent banking legislation, a number of changes have occurred in the seasonal characteristics of interest rates. Similar examples are available in all lines of business. On the other hand, it would be deceptive to compute an index of seasonal variation for interest rates if we used only three or four years, for the data are so affected by the business cycle and stock speculation that such a short period would not be representative. For some data, three years may be enough; for other series, eight, ten, or even more years may be necessary.

To guide the statistician in the selection of the proper period (or periods) for his index, both qualitative and quantitative facts are usually available. Facts about changing economic circumstances will provide the first, and a study of the original data and of the link relatives or other basic ratios will give the second.

Methods of Measuring Seasonal Variation.—There are many different methods of measuring seasonal variation, some of which are quite accurate and some of which are only approximate. The following methods, which are practical for business use, will be discussed somewhat in detail:

1. Method of monthly totals or monthly averages.
2. Ratios-to-trend method.
3. Ratios-to-moving-average method.
4. Link-relative method.

Other specialized methods for measuring changing seasonal variation, and for applying seasonal indexes to unadjusted data, etc., will be discussed briefly.

Method of Monthly Totals or Monthly Averages.—The method of monthly totals or monthly averages is one of the simplest methods of obtaining a seasonal index. The procedure is as follows:

1. Arrange the unadjusted data by years and months as in the *A* columns of Exhibit 150.
2. Find the totals of the Januarys, Februarys, etc. (see column *B*, Exhibit 150).
3. Express the total of each month as a percentage of the average of the monthly totals. This is the measure of seasonal variation (see column *C*, Exhibit 150).

Month	Sales (in thousands of dollars)				*C* Percentage of each monthly total to average monthly total
	A 1936	*A* 1937	*A* 1938	*B* Total of three years	
January	132	172	207	511	99.5
February	104	141	167	412	80.2
March	96	103	203	402	78.3
April	86	110	182	378	73.6
May	91	160	170	421	82.0
June	100	155	216	471	91.7
July	112	186	220	518	100.9
August	113	189	261	563	109.6
September	106	178	274	558	108.7
October	117	168	289	574	111.8
November	131	209	312	652	127.0
December	146	232	324	702	136.7
Total	...	...	...	6162	1200.0
Average monthly total	...	...	...	513.5	100.0

Exhibit 150.—Computation of seasonal variation by the method of monthly totals. (To simplify the illustration, only three years have been used. Ordinarily the period should be longer in a practical problem.)

If, instead of the totals of each month, the average of each month is used in column *B*, the same results are obtained.

The method of monthly totals of computing seasonal variation gives undue weight to the data of the years in which the greatest amount of activity occurs, for those years will have the greatest effect on the monthly totals. Thus, in the example just given, the year 1936 was a very poor year, especially during the first half, and as a result the monthly totals for the early months are less than they would be under more normal circumstances. Again, the last months of 1938 were much higher than usual, so that these monthly totals are larger than they would be under more normal circumstances.

This method does not allow for the trend. Consequently, in the above case, there is an "annual trend increment" in the monthly totals which causes the first months of the year to be less than they would otherwise be, and, similarly, which causes the last months of the year to be larger than they would otherwise be. In applying this index to determine what the sales would have been if no cyclical influences were present (that is, if seasonal and trend influences alone had been present) it is necessary to adjust the datum of the average month of the year instead of the actual datum of each month. The application of seasonal indexes in this manner is discussed on pages 336 to 339.[1]

The advantages of this method are that it is simply computed, easily understood, and may be readily interpreted. It should be appreciated that for purposes of illustration, a very simple example is used in Exhibit 150, and that ordinarily a three-year period is too short for practical purposes. Three-year periods are usually influenced too much by erratic and cyclical influences to give dependable seasonal indexes.

Ratios-to-trend Method.—The method of monthly totals or monthly averages does not directly give any consideration to the trend which may be in the data. The ratios-to-trend method is one of the simplest of the commonly used devices for measuring seasonal variation which takes the trend into consideration. The steps in the calculation of seasonal variation by this method are as follows:

1. Arrange the unadjusted data by years and months as in the *A* columns of Exhibit 151.
2. Compute the trend value for each month (quarter, week, etc.) as in the *B* columns of Exhibit 151.
3. Express the data for each month as a percentage ratio of the corresponding trend value (see *C* columns, Exhibit 151).
4. Total the January ratios, the February ratios, etc., found in Step 3 (see column *D*, Exhibit 151).
5. Find the average ratio for each month (see column *E*, Exhibit 151).
6. Adjust the average ratios found in Step 5 so that they will themselves average 100 per cent (see column *F*, Exhibit 151). (The method of doing this is described in the succeeding section, pages 319 and 320.)

Instead of finding the arithmetic average of the ratios to trend, sometimes the median is found. The choice may depend upon circumstances, but there is a preference for the median if several erratic ratios are found. In fact, if a fairly large number of years is used in the computation, it is not uncommon to omit extremely erratic ratios from the computation of the average of monthly ratios. In Exhibit 151, only three years have been used (to simplify the illustration). Only the arithmetic average should be used for a small number of years.

[1] In using this reference, note carefully the footnote on p. 337.

The ratios-to-trend method has, like the monthly totals method, the advantage of simplicity and ease of interpretation. Although it makes allowance for trend, it may be influenced by errors in the calculation of the trend. Thus, if the trend had been erroneously computed in the preceding example, some or all of the ratios would be affected. If the error is in the slope of the trend, then an error will appear in all the ratios; if the error appears in the computation of certain trend points, then only those ratios are affected which are based upon the erroneous trend

Month	*A* Actual sales (thousands of dollars)			*B* Trend values (thousands of dollars)			*C* Percentage ratios to trend			*D* Total	*E* Average ratios (per cent)	*F* Seasonal index adjusted (per cent)
	1936	1937	1938	1936	1937	1938	1936	1937	1938			
January....	125	213	275	120	180	240	104	118	115	337	112	113
February...	104	176	235	125	185	245	83	95	96	274	91	92
March......	101	163	185	130	190	250	78	86	74	238	79	80
April.......	95	151	216	135	195	255	70	77	85	232	77	78
May.......	105	172	149	140	200	260	75	86	57	218	73	74
June.......	120	193	242	145	205	265	83	94	91	268	89	90
July........	240	226	277	150	210	270	160	108	103	371	124	126
August.....	147	236	203	155	215	275	95	110	74	279	93	94
September..	143	230	286	160	220	280	89	105	102	296	99	100
October....	164	222	315	165	225	285	99	99	111	309	103	104
November..	190	250	342	170	230	290	112	109	118	339	113	114
December..	219	326	396	175	235	295	125	139	134	398	133	135

EXHIBIT 151.—Computation of seasonal ratios by ratios-to-trend method. (In order to simplify the illustration only three years have been used. Ordinarily, a seasonal index should be based upon a longer period.)

values. The ratios-to-trend method also may be undesirably influenced by pronounced cyclical and erratic influences. This source of possible error is theoretically eliminated by the selection of a period of time in which depression is offset by prosperity, so that the repeated observations establish average ratios which approximate normal.

Adjustment of Seasonal Ratios to Average 100 Per Cent.—Seasonal ratios, since they represent deviations from normal conditions, represent a complete seasonal cycle which recurs each year; that is, they give a numerical expression of a decline below the basic trend, a return to trend, a rise above trend, and a second return to trend according to the seasons of the year. As a result of this pendulum-like movement, they should

average 100 per cent. If the seasonal ratios do not average 100 per cent, it is customary to adjust them, however they may have been obtained, so that they will average 100 per cent. This is done by dividing the seasonal ratio for each month (*i.e.*, 112, 91, 79, etc., in Exhibit 152) by the *average* seasonal ratio of all months (98.8 per cent in the example of Exhibit 152). The resulting *adjusted seasonal index* is shown in Column *B* of the table in Exhibit 152.

The Ratios-to-moving-average Method.—The computation by this method is identical with the computation of the ratios-to-trend seasonal index just described, except that a moving-average trend is substituted

Month	*A* Seasonal ratios (per cent)	*B* Adjusted seasonal index (per cent)
January	112	113.3
February	91	92.1
March	79	79.9
April	77	77.9
May	73	73.8
June	89	90.1
July	124	125.5
August	93	94.1
September	99	100.2
October	103	104.2
November	113	114.3
December	133	134.6
Total of seasonal ratios	1,186	1200.0

Average seasonal ratio 98.8 per cent (1,186 ÷ 12)

EXHIBIT 152.—Adjustment of seasonal ratios to average 100 per cent.

for the fitted trend used in the previous calculation. The moving-average trend values are found by computing moving averages on a basis of 12[1] months (or four quarters for quarterly data), see *B* columns, Exhibit 153. In centering the average of each year interval, it will be found that the central month includes one-half of two months. For example, in centering the average of 12 months beginning with January, it will be found that the central month includes one-half of June and one-

[1] Obviously, a trend computed on the basis of a 12-month period does not meet the requirements of a basic trend, as discussed in Chap. XIV. In reality, the 12-month moving average is a composite of all the factors but the seasonal factor—that is, it includes trend, cyclical, and erratic factors. It is adequate as a base, however, upon which to determine the seasonal movements.

half of July. To center the moving averages on the full months, quarters, etc., it is necessary to average the computed moving averages in pairs, as was explained in Chapter XIV (pages 294 and 295) in the discussion of recentering the moving average of an even number of items.

After the moving average has been obtained, the actual items in the original data are expressed as percentages of the corresponding moving-average items as in the *C* columns, Exhibit 153, and averages of all the January ratios, all the February ratios, etc., are obtained as illustrated in column *E*. As a final step, these averages or typical ratios are adjusted proportionally so that they average 100 as in column *F*.[1]

Month	*A* Monthly sales (index numbers)			*B* Moving average (recentered)			*C* Ratios to moving average (per cent)			*D* Total	*E* Average ratio (per cent)	*F* Seasonal index adjusted (per cent)
	1926	1927	1928	1926	1927	1928	1926	1927	1928			
January....	87	81	85	92	100	100	95	81	85	261	87	86.7
February...	72	74	69	93	101	100	77	73	69	219	73	72.8
March	85	88	90	94	101	101	90	87	89	266	89	88.7
April.......	106	122	116	95	101	101	112	121	115	348	116	115.6
May.......	99	97	96	95	102	101	104	95	95	294	98	97.7
June.......	106	115	119	97	102	101	109	113	118	340	113	112.6
July........	79	83	84	98	101	100	81	82	84	247	82	81.7
August.....	68	75	72	98	101	99	69	74	73	216	72	71.8
September..	96	100	107	98	101	100	98	99	107	304	101	100.7
October....	105	103	100	99	101	101	106	102	99	307	102	101.6
November..	102	108	110	100	100	101	102	108	109	319	106	105.6
December..	175	166	158	100	100	102	175	166	155	496	165	164.5

EXHIBIT 153.—Computation of seasonal variation by ratios-to-moving-average method. (In order to simplify the illustration, only three years have been used. Ordinarily a seasonal index should be based upon a longer period.) The data in the *A* columns and the moving average in the *B* columns are shown graphically in Exhibit 157 on page 327 (see years 1926 to 1928 on the chart).

A seasonal index computed by the ratios-to-moving-average method shows the average difference between the unadjusted data and the moving average. Such an index for the data of Exhibit 157 (page 327), for instance, would show the average difference between the sales curve and the moving average.

The index obtained by the ratios-to-moving-average method ordinarily does not fluctuate so much as the index based on straight-line trends. Mathematical methods of avoiding the effects of the business

[1] For explanation of the adjustment to average 100, see discussion of Exhibit 152.

cycle are not usually needed, for the 12-month moving average follows the cyclical course of the actual data quite closely. Therefore, the index ratios are often more representative of the data from which they are obtained than is the case in the ratios-to-trend method.

The Link-relative Method.—One of the best known methods of computing seasonal indexes is the link-relative method.[1] The process,

MONTHLY PIG-IRON PRODUCTION IN THE UNITED STATES
Seasonal Variation by Method of Link Relatives

Year	Jan. %	Feb. %	Mar. %	Apr. %	May %	June %	July %	Aug. %	Sept. %	Oct. %	Nov. %	Dec. %	Jan. %
1903	...	94	114	101	106	98	92	102	99	92	73	81	
1904	109	131	120	107	99	84	86	106	116	107	102	109	
1905	110	90	121	99	102	91	97	106	103	108	98	102	
1906	101	92	113	96	101	94	102	96	102	112	100	102	
1907	99	92	109	100	104	97	101	100	97	107	78	67	
1908	85	103	114	94	101	94	112	112	104	111	101	110	
1909	103	95	107	95	108	103	109	107	106	109	98	103	
1910	99	92	109	95	96	95	95	98	98	102	91	93	
1911	99	101	122	94	92	95	100	107	103	106	95	102	
1912	101	102	114	99	106	97	99	104	98	109	98	106	
1913	100	92	107	100	103	93	97	100	99	102	88	89	
1914	95	100	124	97	92	92	102	102	94	94	85	100	
1915	106	105	123	102	107	105	108	108	103	110	97	105	
1916	99	97	108	97	104	96	100	99	100	110	94	96	
1917	99												
Medians	99.5	96	114	98	102.5	95	100	103	101	107.5	96	102	
Chain relatives	100	96	109.4	107.2	109.9	104.4	104.4	107.5	108.6	116.7	112	114.2	113.6
Adjusted	100	94.9	107.1	103.8	105.4	98.8	97.6	99.6	99.6	106.5	100.7	101.8	100
Seasonal index	98.7	93.7	105.7	102.5	104.0	97.5	96.3	98.3	98.3	105.1	99.4	100.5	

EXHIBIT 154.—Computation of seasonal variation by the link-relative method.

however, is somewhat more complex than those just described. The computation (assuming monthly data) is as follows:

1. Divide each January figure (actual data) by that for the preceding December. Divide each February figure by that for the preceding January. Divide each March figure by that for the preceding February, and so on throughout the entire series. These ratios or percentages are called "link relatives." Place the link relatives in 12 columns, one containing all the ratios of January to December, another containing all those of February to January, and so on, as shown in Exhibit 154.

2. The next step is to select a typical link-relative item which will represent each monthly group. The item selected is usually the median, although an arithmetic average is ordinarily more appropriate when the number of observations is small. Median link relatives are shown in the row designated "Medians" in Exhibit 154.

3. Each median link relative shows the typical relation of each month to the preceding month. The next step is to relate the median link relatives to a common base,

[1] Link relatives may be compared with percentage relatives in Exhibit 60, p. 119.

for which the month of January may be chosen; this month then becomes the starting point or 100 per cent. The resulting percentages, obtained by relating each median link relative to the common basis of reference, are known as "chain relatives." The January chain relative will, of course, be 100 per cent, since this is assumed as the starting point or basis of reference. The February chain relative will be the same as the February median link relative (being the February median link relative multiplied by the January chain relative, or 100 per cent). The March chain relative is the March median link relative multiplied by the February chain relative. The April chain relative is the April median link relative multiplied by the March chain relative. Continue this process of multiplying the median link relative for each month by the chain relative for the preceding month. Chain relatives are shown in Exhibit 154 in the row designated "Chain relatives."

4. The new January chain relative, obtained by multiplying the old January median link relative by the December chain relative (see last column in the row of chain relatives in Exhibit 154), usually does not equal the original January chain relative of 100 per cent; this is due to long-time trend and other influences. If the trend is in arithmetic progression, the discrepancy may be corrected as follows: Divide the discrepancy by 12. (This discrepancy is 13.6 per cent in Exhibit 154.) If the trend is upward, subtract $\frac{1}{12}$ from February, $\frac{2}{12}$ from March, $\frac{3}{12}$ from April, etc., until finally $\frac{11}{12}$ of the increment is subtracted from December. (See row designated "Adjusted" in Exhibit 154.) The adjustment is made by addition if the trend is downward. If the adjustment must be made in geometric progression,[1] or some other curve form, an accurate adjustment is more difficult, but the process just described may be used as an approximation which is satisfactory for most practical business purposes.

5. The series of adjusted chain relatives gives each month as a percentage of January. The next step is to reduce these values to percentages of the average monthly figure in a normal year. To do this, merely average arithmetically the figures for the 12 months (in the row designated "Adjusted" in Exhibit 154) and divide each monthly figure by this average, as described previously on pages 319 and 320. The final result in each case will be the index of seasonal variation for that month. The final seasonal indexes are shown in the last row of Exhibit 154.

This method gives reasonably satisfactory results, as its popularity attests. Probably the greatest points in its favor are that it is not dependent upon the trend calculations, and the influence of the business cycle is minimized by the use of month-to-month ratios, as there is not likely to be marked cyclical change from one month to the next. That is, the use of link relatives places the prosperity relationships on a basis comparable with the depression relationships. The entire process, after the period has been selected, is mechanical to a marked degree, making the computations simple to perform, although the process is rather cumbersome. It should be appreciated that the chain multiplication

[1] A more accurate adjustment for geometric progression is as follows: If the amount of the error in each median link relative is d, and the new January chain relative is A (expressed in decimals, and not as a percentage—that is, 110.5% is written 1.105), then $A = (1 + d)^{12}$. By the use of logarithms $(1 + d)$ is found readily from the above equation. The February chain relative is now divided by $(1 + d)$, the March chain relative by $(1 + d)^2$, the April chain relative by $(1 + d)^3$, etc., until finally the December chain relative is divided by $(1 + d)^{11}$.

makes cumulative errors possible to an extent which cannot be estimated or allowed for, because the errors themselves may be inherent in the data used.

Changing Seasonal Variation.—As previously implied, the problem of gradually changing seasonal distributions is often of considerable importance. Within a period long enough to give reliability to seasonal measurements, the seasonal movement may have noticeably changed (see Exhibit 155). The seasonal variation in gasoline consumption, for instance, is relatively less at the present time than it was 20 years

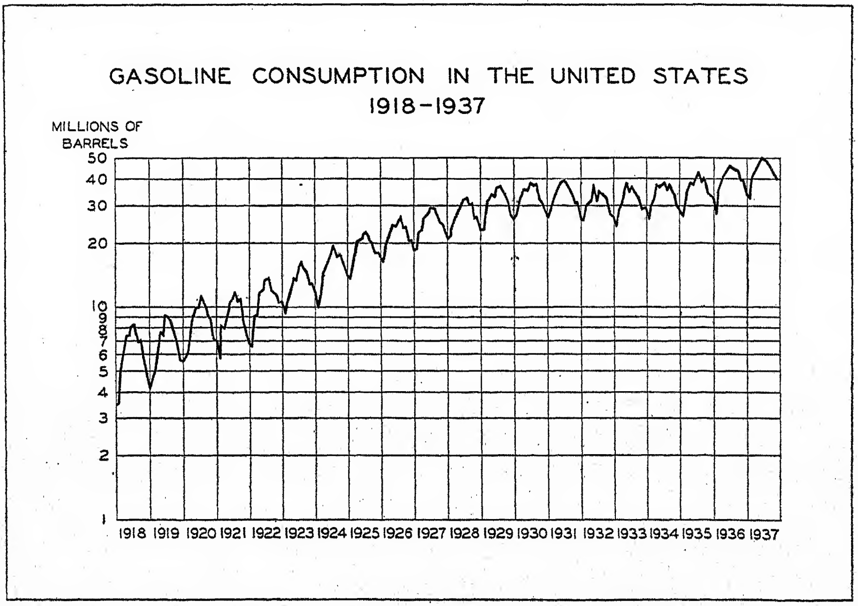

Exhibit 155.—It can be seen from the above unadjusted data that the seasonal variation in gasoline has been decreasing relatively over the period.

ago because of the improvement of roads, which allows relatively more winter driving than was formerly possible. In an analysis of very recent developments or in a forecast, it is important to have the seasonal index up to date. This may make necessary the computation of a *changing* seasonal index, or the computation of a separate set of seasonal percentages for each year.

The determining of a changing seasonal index is not so difficult as it sounds, for the same link relatives, ratios to trend, etc., may be used as the basis for this new type of index. For example, let us suppose that the ratios of the actual December sales to the trend points for December in a certain department store have steadily risen since 1920, but that the similar ratios for the summer months have steadily declined. This

would indicate that December is becoming seasonally more important, and summers less so. To measure a possibly changing seasonal index we may chart the ratios for each separate month, and determine their trend by a free-hand curve, or moving average. These "trends" would then indicate the values of the ratios which were "typical" for each year, and the process of adjusting them to an average of 100 per cent could be carried out for each year separately. The actual-to-moving-average ratios are probably better suited to this process than are those of the ratios-to-trend method because of the disturbing influence of the business cycle in the latter. Link relatives may also be successfully used to measure shifting seasonal distributions, although the necessary separate chain multiplications, trend corrections, etc., are laborious.

Use of the Seasonal Indexes.—The seasonal indexes themselves, once obtained, may be used in an intensive study of the seasonal movements. Seasonal movements may be examined (1) as to their causes, (2) as to the likelihood of change, or (3) as to the possibility of changing them, perhaps by advertising pressure (*e.g.*, "Do your Christmas shopping early"). Or it may be that the results of the business cycle are being studied, in which case it is ordinarily desirable to eliminate the purely normal seasonal effects entirely from the data, so that other effects stand out in bold relief.

Synthetic use of the seasonal index appears in the adjustment of computed trend ordinates to "normal." The normal values are computed by multiplying the trend figures by the proper seasonal percentages. Thus, if the computed trend ordinate for January shows sales of $100,000, and the seasonal percentage is 110, the normally expected January would be $110,000. Such a process is an essential part of many forecasts, as will be explained in succeeding chapters.

Eliminating Effects of Seasonal Variation.—As indicated previously, it is often desirable to study data without the influence of seasonal variation, and in order to do so, it is necessary to eliminate the seasonal effects. This is done in various ways. Two of the most common methods are: (1) by dividing by the seasonal index; and (2) by use of a moving average. Each of these methods of eliminating the influence of the recurring seasonal variation will now be briefly considered.

1. *Division by the Seasonal Index.*—This method of eliminating the effect of seasonal influences is to divide each actual observed item by its corresponding seasonal factor. Thus, in Exhibit 156, the actual data shown in column *A* are divided by the seasonal indexes shown in column *B* and the results are the corrected data, as shown in column *C*. This method of eliminating the seasonal variation is simple. Moreover, it possesses the sometimes desirable characteristic of leaving the trend undisturbed. In Exhibit 156, it is easy to see from the last column,

which shows the corrected data, that the business progressed up until May, and that it declined somewhat during the remainder of the year. (It will be noted that if the seasonal index is greater than 100 per cent, the corrected value is less than the original data, and *vice versa.*)

2. *Correction by Moving Average.*—It is possible to eliminate seasonal influence from a series by computing a moving average of exactly one year's duration throughout the data. The method is somewhat more laborious than the preceding method, but possesses the desirable characteristic of being flexible. Since the computation of the moving average has been illustrated previously, it need not be repeated again at this point (see page 294). It should be noted that the moving average *is*

Month	*A* Actual data	*B* Seasonal index (per cent)	*C* Corrected data
January	$381,000	117.6	$324,000
February	313,000	94.6	331,000
March	283,000	84.9	333,000
April	268,000	79.6	337,000
May	302,000	86.7	348,000
June	312,000	91.1	342,000
July	332,000	96.4	344,000
August	355,000	104.3	340,000
September	331,000	99.0	334,000
October	337,000	102.6	328,000
November	362,000	112.3	322,000
December	328,000	130.9	251,000

EXHIBIT 156.—Eliminating seasonal effects by dividing through by an index of seasonal variation.

the corrected data (see Exhibit 157). There are no subtractions, nor divisions, as was necessary in the first method for eliminating the effect of the seasonal variation.

Seasonal Indexes Usually on a Monthly Basis.—A very large proportion of the studies involving measurement of seasonal variation are on a monthly basis. The month is ordinarily short enough to reflect successively the changes that take place in business conditions, without concealing the timing of such changes or omitting their minor turns; and it is usually long enough to give a representative picture of current business. Again, handling monthly data is not too laborious a process, and the methods available are well adapted for use on monthly data.

However, there are times when time series should be studied on a weekly, biweekly, bimonthly, or quarterly basis; and there are cases in which analysis must be based on an average working day, rather than

on the month as a unit. All of the methods described in this chapter will work satisfactorily on most monthly, bimonthly, or quarterly data. However, both laboriousness and opportunities for error increase rapidly in these methods as the periods studied become shorter.

Seasonal Indexes for Weekly Data.—There is a demand for up-to-the-minute analysis in some types of work, and weekly seasonal indexes are needed. Weekly periods are so short that unexpected weather conditions, a special holiday, or even a regular holiday which does not appear

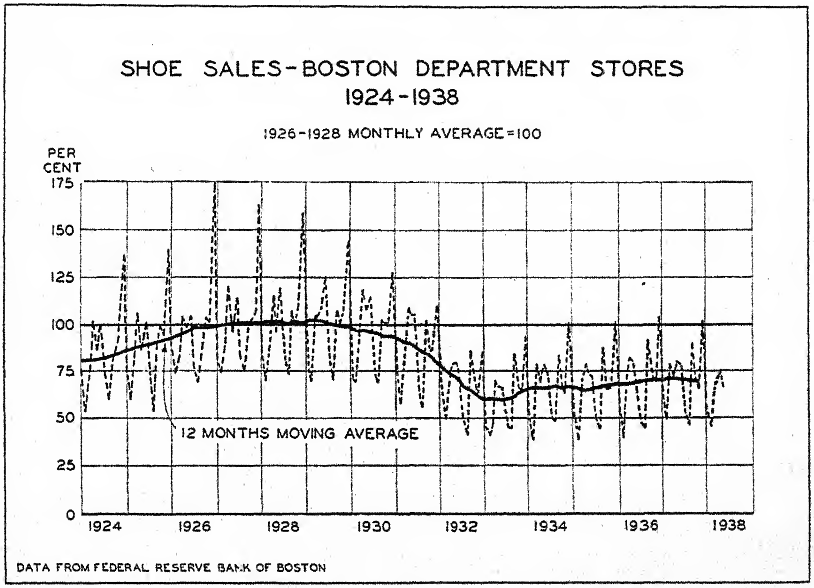

EXHIBIT 157.—Eliminating seasonal variation by use of a 12-month moving average.

the same week each year often requires a current adjustment in the seasonal index for the week.

The usual methods will work fairly well on 13 four-week periods, covering the year; and it is not difficult to estimate roughly what proportion of a given four-week period should fall in each of the weeks. It is not inconceivable that months should be subdivided in this way, too, when necessary.

Questions and Problems

1. Name several industries and lines of business that are particularly affected by the characteristics of different seasons of the year.

2. In obtaining a seasonal index, can cyclical and erratic influences be largely eliminated? How?

3. How do variations in lengths of months affect the determination of a monthly seasonal index? How can the influence of trend be eliminated from the seasonal index?

4. Discuss the problem of choosing the proper period from which to obtain the seasonal index.

5. Name four methods of measuring seasonal variation.

6. State the procedure in obtaining a seasonal index by the method of monthly totals or monthly averages. What are two important shortcomings of this method?

7. How is the seasonal index calculated in the method of ratios to trend? Discuss the strong and weak points of this method.

8. What is the purpose of adjusting computed seasonal ratios to average 100 per cent? How is this done?

9. How is the ratios-to-moving-average seasonal index computed? How does this method differ from that of ratios to trend?

10. State each step in the method of link relatives as used in seasonal-index computation.

11. Can changing seasonal variations be measured and a changing index be obtained? How?

12. How can seasonal indexes be used in analyzing business activities?

13. Describe two methods of eliminating the effects of seasonal variation from time-series data.

14. Discuss the advisability of using monthly, weekly, biweekly, or bimonthly indexes of seasonal change.

15. Sales of the Means Shoe Company for the years 1930–1937 were (in thousands of dollars):

Year	1st quarter	2d quarter	3d quarter	4th quarter	Total
1930	46	52	37	66	201
1931	34	38	43	60	175
1932	28	41	42	71	182
1933	41	56	48	73	218
1934	50	58	44	76	228
1935	42	70	71	94	277
1936	54	83	66	98	301
1937	63	80	58	94	295

Compute the seasonal variation:

a. By the method of quarterly (monthly) totals.

b. By the ratio-to-trend method. (Use yearly totals to compute trend ordinates by method of least squares.)

c. By the ratio-to-moving-average method. (Use a 4-quarter moving average.)

d. By the link-relative method.

e. Eliminate the effects of the seasonal influence from the actual data (*i.e.*, correct for seasonal influence), using successively each index obtained.

NOTE.—Quarterly indexes ordinarily are not so desirable as monthly indexes when analyzing seasonal influences in business data. Quarterly data are used in this problem, however, so that the student may practice all four methods described in the chapter, without performing the volume of computations which would be necessary if monthly data were used.

16. From the monthly sales data (in millions of dollars) of Jamison Department Store compute the index of seasonal variation:

a. By the method of monthly totals.

b. By the link-relative method.

Month	1933	1934	1935	1936	1937
January	16	24	26	30	33
February	17	23	25	29	31
March	17	27	34	38	40
April	19	27	36	41	44
May	24	32	37	47	46
June	24	29	32	49	48
July	21	23	30	41	37
August	27	27	32	42	40
September	29	32	37	50	46
October	33	36	46	60	54
November	34	35	43	51	52
December	39	44	53	68	63

In part *a* what factor influenced the data for which no adjustment was made?

17. From the data given in Problem 16, Chapter XIII, page 284, compute the seasonal variation in the sales of Sears Roebuck and Co. by the link-relative method.

18. From the following data compute the seasonal variation in the sales of electric power for domestic use. Use the method of ratio-to-trend and compute the trend by the method of least squares.

Sales of Electric Power for Domestic Service
(In billions of kilowatt-hours)

Month	1934	1935	1936
January	1.24	1.32	1.47
February	1.12	1.21	1.34
March	1.06	1.13	1.25
April	1.03	1.10	1.22
May	.97	1.06	1.17
June	.97	1.06	1.14
July	.96	1.05	1.16
August	.96	1.07	1.18
September	1.02	1.14	1.26
October	1.08	1.19	1.32
November	1.17	1.29	1.42
December	1.22	1.37	1.48

Average Monthly Sales
(In billions of kilowatt-hours)

1930	.92
1931	.98
1932	1.00
1933	1.00
1934	1.07
1935	1.17
1936	1.28

Source: The Edison Electric Institute.

Carry all computations in billions of kilowatt-hours to two decimal places and in percentages to one decimal place.

CHAPTER XVI

TIME SERIES—BUSINESS CYCLES AND ERRATIC MOVEMENTS

In the two preceding chapters the methods have been described for measuring *trend* and *seasonal variation*—two of the four general movements found in business time series. These two movements are assumed to be regular in their characteristics. Together they establish a "normal" for the series. The other two movements, the *cyclical* and *erratic*, will be discussed in this chapter.

Good and Bad Times or Business Cycles.—Previous illustrations have indicated in a general way how business passes from periods of good times or prosperity, into periods of bad times or depression, and back again. The shaded curve in Exhibit 158 shows this continued recurrence of changes from good to bad times and back again in general business for the period from 1830 to 1937, inclusive. This constantly recurring movement, swinging from a period of prosperity with its optimism, speculation, and expansion, down to the trough of depression, with its pessimism, low profits, business failures, and unemployment, and back again, is commonly referred to as the "business cycle." Business cycle is simply a convenient term for expressing the recurrence of good and bad times.[1]

There are four well-defined periods or phases in the business cycle—namely: (1) prosperity, (2) decline, (3) depression, and (4) improvement (Exhibit 159). Each phase changes gradually into the phase which follows it, in the order given.

In the *prosperity* phase of the business cycle the public is optimistic. Business is booming, prices are high and profits are easily made. The inferior business man believes himself to be a business genius. Money is relatively plentiful for commercial transactions, though the speculator usually finds it increasingly difficult to raise funds because banks ordinarily withdraw money from speculative uses to meet the growing demands of business men.

This expansion of business eventually leads to an overdevelopment. It is then difficult to secure deliveries, and there is a shortage of transportation facilities, which has a tendency to cause large inventories

[1] For references to specialized information on business cycles see "A Selected Bibliography of the Literature on Economic Fluctuations, 1930–1936," *Review of Economic Statistics*, Vol. XIX, No. 1, Part 2, February 1937.

GENERAL BUSINESS CYCLES COMPARED WITH BUILDING CYCLES, UNITED STATES, 1830–1937

GENERAL BUSINESS INDEX ADAPTED FROM CLEVELAND TRUST COMPANY INDEX—TREND AND CURVE REVISED 1920–1937

NUMBER OF CITIES IN BUILDING INDEX BUILT UP FROM 3 IN 1830 TO 20 BY 1875 AND 65 BY 1900; 65 CITIES FROM 1900 TO 1937

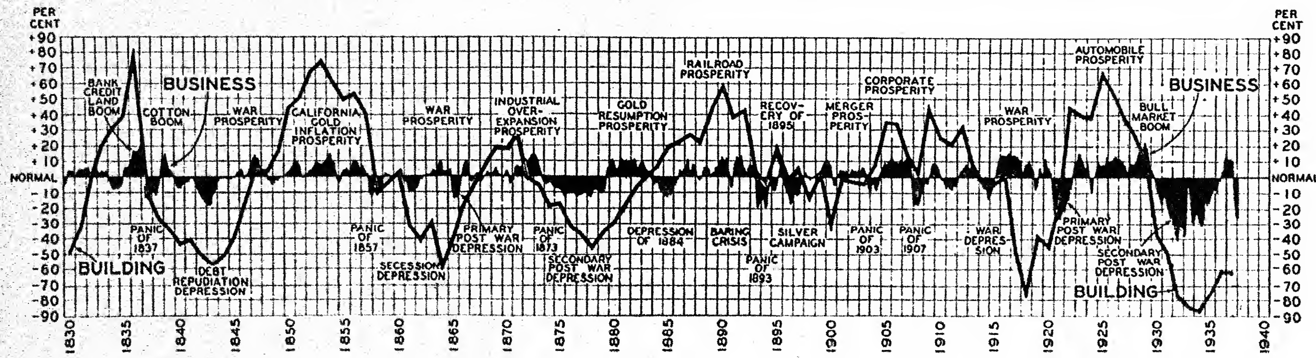

EXHIBIT 158.—The shaded curve shows how general business has swung above and below the normal expectations for each year from 1830 to 1937, inclusive. Note, especially, the boom of the middle 1830's and the panic of 1837; the "golden age" of the early 1850's and the panic of 1857; the overexpansion of the early 1870's, the panic of 1873, and the long depression of the 1870's; the prosperity of the early 1880's and the depresssion of 1884; the prosperity of the early 1890's and the great panic of 1893; the decline in 1896; the minor panic of 1903; the prosperity from 1905 to 1907 and the panic of 1907; the War prosperity; the primary postwar depression; the automobile prosperity and bull market boom of the 1920's; and the great depression of the 1930's. The index of building cycles is indicative of the much greater swings that characterize building activity and certain other lines of business that do not move in short cycles like those of general business. (For descriptions of indexes, see Appendix VI, pages 719–722.)

to be accumulated during the time of highest prices. Wages increase and labor efficiency decreases. The strong demand for money causes interest rates to rise to a high level, while doubt enters the banker's mind as to the advisability of granting further loans. This situation causes business men to make price concessions in order to secure the necessary cash. Then follows the expectation of further reductions and the situation becomes worse instead of better. Buyers wait for lower prices, and orders booked at high prices are canceled. Inventories become frozen and loans cannot be paid. Interest rates continue to increase. Prices drop, and business experiences a *decline*. If we have a very severe financial strain and an unusually drastic liquidation of securities and commodities, we experience a *panic*.

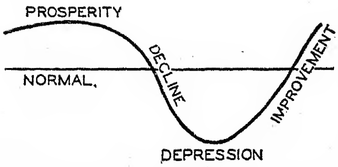

EXHIBIT 159.—Periods or phases in the business cycle.

The decline is followed by pessimism in trade and industry; factories close, businesses fail, there is widespread unemployment, while wages and prices are low. This characterizes the period of *depression*. If the decline in business is only slight, the situation is ordinarily referred to as a *recession* instead of a depression.

After a period of rigid economy, liquidation, and reorganization, money accumulates and seeks a use. Then follows a period of increasing business activity with rising prices, or a period of *improvement* or *recovery*.

The improvement period gradually develops into the prosperity period, and we have completed a business cycle. The movements discussed above are constantly repeated in the order given as the cycle completes its swing every few years.

It is important at the outset that the student of business cycles appreciate the cumulative character of decline and recovery—that is, unemployment leads to reduced consumption, reduced consumption to increased unemployment, etc., and *vice versa*. It is because of this characteristic of cyclical movements that efforts to stabilize business, even though relatively small volumes are brought under control, may be more effective than is now generally realized.

The "General Business" Cycle.—This discussion of cyclical characteristics and tendencies is based on a study of "general business" cycles as distinguished from cycles of certain types of business activities. "General business" may be said to refer to an average of business as a whole. Cycles of individual industries may vary considerably from the average in both length and amplitude, and this should be taken into consideration when analyzing a particular business or line of business.

Major and Minor Cycles; Length of Business Cycles.—For many purposes, the fluctuations in general business activity should be con-

sidered from the point of view of major and minor cycles. Part of our business activity is characterized by great swings of from 15 to 20 years in length, while part of it is characterized by short swings of only a few years in length. The net effect of this condition is that we have an unusually prosperous period and an unusually severe depression about once every business generation.

The shaded curve in Exhibit 158 shows the minor cycles swinging from prosperity to depression and back again every few years. An index of building cycles is also shown on the chart, and this index may be taken as indicative of the major cyclical movements in business activity. When like movements in major and minor cycles coincide, we have an extreme situation, whether it be prosperity or depression. The severity of the depression during the years from 1930 to 1935 and in 1937 and 1938, as well as the weakness of the prosperity of 1936 and early 1937, may be said to be due to the fact that these minor cycles occurred during a general depression period in the major cycle.

Major cycles have varied in length, from one depression to another, during the past 100 years about as follows (in years): 13, 21, 14, 22, 18, and 17 (see Exhibit 158). The shaded curve in Exhibit 158 indicates that, in the case of minor cycles, it is, as a rule, from three to four years from one depression to another.

Causes of cyclical movements will be discussed from time to time, but at this point the student should appreciate that, whatever they are, the causes favorable to business and the causes unfavorable to business accumulate in such a way that they cause the recurring fluctuations described above.

Psychology of the Business Cycle.—A depression is often said to be caused by a "loss of business confidence" which has followed a boom period of "overconfidence." It cannot be denied that popular psychology has a lot to do with business cycles. Undue optimism attends a boom and undue pessimism prolongs a depression. But what causes the recurring optimism and pessimism? What is back of men's opinions regarding the economic situation?

In a period of depression, pessimism prevails and the tendency is to underexpand. The crowd is very cautious, prices are low, and transactions are few. In a period of prosperity, the tendency is to overexpand; prices are high and transactions are many. There are, however, always a few far-sighted business men who understand the psychology of the crowd, and who are able to make their decisions unbiased by its influence.

When prices are rising and business is increasing, the crowd is optimistic and believes that prosperity is ahead, even after business has become greatly inflated. When prices are falling and business declines,

the crowd is pessimistic long after the bottom has been reached. The crowd, therefore, is always wrong in judging its position in the business cycle.

The fact that the crowd is always wrong in judging its position in the business cycle has much to do with the fact that there is a business cycle. Why is the crowd always wrong? Why are only a few men right? It is not chance that determines who is right and who is wrong. The principal reason that the crowd is wrong is that it is misinformed, which causes grave errors to be made regarding present and future production and markets. Lack of information and wrong information are the chief difficulties. With the proper information, however, the individual executive must be able and willing to make the right application in his own particular business.

Control of the Business Cycle.—One of the most noteworthy of modern economic objectives is the partial control of business cycles by manipulating the causes—that is, by offsetting unfavorable tendencies in one direction with compensating movements in the other. During recent years, interest in economic and business planning on a broad scale has rapidly increased with the many efforts of individual business men, trade associations, Federal and state governments, civic organizations, and others to at least reduce the fluctuations of the cycle of business as a whole with the general objective, in turn, of reducing especially the unemployment and failures in business during periods of depression.

The severity of the depressions of the 1930's emphasizes that such control is becoming more and more important. The higher proportion of our annual expenditures now spent for luxury and semiluxury goods, as compared with former periods, increases the sensitiveness of our economic structure. It has not been possible, however, for national business planning to advance much beyond the theoretical stage, and one of the greatest impediments to its progress is the lack of adequate statistical information. To make the necessary statistical analyses requires data extending over considerable periods of time, and the increasing activity of business concerns, trade associations, trade papers, Federal and state governments, and other organizations, in compiling and distributing statistical information will make possible some much needed analyses and indicate the nature and extent of the needs for further data. And then, if control is to be effective, business men and others must take action in accordance with the indications of the data, accelerating production, for instance, when stocks are declining and unfilled orders are increasing, and holding back production when the data indicate that production is getting too far ahead of the market.

In connection with the above statement, it should be emphasized that the problem of control or stabilization is not merely that of accelerating certain lines of business during a period of depression. The chief

problem is, in fact, that of retarding activity at certain times in order to prevent overproduction and inflation. When conditions are allowed to develop in such a way that a "house cleaning" becomes necessary and inevitable, the acceleration of public expenditures may only "prolong the agony." There are, of course, many factors which are difficult, if not impossible, to control, and these vary in number and importance from one period to another. It is obvious, however, that regulation of the factors that can be controlled will be very effective in reducing the amplitude of the waves in business conditions.

Erratic Movements.—It is necessary to differentiate between the business cycle and movements which are variously termed "erratic," "random," or "accidental." Random movements are those changes in business activity caused by such events as wars, strikes, floods, earthquakes, and the like. Thus, increased activity was noted in the lumber industry immediately after the Japanese earthquake in 1923. Other examples are the unprecedented drought of 1930, the uncertainties caused in the same year by new uprisings in China, revolutionary disturbances in India, and the overthrow of several South American governments, and the more recent difficulties in Germany, Italy, and Spain. Erratic movements are very irregular, and they are not recurring by nature.

Quantitatively, it is very difficult to disassociate the erratic movements and the business cycle as pointed out previously. The reasons for recognizing the erratic movements are: first, to suggest that on occasion it may be possible to explain certain movements in the data as due to specific causes, and thus to simplify further analysis; and second, to emphasize the fact that predictions of economic conditions are always subject to a degree of error owing to the unpredictable erratic influences which may enter. Early in 1930, for instance, analysts who were forecasting conditions for the year did not predict as severe a depression for the later months of the year and for the year 1931 as actually occurred, due (in part at least) to the fact that the unusual drought and the difficulties in China, India, and South America, mentioned above, and other difficulties abroad could not be foreseen.

Business Cycle Measurements.—Nearly every published curve or series which is supposed to measure the business cycle is really a combination of cyclical and erratic movements. This is usually unavoidable. The curves are generally used as measures of the deviation of business from normal levels, and therefore must show both movements. Labeling such indexes "business cycles" is perhaps unfortunate, as their movements are not exclusively cyclical; they are a mixture of both cyclical and erratic tendencies.[1]

[1] If adverse erratic movements occur at a time when business activity is on the decline or at a low level, further declines and a general loss of confidence are likely to

Separate measurement of cyclical and erratic fluctuations is seldom attempted. There is no definite line of demarcation between cyclical and erratic movements, and there are no adequate methods of separating them. To obtain a better idea of the major movements, a moving average is frequently used to "scatter" unusual peaks or valleys. However, this does not eliminate the erratic movements.

Business Cycles Measured from "Normal."—Business cycles (including erratic movements) are commonly measured in relation to "normal." Normal, as has been explained previously, is the basic trend with the corresponding seasonal variation. Though the term normal is sometimes applied to the basic trend alone, what the business man ordinarily has in mind when he speaks of normal is not only the position of the long-time growth (or change), but also the condition due to seasonal factors. Normal, therefore, is the basic trend combined with the usual seasonal variation. It is the course that business would have taken had the only influences been those that caused long-time growth and seasonal changes. The deviations from normal are business cycles and erratic movements. In measuring cyclical and erratic movements, therefore, it is practical and convenient to state them in percentages of, or in deviations from, normal. In Exhibit 160, the upper graph shows the normal for Los Angeles building permits. In the lower graph, each month is stated as a percentage of the corresponding point on the normal curve. The lower graph, therefore, shows the cycles of Los Angeles building permits (including erratic movements). This procedure of stating cycles in terms of normal is ordinarily referred to as "eliminating trend and seasonal variation."

Computing Deviations from Normal.—Let us assume that in a particular month a certain railroad loaded 9,900 cars of freight. Analysis of the records showed that the proper trend item for that month was 10,000 cars, and that the seasonal index was 90 per cent. Obviously, a normal month would have meant the loading of 9,000 cars (90 per cent of 10,000). The actual figure of 9,900 cars is 10 per cent above normal; cyclical and erratic factors having accounted for the extra 900 cars. Similar analysis may be made for each month in the entire series.

Another illustration is shown in Exhibit 161. The columns headed "Trend" and "Seasonal index" include trend and seasonal items computed from data covering a period of several years. For purposes of illustration, data for two years only are shown. Normal figures for

follow. Then it is particularly difficult to distinguish the extent to which the low levels are due to the business cycle and the extent to which erratic movements have influenced business activity. Although the depression of the early 1930's was a stage of the business cycle, it is probable that the extent and depth of this depression was caused in part by adverse erratic occurrences.

each month are obtained by multiplying the proper seasonal and trend[1] factors together; in the example each monthly item in column *B* is multiplied by the corresponding value in column *C* and the resulting normal values are shown in column *D*. The actual data are then expressed as percentages of normal, as shown in column *E*. The percentage

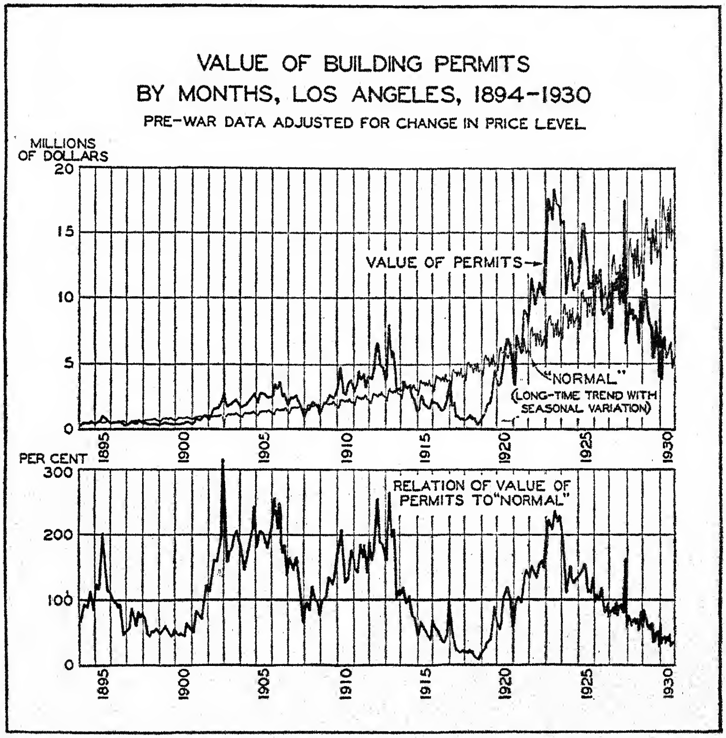

EXHIBIT 160.—"Normal" with actual data (upper graph); and "business cycles" expressed in relation to normal (lower graph).

deviations of the actual data above or below normal are shown in column *F*, which is the final measure of the cyclical and erratic movements in the series.

Precisely the same result as was obtained above might be secured by successively dividing out trend and seasonal variation from the observed values. A third process, which is somewhat of a short cut, is to reduce

[1] Where the method of monthly (quarterly) totals or monthly (quarterly) averages is used in obtaining the index of seasonal variation, each seasonal-index item should be multiplied by the average monthly (quarterly) trend ordinate of the year, instead of by each monthly (quarterly) ordinate of trend, because the average month (quarter) of the year is the basis of reference in computing the index.

the actual items to percentages of the trend, and then subtract the seasonal percentages to obtain percentage deviations from normal.

Year and month	*A* Actual data (thousands of dollars)	*B* Trend (thousands of dollars)	*C* Seasonal index (per cent)	*D* Normal (thousands of dollars)	*E* Actual data as percentages of normal	*F* Percentage deviations from normal
1936						
January	38	37	95	35.2	108.0	+ 8.0
February	41	38	97	36.9	111.1	+11.1
March	46	39	101	39.4	116.8	+16.8
April	50	40	102	40.8	122.5	+22.5
May	45	41	101	41.4	108.7	+ 8.7
June	42	42	99	41.6	101.0	+ 1.0
July	41	43	96	41.3	99.3	− .7
August	39	44	95	41.8	93.3	− 6.7
September	47	45	102	45.9	102.4	+ 2.4
October	49	46	103	47.4	103.4	+ 3.4
November	46	47	103	48.4	95.0	− 5.0
December	47	48	106	50.9	92.3	− 7.7
1937						
January	42	49	95	46.6	90.1	− 9.9
February	44	50	97	48.5	90.7	− 9.3
March	51	51	101	51.5	99.0	− 1.0
April	52	52	102	53.0	98.1	− 1.9
May	51	53	101	53.5	95.3	− 4.7
June	48	54	99	53.5	89.7	−10.3
July	43	55	96	52.8	81.4	−18.6
August	45	56	95	53.2	84.6	−15.4
September	52	57	102	58.1	89.5	−10.5
October	56	58	103	59.7	93.8	− 6.2
November	56	59	103	60.8	92.1	− 7.9
December	62	60	106	63.6	97.5	− 2.5

EXHIBIT 161.—Determination of cylical fluctuations of (or elimination of trend and seasonal variation from) monthly sales of Smith & Company. (*Data hypothetical.*)

This process is not sufficiently accurate, however, unless the seasonal items are all very close to 100 per cent.

The effects of cyclical and erratic fluctuations can be measured either in absolute quantities or as percentages of normal. The latter device

is the one usually adopted, for two reasons: First, it gives a measure of the intensity of the cyclical and erratic influences which is comparable to those of the same series at other times, irrespective of whether the amount of business has grown or declined. Second, there are reasons for believing that cyclical and erratic influences of equal intensity tend to add or subtract similar percentages of the normal business. For example, assume a case of department store sales for November and December, where the November normal may be $100,000 and that for December $150,000. If the prevailing cyclical tendency is toward prosperity, and both months are above normal, December will tend to gain $150 for every $100 gained by November. In other words, similar cyclical influences tend to increase the volume by the same percentage, rather than by the same amount. It is, therefore, ordinarily desirable to follow the practice of measuring cyclical and erratic movements as percentages of normal.

Comparing Cyclical Relationships between Different Series.—In the preceding discussion, the cyclical and erratic movements were stated in terms of percentage deviations from normal. In analyzing cycle relationships between different series, it is advantageous to chart both series on a comparable basis, in order to observe the degree of similarity in their movements.

Some series are more sensitive to economic conditions than are others. Stores handling luxuries show higher extremes of prosperity and depression than grocery stores, which deal largely in necessaries. It would be quite difficult to compare the cyclical fluctuations in two series closely when one fluctuates 50 per cent and the other 10. To compare such series, it is necessary to reduce their fluctuations to about the same average amplitude. This can be done by reducing the percentage deviations of each series to terms of the average deviation or the standard deviation.[1]

The reduction of series to terms of its average deviation or the more commonly used standard deviation is a simple process. To convert percentage deviations to terms of their average deviation or standard deviation, divide each of the percentage deviations by the corresponding average deviation or standard deviation, as in Exhibit 162.

The difficulty of accurately comparing curves of widely varying percentage deviations when studying business cycles is illustrated in Part *A*, Exhibit 163, which shows two series in terms of their percentage

[1] When data are in terms of percentage deviations, A. D. is obtained by taking an arithmetic average of the percentage deviations from normal, irrespective of plus or minus signs. S. D. is obtained by squaring each percentage deviation, averaging the squares, and taking the square root of that average (see pp. 217–221, particularly the second footnote p. 219).

Year and quarter	Percentage deviation from "normal"	Percentage deviation divided by average deviation (A.D. = 8.4)	Percentage deviation divided by standard deviation (S.D. = 8.7)
1937			
First	+ 8	+ .95	+ .92
Second	+12	+1.43	+1.38
Third	+10	+1.19	+1.15
Fourth	+ 5	+ .60	+ .57
1938			
First	− 5	− .60	− .57
Second	− 7	− .84	− .80
Third	−10	−1.19	−1.15
Fourth	−10	−1.19	−1.15

Exhibit 162.—Stating a series in terms of its A.D. and S.D.

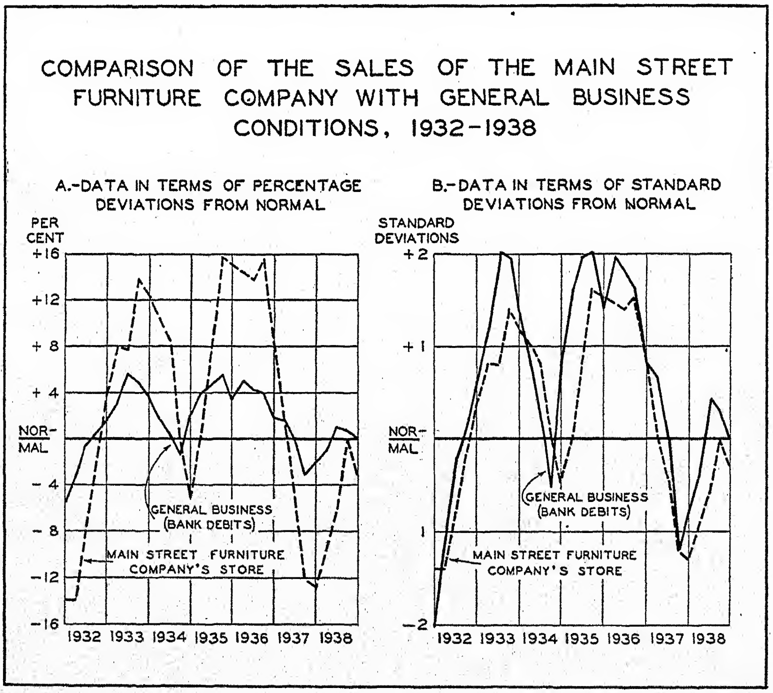

Exhibit 163.—Series expressed in terms of percentage deviations (*A*) and standard deviations (*B*). (*Data adapted.*)

deviations. Part *B*, Exhibit 163, illustrates how much easier it is to compare cycles when they are expressed in terms of their standard deviations.

Correcting Time Series for Trend and Seasonal Variation in Correlation Analysis.—Inspection of Part *A*, Exhibit 164, shows that there is probably some relation between sales of the Williams Department Store and bank debits, but it is not clear. To make it easier to judge the degree of correlation between the cyclical movements of the two series, several adjustments or corrections are necessary. First of all, one of the series—Williams Department Store sales—has a decidedly upward trend over the 11-year period, while the other curve—bank debits—does not have such a marked trend. Accordingly, the trend should be removed from both series. The correction for trend has been made in Part *B*, Exhibit 164.

After the removal of the trend from both curves the degree of cyclical correlation is still not clear. Further corrections must be made before the cyclical movements can be properly correlated. Study of the two curves will indicate that there is a decided seasonal variation in Williams Department Store sales and some seasonal variation in bank debits. If these seasonal variations are alike, the degree of cyclical correlation is likely to be overestimated in curves from which the seasonal variation has not been removed. If the seasonal variations are greatly different, it is easy to conclude, when studying curves from which the seasonal variation has not been removed, that there is no correlation present when there may be a considerable degree of correlation in the cyclical movements. Part *C*, Exhibit 164, shows the two curves after the seasonal variation has been removed.

Removal of the seasonal variation results in a clearer representation of the relation between the two curves. One more adjustment is necessary, however, before making a final estimate of the degree of correlation between the cyclical movements of the two series. The amplitude of fluctuation in bank debits is much less than the amplitude of fluctuation in department store sales (see Part *C*, Exhibit 164). The two series should be adjusted in such a manner that their fluctuations are about equal in amplitude.[1] In Part *D*, this has been done by reducing the data to terms of their standard deviations.

The degree to which two curves, when the corrections above noted have been allowed for, coincide with each other and tend to move in

[1] The scales of two historical curves may be so adjusted that their fluctuations will appear equal by using the method described in connection with constructing a scatter diagram. Any basis which is satisfactory for one type of chart is also satisfactory for the other. The range, interquartile deviation, average deviation, or standard deviation are proper, with the degree of scientific exactitude increasing for the different methods in the order named.

the same (or the opposite) directions at the same time and to the same extent, determines the degree of association between their cyclical movements. If one of the curves practically coincides with the other there

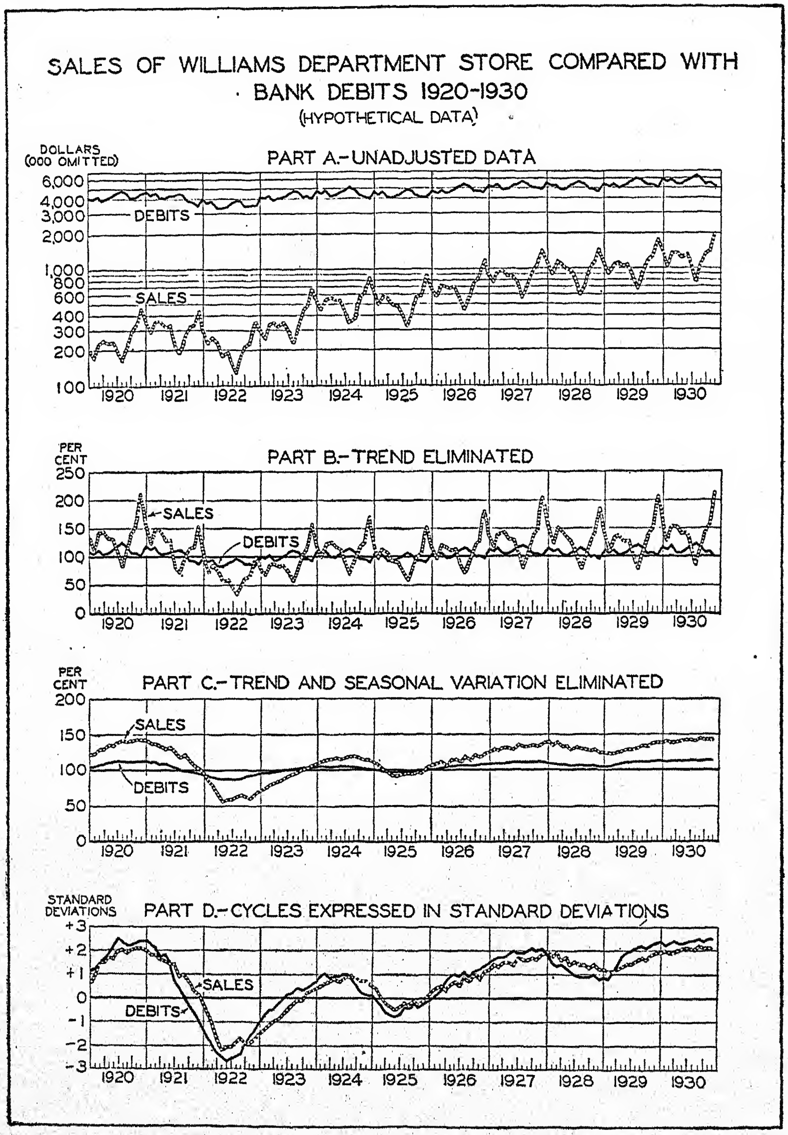

Exhibit 164.—Adjusting time series in correlation analysis of cyclical variations.

would be a high degree of direct correlation; if the curves appear to be unrelated in their movements, there is a low degree of correlation or none at all.

Forecasting in Terms of Standard Deviations from Normal.—In estimating future conditions it is often useful to determine a mathematical forecast in terms of the standard deviation (or average deviation) from normal. For example, if a change of one standard deviation is expected in one series, say building permits, the change in a firm's sales that are dependent upon building activity may also be anticipated as one standard deviation. This does not necessarily mean that there will be the same percentage deviation of change in both series. As pointed out previously, the standard deviation for one series may be 5 per cent, while in the other series it may be 10 per cent. Yet the two series may fluctuate together when a change of one standard deviation in the former (5 per cent) is accompanied by a change of one standard deviation (10 per cent, in this case) in the latter. Thus, the similarity in the fluctuations of the two series can best be seen when they are reduced to terms of their own standard deviations, for, in this example, a change of 5 per cent in the one series is relatively just as important as a change of 10 per cent in the other.

After determining the existence of a relationship between two series such as that suggested above, it is often possible to measure the changes taking place in one series in terms of its standard deviation, and then forecast the changes in the other series. Thus, if a decrease in building permits to $7\frac{1}{2}$ per cent below normal occurs, this will be stated as 1.5 standard deviations, if 1 standard deviation is 5 per cent. If it has been found that the business of the individual company in the following period will follow the change in permits and fall off in a similar degree of 1.5 standard deviations, then a change to 15 per cent below normal may be expected in the latter series (assuming that 10 per cent is a standard deviation for this latter series).

In the following chapter (Chapter XVII), other examples of related business data that are useful in business forecasting will be given, and the use of these relationships in forecasting work will be further explained.

Significance of Irregular Residuals.—Not infrequently the deviations from normal in a series will be very irregular, fluctuating violently from month to month, yet maintaining noticeable general cyclical swings. In such cases, it is often desirable to smooth out the irregularities by means of a centered three- or five-month moving average, or a repeated two-month or a repeated four-month moving average. The shorter the period of a satisfactory moving average, the greater its current significance will be, and it is sometimes desirable to give extra weight to the middle item of such a moving average. As a smoothing device the repeated two-month or repeated four-month (centered) moving average is very effective. This can be appreciated by referring to Exhibit 165, in which a four-year repeated moving average (that is, a four-year moving average

of a four-year moving average) was used to indicate the tendencies of the long swings in building activity.[1]

Irregular cyclical residuals are characteristic of some series, where the data are themselves of a fluctuating nature. This is especially likely to exist when the series is composed of the data of a small community or a small business. In data which are wide in scope, to which many businesses or many communities contribute, minor irregularities tend to be averaged out. However, an irregular series of deviations from normal may indicate inadequate or inaccurate statistical work, particularly in computing the seasonal index. The use of an ordinary seasonal index on

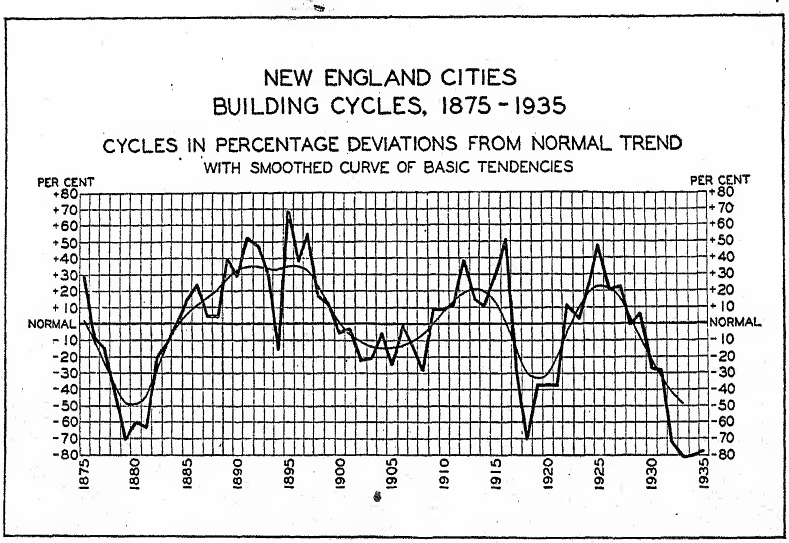

EXHIBIT 165.—Eliminating violent fluctuations by means of a repeated four-item moving average (centered) for the purpose of indicating more clearly the major movements.

a series whose seasonal distribution is changing would cause very noticeable irregularities in the cyclical curve.

Analysis of the data by periods of single days, weeks, or even months in some cases, may cause uneven cyclical-erratic residuals, because of the shortness of the period. In most studies of the general business cycle, a month is long enough to be truly representative; but in exceptional instances weekly and even daily observations must be made. In studying long swings like those of the building cycle (see Exhibits 158 and 165) a year is not too long a unit.

[1] While the illustration in Exhibit 165 presents deviations from normal, it will be appreciated that the moving average will have the same smoothing effect upon actual data.

Construction of Composite Business Curves.—In order to secure representativeness, most of the best known indexes of general business conditions are made up of more than one series of data. The usual process in constructing these composite curves is to analyze separately each series used in making the curves, obtaining the "cyclical" residuals in terms of percentage deviations from normal. These are then converted to terms of their respective standard deviations from normal[1] to provide a sort of "common denominator" for the different series, and the deviations may then be averaged or weighted and averaged readily, month by month, week by week, etc.

A COMPOSITE BUSINESS CURVE BASED ON BANK DEBITS AND CAR LOADINGS FOR BLANK CITY

Year and quarter	Series in percentage deviations from "normal"		Series in units of standard deviations		Composite curve in units of standard deviations from "normal"
	Bank debits	Car loadings	Bank debits S. D. = 4.1%*	Car loadings S. D. = 5.9%*	
1935					
First	−5.2	−8.1	−1.27	−1.37	−1.32
Second	−4.7	−5.7	−1.15	− .97	−1.06
Third	−3.2	−2.2	− .78	− .37	− .58
Fourth	−0.8	+1.0	− .20	+ .17	− .01
1936					
First	+1.6	+2.2	+ .39	+ .37	+ .38
Second	+4.2	+5.1	+1.02	+ .86	+ .94
Third	+6.1	+7.8	+1.49	+1.32	+1.40
Fourth	+3.4	+6.3	+ .83	+1.07	+ .95

* S. D. for each series is based upon a longer period than that shown in the table.

EXHIBIT 166.—Computation of a composite curve.

The example of Exhibit 166 will help to clarify the process. In this case we are attempting to get an accurate measure of business conditions by averaging the deviations from normal (1) of bank debits and (2) of railway car loadings. The two years' data shown in the table are taken from an analysis covering a period of nine years, and the standard deviations shown have also been computed over a nine-year period. A glance at the two series of percentage deviations shows that their fluctuations are very similar as to direction and time, but that the car-loading series fluctuates to a somewhat greater extent. If the percentage series were

[1] Or average deviations when desirable. See footnote on p. 219 relative to the term "standard error of estimate."

averaged directly, the car-loading series would have a greater influence on the composite curve than would bank debits. Although it may sometimes be desirable to give extra weight to one series or another, such weighting should be done only after careful consideration.

Computation of a Business Index.—For the purpose of reviewing the entire process of computing and eliminating trend and seasonal variation in analyses of cyclical movements, the computation of a simple index of business conditions for the United States, based upon automobile production and steel ingot production, will now be described and illustrated. The computation will be illustrated for the automobile production series only, and the combination of the two series will be illustrated graphically in Exhibit 172.

The first step was to determine the *basic trend*. This was done by fitting a straight line by the method of least squares to the annual averages of the monthly items, as illustrated in Exhibits 167 and 168, on pages 349 and 350. Exhibit 167 shows the computation for an odd number of years, while Exhibit 168 shows the computation for an even number of years. The increment computed in Exhibit 168 was used in computing the index in Exhibit 171. In Exhibit 168, the annual averages of the monthly items were listed chronologically in column Y. The mid-point of the series in units of one year was found to be the last half of 1927 and the first half of 1928, and the distance from the mid-point, in units of one year, was recorded as shown in column x. Then these items were squared as shown in column x^2. Next the products of the x items and the Y items were listed in the xY column and their algebraic sum was found. The sum of the xY column was then divided by the sum of the x^2 column, and the quotient (7,084.95) is the annual increment, as shown at the bottom of the table. As the algebraic sum is positive, the slope of the trend is upward. The monthly increment is one-twelfth of the annual increment,[1] or 590.41, as shown. The mid-point value was found simply by determining the arithmetic average of column Y as shown at the bottom of the table.

As shown in the table, the last half of 1927 and the first half of 1928 represent the mid-point year. The mid-point month obviously lies at the middle of this year, which is between December and January. In other words, the middle month of the year is from December 16 to January 15, if the number of years is even, as is the case in Exhibit 168.

[1] This is the annual increment from the monthly average of one year to the monthly average of the next, and it should not be confused with the annual increment of the total yearly figures. If the annual totals are used instead of the annual averages of the monthly items, the annual increment will have to be divided by 144 instead of 12 to obtain the monthly increment. In the illustrations throughout this discussion the averages were used.

Therefore, it is necessary to adjust the mid-point values so that they will be representative of the full months of December and January. This was done as follows: From the mid-point of the series, one-half a monthly increment was subtracted to find the ordinate for December, and one-half an increment was added to find the ordinate for January. Then, starting with December (or January), the monthly increment was subtracted successively to get the ordinates for the previous months, and was added successively to get the trend ordinates for the following months. This arithmetic straight-line trend is illustrated graphically in Exhibit 133, page 287.

If the number of years used in determining the trend had been odd, the computation would have been made as illustrated in Exhibit 167. The principal differences between computing a trend for an even number of years and for an odd number of years appear in column x and in the computation of the mid-point. In this illustration (Exhibit 167) the mid-month is from June 16 to July 15, instead of December 16 to January 15, as was the case in Exhibit 168.[1]

If it has been desirable to use a compound-interest curve, the trend would have been determined as illustrated in Exhibit 169. In this exhibit, the logarithms were substituted for the data and the computations were carried through in the same manner as described in connection with Exhibit 168. Then, after the trend was stated in terms of logarithms, the data were converted back to natural numbers. This is illustrated for June in the last two columns of Exhibit 169.[2]

The next major step was the determination of the *seasonal variation*. In this case, two different seasonal indexes were computed for the automobile production series—one based on the period from January 1918 to December 1934, inclusive, and one based on the period from March 1935 to March 1938, inclusive. The two indexes are necessary because of the abrupt change that took place in the seasonal variation in 1935. Both indexes were computed by the method of link relatives. The percentage that each month is of the preceding month is shown in the upper part of the table of Exhibit 170, page 352. These link relatives were arranged so that all of the Januarys are in one column, all of the Februarys in another, and so on. Then, to determine the 1918–1934

[1] It is interesting to note that the annual increment determined in Exhibit 167 is 5,313.92, whereas in Exhibit 168, the annual increment is 7,084.95. Likewise, the monthly increment is 442.83 in Exhibit 167 and is 590.41 in Exhibit 168. This illustrates the possible effect of the addition of data for one year (1918 in this case) upon the slope of the trend line, and emphasizes the necessity for exercising careful judgment in determining basic trends.

[2] It will be noted that a compound-interest trend, which is a straight line on a semi-logarithmic chart, has as its mid-point a geometric mean rather than an arithmetic mean.

index, the median link relatives were found and recorded in the row designated "Medians." These median link relatives show the average relation of each month to the preceding month. In order to show the relation of each month to the January median link relative, the ratios were converted to chain relatives, as shown in the row designated "Chain relatives." It will be remembered that the process of converting the link relatives to chain relatives is to take January as 100 and then multiply each median link relative by the chain relative determined for each preceding month. That is, the chain relative for January is 100, and for February is, of course, the same as the median link relative. Then, in Exhibit 170 (first index), the March chain relative is the product of the February chain relative multiplied by the March link relative, or 118 times 126, which is 148.7, and so on.

In computing the first index in Exhibit 170, it was found that the chain relative at the end of the year was 4.0 per cent too high, because of trend, etc. That is, when the new January was determined by multiplying the December chain relative by the old January link relative (89.7 × 116), the result was 104.0 instead of 100. This discrepancy was corrected according to the method previously explained—that is, by deducting from February 1/12 of the discrepancy; from March 2/12; and, finally, from December 11/12, so that the adjusted indexes appear as in the row designated "adjusted," and the new January is 100, which is the same as the first January. The adjusted chain relatives give each month as a percentage of January. To reduce these values to percentages of the average monthly figure in a normal year, the 12 items were averaged,[1] and each item was then divided by this average, which gave the final index of seasonal variation, as expressed in the upper one of the two rows designated "Seasonal index" (Exhibit 170).

The same procedure was used in determining the March 1935–March 1938 index, except that arithmetic averages were used instead of medians because of the shortness of the period (see Exhibit 170).

The third major step was to *eliminate the trend and seasonal variation*, or, in other words, to *express the series as a comparison with normal*. This was done as illustrated in Exhibit 171, page 353, which presents the computation in detail for the first two years and the last two years of the 1918–1937 period. First, the actual data were listed as shown in column *A*. Then, the monthly trend items were recorded in column *B*, and the seasonal indexes presented in Exhibit 170 were listed in column *C*. Normal was then computed, by multiplying the items in column *B* by the items in column *C*, and entered in column *D*. The percentage

[1] The "new January" 100 as shown in Exhibit 170 (see last figure in the row designated "Adjusted") is, of course, not included in the average.

BASIC TREND—ARITHMETIC STRAIGHT-LINE TREND BY METHOD OF LEAST SQUARES
(Odd number of years)
Automobile Production in the United States

Year X	Monthly average (number of vehicles) Y	x	x^2	xY
1919	161,133	−9	81	−1,450,197
1920	185,612	−8	64	−1,484,896
1921	133,069	−7	49	− 931,483
1922	212,015	−6	36	−1,272,090
1923	336,168	−5	25	−1,680,840
1924	300,212	−4	16	−1,200,848
1925	355,486	−3	9	−1,066,458
1926	358,411	−2	4	− 716,822
1927	283,444	−1	1	− 283,444
1928	363,230	0	0	
1929	446,535	1	1	446,535
1930	279,666	2	4	559,332
1931	199,145	3	9	597,435
1932	114,223	4	16	456,892
1933	160,005	5	25	800,025
1934	229,426	6	36	1,376,556
1935	328,911	7	49	2,302,377
1936	371,211	8	64	2,969,688
1937	400,797	9	81	3,607,173
Total	5,218,699	...	570	3,028,935

Mid-point........................	5,218,699 ÷ 19 = 274,668.37
Annual increment..................	3,028,935 ÷ 570 = 5,313.92
Monthly increment................	5,313.92 ÷ 12 = 442.83

Mid-point of series lies midway between June and July of the middle year of the series, therefore,

$$\text{June 1928} = 274{,}668.37 - \frac{442.83}{2} = 274{,}446.95$$

$$\text{July 1928} = 274{,}668.37 + \frac{442.83}{2} = 274{,}889.79$$

EXHIBIT 167.—Computation of arithmetic straight-line trend for automobile production in the United States from 1919 to 1937, inclusive (odd number of years). Compare with Exhibit 168 which shows this computation for an even number of years for the same series.

that each monthly item in column A varied from the normal item in column D was then computed and recorded in column E. In Part A, Exhibit 172, page 354, normal for this series is shown with the actual data. In Part C, the percentage deviations from normal are presented. The next step in the computation illustrated in Exhibit 171 was to reduce the percentage deviations to terms of their standard deviation.

Basic Trend—Arithmetic Straight-line Trend by Method of Least Squares
(Even number of years)
Automobile Production in the United States

Year X	Monthly average (number of vehicles) Y	x	x^2	xY
1918	97,557	−9.5	90.25	− 926,791.5
1919	161,133	−8.5	72.25	−1,369,630.5
1920	185,612	−7.5	56.25	−1,392,090.0
1921	133,069	−6.5	42.25	− 864,948.5
1922	212,015	−5.5	30.25	−1,166,082.5
1923	336,168	−4.5	20.25	−1,512,756.0
1924	300,212	−3.5	12.25	−1,050,742.0
1925	355,486	−2.5	6.25	− 888,715.0
1926	358,411	−1.5	2.25	− 537,616.5
1927	283,444	−0.5	.25	− 141,722.0
1928	363,230	0.5	.25	181,615.0
1929	446,535	1.5	2.25	669,802.5
1930	279,666	2.5	6.25	699,165.0
1931	199,145	3.5	12.25	697,007.5
1932	114,223	4.5	20.25	514,003.5
1933	160,005	5.5	30.25	880,027.5
1934	229,426	6.5	42.25	1,491,269.0
1935	328,911	7.5	56.25	2,466,832.5
1936	371,211	8.5	72.25	3,155,293.5
1937	400,797	9.5	90.25	3,807,571.5
Total	5,316,256	.....	665.00	4,711,493.0

Mid-point........................ 5,316,256 ÷ 20 = 265,812.80
Annual increment.................. 4,711,493 ÷ 665 = 7,084.95
Monthly increment................. 7,084.95 ÷ 12 = 590.41

Mid-point of the series lies midway between December and January of the middle year of the series, therefore,

$$\text{December 1927} = 265{,}812.80 - \frac{590.41}{2} = 265{,}517.59$$

$$\text{January 1928} = 265{,}812.80 + \frac{590.41}{2} = 266{,}108.01$$

Exhibit 168.—Computation of arithmetic straight-line trend for automobile production from 1918 to 1937, inclusive (even number of years). Compare with Exhibit 167 for odd number of years for same series. (This increment is used in computing the business index in Exhibit 171, which is presented graphically in Exhibit 172. The straight-line trend computed above is shown graphically in Exhibit 133, page 287.)

Percentage deviations are reduced to standard deviations as follows: Each of the percentage deviations is squared (column F). The sum of these squares is divided by the number of months in the series and the square root of the quotient is extracted. The resulting figure is the

Basic Trend—Compound-interest Curve by Method of Least Squares
(Odd number of years)
Automobile Production in the United States

Year X	Monthly average (number of vehicles) Y	Log Y	x	x^2	x (log Y)	Log trend June figures	Trend June figures
1919	161,133	5.20718	−9	81	−46.86462	5.33107	214,323
1920	185,612	5.26861	−8	64	−42.14888	5.33960	218,575
1921	133,069	5.12407	−7	49	−35.86849	5.34813	222,910
1922	212,015	5.32637	−6	36	−31.95822	5.35666	227,334
1923	336,168	5.52656	−5	25	−27.63280	5.36519	231,841
1924	300,212	5.47744	−4	16	−21.90976	5.37372	236,440
1925	355,486	5.55082	−3	9	−16.65246	5.38225	241,130
1926	358,411	5.55438	−2	4	−11.10876	5.39078	245,910
1927	283,444	5.45247	−1	1	− 5.45247	5.39931	250,790
1928	363,230	5.56019	0	0		5.40784	255,764
1929	446,535	5.64985	+1	1	5.64985	5.41637	260,838
1930	279,666	5.44664	+2	4	10.89328	5.42490	266,011
1931	199,145	5.29917	+3	9	15.89751	5.43343	271,288
1932	114,223	5.05775	+4	16	20.23100	5.44196	276,669
1933	160,005	5.20413	+5	25	26.02065	5.45049	282,156
1934	229,426	5.36064	+6	36	32.16384	5.45902	287,755
1935	328,911	5.51708	+7	49	38.61956	5.46755	293,460
1936	371,211	5.56962	+8	64	44.55696	5.47608	299,288
1937	400,797	5.60292	+9	81	50.42628	5.48461	305,218
Total	....	102.75589	...	570	4.86247		

Log mid-point........................ 102.75589 ÷ 19 = 5.40820
Log annual increment................. 4.86247 ÷ 570 = .00853
Log monthly increment................ .00853 ÷ 12 = .00071

Mid-point of series lies midway between June and July of the middle year of the series, therefore,

$$\text{Log of June 1928} = 5.40820 - \frac{.00071}{2} = 5.40784$$

$$\text{Log of July 1928} = 5.40820 + \frac{.00071}{2} = 5.40856$$

Exhibit 169.—Computation of a compound-interest trend for automobile production from 1919 to 1937, inclusive. (Same series as shown in Exhibits 167 and 168.)

standard deviation. Then each of the percentage deviations (column *E*) is divided by this standard deviation, which expresses the series in terms of its standard deviation from normal, as shown in column *G*.

The curves in Part *C*, Exhibit 172, page 354, show not only the elimination of trend and seasonal variation from automobile production, for which the computations have been shown in Exhibits 167 to 171, inclusive,

SEASONAL VARIATION BY THE METHOD OF LINK RELATIVES
Automobile Production in the United States

	Jan. %	Feb. %	March %	April %	May %	June %	July %	Aug. %	Sept. %	Oct. %	Nov. %	Dec. %	
1918	61	116	124	116	93	83	99	76	67	84	86	146	
1919	149	122	127	113	107	94	99	100	109	115	92	85	
1920	116	100	122	76	121	107	93	98	92	88	82	81	
1921	49	133	159	135	103	122	93	102	88	93	79	68	
1922	112	132	145	128	117	109	88	110	75	114	100	97	
1923	109	113	129	105	104	96	87	105	94	112	86	98	
1924	104	116	103	98	84	80	107	106	104	99	80	88	
1925	116	118	132	116	97	95	100	65	125	136	84	85	
1926	98	118	119	101	97	91	93	119	93	84	77	66	
1927	142	128	129	103	100	80	83	115	84	84	61	99	
1928	173	140	128	99	104	93	99	118	90	96	65	91	
1929	171	116	126	106	97	90	92	100	83	91	57	55	
1930	228	121	120	112	95	80	79	84	98	70	89	114	
1931	110	128	126	122	94	79	87	86	75	57	86	176	
1932	98	98	101	125	124	99	60	83	93	58	122	180	
1933	120	82	109	153	122	116	92	102	82	70	45	133	
1934	193	148	147	104	94	93	86	89	72	78	63	184	
1935	189	115	128	106	80	99	93	71	37	311	145	102	
1936	90	79	146	119	92	98	97	62	50	166	176	126	
1937	76	96	136	109	96	96	88	90	43	193	109	91	
1938	64	89	119										
1918–1934 medians	116	118	126	112	100	93	92	100	90	88	82	97	
Chain relatives...	100	118	148.7	166.5	166.5	154.8	142.4	142.4	128.2	112.8	92.5	89.7	104.0
Adjusted.........	100	117.7	148.0	165.5	165.2	153.1	140.4	140.1	125.5	109.8	89.2	86.0	100.0
Seasonal index....	77.9	91.7	115.3	128.9	128.7	119.2	109.4	109.1	97.8	85.5	69.5	67.0	
March 1935–March 1938 average...	76.7	88.0	132.2	111.3	89.3	97.7	92.7	74.3	43.3	223.3	143.3	106.3	
Chain relatives...	100.0	88.0	116.3	129.4	115.6	112.9	104.7	77.8	33.7	75.3	107.9	114.7	88.0
Adjusted.........	100.0	89.0	118.3	132.4	119.6	117.9	110.7	84.8	41.7	84.3	117.9	125.7	100.0
Seasonal index....	96.6	86.0	114.3	127.9	115.5	113.9	106.9	81.9	40.3	81.4	113.9	121.4	

EXHIBIT 170.—Computation of automobile production seasonal variation by method of link relatives. Because of a decided change in seasonal variation early in 1935, two indexes have been computed for the period. The first index is based on the period from January 1918 to December 1934, and the second on the period from March 1935 to March 1938.

but also the elimination of trend and seasonal variation from steel ingot production. Both series are shown in terms of their standard deviations from normal in Part *D*. The standard deviations are then averaged together as shown in Part *E*. The curve presented in Part *E* is the desired index of business conditions.

The reader should fully appreciate that the purpose of calculating this simple index is to illustrate the method of computation, rather than to develop a practical indicator of general business conditions. An index of general business ordinarily should include a more representative group of series, and these require many preliminary corrections, adjustments, and analyses before they are combined. The question of trend is

"ELIMINATION" OF TREND AND SEASONAL VARIATION
Automobile Production in the United States

Year and month	Production (number of vehicles) A	Trend (number of vehicles) B	Seasonal index (per cent) C	"Normal" ($B \times C$) D	Percentage deviations from normal ($A \div D$) − 100 E	Percentage deviations squared F	Cycles in S.D. ($E \div \sigma$) (σ = 40.98 %) G
1918							
January	92,208	195,259	77.9	152,107	−39.4	1,552.36	− .961
February	106,962	195,849	91.7	179,594	−40.4	1,632.16	− .986
March	132,142	196,440	115.3	226,495	−41.7	1,738.89	−1.018
April	153,673	197,030	128.9	253,972	−39.5	1,560.25	− .964
May	143,003	197,620	128.7	254,337	−43.8	1,918.44	−1.069
June	118,859	198,211	119.2	236,268	−49.7	2,470.09	−1.213
July	118,060	198,801	109.4	217,488	−45.7	2,088.49	−1.115
August	89,542	199,392	109.1	217,537	−58.8	3,457.44	−1.435
September	59,555	199,982	97.8	195,582	−69.5	4,830.25	−1.696
October	50,311	200,573	85.5	171,490	−70.7	4,998.49	−1.725
November	43,244	201,163	69.5	139,808	−69.1	4,774.81	−1.686
December	63,127	201,753	67.0	135,175	−53.3	2,840.89	−1.301
1919							
January	93,779	202,344	77.9	157,626	−40.5	1,640.25	− .988
February	114,598	202,934	91.7	186,090	−38.4	1,474.56	− .937
March	146,091	203,525	115.3	234,664	−37.7	1,421.29	− .920
April	165,229	204,115	128.9	263,104	−37.2	1,383.84	− .908
May	176,484	204,705	128.7	263,455	−33.0	1,089.00	− .805
June	166,575	205,296	119.2	244,713	−31.9	1,017.61	− .778
July	164,831	205,886	109.4	225,239	−26.8	718.24	− .654
August	165,414	206,477	109.1	225,266	−26.6	707.56	− .649
September	180,786	207,067	97.8	202,512	−10.7	114.49	− .261
October	207,222	207,657	85.5	177,547	+16.7	278.89	+ .408
November	190,321	208,248	69.5	144,732	+31.5	992.25	+ .769
December	162,265	208,838	67.0	139,921	+16.0	256.00	+ .390
(1920–1935 omitted from this table)							
1936							
January	364,004	322,787	96.6	311,812	+16.7	278.89	+ .408
February	287,606	323,378	86.0	278,105	+ 3.4	11.56	+ .083
March	420,971	323,968	114.3	370,295	+13.7	187.69	+ .334
April	502,775	324,559	127.9	415,111	+21.1	445.21	+ .515
May	460,565	325,149	115.5	375,547	+22.6	510.76	+ .551
June	452,955	325,739	113.9	371,017	+22.1	488.41	+ .539
July	440,999	326,330	106.9	348,847	+26.4	696.96	+ .644
August	271,291	326,920	81.9	267,747	+ 1.3	1.69	+ .032
September	135,130	327,511	40.3	131,987	+ 2.4	5.76	+ .059
October	224,628	328,101	81.4	267,074	−15.9	252.81	− .388
November	394,890	328,691	113.9	374,379	+ 5.5	30.25	+ .134
December	498,721	329,282	121.4	399,748	+24.8	615.04	+ .605
1937							
January	380,055	329,872	96.6	318,656	+19.3	372.49	+ .471
February	363,995	330,463	86.0	284,198	+28.1	789.61	+ .686
March	494,277	331,053	114.3	378,394	+30.6	936.36	+ .747
April	536,339	331,644	127.9	424,173	+26.4	696.96	+ 644
May	516,919	332,234	115.5	383,730	+34.7	1,204.09	+ .847
June	497,311	332,824	113.9	379,087	+31.2	973.44	+ .761
July	438,971	333,415	106.9	356,421	+23.2	538.24	+ .566
August	394,330	334,005	81.9	273,550	+44.2	1,953.64	+1.079
September	171,203	334,596	40.3	134,842	+27.0	729.00	+ .659
October	329,876	335,186	81.4	272,841	+20.9	436.81	+ .510
November	360,055	335,776	113.9	382,449	− 5.9	34.81	− .144
December	326,234	336,367	121.4	408,350	−20.1	404.01	− .490

EXHIBIT 171.—In the above table the data are given for four years only as this will be sufficient to illustrate the method of eliminating the trend and seasonal variation from the automobile production data shown by the curve in Part *A* of Exhibit 172. The years included in this table are the first two and last two of the 1918–1937 period. The same procedure was, of course, followed for the other years and the resulting curves for the entire period for columns *D*, *E*, and *G* are shown in Exhibit 172. (See Exhibit 170 relative to the two seasonal indexes used above.)

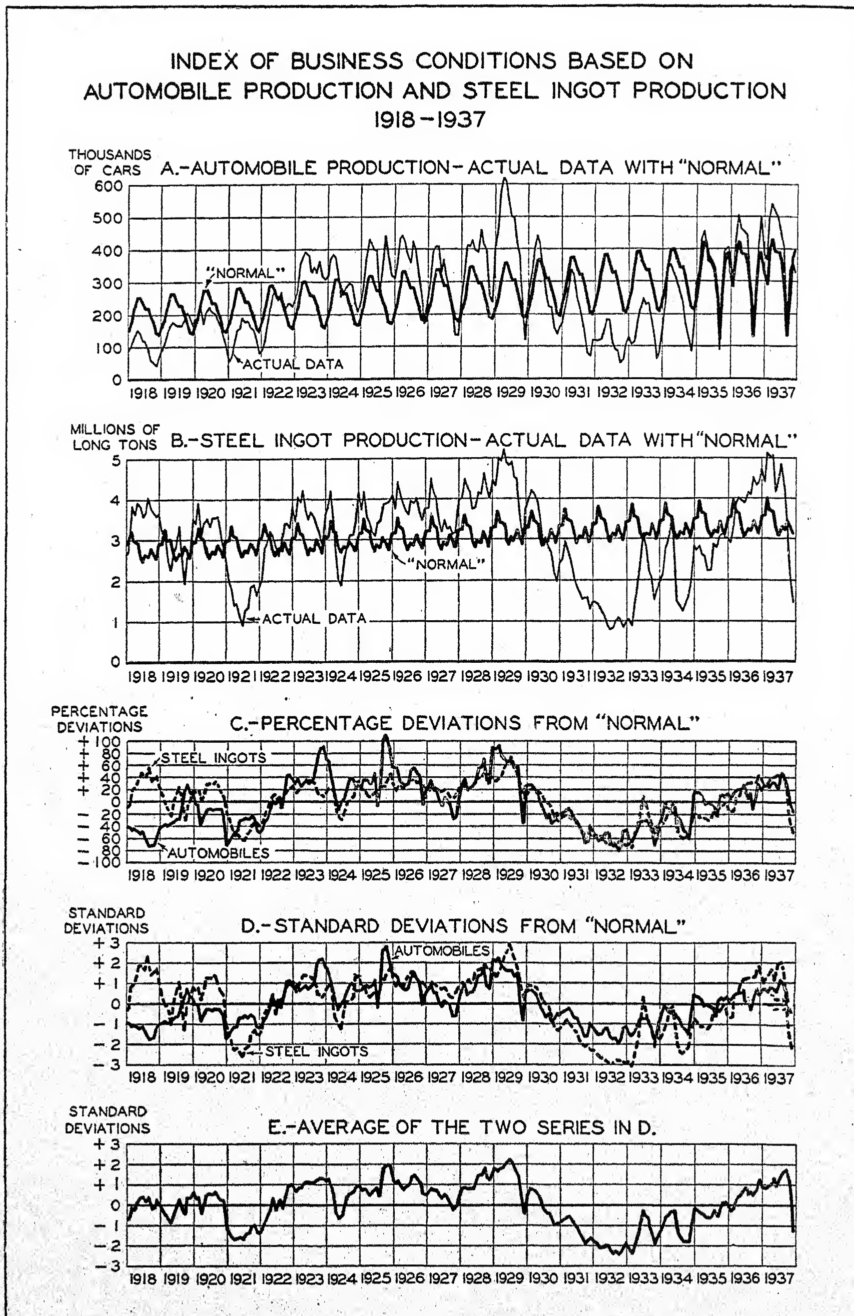

EXHIBIT 172.—Development of a simple index of business conditions. The index curve shown in Part *E* is an average of automobile production and steel-ingot production when expressed in terms of standard deviations from normal.

particularly important. In the illustration just presented (Exhibit 171), a computed trend was used. Very often, however, a computed trend will not actually be the long-time trend, because of random variations during the period. In fact, the computed trend in the illustration just presented is open to serious question because of the effect of the severe depression of the 1930's.

It has been the purpose of the last four chapters to describe some of the technical methods which are most useful in analyzing time series. The practical usefulness of these methods in making analyses of economic changes will be evidenced from time to time throughout the remainder of this book, especially in the following chapter on Business Forecasting.

Questions and Problems

1. What is meant by the term "business cycle"?

2. What are the four phases, or periods, of the business cycle? Describe each. Are the phases of equal duration? Are the entire cycles regular as to length?

3. Distinguish the "general business" cycle from cycles in certain industries.

4. Distinguish between major and minor cycles.

5. How does overconfidence and loss of confidence affect the business cycle? Why is the "crowd" always wrong in its judgment of the position in the business cycle?

6. Discuss the relation of erratic movements to cyclical movements. Are they usually measured separately?

7. Can business conditions in general be controlled or stabilized? Explain.

8. What is meant by "normal" business conditions and how may mathematical measurements of normal be made?

9. Describe the process of obtaining percentage deviations from normal. What is the purpose of the process of reducing percentage deviations to units of standard deviation? How is this done?.

10. Why is it sometimes desirable to forecast an individual concern's business in terms of standard deviations?

11. Discuss the correction of time series for trend and seasonal variation in correlation analysis.

12. What is the significance of irregular residuals after trend and seasonal influences have been eliminated?

13. Explain how composite business curves composed of two or more series of data may be prepared. Describe the computation of a simple business index for the United States based on automobile production and steel-ingot production.

14. The quarterly sales (in thousands of dollars) of the Dandy Grocery Company for the years 1935–1938 were:

Year	1st quarter	2d quarter	3d quarter	4th quarter
1935	95	121	124	97
1936	108	120	135	120
1937	126	135	136	126
1938	130	145	142	120*

* Estimated.

From previous experience Mr. Dandy has determined the seasonal variation of his sales, computed by the link-relative method, to be:

Quarter	*Per cent*
First	92
Second	106
Third	109
Fourth	93

He also has found that the quarterly trend of his sales has a quarterly increment of $2,000 and that its ordinate for the year 1936 was $116,000.

a. Compute the expected normal sales for the years 1935–1938 and express the actual sales as percentage deviations from normal.

b. Follow the instructions given in *a*, assuming that the index of seasonal variation was computed by the method of quarterly totals.

(Quarterly data are used in this and some succeeding problems in order to limit the number of computations. Ordinarily, monthly data are more satisfactory for time-series analyses of this character.)

15. Annual sales (in millions of dollars) of James and Co. for the years 1930–1938 were:

Year	*Sales (millions of dollars)*
1930	45
1931	46
1932	35
1933	32
1934	28
1935	34
1936	40
1937	46
1938	54*

* Last quarter estimated.

The quarterly sales (in millions of dollars) for the years 1935–1938 were:

Year	1st quarter	2d quarter	3d quarter	4th quarter
1935	8	6	9	11
1936	9	6	11	14
1937	11	8	12	15
1938	14	11	13	16*

* Estimated.

a. Computing trend by the method of least squares and seasonal variation by the link-relative method, determine the normal sales for the years 1935–1938 and express the actual sales as percentage deviations from normal.

b. Reduce the percentage deviations from normal to units of standard deviation from normal.

c. If Mr. James expects sales in the first quarter of 1939 to be .8 of one standard deviation above normal, what volume of sales may he expect in the first quarter of 1939?

16. Quarterly sales (in millions of dollars) of Best Motor Company for the years 1935–1938 were:

Year	1st quarter	2d quarter	3d quarter	4th quarter
1935	14	17	10	13
1936	15	19	12	18
1937	23	21	14	22
1938	31	29	17	24*

* Estimated.

From previous records it has been determined that the quarterly trend of the company's sales has an annual increment of 2 (millions of dollars) and an ordinate for the year 1938 of 22 (millions of dollars).

a. Computing seasonal variation by the ratio-to-trend method, determine the "normal" sales for the years 1935–1938. Express the actual sales as percentage deviations from normal.

b. Estimate the sales for the first and second quarters of 1939, assuming the sales are expected to be +.6 and +.4 standard deviations from normal, respectively.

17. Bank debits (in thousands of dollars) of Brown City for the three years 1936–1938 were:

Month	1936	1937	1938
January	24	39	64
February	21	34	61
March	25	40	69
April	24	38	76
May	29	41	75
June	28	46	83
July	31	50	80
August	34	56	84
September	32	54	88
October	37	61	95
November	36	61	93*
December	43	87	114*

* Estimated.

The monthly trend line has been found to have an annual increment of $24,000 and an ordinate for 1937 of $52,000.

Finding the seasonal variation by the method of monthly totals, determine normal expected bank debits for the first six months of 1939.

18. Actual and computed normal sales (in thousands of dollars) of the Johnson Motor Co. for the years 1935–1938 are given below:

Sales (in thousands of dollars)

Quarter	1935		1936		1937		1938	
	Actual	Normal	Actual	Normal	Actual	Normal	Actual	Normal
First..........	93	103	106	110	125	118	130	125
Second.........	122	118	122	126	137	135	145	143
Third..........	127	122	133	130	138	139	143	147
Fourth.........	95	106	118	113	124	120	119*	126

* Estimated.

A business service in the area informs the management of the Johnson Motor Co. that general business in the area (expressed in units of standard deviation from normal) has been, for the past four years:

Year	1st quarter	2d quarter	3d quarter	4th quarter
1935	−1.6	.4	.6	−1.1
1936	−1.0	− .6	.2	1.3
1937	2.1	1.1	.1	1.8
1938	1.2	.6	−.6	−1.4*

* Estimated.

The business service estimates general business in the area for the first two quarters of 1939 to be −.6 and +1.2 standard deviations from normal, respectively.

a. Determine if there is enough correlation between the sales of Johnson Motor Co. and general business in the same area to warrant Mr. Johnson's using the estimates of the business service for the first two quarters of 1939 in making a forecast of his own sales for the same period.

b. If the relationship in *a* is found to be sufficiently close, estimate the sales of the Johnson Motor Co. for the first two quarters of 1939. The estimated computed normal sales of the Johnson Motor Co. for these two quarters would be 133 and 152 (in thousands of dollars), respectively.

19. Sales of Smart and Co., retail merchants, are given below for the past seven years (data in thousands of dollars).

Retail Sales (in thousands of dollars)

Quarter	1932	1933	1934	1935	1936	1937	1938
First..........	102	96	76	96	96	102	121
Second.........	107	78	84	92	88	114	118
Third..........	104	83	93	101	104	107	132
Fourth.........	98	80	100	107	113	132	146
Total........	411	337	353	396	401	455	517

The retail sales for the entire area (expressed as a percentage of normal) are computed by the Retail Merchants' Association and are as follows:

Retail Sales (per cent of normal)

Quarter	1932	1933	1934	1935	1936	1937	1938	1939
First...	136.2	87.6	77.4	94.8	88.8	97.6	109.1	114.0*
Second.	127.5	93.8	89.7	95.0	86.4	102.1	110.2	102.0*
Third..	121.3	85.4	90.8	96.9	92.4	103.1	107.1	
Fourth.	101.6	75.6	90.9	93.8	93.7	107.8	111.3	

* Estimated.

a. Mr. Smart would like to know:

1. The rate of long-time growth in his business (use method of least squares).
2. The seasonal variation in his business (use link-relative method).
3. How closely the cyclical fluctuations in his business followed the cyclical fluctuations in the data presented by the Retail Merchants' Association.

b. What sales would you forecast for Smart and Co. for the first half of 1939?

CHAPTER XVII

BUSINESS FORECASTING

When definite estimates of future conditions are made on a systematic basis, the process is referred to as "forecasting," and the figure or statement obtained is known as a "forecast." During the last twenty years there has been a great increase in the number of organizations which definitely plan their buying, production, selling, financing, and building in relation to a scientific forecast of future conditions.

Object of Forecasting in Business.—It should be realized at the outset that the object of business forecasting is not to determine a curve or series of figures that will tell exactly what will happen, say a year in advance, but it is to make analyses, based on definite statistical data, which will enable an executive to take advantage of future conditions to a greater extent than he could without them. Some predictions are based upon relatively complicated statistical procedure, but many of the most valuable forecasts are those which are relatively simple and which the business man can check directly with his own judgment and experience.

Business forecasting does not depend upon the determination of a certain figure for a certain future situation through the application of some mysterious or magic process, although there are some so-called experts who would have us believe that they are in possession of highly valuable secret tricks and formulas. The business man, however, ordinarily does not have confidence in this type of forecast.

Business forecasting depends upon practical analyses of the past and present conditions which indicate the nature of probable future conditions. Suppose, for instance, that while conditions are still prosperous, a cement manufacturer finds that, in his market area, building is overexpanded, public funds are not available for paving and similar construction, and that the other users of cement have built up an excess of construction in relation to the demand. This cement manufacturer then knows that the future market is in a precarious condition. He cannot foretell the exact day, week, or month when sales will decline, but he knows that his market is unsafe. Consequently, he knows that he must experience a distinct decline in his sales in the near future, and that he should conduct his business accordingly. Suppose, on the other hand, that during a period of quiet business, a shortage of buildings has developed, and that bonds have been floated for extensive municipal improvements, such as paving, sewers, bridges, and the like. Under such

conditions, he can reasonably expect an improved market in the following season, and he will prepare his plant and organization to handle an increased demand for his product because of what is shown by such statistical indicators of the future.

The more accurately the above cement manufacturer can determine his cyclical position in the present, the better idea he will have of how he should prepare his concern for the future. In this connection, the point should be appreciated that the principal reason for determining one's position in the business cycle is to determine when one is in a sound position and when one is in an unsound position. If a person knows that he is in a boom period, he knows that he must be cautious in assuming obligations, in expanding plant or inventory, etc., for he must look forward to contracted business. If, on the other hand, one knows that a depression has run its course, he can make preparations for expanding business. Of great assistance in this connection, when one can find it, is a series of related business data which will *indicate* the cyclical situation in a particular market when exact data on the market itself are not available.

Limitations of Statistical Analysis.—The primary purpose of the preceding chapters has been to develop a background of statistical technique that would be useful in analyzing practical business problems. In the present and remaining chapters, the purpose is to indicate more specificially and completely the types of statistical problems that confront business men, and how such problems can be solved in a practical way with the aid of statistics. The methods that have been described will be used as tools when they are needed and when it is expedient to use them, but the emphasis will be placed upon securing practical results. However, before proceeding with the discussion of specific applications to business problems, it is desirable to review the more important limitations of statistical analyses, for, important as such analyses are, there is often a tendency for the student and business man to expect too much from statistics. In fact, one of the greatest difficulties in present-day statistical work is the failure to realize or properly appreciate the limitations involved in the application and the interpretation of results obtained by statistical methods. Four of the most common limitations, each of which may be the proper subject for no small amount of reflection by the practical statistician, are as follows:

1. Failure to realize that statistical methods are applicable only to quantitative data when, in fact, quantitative data practically never constitute more than a part of the information on a given business problem.

2. The assumption in sampling that all of the data are equally representative of and equally affected by the infinite number of independent forces presumed to be acting upon the data.

3. The imperfections (primarily because of rigidity) of the mathematical methods used in statistical technique.

4. Inadequate statistical material and the use of methods too refined for the data at hand.

These various groups of limitations will now be discussed briefly.

Statistical Methods Limited to Quantitative Data.—The failure to realize that statistical methods are applicable only to quantitative data and that quantitative data can probably never constitute more than a fraction of the information necessary in a given business problem is undoubtedly the cause of more erroneous conceptions and improper deductions than any other one difficulty in the mechanical use and application of statistical methods. Altogether too frequently the so-called statistician overlooks or ignores those multitudinous and varied forces and phenomena which, although they are not easily reduced to a numerical basis nor expressable in quantitative terms, do have an important bearing upon the solution of the immediate problem in hand. One reason why professional statisticians ordinarily do a considerable amount of field work themselves when carrying on an extensive practical investigation is that the casual observations and non-statistical information gathered are very important as a background and supplement in interpreting the statistical data.

Limitations of Sampling.—In almost all statistical investigations, as was previously pointed out in Chapters II and XI, it is not only impractical but quite impossible to obtain a statement of all the numerical data bearing upon a given problem. In order to save time and expense, it is necessary to resort to the sampling process, which consists of utilizing only a part of the possible data and of making the assumption that the results are substantially the same as if complete data had been available. It is imperative in interpreting such data that the limitations of the sample be understood as far as possible and that all conclusions be properly qualified.

Rigidity of Mathematical Methods.—The mathematical methods which underlie statistical processes are based upon certain ideal conditions. Since these ideal conditions are seldom if ever present in statistical data, it is commonly necessary to "idealize" the data; that is, it is customary to assume that the data are ideal, and, if necessary, to make qualitative reservations or to provide corrections to offset the possible error. Thus, certain statistical methods presuppose that the frequency distributions involved are normal probability curves—a condition quite difficult to find in the economic world. Such distributions in business are almost always skewed and not infrequently they are quite erratic. The art of judging just how far such statistical methods can be applied to data which deviate from a normal probability curve

marks the difference between a dependable and a non-dependable statistical analyst.

Although most statistical methods can be mechanically applied without an understanding of the mathematics involved, safe interpretations can be made only when the mathematical processes are understood and qualitative information is considered. Dogmatic interpretations often result from blind reliance upon mathematical methods. The trained statistician uses the same mathematical methods (which need not necessarily involve the higher mathematics), but he realizes their distinct limitations and checks his interpretations in other ways.

At an earlier point in the discussion it was noted that several methods tended to be biased in one way or another. Thus the arithmetic and harmonic means are biased by small and large numbers in index-number construction. The coefficient of correlation may be biased by non-linear relationships. The former type of bias may be said to grow out of the imperfections of our numerical system rather than from the way in which these numbers are treated; the latter type of bias comes from the use of a mathematical method based upon one set of assumptions, where another and different set of conditions obtains. It is obvious that care should be taken to observe any such bias when it exists.

Use of Too Highly Refined Methods.—Another of the dangers in statistical analysis is the use of methods which are too highly refined for the data at hand. Since all statistical methods are not equally refined, they possess varying degrees of appropriateness when applied to different types of data, and when the data are in various forms or of different degrees of accuracy. Thus, it may not be proper or practical to compute a coefficient of correlation when only a small number of items are available, or when the data are given merely by arbitrary classes.

Another difficulty arises in determining the extent to which the calculation should be carried out, when the extent of accuracy in the original data is known. Cases are on record where weekly earnings with a probable accuracy of each individual case not nearer than $5 a week have been averaged to three decimal places. Spurious accuracy of this kind only serves to place good statistical work in disrepute. A case recently observed was one in which the amount expended annually by the average citizen in the United States for shoes was stated as $7.1124 and the amount of sugar consumed as 65.3185 pounds. In this instance the original data probably consisted of estimated total amounts divided by an estimated population—a circumstance certainly not requiring computation to four decimal places. A good general rule is to compute the statistical measure to one greater decimal place than the accuracy of the data. Thus, if one had data which were accurate to 1 pound, it would be safe to compute an average to .1 pound.

Statistical Checking List.—Experience soon teaches the statistical investigator that there are many checks that should be made of his methods and of his analyses. The following list includes some of the more important questions which should be answered before an investigation may be considered trustworthy:

1. Are the results reasonable?
2. Is it logically possible to reach any other conclusion?
3. Is a solution based upon past conditions sound as a basis for predicting future conditions?
4. Is the group of data sufficiently large to give a dependable indication of the characteristics as they apply to the problem in hand?
5. Are the methods applicable to the data? That is, is there too much "idealization"?
6. Are the data accurate and complete enough to justify the application of the methods used?
7. Are there cumulative or other errors in the compilations or calculations? Or will the application of the results necessarily involve cumulative errors?
8. Has there been comparison of non-comparable data?
9. Have percentages been erroneously used?
10. Have previous convictions influenced either the selection of the data or the application of statistical methods to them?
11. Have any important factors, which may be discovered by logical reasoning, been neglected in the analysis?
12. Has there been a repeated use of methods which will introduce error (such as ratio of ratios or average of averages)?

Obviously, this list is not complete, but it will be of assistance in avoiding errors.

Probably no check in statistical analysis is more important than that of determining the reasonableness of the results in relation to other information. "The verification of conclusions by means of controlled statistical experiments is very rarely possible in the social sciences, and the impracticability of such experiments results too often in excessive reliance upon theory and pure correlations. Nevertheless, there are many direct verifications of cause and effect relations in economic phenomena that, while fragmentary and incomplete in themselves, may still be of convincing value in combination with the results of other analyses.

"As a specific though somewhat odd illustration, we may take the correlation between daily weather conditions and the lunch time patronage of restaurants. On first examination, the situation is confusing. Some restaurants show a positive correlation between fine weather and the number of lunches served. Others show a negative correlation, and still a third group reveal no definite correlation whatever. From the standpoint of pure statistics, the problem is perhaps insoluble. Yet a few direct inquiries will not only reveal the real facts, but add an unex-

pected significance to the data. Stormy weather reduces somewhat the total number of lunches eaten in restaurants. But its more important effect is to drive customers into unattractive near-by restaurants, who in fair weather would go to more attractive restaurants at a distance. The attractive restaurant shows a positive correlation between trade and sunshine. The unattractive restaurant profits from cloudy skies.

"Another example of this practical form of verification is in connection with the assumption that there is a direct cause and effect relation between interest rates on second mortgages and the volume of building construction. It has been stated that the reduced building activity of the last few months of 1930 was directly ascribable to the high cost of second mortgage financing. Nevertheless, even the most sketchy inquiry of those directly concerned with the handling of second mortgages would have shown that the difficulty was not a shortage of second mortgage money, but a shortage of second mortgage confidence. The real obstacle to construction at that time was the fear that building investments made at the then current levels of cost, and in the face of apparent building surpluses, might be wholly unprofitable.

"It is this checking of circumstantial evidence by the forcing of a direct confession from the statistical culprit that may properly be described as the application of the 'statistical third degree.' And no professional sentimentalism should be allowed to prevent a full development of the possibilities of this procedure. The statistician, as the chief pathfinder among scientific pioneers, must necessarily combine that vision which comes from fertility, breadth, and incisiveness of hypothesis, with the balance and sureness of step of the scientific method as a whole. But, above all, he must possess something of the rude spirit of the frontiersman and must seek his results in every legitimate way, regardless of refinements of method and rigidity of conventions."[1]

Predominance of Forecasting in Business Statistics.—Questions of the future commonly predominate in business investigations. As has been mentioned previously, the business man must constantly plan for the future. Questions of expanding production, increasing inventories, constructing new buildings, acquiring land, extending credit, curtailing loans, decreasing interest rates, raising prices, developing new markets, and the like usually must be decided upon the basis of present indications of future conditions. Even when one makes a frequency distribution study of, say, factory wages, it is usually done with the object of improving one's position in the future.

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[1] Quoted with slight changes from M. C. Rorty, "Statistics and the Scientific Method," Presidential Address, Ninety-second Annual Meeting of the American Statistical Association, December 31, 1930; published in the *Journal of the American Statistical Association*, pp. 1–11, March 1931.

Some estimates of the future take the form of long-range forecasts, covering several or even many years, while others involve only a few weeks or days. Probably the most common problems are those that involve planning ahead for 1 to 18 months. Sometimes refined methods of measuring trend, seasonal variation, etc., are necessary, as when economic and statistical services forecast the movements of the business cycle, but the statistical methods used in most estimates regarding the future are much more simple. An illustration of this simple type of forecasting is given in the following: A certain San Francisco apple dealer, learning that the apple market was unusually good in British Malaya, had to estimate future conditions because of the time involved in reaching that market. A brief investigation showed him that 5,000 cases were already on the way, and this quantity was enough to overstock the market before it would be possible for his shipments to arrive there. Having this simple but significant datum at hand, he knew that future conditions would be such that it would not be advisable to ship apples to Malaya.

In forecasting the future in business, the value of statistics depends upon the extent to which they serve as bases for making better forecasts than could be made without them. Statistics alone ordinarily do not forecast future conditions. In fact, purely mechanical statistical forecasters are not reliable. Judgment, experience, and knowledge of the particular industry are necessary in making the proper interpretations and analyses and in reaching sound conclusions.

Local, Regional, and National Points of View.—Some businesses are concerned principally with local conditions while others are concerned with regional, national, and even international conditions. For some products (such as raw materials like wheat and cotton) a wide market exists, not only throughout the United States, but largely throughout the world. Conditions in such markets do not vary much from place to place, and if temporary differences arise, they soon adjust themselves. For most consumers' goods, however, sales effort is necessary, and the relation between demand and supply varies greatly in different parts of the country. Consequently, if we are considering the cyclical situation in connection with forecasting or planning business operations, we must recognize that local business cycles may be distinct from national business cycles. Sometimes cycles in one area move with national cycles, sometimes there is a lag, and sometimes they move in opposite directions. Much of our trade, industry, and agriculture is affected by both national and local conditions, some is affected almost entirely by national conditions, and some is affected almost entirely by local conditions.

The importance of recognizing the differences between local and national cyclical situations may be illustrated by the following examples:

In 1921 a Los Angeles dry-goods wholesaler bought his merchandise in the national market which was in a period of depression. He sold it in the local market where conditions were prosperous; this was an ideal situation for the wholesaler. On the other hand, a Los Angeles manufacturer at the same time (1921) sold his products in a depressed national market at a loss because local labor and material costs were high, on account of the prosperous conditions in Los Angeles at that time. Similar instances could be cited in connection with the Florida, Iowa, and other

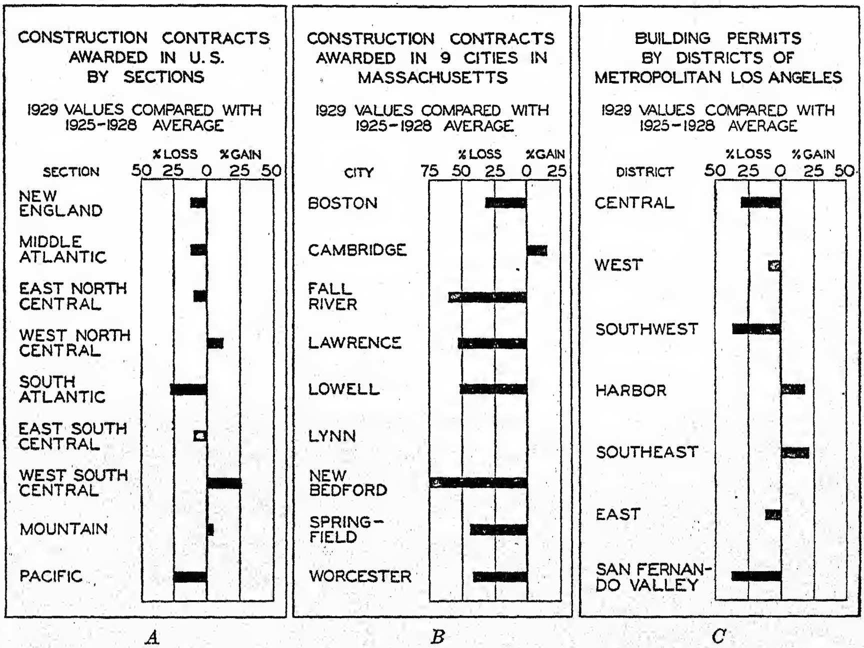

EXHIBIT 173.—Not only do conditions vary widely from one major part of the nation to another (*A*), but there are wide variations within small divisions of the country (*B*), and also within local areas (*C*), even when conditions in general are highly prosperous. (See Exhibit 158, page 331, for curves showing the general business boom and fairly high building activity in 1929.)

booms. How construction activity (as indicated by new contracts) varied in nine sections of the nation during the generally high prosperity of 1929 is shown in Part *A*, Exhibit 173.

Differences in national and local conditions are pointed out above. Often it is necessary to break local areas down into still smaller parts. That is, conditions in a small section of the country, or conditions in a metropolitan city area often differ greatly from one part of the area to another. This makes it necessary to study these subdivisions separately if one is to take advantage of the conditions as they exist. Illustrations of how these variations occur, even when conditions in general are highly

prosperous, are presented in Exhibit 173, Part *B*, which shows the variations in construction contracts in nine cities of Massachusetts, and in Exhibit 173, Part *C*, which shows the variations in building permits in seven different districts of metropolitan Los Angeles. Certain business concerns, especially wholesalers and manufacturers, find such studies of small areas very valuable checks upon the various parts of their market. In a certain case, for instance, a salesman was making but few sales in his district. A report showing active business conditions in one of these small areas, which had practically the same boundaries as his territory, was called to his attention. Realizing that somebody was actually getting the business, the salesman renewed his efforts and substantially increased his sales.

A Practical Example of Local Business Forecasting.—In order to illustrate the problem of local business forecasting in greater detail, let us refer to an actual case of forecasting office-building conditions in Los Angeles. Early in 1925, it was found advisable to make a careful analysis of the future office-building situation for the use of bankers, realtors, building-material manufacturers and dealers, and other business men.[1] It was found that there was a fairly definite relation between occupied office space and population. From 1909 to 1924, the number of square feet of office space per person per year varied from 6.1 to 7.6, excluding war years. Both of these extremes occurred in the early part of the period. The more rapid growth of the metropolitan population, as compared with that of the city proper, did not seem to affect the ratio. In fact, it appeared in this survey that any increase in the ratio, due to the metropolitan population, was more than offset by the increased efficiency in the use of office space. Consequently, in determining the relation of present space to future population, it was deemed advisable to use the latest factor instead of the average for the period. This latest factor was 6.27 square feet per capita.

Population forecasts for January 1, 1927, and January 1, 1930, were then used as the bases of future demand estimates. In making these population forecasts, careful estimates were made for the intercensus years back to 1900, so that the cycles could be studied. A study of the long-time tendencies for the period since 1900 indicated a compound-interest trend. The forecasts were made on the basis of projecting this trend of 1930, and then making a cross-section analysis to determine whether such factors as growing industry, oil developments, trade expansion, tourist traffic, and the like would operate to change this trend. It appeared that the average rate of development was still fundamentally the same and that there were no good reasons for expecting a substantial

[1] Eberle & Riggleman Economic Service, Vol. II, No. 13, March 30, 1925.

change in the rate of growth for the period under consideration. Since there appeared to be no fundamental factors which indicated that the population in 1927 and 1930 would be greater or less than the trend points, these figures were taken as the estimates of future population for these dates. It should be emphasized in this connection, however, that such trend figures should not be used as forecasts unless the fundamental factors influencing population growth are analyzed. It is this cross-section analysis of fundamental conditions, rather than the computation of the trend figures, that is of greatest importance.[1]

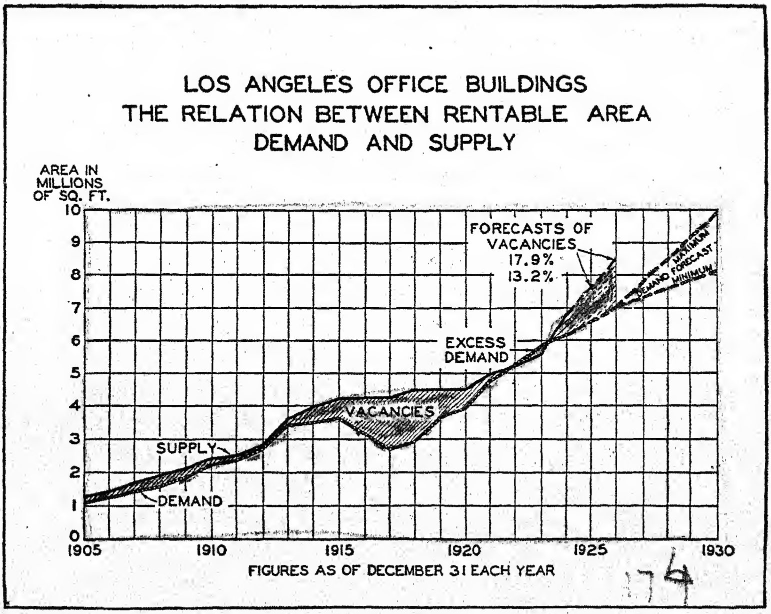

EXHIBIT 174.—This chart, which was made early in 1925, showed the current and past relations of supply to demand, and forecasts of future conditions.

On the basis of a forecast of population for January 1, 1927, of 1,117,291 multiplied by the space per capita factor of 6.27 square feet, it was estimated that there would be a demand for approximately 7,005,415 square feet on December 31, 1926 (see demand curve in 1926 in Exhibit 174). For December 31, 1929, maximum and minimum demand forecasts were made on the basis of population forecasts for January 1, 1930. The maximum population estimate for January 1, 1930 (1,300,000) was multiplied by the average of the six highest factors taken from the 16-year period which gave the maximum demand estimate of 9,165,000 square feet. The minimum population estimate (1,250,000) was multiplied by the average of the six lowest factors (excluding war years) which

[1] For detailed methods of making population estimates, see pp. 431–443.

gave a minimum demand estimate of 7,825,000 square feet. These estimates for December 31, 1929, are shown by the maximum and minimum curves in the chart just referred to (Exhibit 174), and the curves are projected one year to the end of 1930.

At the same time that the survey of vacant space was made, a survey was made of buildings under construction and projects that probably would be completed within the following two years. This investigation indicated that by the end of 1926 the supply would be increased to approximately 8,528,284 square feet. Since, by this date, a normal demand would increase to only 7,005,415 square feet, it was probable that there would be a vacancy of 1,522,869 square feet or 17.9 per cent

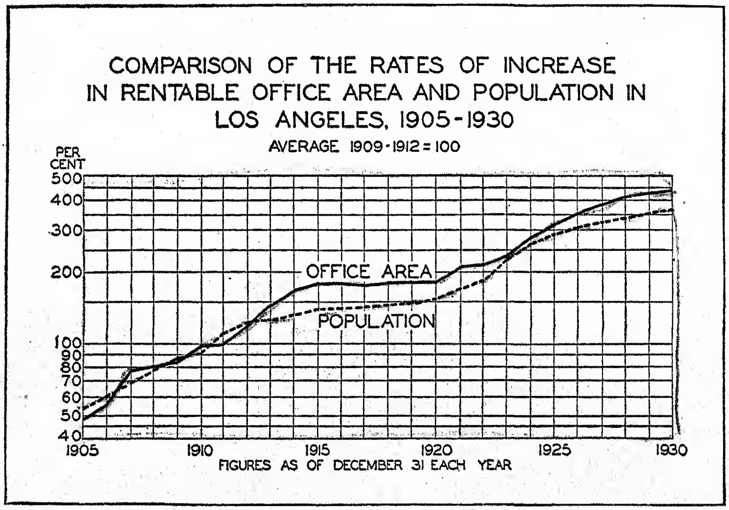

EXHIBIT 175.—When relations between population and office space exist as in the above chart, a diverging of the curves indicates an unsound condition.

(see Exhibit 174). A comparison of the supply expected for 1926 with the demand forecasts for 1930 (see Exhibit 174) indicated at the time this report was made in 1925 that there probably would be no serious shortage of office space by January 1, 1930, and there might be a vacancy as high as 8 per cent, even if no further office buildings, except those under construction or definitely projected, were added to the supply. The accuracy of this prediction is indicated by the fact that the actual demand on January 1, 1930, for office space of the type included in the survey was approximately 8,600,000 square feet—a point about midway between the maximum and minimum forecasts published in 1925.

If actual demand figures are not available, it is sometimes possible to make a comparison of total supply figures with population data that will be of great value in analyzing such situations. In Exhibit 175,

these figures for Los Angeles are reduced to percentage relatives, with the period 1909 to 1912 as a base. It will be noted that in the period following 1913 and in the period following 1924 the curves diverged widely. We know from Exhibit 174 that these were periods of high vacancies. Where such relations are known to exist, a population series can be used as an *indicator* of demand for office space, and when the curves diverge, as in Exhibit 175, an unsound condition is indicated.

While the relation shown in Exhibit 174 indicates that the procedure described in connection with Exhibit 175 could be used in this particular case if there were no demand data, it does not prove that the principle can be used in all cases where demand data are not available. Some kind of a check must be made to find whether or not the plan is reasonable and reliable. Suppose, for instance, that Exhibit 175 represents some other city and that demand data are not available. If it were known in a general way that a sound relation existed between the supply and demand for space in 1908, 1909, 1910, 1922, and 1923, such divergings as those following 1913 and 1924 would be distinct indicators of unsound conditions. Through the use of building permit data and other available data on projected construction, the actual supply of space can be anticipated for several months, and in some instances for a few years, thus improving the accuracy of the forecast.

In connection with the application of the above information, let us consider the cases of three business organizations which planned the construction of office buildings in a new business section of the city at the time the studies were made. Realizing that a large vacancy in office buildings might be expected during the succeeding few years, and that the trend of business movement was no longer in the direction of this newly developed area, one concern abandoned its project. The second went ahead with its original building plans, in spite of the unfavorable outlook indicated by the survey, and put up a structure of the type for which there was very little additional demand, suffering a severe loss. The third organization made a detailed statistical survey and found that there was a good market for a specialized medical building in a particular location, even though the general market for office building space was over-supplied.

Similar studies can help at times in forecasting market values as well as demand. In Cleveland, for instance, the ratio of property values to annual wages and salaries paid doubled in the five years from 1919 to 1924. Such a rate of increase obviously could not be sustained. A short time later values began to drop sharply and the rate of building activity began to fall. Another example is the situation in Washington in the late 1920's. When mortgage men decided not to advance further funds on the assumption that the city was over-built and that about 10 per cent of the

houses were standing idle, a house-to-house survey was made which revealed a vacancy of about $2\frac{1}{2}$ per cent. As a result of this indication and further studies of demand, money was soon made available by the bankers. Supply was kept within the limits set by demand for the time being, and values remained relatively stable.

Major Classes of Methods.—From the foregoing discussion, the reader will appreciate that there are two major classes of methods involved in forecasting. The first may be designated *historical analysis of past conditions*, and the second may be designated *cross-section analysis of current situations*. Both classes are used in combination by all reputable forecasters, but the emphasis placed upon each will vary.

In the first class, there are two outstanding theories. One is that when business is inflated or depressed, it tends to come back to normal; the other is that there are definite cyclical sequences between different types of business activity. The application of the first of these theories depends upon determining what constitutes normal activity and upon determining the significance of variations from this normal. One well-known application of the theory adapts the physical law that "for every action there is always an opposite and equal reaction" to economic analysis—that is, over any long period of time, a period of prosperity with a certain product of duration times intensity must eventually be balanced by a depression with an equal product of duration times intensity.[1] The second theory depends upon discovering cyclical sequences so that changes in certain classes of activity can be determined on the basis of how they follow or precede the movements of other classes. The sequence of events is then used in such a way that one type of series acts as a forecaster of those which occur later. The general tendency of the upturn in the prices of industrial stocks to precede the upturn of commodity prices, or the general tendency of the velocity of circulation of currency and bank credit to reach its highest point a few months before the decline in commodity prices and to reach its lowest level shortly before commodity prices start upward, are illustrations of relations that are used as bases for predictions according to this theory.

Cross-section analysis of current situations depends upon finding what important forces are operating in a particular situation, measuring the significance of each, and determining their net effect. The elements considered in a particular situation will vary as any specific situation is

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[1] It should be appreciated that, in adapting this law to economic analysis, the analogy is not a perfect one. Action and reaction, as the terms are used in physics, are not necessarily subsequent in point of time, but the reaction may be cotemporaneous with the action. If not too literally applied, however, the analogy is undoubtedly helpful in explaining the tendency of business to deviate from and return to normal.

considered to be different from all others. New relations of supply to demand, new discoveries, new inventions, unusual attitudes, styles, and fads are representative of the factors to be analyzed.

The office-building forecast previously described was based upon a combination of historical analysis and cross-section analysis. The problem is a representative one, and illustrates how both types must be used in combination to secure practical results. Other illustrations of how the methods are used will be presented from time to time throughout the discussion.[1]

Basic Indicators—Business Barometers.—Of great assistance in practical forecasting is information that can be used as an "index" or "indicator" of the basic conditions related to the industry. The term "barometer" is also widely though loosely used in business statistics; sometimes the term is used to mean simply an indicator of the present economic situation, and sometimes it is used to designate an indicator of future conditions.

To illustrate the use of a statistical index or indicator, suppose that estimates made on the basis of probable crops show that farmers in eastern Nebraska should have 25 per cent less income this year than last, while farmers in eastern Iowa should have 50 per cent more. The manufacturer of agricultural implements can use such data as an indicator of market conditions, and he can have his salesmen concentrate on the eastern Iowa market, as far as these two areas are concerned. By using information secured from the United States Department of Agriculture, the state departments of agriculture, local farm bureaus, and from other sources, the manufacturer or other business man can analyze the probable farm income in different parts of the country in relation to his own particular business. Similar indicators in other lines of activity will be discussed from time to time throughout the chapter.[2]

Statistical indicators or barometers are usually presented as graphic charts, but they may also be presented in tabular or textual form.

Mathematical Projection.—Mathematical trends projected into the future often serve as valuable checks on other types of forecasts, especially when the numbers are large.[3] An illustration of how future tendencies can be estimated by this method is furnished by a projection of automobile registrations which was made early in 1934. This projection,

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[1] For a more detailed discussion of basic assumptions and methods, see Charles O. Hardy and Garfield V. Cox, "Forecasting Business Conditions," especially Chap. III, The Macmillan Company, New York, 1927.

[2] See also Lewis H. Haney, "Business Forecasting," Ginn and Company, Boston, 1931, for other suggestions of business data that can be used in measuring and in forecasting activity.

[3] See pp. 21 and 22 on the principles of statistical regularity and large numbers.

made to 1940, is presented graphically in Exhibit 176, together with a curve of the data for the period from 1912 to 1933 to which the basic curve was fitted. A curve of actual registrations for the period from 1934 to 1937 has also been added to the chart in order that the accuracy of the projection as an indicator of future conditions may be checked. If a concern's market is forecast on the above basis, it is obvious that it must be checked constantly with information which would indicate promptly, if possible, any changes or faults in the trend.

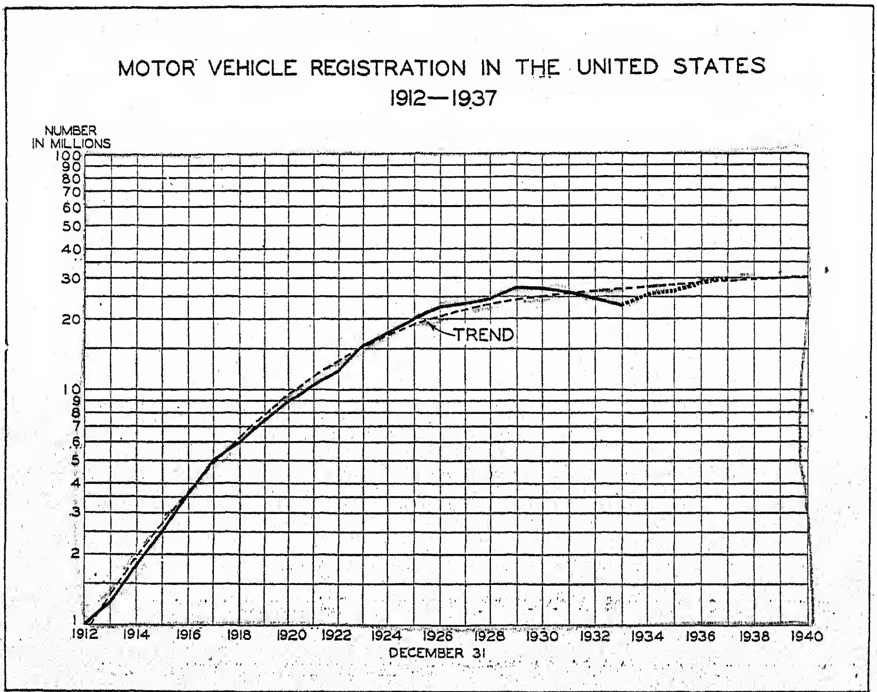

EXHIBIT 176.—The normal curve above was computed early in 1934 on the basis of 1912 to 1933 data and projected to 1940. This indicates how mathematical projections may serve as an aid in forecasting. (The data for the period from 1934 to 1937 were later added to the chart for comparison with the projection.)

The value of projections of constant or changing rates of change is indicated in another instance in which a branch manager in a certain territory wished to move into larger quarters and increase his staff. After employing mathematical methods to determine not only the rate of increase of the firm's sales, but also the rate of change in potential sale possibilities, it was found that the firm's sales were increasing at about 2 per cent per year, while the total sales of that particular product were declining at the rate of 3 per cent per year. In other words, the company was securing an increasing percentage of a decreasing volume

of business. Consequently, the office was not enlarged nor was the staff increased.[1]

The Stock Market as a Forecaster.—The stock market has been quite generally considered to be a barometer or forecaster of business conditions. Its movements are said to indicate what is going to happen weeks and even months in advance. In view of this widespread belief, it is deemed advisable to point out some of the limitations of this indicator before proceeding farther, because it does not always forecast changes in general business. It reflects a combined condition based on business, money, speculative, psychological, and other situations; and the proportions of these factors vary widely from time to time. The stock-market situation is an excellent indicator of speculative conditions, but it should not be used alone as a forecaster of general business conditions. It is only *one* of the indexes that should be studied in analyzing the general business situation.

The prominence that is given to this indicator by some of the better forecasters sometimes confuses the business man and leads him to believe that more importance is attached to it than actually is the case. The supposed value of the stock market as an indicator of future business conditions is based upon the theory that stock speculators are quicker to sense the conditions that will lead to increases or decreases in earnings than most men in the general lines of trade and industry, and that for this reason they will forge ahead or hold back in advance of the actual changes in general business activity.

One of the principal difficulties, however, is that a large proportion of the stock speculators cannot be trusted to sense these conditions, and a boom or a slump may develop in the stock market which has no fundamental counterpart in actual business. The great stock-market boom which reached its height in September 1929, for instance, was not followed by a similar period of inflation in business. And the abrupt stock-market decline of November 1929 was not accompanied by the same kind of a decline in business. This brings out the point that the stock market is only one of the indexes that should be watched, and that other indicators may qualify to a great extent the significance of the stock-market situation. The recognized forecasting services always use other indicators in conjunction with the stock-market index when attempting to interpret its significance.

Analyzing and Forecasting General and Special Conditions.—In this discussion, forecasting will be considered both from (1) the point of view of common interest forecasting for the use of business men in general, and

[1] See Ray B. Prescott, "Knowing Next Year's Increase in Advance," *Manufacturing Industries*, p. 53, July 1927.

(2) the point of view of specific problems of individual concerns. The most extensive work in forecasting *general business conditions* is carried on by special service firms organized for the purpose. The best work in forecasting *business conditions for individual concerns* is ordinarily carried on within the concern's own organization. Many of the larger banks, manufacturers, public utilities, and other business organizations have elaborate statistical departments for carrying on such work.

Harvard Review of Economic Statistics and Index Chart of General Economic Conditions.—As a background to the discussion of current

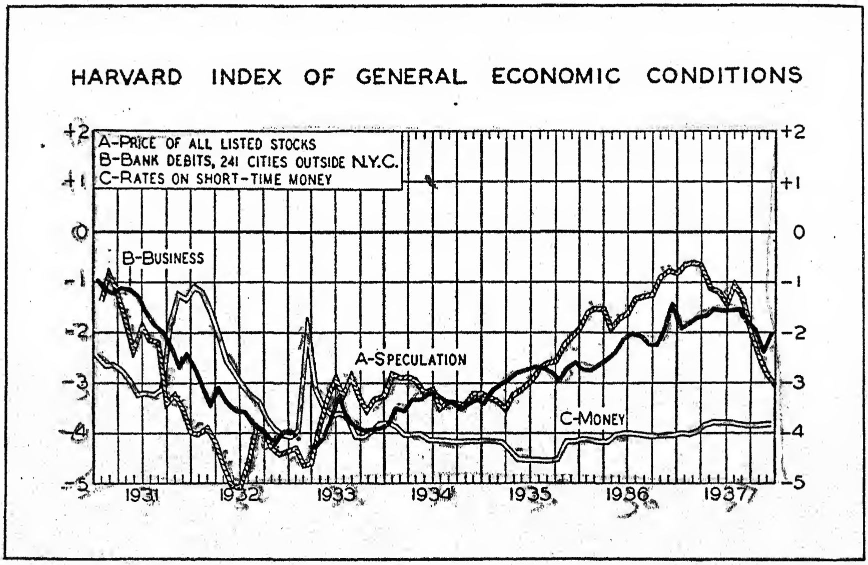

EXHIBIT 177.—Harvard Monthly Index Chart of economic conditions. In this chart, the time and direction of the major swings are important. The amplitudes of the fluctuations may not be significant unless analyzed in relation to outside checks. (*Courtesy, Review of Economic Statistics.*)

Economic Services, a brief reference will be made to the work of the Harvard *Review of Economic Statistics* and the former Harvard Economic Society. Part of the work formerly carried on by the Harvard Economic Society has been continued by the Harvard University Department of Economics through the quarterly *Review of Economic Statistics*. Each issue contains articles of a statistical, historical, and theoretical nature dealing with problems of the business cycle. A feature of each issue is the review of economic developments in the United States during the preceding quarter together with a chart of the Harvard index of general economic conditions (see Exhibit 177). This article reviews and appraises developments of recent months, and while no forecasts of conditions are included, it serves as an aid to the user in making his own forecasts.

The effort of the Harvard Economic Society represents one of the most thorough and painstaking statistical investigations of economic factors yet made by any forecasting agency. Although the sequence of events is similar to that formerly relied upon by The Brookmire Economic Service, the relations used in the Harvard studies were independently arrived at and based upon statistical analyses much more extensive than were the earlier conclusions of other investigators.

In studying the cycles of different series, it was found that they fell into three groups. Curves representing these three groups from 1903 to 1914 are shown in Exhibit 178. The first, or speculative group, consists of the yield of railroad bonds (reversed in sign), the price of industrial stocks, the price of railroad stocks, and New York bank clearings. The second, or business group, is composed of pig-iron production, bank clear-

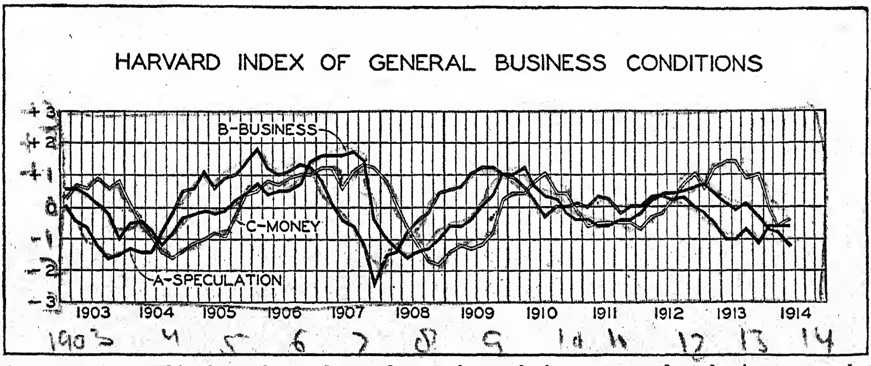

EXHIBIT 178.—This chart shows the tendency of speculation to move first, business second, and money third during the period 1903 to 1914.

ings outside New York City, Bradstreet's index of wholesale prices, and the Bureau of Labor Statistics index of wholesale prices. The third, or money group, consists of the rate on four- to six-months paper, the rate on 60- to 90-day paper, loans of New York banks, and deposits of New York banks, the last two being reversed in sign.

The time relationships of these three groups (see Exhibit 178) were found to be as follows: (1) The major fluctuations of speculation anticipated those of business by four to ten months. (2) Major fluctuations of business preceded those of banking by two to eight months.

The principles developed from these investigations may be applied to the Harvard Index Chart, but the interpretation of this chart is by no means a mechanical matter. Any forecasts that may be based upon the index of general business conditions must be supplemented by careful studies of data lying back of the index, and cross-section analyses must be made as an aid in interpreting the movements of the different curves.

The curves, "Speculation," "Business," and "Money" on the Monthly Index Chart (see Exhibit 177) are now constituted somewhat differently from those used in the preliminary investigations, but the principles underlying their construction are the same. Each curve is a representative index based upon selected series, each of which, when it is appropriate, has been corrected for long-time trend and seasonal variation. The resulting deviations from normal have been expressed in terms of standard deviations for the purpose of eliminating differences in the intensities of fluctuations. Curve *A* (*Speculation*) is based upon the price of all listed stocks. Curve *B* (*Business*) is based upon bank debits in 241 cities outside of New York City. Curve *C* (*Money*) is based upon short-time money rates. Although the Index Chart must be revised from time to time, the methods of interpreting it have not changed materially. The relationships of time, and the amplitude between the movements of the three curves provide an important, but not the only basis of making forecasts from this chart.

In interpreting the curves on the Index Chart, one must consider (1) the direction of the movements of each curve in relation to the movements of the other curves; (2) the direction of the immediately preceding movement; and (3) the magnitude of such movement. The rise or fall of the speculation curve tends to forecast a similar movement in the business curve, and the movement of the speculation curve is doubly significant if it is accompanied by an opposite movement of the money curve. If the speculation curve is rising while the money curve is falling, one should ordinarily expect an advance in business in ten to twelve months. If the speculation curve is declining while the money curve is rising, a business depression should be expected in approximately six to ten months. It should be appreciated that the general trends of the curves, and not their monthly fluctuations, are significant, and also, that the magnitudes of their fluctuations are much less important than the direction and persistence of the movements. Furthermore, on the basis of other data and analyses, careful checks must be made to see if the relations between the series are being disturbed, and the interpretations must be made accordingly. Such checks have been especially important during the past few years.

Since the Harvard Index Chart has been subject to much misunderstanding, it is believed well to emphasize that the interpretation of this chart is much more than a mechanical matter. Analyses must be made of the economic factors lying back of the curves on the chart. Forecasts have required, for instance, consideration of such factors as the Federal Reserve policy, unprecedented gold imports, the large volume of available credit coincident with high money rates, the effects on commodity prices of the return of European countries to the gold basis, and the significance

of declining commodity prices and hand-to-mouth buying during periods of prosperous business. The relationship between the curves is not fixed, but depends upon the effects of such factors.

The work of the forecasting services in forecasting conditions for business men in general will now be discussed.

Forecasting Services.—A number of institutions have been established and developed, which not only gather and furnish statistical information, but which also make intensive analyses of economic data and forecast the changes in business conditions. These services are very valuable to the small business concern, since it cannot afford to carry on extensive research work, and must do without such analyses unless it can secure them from outside sources. And they are of even greater value to the large organization which has its own statistical department to interpret and apply the services' reports to its specific line of business. Among the more important professional services which make business forecasts are:

The Brookmire Economic Service, published by The Brookmire Bulletins, Inc., New York.

Babson's Reports, published by Babson's Statistical Organization, Wellesley Hills, Massachusetts.

The Standard Trade and Securities Service, published by Standard Statistics Company, Inc., New York.

Moody's Investors Service, published by Moody's Investors Service, New York.

The Real Estate Analyst, published by Real Estate Analysts, Inc., St. Louis.

The United Business Service, published by The United Business Service, Boston.

The purpose of the present discussion is to describe the scope and methods of the preceding services so that the reader will appreciate the possibilities in their use, and so that he will be sufficiently familiar with them to remember them later on in connection with the practical business problems that he may have to solve. In judging the relative merits of these and other services, the reader should appreciate that forecasting work must not be rated on the basis of successful predictions alone, for there may be a considerable amount of guessing correctly by accident, the significance of an indicator may vary, and personnel may change. Consequently, in selecting an economic or statistical service, it is important to observe whether or not the methods that are used are fundamentally sound.

In addition to the preceding services, references will also be made to the forecasting work of bank letters, financial magazines, and local economic services.

The Brookmire Economic Service.—The Brookmire Economic Service, which was one of the pioneers in business forecasting, makes a

wide range of practical economic analyses for both the business man and the investor. It not only forecasts conditions for specific activities, but it gives detailed advice on the action that should be taken.

The scope of The Brookmire Economic Service is indicated by the following descriptive list of bulletins: *The Brookmire Forecaster*, published monthly, which discusses developments and the future probabilities in general business activity; *The Brookmire Purchaser*, published monthly, which analyzes commodity prices and purchasing problems in general, and which gives definite buying advice for basic commodities; *The Brookmire Income Map* (monthly), which forecasts consumer purchasing power by cities, states, and industries; *The Brookmire Counselor* (weekly) and *The Brookmire Analyst* (monthly), which analyze investment conditions, recommend specific purchases, and advise sales when necessary; *The Brookmire Investor* (bimonthly), which analyzes and recommends bonds and preferred stocks; and a list of special reports, which are furnished to interested clients.[1] The service also furnishes assistance in conference or by mail in solving the purchasing, financial, marketing, or other business problems of the client, as well as assistance in solving investment problems.

In The Brookmire Service's forecasting, emphasis is placed upon cross-section analysis of the situation at any particular time, as each situation is considered to be a special combination of factors. New factors appear, old ones disappear, and their relative importance changes. This emphasizes the difficulty of setting up a rigid index that will always be representative. The Brookmire Service does not base its forecasts upon a rigid index or predetermined set of factors, but proceeds on the theory that the major forces which control the situation at any particular time must be discovered, analyzed, and appraised in relation to the particular set of circumstances. The statistical series used are changed from time to time. Thus, if the Brookmire analysts were forecasting the trend of business in the later 'eighties and early 'nineties, they would watch the traffic and earnings of the western railroads; the miles of new railway track laid; the acreage brought under cultivation each year; the crop reports; the exports of wheat, livestock, livestock products, and cotton; the building of refrigerator cars (one of the new inventions of that time); the coinage of silver dollars; and, of course, the national bank statements. During the period since 1920, they have watched gold imports and exports, building contracts, motor-car output and registration, steel activity, exchange rates, money rates and commodity prices in foreign countries, Federal Reserve statements, and industrial-employment reports.

[1] The annual fee for the bulletin service is $120 annually.

The Brookmire Service carefully distinguishes between basic trends, seasonal variations, and business cycles in connection with its studies of the business outlook. Trends are analyzed in relation to growth in population, accumulation of capital, progressive improvements in technology, exploitation from time to time of hitherto unused resources, shifting of population from extractive to urban industries with the attendant increase in interchange of goods, and other important economic factors.

In order that the composite long-time growth may be understood, business activity is broken down into many parts, and the trends of individual industries and activities are studied separately. Rates of growth in new industries are not the same as and do not have the same significance as the trends in the older and more seasoned industries. The typical growth of an industry is characterized by small gains at first, then a period of very rapid expansion followed by a flattening and perhaps eventually a decline. The motor industry, for instance, is considered to have become a mature, rather slow-growing industry, although—owing to the character of its product—it is still subject to sharp cyclical fluctuations. Steel production and department store merchandising are other examples of mature activities. Businesses in the rapid growth stage at present are exemplified by aeroplane and radio manufacture. Industries in the declining stage are represented by lumber, which has been receding over twenty years, and the domestic newsprint industry, which has passed its zenith in the last few years. It is only after the different factors which are reasonably homogeneous in themselves have been analyzed, that The Brookmire Service arrives at its opinion of the composite basic trend.

In analyzing cycles, The Brookmire Service also breaks down a particular situation into its factors. The service recognizes that there are a great many intricately effective forces, and that not all of them are discoverable by any certain method. They vary in intensity, and they wane and die out. Predominant forces of one cycle may play a minor role in the next. This service considers the business cycle a permanent and unavoidable phenomenon for the simple reason "that human beings will never be able to make any set of economic machinery operate smoothly at all times." It points out that periods of prosperity will always generate weaknesses, abuses, extravagance, overspeculation, graft, and despoliation, and that periods of depression will see these evils abated and foundations laid for new advances. While The Brookmire Service clearly recognizes that direct active causes of a business expansion and subsequent decline baffle complete analysis, yet it finds that three influences stand out as dominating the major cycles of the past. These are:

1. Unusual accretions to the monetary gold stock.
2. Opening up of vast new resources.
3. Epochal improvements in industrial or financial technique.

Seasonal fluctuations are carefully considered from the point of view that in general business forecasting they are chiefly of interest in order that they may be "eliminated." It is important to be able to say whether a spring rise or a midwinter lull is merely seasonal, or more or less than seasonal.

In order to indicate the present and past composite business situations, The Brookmire Panorama Chart (see Exhibit 179) is presented regularly in the *Forecaster*. This index is based on a combination of copper production, pig-iron production, steel-ingot production, construction activity, cotton consumption, car loadings, department store sales, and other representative series. While this index helps in determining the position of business in the cycle, it is, however, in no sense a forecasting curve, and is simply an indicator of past and present conditions. The changes that are expected to take place in the future are determined by the cross-section analyses previously discussed.

Babson's Reports.—One of the oldest and most widely known of the business forecasting services is Babson's Reports. This service is easy to understand, and it has probably caused more business men to make cyclical analyses of economic conditions than any other single institution.

The scope of the Babson service is indicated by the following descriptive list of bulletins: the *Babsonchart* and *Barometer Letter* (weekly), which analyze and forecast the general business situation; the *Graphic Outlook* (monthly), which indicates changes in the major lines of business; the *Management Supplement* (monthly), which furnishes current information on management problems and labor conditions; the *Commodity Bulletin* (monthly), which includes forecasts for the major lines of commodities and makes specific purchasing recommendations; the *Sales Bulletin* (monthly), which furnishes information indicative of sales opportunities; the *Sales and Credit Map* (monthly), which indicates the sales and credit outlook by states and cities for various lines of activity; the *Financial Supplement* (biweekly), which analyzes the security situation and makes specific recommendations as to what to buy and sell; business supplements on problems of special interest; and regular monthly reports on Canadian business conditions. In addition, the organization issues various other special reports, including confidential releases and special opinions on investments.[1]

[1] The annual subscription rates are as follows: for the business service, \$120; for the supervised investment service, \$120; for the special commodity service, \$48; and for the complete combined service, \$170.

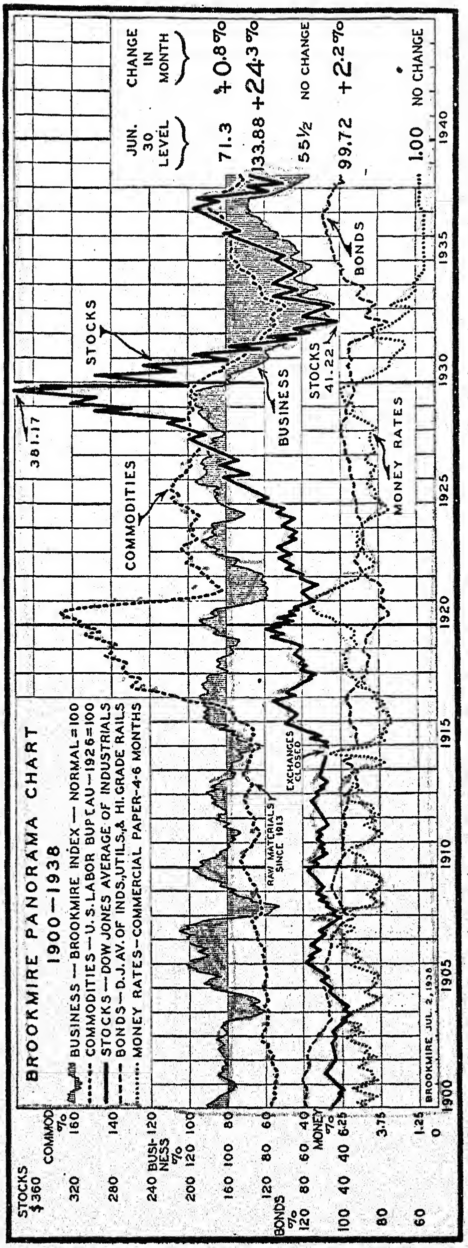

EXHIBIT 179.—The Brookmire Panorama Chart of Business Conditions. The curve is designed to show the present situation in relation to past conditions.

In its forecasting work, this organization places considerable reliance upon the assumption that the law of action and reaction applies to economics and human relations, *en masse*, as it applies to mechanics—that is, that to every action there is always an opposite and equal reaction in business activity as truly as in the realm of physics. This assumption is graphically shown by the *Babsonchart* (Exhibit 180), on which the areas above the trend line approximately equal the areas below. Under this area theory, a long, mild period of prosperity might be offset by a deep, short depression as well as by a long, mild depression, but the areas should be approximately equal.

The Babsonchart of United States Business Conditions was especially prepared to present a measure of the actual physical volume of business activity on a monthly basis over a sufficiently long period of time to show the growth factor and also to make possible important comparisons between current and earlier periods. It was prepared with the objective of indicating changes in the nation's real (as against money) income. The business index comprises 54 series in 7 groups as follows: (1) Manufactures (flour production; sugar refining; creamery butter; cattle, hogs, and sheep slaughtered; cotton, wool, and silk textiles; shoes; tires; tobacco; cement; automobiles; coke; gasoline; fuel oil; pig iron; steel; paper; news print consumption; and newspaper and magazine advertising). (2) Mineral production (coal, petroleum, natural gas, iron ore, copper, lead, and zinc). (3) Agricultural marketings (receipts of cotton, grain, livestock, poultry, and eggs; and carloadings of apples, oranges, and potatoes). (4) Building and construction. (5) Railway freight. (6) Electric power. (7) Foreign trade.

Since 1921, all series are in terms of physical volume with two exceptions—public works construction and foreign trade, for which no satisfactory quantity series are available. In the case of construction, a specially computed monthly cost index is used to convert the dollar data to an index of physical volume. In cases of industries that did not work 365 days a year, adjustments were made as far as possible for the working-day irregularities by compiling the series on the basis of average daily output.

The original monthly series were individually adjusted for seasonal variation by seasonal indexes determined on a 7-year moving basis. Ratios were computed of each month to the 12-month average in which it centers. Arranging these ratios according to months, the arithmetic mean of the middle three ratios for the respective months in each 7-year period was used as the seasonal index for that month in the middle of the period.

The basic formula used in computing the index is Fisher's "Ideal Formula." Arbitrary weightings of the individual series have been

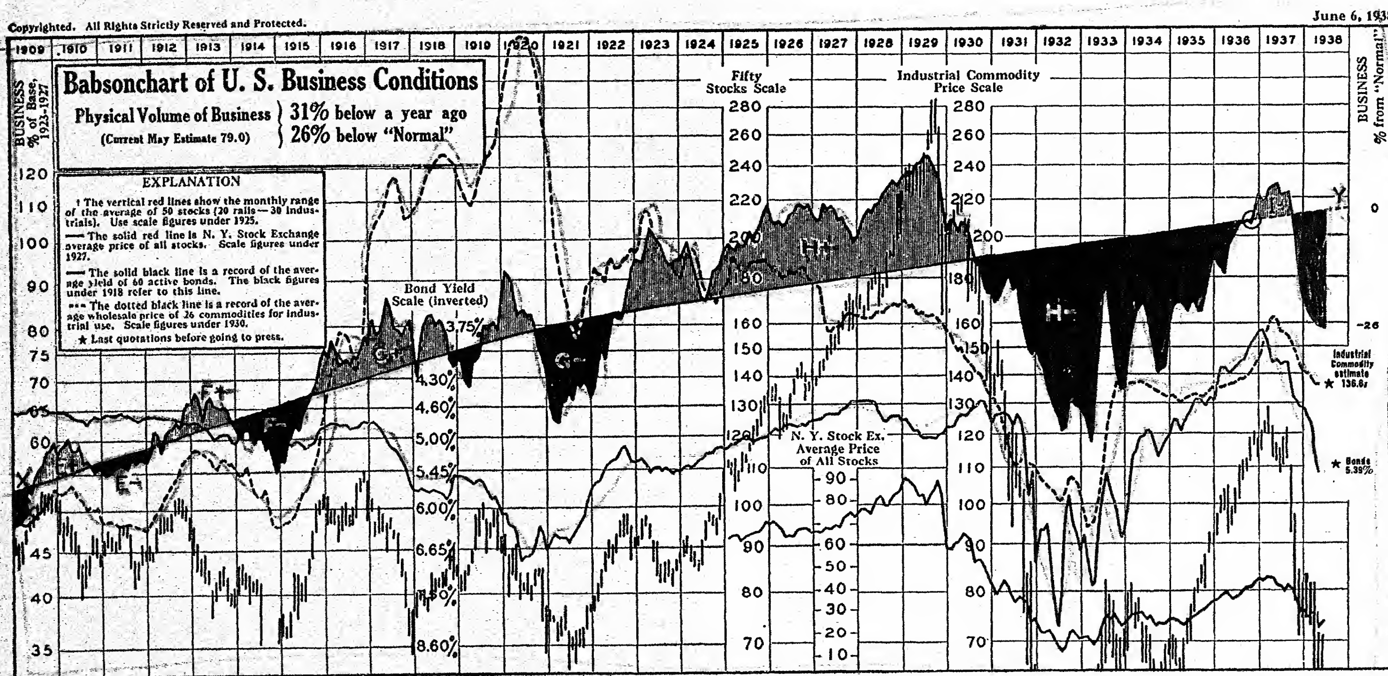

Exhibit 180.—The Babsonchart of Business Conditions. (The vertical bars and the solid curve are in red on the published charts.) The theory underlying this chart is that a certain area of prosperity above the trend will tend to be followed by an equal area of depression below the trend. (*Courtesy, Babson's Statistical Organization.*)

avoided wherever possible. The totals, however, have been weighted according to the value added by manufacture as indicated by Federal census figures. The monthly figures can be compared with the 1923–1927 base by means of the scale on the left of the chart. The scale on the right is for the current business index only and expresses it as a percentage from normal, the current position of the X-Y line being 0.

The long-time trend or X-Y line is not eliminated from the Babson index, but is left in to represent the country's net gain or growth. In the chart of Exhibit 180, the X-Y line, being on a logarithmic scale, indicates a decreasing rate in the long-time growth. (It is of interest to compare the position of this trend relative to the cycles, especially during the last major depression, with the position of the normals in Exhibits 179 and 181, and also in Exhibit 158 on page 331.)

In its forecasting, the Babson organization considers that the size of a prosperity area tends to forecast the size, though not the shape, of the depression area which is to follow. An area of depression, however, does not indicate the size of the following prosperity area. The area curve does not indicate when a prosperity period will end, but it does help in determining the degree of inflation.

While the area index is used by the Babson organization as its basic forecasting device, it uses many other indicators in conjunction with this index in making its forecasts. Special emphasis is placed upon studying stock prices, bond yields, and commodity prices in relation to the area index. These series are included on the *Babsonchart*. The high points of the stock market have been found to come in the early part of the overexpansion areas, and the low points have been found to come in the early part of the depression areas. Low money rates and high bond prices have usually (but not always) occurred at about the end of the depression areas, and high money rates and low bond prices at about the end of the overexpansion areas. The major swings of money rates sometimes modify the money and security relationships in the area movements. Consequently, when studying prices of stocks, their yield in relation to interest rates and bond yields are studied along with the shaded areas of the *Babsonchart*. Each industry is analyzed not only in relation to the areas indicating general business conditions, but also in relation to certain other specific factors applicable in that particular industry. Careful consideration is also given to commodity prices and merchandise sales in connection with the *Babsonchart* areas. The commodity index curve on the chart shows this relationship.

In addition to the foregoing historical analyses, the Babson organization makes detailed cross-section analyses of current conditions in the various lines of activity. Since neither the intensity nor duration of a period of prosperity can be foretold by the size of the preceding area of

depression, less emphasis is placed upon the area chart during such a prosperity area and more is placed upon the sequence in movements of different classes of activities and on cross-section analyses of current conditions in the different industries.

The Standard Trade and Securities Service.—Although the organization now known as the Standard Statistics Company has been in the business of collecting and analyzing statistics since 1906, it was not until 1921 that it undertook the task of business forecasting. It has been one of the most successful of the major services in forecasting business conditions.[1] The company has about 35 business and investment services in all. The one that is of greatest interest in connection with forecasting general business conditions is the Standard Trade and Securities Service.

The Standard Trade and Securities Service analyzes and forecasts business and financial conditions and security price trends. It includes the following weekly and monthly bulletins: (1) *The Business Prospect,* which is designed to keep the business man abreast of current business developments and prospects, and to provide a background analysis of the trends of volumes of production, prices, wages, taxes, legislation, and other business factors; (2) *Outlook For The Security Markets,* which provides a concise expression of the broad market policy currently advised, together with specific investment suggestions; (3) current and basic industrial surveys of about 40 major industries and the leading companies in each; (4) the monthly market ratings bulletin which provides the subscriber with an up-to-date opinion at all times of the relative market attractiveness of several thousand stocks and bonds; (5) the stock earnings guide which provides facilities for quick reference to facts regarding earnings, financial position, and dividend and price records of every important common or preferred stock; (6) statistical bulletins (*Basic Statistics*) which provide complete and up-to-date statistics on prices, production, employment, credit volume, new financing, bank debits, foreign trade, etc., with long-term comparisons; (7) special surveys which are issued as the occasion warrants and present the results of extensive research undertakings upon which the staff is constantly engaged, including economic analyses and forecasts covering a wide range of topics; and (8) indexes which are provided for the purpose of facilitating reference to the analyses and forecasts presented in the various bulletins. Subscribers to this service are privileged to call upon the company to a reasonable extent for advice upon their investments and upon their financial business problems.[2]

[1] See Garfield V. Cox, "An Appraisal of American Business Forecasts," revised edition, The University of Chicago Press, 1930.

[2] The subscription rate is $210 per year for the complete service.

The reports of the Standard Statistics Company indicate that two general principles predominate in making the forecasts published in the Standard Trade and Securities Service. "One is that the company does not commit itself to a single fixed system as the basis of its general forecast; the other is that it is giving progressively less attention to evidences of rhythm in general business activity and more to the analyses of each situation as a new one."[1] It is held that the conditions in all major business swings are different, and that these constantly changing conditions, rather than theoretical expectancies, should be given the greatest consideration.

Although the Standard forecasts are based upon many factors, no factor appears to have been given greater weight than the condition of the credit supply. When "A New Indicator of Stock Market and Business Activity" was announced in 1921, it was accompanied by a statement that "thirty years of charted experience demonstrates beyond reasonable doubt that a condition of plentiful credit invariably precedes every upward movement in stock-market and general business volume; and that credit stringency is the unfailing precursor of every great downward swing in security prices and business activity." This indicator consisted of three curves showing the major swings of the credit supply, stock prices, and business activity. The credit curve was considered to be an indicator of later movements in the other curves. The movements were reflected first in the curve for stock prices, and still later in the curve for general business. Fundamentally, this indicator resembled somewhat the Harvard Index Chart and the former United States Barometer Chart of the Brookmire service. This barometer of the Standard Service was discontinued in 1923 as it was found that it did not adequately forecast minor industrial cycles, but the service still holds that ordinarily a major depression should not be expected so long as there is an ample supply of bank credit readily available for commercial and industrial use.

Although the Standard Service has given the attention indicated above to historical comparison, it places its chief reliance on cross-section analyses of current situations. For instance, "in May 1923, when business activity showed signs of weakening, the service arrayed on one side the causal forces which it held responsible for the lull, and on the other, the fundamental elements of strength in the situation, and decided that the business recession would be mild and temporary. Among the factors most often given consideration in these studies are stocks of goods on hand, prospects for yields and prices of crops, foreign demand, employment, prospects for building construction, and business sentiment."[2]

[1] Hardy, Charles O., and Garfield V. Cox, "Forecasting Business Conditions," p. 108, The Macmillan Company, New York, 1927.

[2] *Ibid.*, p. 116.

These cross-section analyses of the outlook are usually made for each of some 42 separate lines of business activity, and the forecasts of *general business* are built up as a composite from these forecasts of individual industries. In emphasizing the importance of this procedure, the service has stated that: "Every period of prosperity differs in its fundamental characteristics. Prosperity is never a uniform, but always a zig-zagged uneven phenomenon, affecting various industries in varying degrees at different times. The task of those who would properly appraise business conditions at any time is to learn what individual industries are the princes and what ones are the paupers—and to understand why."

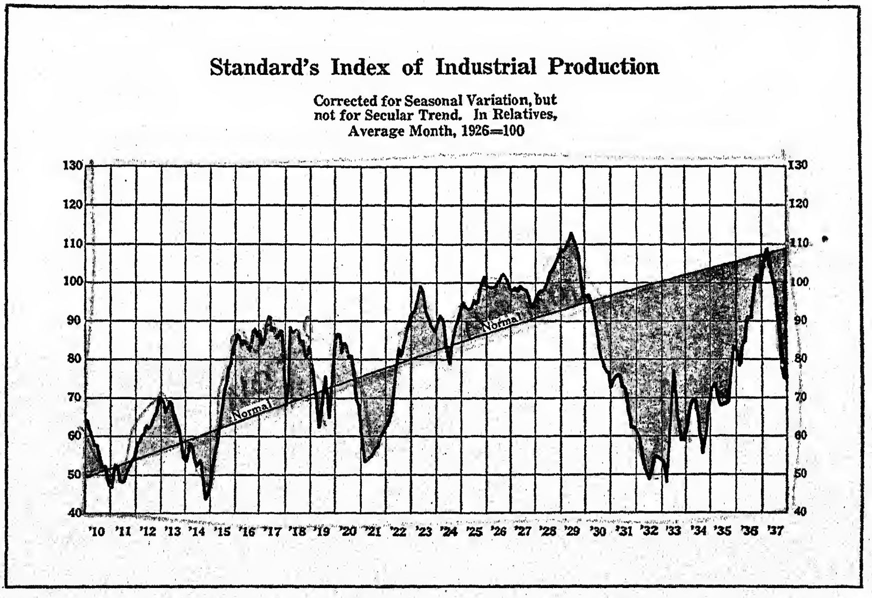

EXHIBIT 181.—One of the many charts used by the Standard Statistics Company in its forecasting work.

One of the graphic devices used by the Standard Service is its index of industrial production (Exhibit 181), but this is not in itself a mechanical forecaster. The course of this curve is predicted on the basis of analyses of the current factors which influence each of its components. The accuracy of the service's forecasts depends upon how well the component forces are identified, measured, weighted, and interpreted.

Finally, it should be noted that the Standard Service insists that there is no scientific basis for forecasting business conditions very far into the future. It seldom forecasts farther than several months, and there are times when it frankly declines to make general forecasts.

Moody's Investors Service.—Though primarily an organization devoted to the analysis of securities, Moody's Investors Service is one

of the leaders in analyzing business conditions. This service does not rely on any fixed mathematical forecasting device or formula. Although there may be some uncertainty, due to the freedom of determining at a given moment what factors in the situation are the most powerful and the most likely to bring about a change, the Moody organization feels, after many years of trying to perfect a set of forecasting indexes, that the selection of the components of any barometer involves just as much uncertainty and chance of error.

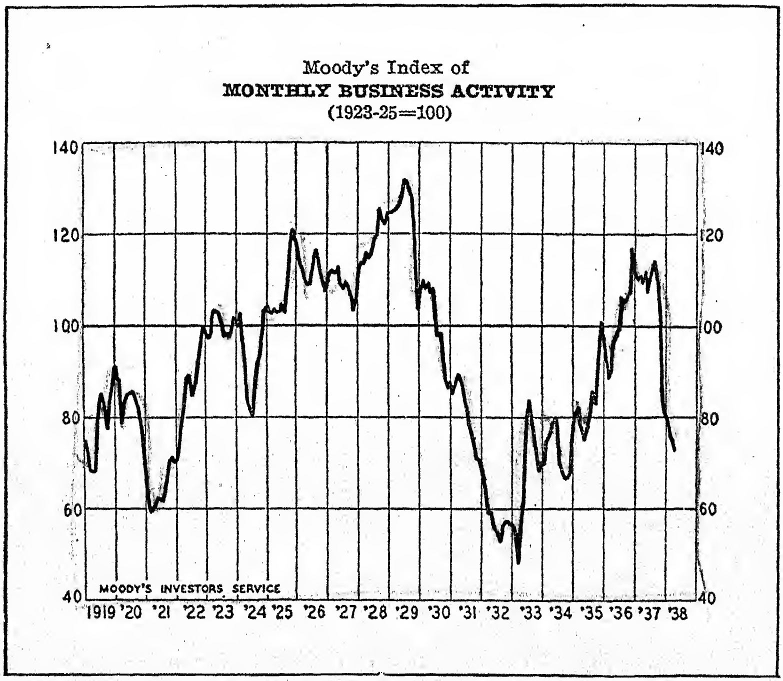

EXHIBIT 182.—The above index is published in tabular and graphic form in the "Business Review and Outlook" letter of Moody's Investors Service.

The main body of information used by Moody's Investors Service in appraising the business outlook comes from its wide contacts with industry, trade, finance, and other lines of business. The studies made of the positions of individual industries and companies, with their activities, plans, and possibilities, bring the Moody analysts into contact with a great many elements, the composite of which is largely responsible for making what is called "general business conditions." The cross-section analysis of these varying elements at a given time constitutes the principal basis of Moody's predictions of business conditions.

When making the various cross-section studies, "the theory which seems to be most influential in shaping the character of Moody's fore-

casting analyses is that the course taken by business activity is determined by business men's anticipations of profits. The basic question to which Moody's seeks the answer is not how the present rate of activity compares with some computed normal, nor whether a given statistical barometer has turned downward or upward, but whether in the eyes of business men the prospects for profits are growing darker or brighter. This practice of taking the enterpriser's viewpoint and of regarding profit prospects as an important clue to business fluctuations accords with the views of a number of the most eminent students of business cycles."[1]

One of the published statistical devices used by Moody's Service in analyzing the business outlook is Moody's Index of Monthly Business Activity (see Exhibit 182). This index is a weighted average of bank debits outside New York City, automobile production, building contracts awarded, railroad carloadings, department store sales, electric power consumed in industry, and steel production. Moody's Service also compiles a weekly index of business activity which is an unweighted composite of seasonally adjusted carloadings, electric power output, and steel operations. The principal value of these indexes is that they help in detecting inflation and deflation in general business whenever it rises or falls below certain levels. They are not used, however, as mechanical forecasting devices, but merely as indicators of one of the many factors that must be considered at a given time in analyzing the outlook.

The Real Estate Analyst.—A very valuable aid in analyzing real estate conditions is the economic service known as the *Real Estate Analyst,* published by Real Estate Analysts, Inc., at St. Louis. This service is devoted exclusively to the measurement, analysis and forecasting of real estate and construction activity. It includes a number of monthly and special bulletins on the latest developments in real estate, and on current construction, agriculture, taxation, and other conditions in relation to the real estate situation. All analyses and forecasts are presented clearly and concisely in textual, tabular, and graphic form in such a manner that they can be easily understood by the business man and used for such purposes as closing sales with prospects who have some appreciation of economic analyses. The service includes an index of real estate activity which extends back to 1795. A curve of this index since 1800 which shows eight great cycles of real estate activity averaging 18 years in length is presented in Exhibit 207, page 533, in the chapter on Real Estate Analysis.

The United Business Service.—A service which is compact in form and very convenient in practical use is The United Business Service. "Forecasts and opinions are based on a weighted average of 'United

[1] See Garfield V. Cox, "An Appraisal of American Business Forecasts," revised edition, p. 66, The University of Chicago Press, 1930.

Opinion' as expressed by such authorities as Alexander Hamilton, Babson, Brookmire, Ayres, Gartley, Gibson, Investors Research, Kiplinger, Poors, etc." The Service uses an index chart of business conditions which is called "The United Business Thermometer." This index is similar to the curve in Exhibit 181 when adjusted for trend but it is independently computed to represent average conditions.

Minor Importance to Economic Services of Rigid Devices and Systems.—In the preceding discussion of economic and statistical services, it was pointed out that the forecasts of these services depend principally upon the application of good judgment in analyzing the probable effects of many and various factors. Because of the fact that considerable space was given to describing a number of somewhat rigid devices, it is believed well to emphasize the point that, valuable as these devices may be when properly used, the approaches which might be characterized as rigid or mechanical play only a minor part, as a rule, in the forecasting work. In other words, these services do not depend to any great extent upon any mechanical system or, in fact, upon any definite system at all.

All of the services mentioned attach considerable importance to contacts with bankers, manufacturers, merchants, and other business men. Babson's Statistical Organization has a well-developed system of contacts with various financial, industrial, commercial, and other groups. The Standard Statistics Company maintains a field force of considerable size for the purpose of keeping in touch with developments in various sections of the country. Moody's Investors Service frequently sends men out from the home office to accomplish the same general purpose. The Brookmire Economic Service also keeps in close touch with business developments through its contacts with the different groups of business men. This emphasis on contacts indicates the importance of other than statistical procedures or even numerical information in forecasting business conditions.

Bank Letters' Forecasts.—In connection with this discussion of forecasting services, mention should be made of the monthly letters published by the leading banks. Some of these letters evidence very good forecasting work. Two of the outstanding letters of the country are *Economic Conditions* published by the National City Bank of New York and the *Business Bulletin* published by the Cleveland Trust Company. Both of these letters are furnished gratis to those who are interested. In neither of these reports is as much emphasis placed upon forecasting as in the services previously mentioned, but both letters present and appraise the favorable and unfavorable factors in each current situation, and, after analyzing the relative strength of the opposing elements, indicate the outlook in a straightforward, practical manner, which is readily understood by intelligent business executives. One

outstanding advantage that the larger banks in the country have over private forecasting agencies is that, because of their direct confidential relationships with business firms, they undoubtedly have more real inside information on the actual conditions and the probable activities of the important industries than is possible on the part of any other type of institution.

Financial Magazines.—The better financial magazines and trade papers also furnish reliable analyses of the business outlook. Such magazines as the *Commercial and Financial Chronicle*, *Dun's Review*, *Barron's*, and *The Annalist* are among the publications that should not be overlooked. *The Annalist* weekly article on "The Business Outlook" is a compact, impartial, and clear-minded analysis of the economic situation. *The Annalist Index of Business Activity* and the *New York Times Weekly Index*, both of which are features of *The Annalist*, are widely accepted and used by business men. An especially valuable feature of *The Annalist* monthly index is that the different series composing the index are also presented separately with their relative weights. This enables the user of the major index to make better applications to his own business. When, in the early part of 1931, for instance, the index was dropping to new low points for the current depression, the user could see the heavy influence of the drastic decline of automobile production by referring to the automobile component.

Local Business Forecasting.—The national indexes of general business just described represent an *average* of conditions that are widely scattered, both geographically and industrially. It is an obvious fact, however, as pointed out in connection with Exhibit 173, that conditions in specific localities may vary widely from the average. And if a concern having a national market wishes to take advantage of economic conditions, it must know these individual situations by specific localities. Several of the national economic services have made a special effort to furnish this information in a limited way. If more complete information is wanted, intensive first-hand studies must be made in the locality by qualified investigators, and ample opportunity must be given to make first-hand cross-section analyses of the many and varying special factors that so often greatly influence the interpretation of the more readily available indexes. To meet this requirement, there has been a considerable development of local statistical analysis work which is published in the form of local economic service bulletins, bank letters, and university bulletins, or furnished as special reports by consulting firms. Such information is also published by local newspapers and in the promotional literature of private concerns and civic organizations.

One of the most valuable economic services for use in analyzing local conditions is *The Real Estate Analyst* mentioned on page 391. This

service not only gives specific information on local real estate and construction conditions in most of the large cities and important regional areas of the United States, but analyzes many fundamental characteristics which the business man can apply in studying his own relationships to local real estate and construction activity or to conditions which are dependent upon these types of activity.

Necessity for Analyzing Industries Separately.—Not only should conditions sometimes be analyzed by specific localities, but they should also be analyzed by separate industries for the reason that certain industries do not follow general business activity in a consistent manner. It has been established that price-movement curves for agricultural products, industries manufacturing food products, and the stone, clay, and glass-manufacturing group do not correspond to the curves of general prices and general business conditions. On the other hand, it has been found that price indexes for mining, and for manufactures of iron and steel, leather, chemicals, and textiles show cyclical swings that correspond to the cyclical swings of business. Businesses in the industries first named will therefore need to rely upon crop and trade forecasts and reports upon the products which they use. Companies in the last-named industries can use studies of general economic conditions to a much greater extent in estimating the external business forces which will affect sales.[1] And businesses which follow the long swings of real estate and building (see Exhibit 207, page 533) require a still different type of analysis.

Applying Economic Service Forecasts.—It is obvious from the foregoing brief discussion that the economic and statistical services mentioned provide the business executive with statistical analyses of the outlook at a cost which is relatively low, as compared with what it would cost him to make these studies in his own organization. As previously indicated, if the concern is small, it may be impractical to use other statistical analyses of economic conditions. If it is large enough to have its own statistical department, it may use these services as aids in its own forecasting work.

In using economic services in business, it is of the utmost importance that the executive appreciate their limitations. At the outset, he should appreciate that it is the purpose of economic and statistical services to *help* the business man make better analyses of future conditions than he could without them. Even when they give definite advice as to what to do, the business man must qualify the suggested acts in relation to his own business. In applying the forecasts he should realize that, while they will not always be right, the fact that they are right most of

[1] Based on studies of Warren M. Persons.

the time will enable him to improve his own estimates of the future. For the reason that these services do not always agree regarding the outlook, and regarding the factors which cause a certain situation, it is often desirable to follow more than one service and many organizations subscribe to several.

In making the best practical use of economic services, the executive should appreciate the fact that the best business forecasting is subject to many limitations. The date that a depression will begin ordinarily cannot be forecast closer than from six to nine months, nor can the depth and length of a depression be closely predicted. Likewise, the time of the beginning of a period of prosperity cannot be foretold with certainty, nor can its height and duration be determined in advance. In spite of these limitations, however, the executive finds the economic service reports of great value in planning his operations and in determining his policies. Even if the precise moment of decline or improvement cannot be predicted, it is highly important to know when the conditions requisite for a change in the situation have been fulfilled.

As pointed out previously, none of the reliable economic services depends solely upon mechanical forecasting devices, and the tendency is to place more and more emphasis upon cross-section analyses of factors responsible for future business changes. Conclusions from historical sequences cannot be used alone. Even if certain sequences have been observed to occur with rigid regularity over a considerable period of time, one cannot be sure that conditions will not change and that the significance of the indicator will not also change. No one feels this lack of confidence in rigid mechanical forecasters more than the practical business executive. In using the forecasts, he must be able to visualize causes and effects as clearly as possible, and check them with his own judgment, if he is to make intelligent applications.

Ordinarily the business man will not and should not rely upon a forecast, unless he can understand the reasons for the probable occurrences. For this reason, the economic services' causal analyses are carefully described and presented along with their forecasts so that the executive can make the proper qualifications in applying the forecasts in his own business. The business man must continuously forecast conditions in his own firm, whether or not he makes a *systematic effort* to do so, and the clearer and easier the economic service forecasts are to understand, the more readily he can apply them in improving his own plans for the future.

More specific uses of economic services are illustrated by the applications made by the Walworth Manufacturing Company.

An Illustration of the Practical Use of an Economic Service.—This illustration has been selected not only because it shows how a business

concern made a definite use of a forecasting service, but also because it shows how the experience gained was used in developing improved methods of forecasting the concern's own particular conditions.

To compare statistically one company's business with general business activity as a whole, obviously each will have to be measured or expressed in comparable terms. The usual statistical processes of determining and eliminating trend and seasonal variation may have to be applied. If the general business curves published by economic or statistical services are plotted in terms of standard deviations from normal, the sales of

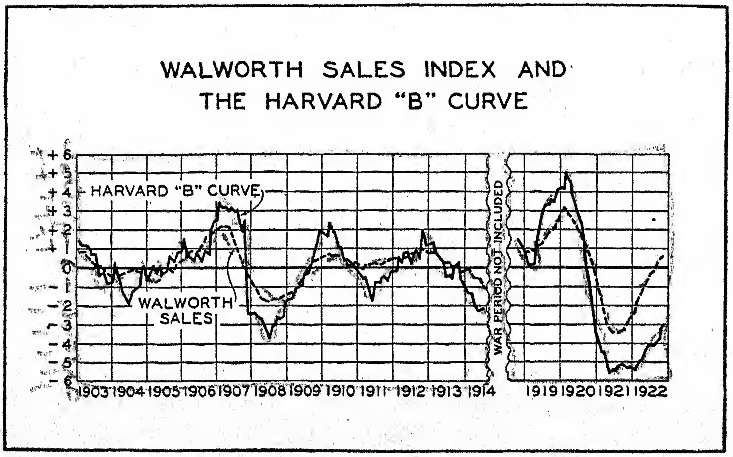

EXHIBIT 183.—In making use of the Harvard Economic Service, the Walworth Manufacturing Company found a close relation between the Harvard Business Curve and the Company's sales.

the individual business concern over a similar period of years ordinarily should be reduced to these terms also to obtain a strictly comparable curve, although rough comparisons can usually be made without making such computations.

When the statistical section of the Walworth Manufacturing Company was inaugurated during the depression of 1921, it was thought at first that the principal problem was simply that of finding out where the Walworth business stood with relation to general business.[1] Accordingly, the Walworth sales were reduced to the proper terms and a comparison was made with the Harvard "Business" curve. The close relation for the

[1] The Walworth Manufacturing Company, with headquarters at Boston, has several factories for producing valves, fittings, and accessories for steam, water, gas, oil, and air piping systems. This output, together with some purchased items, is marketed through jobbers and branches. The information on the use of the Harvard Economic Service by the Walworth Manufacturing Company in 1921 was published in an article by the company's President, Howard Coonley, in the *Harvard Business Review*, Vol. I, pp. 387–391.

period from 1903 to 1914 is shown in Exhibit 183. This study indicated that the Walworth Manufacturing Company not only had a business cycle of its own, but that its cycle had a definite measurable relation to that of general business. The company was, therefore, in a position to use, in a very tangible way, the forecasts which were made by the economic services. That is, the study indicated that the forecasts of the economic services might be taken as general forecasts of the Walworth Company's business.

In the summer of 1921 the Walworth Company had its first opportunity to make use of its discovery. At that time, the Harvard *B* curve gave definite assurance that general business was scheduled for an advance. Even when the company was not previously prepared, its record showed that it was following closely the major movements in general business (see years 1919 to 1921 in Exhibit 183). Accordingly, the company set about to prepare to take advantage of the cyclical changes in business conditions.

Since making the relatively simple analyses in 1921, the Walworth Company has developed a detailed specialized system of its own, which is more satisfactory for its own particular line of business. The point has been made, however, that the experience gained in making definite applications of the economic service forecasts was of considerable value to the company initially in developing its own forecasting system. The relation between the Harvard *B* curve and Walworth sales is still reasonably close, but the company now uses the economic services only as general aids in making its forecasts.

In its forecasting work, the Walworth Company uses a number of carefully developed statistical devices. Obviously, a detailed discussion of the methods used in developing such devices would require much more space than can be devoted to it in this text, and the reader must be referred to the more specialized articles and books for a complete description. One of the most important of these statistical devices, however, the Valve and Fittings Index, will be discussed briefly for the purpose of indicating how these devices are developed and applied.[1]

The Walworth Manufacturing Company's Valve and Fittings Index.—When the Walworth Manufacturing Company decided to take advantage

[1] This discussion of the Walworth Valve and Fittings Index is based upon information furnished by Joseph H. Barber, Assistant to the President and Chief Statistician, Walworth Manufacturing Company. Much use has been made of his published material, especially "Scientific Approach to Forecasting in the Valve and Fittings Industry," a paper presented at the Institute of Management held in Chicago, November 12, 1928, and articles and books on the subject published through The Ronald Press Company, New York. For detailed explanation of the methods used in the company's forecasting work, the reader is referred to these publications.

of economic conditions in 1921, the first step was to make a detailed investigation of its own particular products and markets. When the initial survey for sizing up conditions and needs had been completed, the company came to appreciate that its earlier confusion had resulted from seeing too many elements of its business at once; it was looking at orders, shipments, production, quantities, prices, collections, works, and branches all in the whole and as a mass. Consequently, the next step was to segregate elements and define specific problems for solution, looking at the business as a basic industry, and realizing that the demand for its particular products must be subjected to a rigid analysis of its own.

One of the first discoveries that the statistical department made was that the company's own business was among the first to show the effects of economic changes. For this reason, there could be but little chance of finding any barometer series which would move ahead of the company's own cycle. Even the movements of the stock market did not regularly precede those of the company's own line of business. Consequently, in order to predict future changes, other methods had to be devised.

In looking for barometer suggestions, it was observed that the manufacturer thinks of business in terms of shipments against orders. Although financially he must figure profits against actual shipments, to him business is good when his distributors are ordering in large quantities, and business is bad when the volumes of orders are small. The company became convinced that the front-line merchants, who carry the manufacturers' stocks against the future requirements of the ultimate consumer, are in a position to become expert judges as to future prospects, because, in relation to the consumer market, they weigh conditions, they size up probabilities, they inspect the foundations of sentiment to see whether it is currently optimistic or pessimistic, they justify or condemn the going price level, and in their ordering on the manufacturer, they register their own confidence or lack of it promptly and always in a very tangible way. Of course it is always well to check dealer opinions with independent analyses of fundamental conditions in the principal consumer markets. In accordance with this theory, it was finally decided that order volumes relating to the valve and fittings industry appeared to be the timeliest and most significant index for guiding the Walworth executives in planning for a stabilized manufacturing control.

Accordingly, the Valve and Fittings Index was developed, which finally became an average of 16 separate indicators of national demand for valves and fittings. Eleven of these indicators were selected because they would actually measure the relatively stable volume of ordering or contracting in the building, the equipment, and other national industries in which there is demand for valves and fittings. And, given equal importance in the average, there were five "sensitive" indicators of the

actual quantity of orders placed upon nationally representative producers for sanitary ware, brass faucets, and other products usually associated with the valve and fittings line. As finally developed, the Valve and Fittings Index has these distinctive features: It is a broad sample of conditions; it measures jobber psychology; it is an index of the trend of demand; it measures the quantity of going business; it turns the corner ahead of many other barometers; and it swings higher and lower than other barometers.

Forecasting the Movements of the Valve and Fittings Index.—In developing its own method of interpretation and forecasting, the company considered separately four different methods of forecasting, which the chief statistician designates as follows: (1) by historical analogy; (2) by time lags and cycle sequences; (3) by "reading the averages"; and (4) by analyzing "statistical positions." This classification will be adhered to in the discussion of the index which follows.[1]

The company had good records which showed its business cycles before the war, but it did not have sufficient confidence in the first method (historical analogy) to make it the basis of its forecasts because of the risk that deductions from pre-war precedents might not always be applicable to post-war forecasts.[2] As previously indicated, the second method (time sequences) could not be used because no series could be found which would precede the movement in the company's own curves. According to the company's chief statistician, "all our experience had argued against reposing confidence in anything but demonstrated natural relationships between cause and effect. Analogies and correlations, that were dramatic enough in themselves, would quickly disclose their weaknesses when forced to explain the reason why." This left the company the last two methods with which to forecast changes in its orders index. And it found that it could use both the method of "reading the averages" and the method of "analyzing statistical positions" as aids in predicting the course of the Valve and Fittings Index.

The Statistical Department studied how forecasting by "reading the averages" was utilized (in connection with stock-market activities) in developing predictions by reading meanings into the market price actions or reactions, the changes or lack of changes, and the fluctuations or

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[1] This classification is consistent with the one on p. 372 in which "historical analogy" and "time sequences" are included under "historical analysis," and in which "reading the averages" and "statistical positions" are included under "cross-section analysis."

[2] "Historical analogy" should not be confused with "historical analysis." Briefly stated, the procedure in historical analogy is to find a past situation or situations in which the fundamental factors involved appear to be like those in the present situation. Then the forecast is based on what followed in these similar past situations.

stabilities as revealed by the index curves themselves. For many years, expert traders on the stock exchange had relied upon this method for interpreting the fluctuations of the average prices of stocks. Since they had found nothing that would automatically forecast the general market trend, which they needed to know as a background for judging prospects for their particular stocks, they developed a reasonable theory of interpretation from the action of the curves themselves, and their terms are now familiar to many of us. Such phrases as "bull movements," "resistance point," "intermediate phase," "support level," "forming a line," and so forth, are terms now having definite significance.

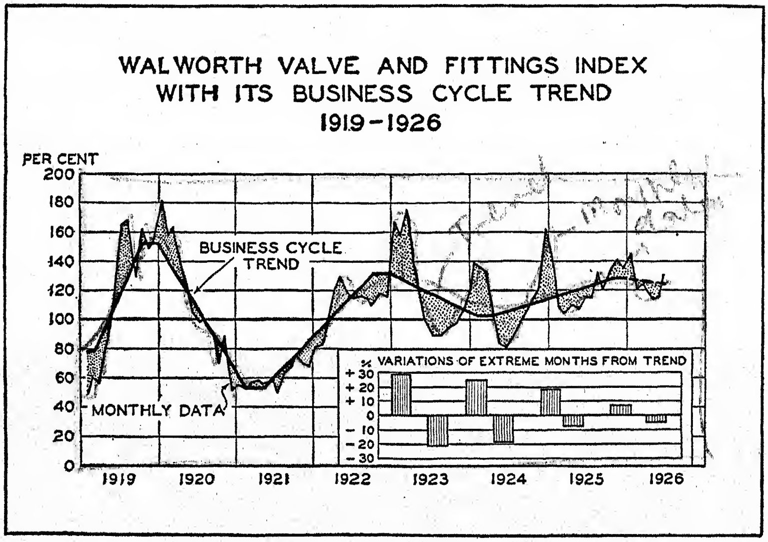

Exhibit 184.—The monthly fluctuations of the Walworth Valve and Fittings Index compared with its cyclical tendencies. (*Courtesy, Walworth Manufacturing Company.*)

To illustrate how the Walworth Manufacturing Company uses the method of reading the averages, let us consider an interpretation made about the middle of 1926, which is shown graphically in Exhibit 184. In this chart, the index curve is smoothed to indicate the business cycles clearly and simply, the curve being located according to the year's moving average of the monthly plottings of the index. The following paragraphs, abstracted from the interpretation published at that time, are illustrative of the method of "reading the averages," as applied in interpreting the order demand index.

"Looking at the heavy smoothed curve (Exhibit 184) we see that, since 1921, there has been a continuing and persistent wave-like up-and-down movement, and that recently we have been riding along the crest of such a wave. In view of what we know of conditions influencing the

cycles since the war, it seems reasonable to assume that we have been riding on the crest of the wave and that the immediate tendency is downward.

"The percentage bars in the inset diagram of Exhibit 184 show how average ordering rates in the high and low four-month periods varied from the conditions represented by the smoothed curve which would have persisted, if there had been no 'fits and starts' in ordering. Now, couple this clear indication with a reasonable projection of the cycle tendency itself and if none of the other checks contradict this indication, you have a reliable basis for fixing budgeting policies according to the business cycle."

The foregoing indicates how an interpretation is made by the method of "reading the averages." Although too much importance should not be attached to this method, it does serve as a valuable check. Much greater emphasis is placed upon the method of analyzing "statistical positions," which will now be discussed.

At the outset, the Walworth Statistical Department studied how forecasting by analyses of "statistical positions" had been applied in such problems as determining when to buy stocks or when to purchase commodities. The method consisted primarily of fact gathering and cross-section analysis of all matters bearing directly upon the particular stock or commodity. Accordingly, a plan was developed whereby each of the elements affecting the Valve and Fittings Index could be analyzed separately and then the several elemental forecasts could be combined to develop average forecasts.

To illustrate the method, let us consider how the statistical position of one of the components was analyzed in 1928. The example selected is "Producing Oil Wells Completed." Much of the oil industry's demand for valves and fittings is to satisfy the needs of new oil-well construction. This industry is sensitive to economic changes, and careful study shows a definite correspondence between the results that occur and the economic factors that should cause such results. In this industry, just as in any other, it is ultimately the adjustment or maladjustment of supply and demand that causes price stability or price variations. Since gasoline, the principal product of crude oil, is the primary foundation on which the whole industry rests, a device measuring the balance of supply and demand for gasoline should prove valuable in revealing the industry's prospects. If this current supply of gasoline exceeds that being consumed in the market, then, at least theoretically, the price of gasoline should fall. As a matter of fact, the gasoline price situation is so closely tied up with crude-oil price conditions that almost coincidentally crude oil prices will also decline.

The upper curve in Exhibit 185 shows the Walworth index of gasoline supply *versus* demand. Note that the curve is plotted on an inverted

scale so that decreasing supply (rising curve) will appear to represent improving conditions. The lower curve shows oil-well completions. A comparison of the two curves shows clearly that the gasoline situation has been definitely improving for a considerable time prior to a corresponding increase in the number of producing oil wells completed. This is a reasonable sequence. As long as gasoline demand lags so that crude oil prices are depressed to unprofitable levels, there is a natural pressure against adding to the supply of crude oil by bringing in surplus produc-

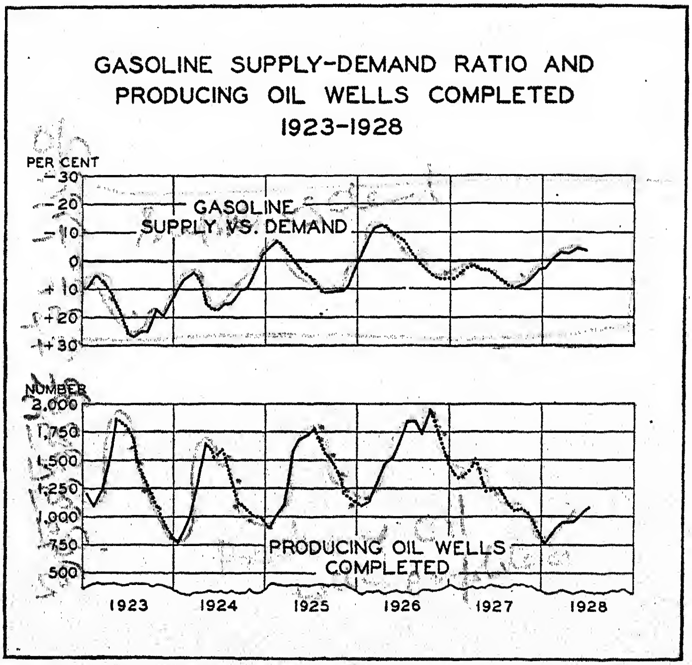

EXHIBIT 185.—Chart used in analyzing the "statistical position" of the petroleum industry. The movements of the upper curve tend to forecast changes in the lower curve. (*Courtesy, Walworth Manufacturing Company.*)

tion. But when gasoline demand increases and oil prices rise again, there is sufficient inducement for producers to "bring in" the new wells. A new cycle of recovery then appears to be under way. And, to the extent that this factor is important in the average of demand as measured by the Valve and Fittings Index, to that extent would this specific indication forecast a new cycle of recovery for that average.

In the same manner, the other elements in the index are analyzed, and finally the combined results of the various analyses indicate the direction that the Valve and Fittings Index is expected to take.

The General Motors Divisional Index.—A brief reference to the General Motors Divisional Index will be helpful in further illustrating

the practical application of business forecasting in a specific industry. It will also emphasize again the importance of making cross-section analyses of current conditions.

The General Motors Corporation must take into account the competitive situation and business conditions in general as they affect its automobile and other lines of business. The relation of developments in competing lines is a part of the competitive situation. Among the important indicators of business conditions, which are considered, are: agricultural conditions as reflected by crop reports and prices of farm products; the livestock situation; the state of general industrial employment; the trend of commodity prices; the volume of industrial production; credit conditions as indicated by bank statistics and interest rates; and the probable effect of fluctuations in the securities markets on business sentiment.

The capacity of the company's dealer organization must be considered in the light of any impediments which would be likely to restrict or hamper its sales or effectiveness. Examples of such impediments are the accumulation of used car stocks, the introduction of a companion car to be sold through the same organization, inability to expand capital and personnel to keep pace with potential demand, inadequacy of dealer's capital, and an uneconomical situation of a dealer (such as a place of business which is too extravagant). Furthermore, the company must study the probable influence upon sales appeal of special mechanical features of the product, as well as the factors of style, general performance, and serviceability.

As a result of the best judgment that can be brought to bear on the problem at a given time, the company arrives at what it calls its "Divisional Index." This Divisional Index means its best guess of what the retail sales for a particular line of cars will be during the ensuing sales year. This does not mean that it sets up an arbitrary quantity in accordance with which the production is determined, regardless of what may afterwards happen. The Divisional Index is under continual scrutiny, and is adjusted immediately when the trend of events demonstrates the desirability of revising the expectations.

Subject to the foregoing adjustments, the Divisional Index at any given time is accepted as dictating the basis upon which production should be scheduled. In other words, production is always scheduled with regard to the expected requirements for the complete sales year. It does not follow, however, that production in any given month should be exactly one-twelfth of the year's production, for there is a seasonal variation in consumer demand which must be taken into account, and it is usually desirable to shade production somewhat during the dull season so as to modify the seasonal accumulation of stock. It is perhaps best to

compromise between the desirability of a level rate of production in the interest of manufacturing economies and a fluctuating rate of production in the interest of minimum stocks in the hands of dealers.

On the twenty-fifth of each month a complete forecast is submitted by each of the company's divisions for the current month and three succeeding months. This includes a forecast of retail deliveries by dealers. The company also has complete information as to actual retail deliveries, by months, in the past. Thus, at all times, it has available the actual records of the past as well as a forecast of retail deliveries for three or four months ahead. The accumulation of these records of retail deliveries by dealers, covering a number of years, and the analysis of state registration data furnish a good means of appraising the ordinary characteristics of the company's business.

The company has found from experience that as it proceeds with the continual analysis of deliveries and forecasts, and measures its performance and prospects in the light of seasonal expectancy, it can detect changes in trends and recognize disappointments or improvements well in advance of what would be possible through any other means.

For example, let us suppose that it is the latter part of November and that the Divisional Index was made as of August 1. Data on actual retail deliveries for August, September, and October are at hand. Let us assume that this record was such as to support the expectancy of the Divisional Index. When the forecast for November, December, and January was prepared a month previous, it appeared to be reasonably in line with past actual experience, seasonal conditions and any other known influences being considered. Since reports of retail deliveries are received from the dealers three times a month, the company knows what these were during the first 20 days of November. If they are in line with the forecast for November there is no need, from statistical observations, to modify the Divisional Index. If the actual deliveries during that period are out of line with the forecast for November, this is the first flash or caution signal indicating that the Divisional Index may require modification. And so the General Motors Corporation proceeds to analyze past performance and forecasts of retail deliveries month after month. During each month, 10-day delivery figures, reported by the dealers, are compared with the latest forecast for the month. Ordinarily, the Divisional Index is not actually modified because of a statistical indication of a new development covering a brief period of time. The indication is taken as a caution signal, and the situation is watched as it develops to see whether there seems to be a sustained change.[1]

[1] This discussion of the General Motors Divisional Index is abstracted from "Forecasting and Planning as a Factor in Stabilizing Industry," by Donaldson Brown,

Specific Planning of Internal Operations.—Finally, it is desired to emphasize the point that business forecasts must be used subject to the limitations that they are often indefinite and sometimes inaccurate. The nature of the forecasting problem is such that these limitations will undoubtedly exist for a long time, if not forever. If the executive properly understands these qualifications, however, the applications that he makes can be definite and his estimates of the future will be more accurate than they would be without them. To apply definitely these and other principles of planning to the internal operations of an organization is the object of the next chapter.

Questions and Problems

1. Explain what is meant by forecasting.

2. Can technical statistical methods of analyzing and forecasting business conditions entirely replace judgment and experience? Explain.

3. Explain the general effect of mathematical training and the knowledge of statistical methods upon the business analyst.

4. Explain four common limitations of statistical methods.

5. Discuss the danger of allowing refined statistical methods involving spurious accuracy in computation to be used in business problems.

6. Give a list of questions that are useful in checking the adequacy of the statistical methods utilized in any given business problem.

7. Discuss the "statistical third degree."

8. Discuss the importance of forecasting in business problems.

9. Can valuable forecasts be made by simple methods? Explain.

10. Do business conditions vary in different large areas? Are variations evident in local areas? For what area should business conditions be studied?

11. Describe a method of estimating office space requirements during the next five years in your particular city.

12. Explain (1) historical analysis, and (2) cross-section analysis in relation to forecasting.

13. What is a "business barometer"?

14. How can mathematical projections of trends be used in forecasting?

15. To what extent can the stock market be relied upon to indicate future general business conditions?

16. What curves are shown on the Harvard Index Chart of General Economic Conditions and what data are included in each at the present time?

17. What is the relation of the Harvard curves to each other and how can they be used in forecasting?

18. Name several economic services that forecast general business conditions in the United States.

19. Describe the scope of The Brookmire Economic Service.

Vice President in Charge of Finance, General Motors Corporation, published in *Sales Management*, January 26, 1929.

20. What statistical methods are used in *The Brookmire Forecaster?* Name three influences which that company believes have dominated past cycles of business.

21. What services are included in *Babson's Reports?* The *Babsonchart* shows physical volume of business as a composite of what groups of data?

22. Is trend eliminated on the *Babsonchart?* Is seasonal variation? Is price inflation?

23. In addition to the physical volume of business index, what price indexes are included in the *Babsonchart?*

24. In relation to general business conditions, when are high and low points usually found in stock-market activity, and when in money rates, according to the Babson and Harvard services?

25. Describe the analysis and forecasting services of the Standard Statistics Company.

26. What general methods does the Standard Statistics Service employ in making forecasts, and which general method is most used?

27. How does the Standard Statistics Company apply cross-section analyses to current situations?

28. What data are used in Moody's Investors Service in forecasting business conditions? Are mathematical devices used?

29. What barometer does Moody's Investors Service publish, and what two curves does it include? How is it used?

30. Describe the service of *The Real Estate Analyst.*

31. Describe The United Business Service.

32. In connection with forecasting, why do economic and statistical services maintain close contacts with business men?

33. Name several publications or organizations, other than those considered above, that measure general business conditions and that comment on the general business outlook.

34. Is the forecasting of local business conditions important? Why? How can work of this type be done?

35. Do price curves for all products move in conformity? What important price indexes fluctuate with the cyclical swings of business?

36. How definitely can business changes be predicted?

37. How can the business man in planning his business make practical applications of information furnished by economic and statistical services?

38. In order to compare the sales of a given business with general business activity, to what statistical treatment should both sets of data sometimes be subjected (*i.e.*, to what terms should they be reduced)?

39. What index of national demand did the Walworth Manufacturing Company develop as an aid in forecasting its business? In using this index of orders, what methods of analysis does the company apply?

40. What sort of analysis of business conditions does the General Motors Corporation make for forecasting its business, and what are some of the business data which it considers to be important indicators? How do conditions with the company's dealers affect sales, and what conditions are important? What is the nature of the General Motors Divisional Index, and how is it used?

41. On the basis of series which indicate business conditions in the city or area in which you live, analyze the outlook for the following year.

42. On the basis of a study of the bulletins of the leading economic services, make up a composite report advising the president of a large bank as to the probable business conditions during the following year.

Note to the Instructor Regarding Problems

Many instructors find it desirable to assign a "term report" or statistical project when the class has covered the material presented in the preceding chapters of this book. At the ends of the question and problem lists which follow the present chapter and those that remain, a number of practical surveys and investigations are suggested, which can readily be assigned as term reports to supplement or take the place of the usual laboratory assignments during a period of from three to six weeks. If it is desired to assign term reports, it is suggested that the student be instructed to look over these problems, selecting one in the field in which he has the greatest interest. It is further suggested that, though the class as a whole may study the chapters in the order presented, the student study the chapter from which his problem is selected, out of order, as a background for preparing his report.

The authors believe that the emphasis in these reports should be placed upon securing practical results. For instance, if the student is analyzing the business cycle, it is suggested that instead of using the laborious methods involved in determining trends by the method of least squares or determining seasonal variations by the method of link relatives, the student should try his hand at a free-hand trend and one of the simpler methods of computing seasonal variation, with the emphasis being placed upon supporting his findings through logical analysis. Care should be taken in outlining the investigation at the start. Appendix IV to this book should be helpful in this connection. The student should also give careful attention to the preparation of his report. Suggestions on preparing a report are given in Appendix V.

CHAPTER XVIII

BUDGETING

If a business concern is to take the greatest advantage of the forecasts and analyses described in the preceding chapter, it must have a specific, detailed plan of future operation definitely worked out and expressed in monetary units. That is, it must have a *budget.* Although the details necessarily vary in different organizations because of differences in their markets, production periods, financial obligations, etc., there are, nevertheless, certain principles that are fundamental in systematic planning or budgeting, and it is the purpose of this chapter to describe these principles and to indicate how a practical business budget is developed through the use of statistical forecasts and analyses.

Business Budgeting Defined.—As the terms are commonly used, there is no definite dividing line between "forecasting" and "budgeting." In fact, when they are loosely applied, there is considerable overlapping, and either of the terms is sometimes used to include the other. In this discussion, however, it is necessary to draw a sharper distinction for the purpose of study and analysis. By *forecasting,* is meant the predicting of what is likely to happen on the basis of analyses of both external and internal conditions. By *budgeting,* is meant the development and application of a definite financial plan for internal operations that is systematically coordinated with the conditions that are expected on the bases of the forecasts. This distinction, however, is principally in how the emphasis is placed, for forecasting includes some of the same work that is done in budgeting, and budgeting automatically results in the making of some new forecasts. Mere planning for the future in a general way does not constitute budgetary methods; definitely planned operations, and the coordination of the parts of the business to carry out the planned procedure, are essential to real budgeting.

The term "budget" is often loosely used in the sense that it is only an estimate. In such instances, failure of actual occurrences to measure up to the estimate is expected, for the budget is treated as a goal or ideal seldom, if ever, to be achieved. Again, the budget is sometimes considered to be only a device for curbing expenditures. The basis for the allotments has often been either the requirements of the departments in past periods, or what these requirements should have been in past periods, according to the judgment of the management. In these instances,

future conditions and changing methods have not been considered in fixing the allotments.

Although budgetary methods do include both the making of estimates and the curbing of expenditures, these are but part of the budgeting process. A real budgeting plan involves something much more definite than an estimate or goal; it is carefully based upon forecasts and analyses, and it is expected that the plan will be carried out in detail. A real budgeting plan also includes much more than the authorization of expenditures; it definitely indicates future income, and coordinates the plans for expenditures with the income plans.

Preparing and Operating the Budget.—Budgeting may be said to include (1) the preparation of detailed plans for future operation; (2) a check-up or comparison of the actual results with the detailed plans; and (3) a continual revision of the plans previously prepared, in order to correct for changing conditions and new developments. By planning all the phases of a concern's activity for an extended period in advance; by checking up on the plans frequently, and by reorganizing them when necessary to meet changing material supplies, labor situations, plant facilities, demand conditions, etc.; and by continually comparing actual performances with those planned—by these methods a practical and effective system of control can be developed and maintained.

Yet it is not to be assumed that it is a simple matter to prepare detailed plans that provide for all emergencies or that even provide for the ordinary course of business activity. The preparation of such plans often requires the careful collection of a considerable volume of statistical information, intensive study of the information obtained, and keen judgment in applying the facts. Nor is this the task of one employee or executive of the business. The cooperation of all departments through the department heads is essential, and these executives must assume their share of responsibility in preparing the plans. Thus, the sales department must assist in collecting information as to the volume of sales which may be expected, the production department must compute its requirements in view of the expected sales, the general office manager, the personnel manager, the credit manager, the sales manager, and all other department heads must prepare detailed statements of anticipated department expenses. Then, meeting as a budget committee, these same executives can work out the final plans under the direction of an executive[1] in charge of the budget.

Also, one should not surmise that the administration or control of a business enterprise, after the budget plans have been prepared, is an

[1] Usually the comptroller (sometimes called "controller") is best fitted for *operating* the budget, whereas the president or general manager assumes the responsibility for the adoption of budget plans, subject to the final action of the board of directors.

easy task. The budget method is only a tool or instrument for control. It is not an end in itself; it is but a means to an end. In fact, the use of budgetary methods requires more careful and continual executive control than where budgets are not employed. The business manager who is content to manage a business on the basis of the policies of the past and who does not carefully plan the details of the future has an easier task than one who employs budgetary methods. Under the budget, the general manager and all department officials must watch for changing conditions, must anticipate new methods and problems, and must cooperate with each other in changing business policies or methods. Thus, executive control under budgetary methods is a more sensitive control in that it provides for the recognition of ever changing forces influencing business activity. But it also requires more analytical ability on the part of the business executive, and the ability to obtain the cooperation of all departments.

In a later chapter (Chapter XXV), the problems of control and the part which statistics may play in executive control will be considered from the standpoint of the executive looking down upon an operating organization. In the present chapter, we shall consider chiefly the first step in budgetary operation—namely, the preparation of the budget plans. Particularly, we shall point out how the business forecasts (discussed in the preceding chapter) are applied, and how certain other statistical information and analyses are used in formulating these plans.

Parts of the Budget.—The budget plans as a whole are called "the master budget." There are five main parts of the master budget. In a manufacturing plant these are:

1. The sales budget.
2. The production budget.
3. The expense budget.
4. The plant-requirements budget.
5. The financial budget.

In a merchandising business, the production budget is replaced by the budget of purchases.

The detailed budget plans are usually prepared on a value or dollar basis. Sales and production plans can be made on a physical quantity as well as on a dollar basis. However, since the other budget plans must be made on a dollar basis, sales and production plans must necessarily use this basis. As a supplementary method, the physical-quantity budget of sales and production can be used where statistics on the quantities of products manufactured and sold are maintained.

Readers familiar with accounting statements will recognize that the master budget consists of two main statements or exhibits and at least

two supporting schedules. The first exhibit is the estimated profit and loss statement, and it is prepared chiefly from the sales budget, the production budget (which includes production costs), and the expense budget. The second exhibit is the estimated balance sheet setting forth the expected condition of the business at the end of the formal budget period. Other balance sheets often are prepared showing expected conditions at the end of intermediate parts of the formal budget period. To obtain the estimated balance sheet, the expected inventories are determined from the production and sales budget (or the *purchases* and sales budget in a merchandising concern), the estimated fixed or plant assets are obtained from the plant-requirements budget, and the cash, receivables, and payables are taken from the financial budget. Thus, the sales, production, and expense budgets comprise the profit and loss exhibit, and the balance-sheet exhibit is obtained by two supporting schedules which present the plant-requirements budget and the financial budget, respectively.

The Sales Budget.—The preparation of a complete budgeting plan usually begins by determining, on the basis of demand forecasts and other market analyses, how much the concern can sell. This sales forecast indicates, in turn, how much the manufacturer should produce, or how much the merchant should buy. In other words, the probable volume of sales determines the probable scope of a concern's operations, and logically the determination of this is the first step in preparing the budget plan. At this point in the discussion, we will assume that we have already determined the conditions of the market. (The details of making such market analyses will be described later in Chapter XXI.)

As indicated above, the making of the budget plan usually begins with making a forecast of the particular concern's sales, but sometimes the volume of sales is determined by factors other than the probable extent of the market. It is very important to recognize these other factors when they exist. There are many local canning factories, for instance, which cannot expand their production if the local crop is poor, no matter how good the market may be. Again, in years when crops are good, they may be obliged to manufacture larger quantities than would be absorbed readily by the market, in order to have the local growers produce large enough quantities during the years that market conditions are favorable. This practical influence of supply and demand is also illustrated in the budget problem of the California and Hawaiian Sugar Refining Corporation. In this concern's budget, the first step is to determine how much sugar will be available for sale, rather than the probable volume of sales. This procedure is necessary because the company refines all of the raw cane sugar which is shipped to it by the planters (who own the corporation), and therefore the sales volume must

be planned in relation to the expected volume of production. This does not mean that the production budget must be prepared first. It means that the forecast of sales depends to a considerable extent upon the expected volume of production in this instance.

The procedure followed by this company illustrates the budgeting of sales under comparatively simple budgeting conditions. "The first problem to be determined is how much sugar will be available for sale within the year. After this, each broker is assigned a quota, and as brokers are segregated by geographic groups, totals for these groups are thereby obtained. The brokers are classified into three main classes, the Coast group, the River group, and the Southern group. An estimate is then made of the quantity of each grade which will go to make up the total to be shipped to each of these groups. The estimates for the three groups are combined to determine what tonnage of each grade will be sold and its percentage in terms of the total of sales.

"After the annual estimates are prepared, monthly sales forecasts for each grade and package are made. This is necessary because the grades are affected by different seasonal influences. So far as possible, the production budget is calculated on the annual grade percentages, despite this seasonal influence, in order that a uniform rate of production may be maintained. Some variations are necessary, however, because the proper balance of the inventory is of major importance.

"At the first of each month, a revised sales estimate is prepared for each grade and package. This estimate is checked with the production budget and stocks on hand, and if conditions have so changed that stocks will not be available to meet sales requirements, revisions are made.

"At the close of each quarter, sales are analyzed and compared with past indices to determine whether revisions in the annual grade and package ratios will be necessary. Irregular fluctuations in consumption from month to month are encountered, but these tend to equalize over the longer periods of time such as three, six, or nine months, necessitating only slight revisions, if any, in the ratios."[1]

Period of the Sales Budget.—It was observed previously that the California and Hawaiian Sugar Refining Corporation prepares estimates or forecasts of annual, quarterly, and monthly sales. Not only is it important to plan the sales of the year in advance, but it usually is necessary to plan them for quarters and months.

Many concerns prepare preliminary forecasts for periods longer than one year in advance. It is particularly common to prepare the sales estimate for two years in advance. The American Telephone and Tele-

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[1] EMMET, BORIS, "The California and Hawaiian Sugar Refining Corporation," *Stanford Business Series* II, p. 150, 1928.

graph Company and its affiliated telephone companies forecast their expected service (sales) requirements for five years in advance, and for even longer periods in certain instances.

Variation in the period covered by the preliminary estimate and in the period for the immediate budget plans depends to a considerable extent upon the products marketed. In the case of the telephone companies, the product developed—namely, service—requires the installation of a large plant with telephone wires and exchanges. This installation must be planned several years in advance to meet the service (sales) needs as they arise, because it requires a considerable period to install telephone wires, exchanges, etc., and in certain respects the plant must be built large enough to meet the increase in the needs for some time to come; this is especially true in rapidly growing communities. However, the service requirements must be estimated first to determine the size and location of the plant. These service or sales estimates can be made on the basis of population forecasts (see Chapter XIX), since telephone service depends to a large extent upon population. When a major plant-installation plan is adopted, it must be on a very definite rather than a preliminary basis, because a change in the plan cannot be made easily without resulting in substantial losses to the company.

For the sugar refining company, a budget of sales for more than one year is not practical, although a rough estimate for a longer period may be desirable in making general plans. For example, more acreage in sugar-cane plantations may be expected, or a loss of supply may appear likely because of planters seeking other refineries. These and other factors would be considered in making the preliminary estimate for later years. Yet the one year is all that can be used for definite sales plans. Then, too, in this type of company, as any one year progresses, revisions in the sales budget may be required to a greater extent than in the sales budget of the telephone companies. Production of sugar may be less than the outlook indicated; price conditions may become unfavorable, owing to general business depression setting in sooner than expected; or other conditions may arise to cause a decision to hold over larger stocks of sugar until later periods. Consequently, in this type of company, the monthly or quarterly sales budget is a more definite basis for plans than the annual one.

In other companies, the budget for the current month may require revision during the course of that month, and a weekly or even a daily budget of sales may be required. In baking companies and in ice-cream manufacturing plants, production and marketing to the retailers often must be completed in 24 to 48 hours. In these companies, the monthly sales budget is continually subject to adjustment, and even the weekly sales estimate may be changed because of sudden weather changes which

materially affect the demand for these products. The daily budget for both bakeries and ice-cream manufacturers is, therefore, an essential part of the sales budget.

It will be noted that the three types of products mentioned—namely, telephone service, sugar, and quickly perishable foods—have very different marketing and production problems, which cause extreme variations in the period for which sales plans can be definitely made. In preparing the sales budget for any manufacturing business, one must first consider such influences as the availability of raw materials, the time required for production, the perishability of the finished product, the effect of weather conditions and seasons of the year upon sales, and the control which the manufacturer has over marketing channels. In all cases, preliminary budgets should be prepared for periods much longer than those allowed for the definitely planned budget, but the preliminary budget will be subject to more revision than the budget for the immediate future. One- to three-month periods are common for the immediate operating budget.

Use of Data on Past Sales in Preparing the Sales Budget.—In forecasting the sales of future periods, one of the first considerations is past performance.

In preparing the sales budget, the estimates or quotas for each geographical territory, for each salesman, and for each type of product must be made only after noting the actual sales for similar periods in the past. Therefore, an analysis of records of the past by years, quarters, months, and sometimes by weeks and days, is the first step in preparing the sales budget. The data for such analyses should be furnished by the accounting department. In order to furnish information necessary to the budget, the accounting records must be maintained to show not only total sales, but also:

1. Sales according to products or types of products.
2. Sales according to geographical territories, where an extended sales area exists.
3. Sales according to salesmen, where salesmen can be of service in estimating their own future sales.

Such information must be made available if a sound basis of sales planning is to be established. Detailed plans are essential to general plans, and the details of past performance often give concrete evidence of what can be done in the future.

In comparing dollars of sales in past periods with expected dollars of sales in the future, cognizance of variations in prices of the products sold is essential. If marked changes have occurred, or if they are expected in the period for which the budget is being prepared, a difference in dollars of sales should occur, even though the physical volume is con-

stant. In other words, if a price decrease of 20 per cent occurs, an increase of 25 per cent in the quantity sold will be necessary to maintain the same total of dollars of sales. To measure price changes, an arithmetic average price at which the sales of the period were made (or are expected to be made, if in the future) should be obtained, and ordinarily the computed average price of one period can best be expressed as a relative of the average price in the period with which it is to be compared. In obtaining the arithmetic average price, however, it is essential that each price be weighted by the approximate quantity sold at that price, or the quantity anticipated, if in a future period.

It should be clearly recognized that consideration of past sales volumes is only one step toward estimating the volume of future sales. Conditions of the past may not continue in the future. In looking ahead, three additional steps in preparing the sales budget appear. First, the market must be analyzed; second, the general and specific business conditions of the marketing area must be studied; and third, the limitations of the business organization itself (in plant, in finances, and in available supplies) must be considered. Only after these analyses are made can the changes from past sales be accurately predicted.

Market Surveys and Analyses in Forecasting Sales.—In other chapters, the methods of making market surveys and business analyses are considered somewhat in detail. It is advisable, however, before proceeding further, to review briefly, from the point of view of budgeting, the sources and methods used in compiling sales data, in order that one may more clearly appreciate what kind of information is required and how it is applied, and in order that one may have the proper background for making or checking the sales forecast.

Although market analyses are usually not undertaken for budgetary purposes alone, they are always helpful in preparing the budget plans. Analyses of sales possibilities, for example, are not directly a part of the budget plans, but they assist in determining sales policies and in estimating sales volumes.

Some surveys are restricted to a local area, while others are national and even international in scope. Manufacturers or distributors of nationally advertised products find it particularly necessary to survey the entire country and obtain sales estimates from each area in which their products are sold. This is true whether the product is sold directly to the consumer, or through numerous wholesalers and retailers, or through a few exclusive distributors who sell to the retailers. Such market surveys require the use of varying methods and combinations of methods. Sometimes representatives of the sales department from the main or branch offices of the company call on the dealers in the market areas and obtain estimates of their merchandise requirements. Again,

the manager may obtain estimates from the salesmen without requiring them to make definite surveys. Where company salesmen are dealing directly with the consumer, their estimates take the place of dealer estimates. In any case, however, the survey should include basic information on population, buying power, number of dealers, agricultural and manufacturing conditions, and other conditions which affect the market. Sometimes this information is collected by salesmen or investigators in the field, and sometimes it is collected directly by the main office.

A certain manufacturer's sales are budgeted on the basis of reports rendered annually by branch-office representatives who call upon the retailers every year. The product is not sold directly to retailers, as a rule, but is distributed through jobbers in the principal cities. Yet each year the representatives of the manufacturer call on the retailers to determine, among other things, the sales expectations of each dealer for the next year. From this information and the personal observation and judgment of the representatives, an estimate of sales for each district is obtained.

In carrying on a survey of the above type for budgeting purposes, a definite questionnaire should be filled in by the company representatives, showing not only the total sales expected by the dealer, but also the estimate of the investigator. Furthermore, this questionnaire should show not only the totals of all sales, but also the sales expectations by grades and varieties of products. The actual sales of the past year, or past two years, and the budget estimates which were made for those years should likewise be entered on the form. Also, provision must be made to show the expected differences in stock holdover. The details involved in making such a survey (which ordinarily, of course, includes much information which is not directly used in budgeting) need not be considered at present. They will be described in Chapter XXI in connection with Exhibit 205 on page 508. The use of salesmen's estimates will also be described in that chapter.

Many concerns do not find it advisable to send representatives into the field to call on dealers or to make estimates. One of the more important reasons for this is that it may be too expensive. Also, unless the salesmen are properly trained, their investigations and estimates may be unreliable. Consequently, it is often necessary to limit the investigation of market conditions to a study that is made at the company's offices. Some of these studies are based upon data that are readily obtainable from the common secondary sources, while in others it is important to make use of primary data collected through letters and questionnaires. This practice will be discussed in greater detail in Chapter XXI (in connection with Exhibit 206 on page 510), where the methods used by the Perfection Stove Company will be described.

In order that the manufacturer may know how much of each individual product he should manufacture, it is obvious that the estimates of future sales must be by specific products. This is true for merchants also, in order that they may know how much of each product they should purchase.

Analyses of Economic Conditions in Sales Budgeting.—It should be emphasized in connection with the foregoing discussion that the information collected may not make proper allowances for conditions in the business cycle, especially when the information is gathered from dealers and from salesmen. Since general economic conditions may influence the sales of a concern to a marked degree, it is obviously necessary to give the proper consideration to this part of the analysis, and studies will have to be made which are independent of the personal opinions or influence of the dealers or salesmen. In such studies, information on the stocks of merchandise accumulated by the entire industry and by the individual concern and other indications of the relation of supply to demand are of utmost importance. When boom conditions are being experienced, those who are not skillful in recognizing the situation will ordinarily estimate that the need for merchandise for the coming year is much greater than the amount that can actually be sold. During conditions of depression, these estimates will tend to be unduly pessimistic and not large enough to meet the demand which actually develops. Consequently, during conditions of prosperity, it is necessary to be cautious regarding the size of the estimate, sometimes reducing it substantially, and during conditions of hard times, it is necessary to expand the figure if an increase in business activity is foreseen.

Business activity must be considered in relation to budgeting particularly from three points of view. These are: (1) the nation as a whole, (2) specific localities, and (3) the specific industry or industries in which the concern operates.

The use that the Walworth Manufacturing Company makes of economic services and other business indicators, which use was described at length in the preceding chapter, represents a practical application of the relation of business activity in the nation as a whole to the budget of an individual concern. Similar analyses of economic conditions in specific localities must be made because of variations from one area to another. These variations must be taken into consideration in budgeting, for the concern will wish to push sales where the opportunities are good and contract where sales possibilities are poor. Likewise, special analyses of conditions in the industry of which the concern under consideration is a part, and in allied industries, are necessary in making budget plans. If competition has increased, if overexpansion has occurred or is appearing on the horizon, if other products are being

developed which give promise of superseding the products now produced—these and similar circumstances will affect the sales volume of the future.

In recent years, business activity has been particularly sensitive to government legislation and regulation. Some legislation, such as the Banking Act of 1933, has been instrumental in effecting changes in the business tempo of the nation as a whole. In other instances, legislation has affected principally specific localities, such as, for example, where specific projects for the development of electric power have been authorized. Legislation affecting wages and hours of labor is of greater direct importance to industries that have not as yet met the standards set by the law, or by the authority administering the law, than to those industries whose labor conditions have already been adjusted to meet or exceed the requirements of the law. In budgeting for any extended period, the effects of possible legislation or other regulation by governmental authorities, as well as the effects of laws already enacted, must be studied carefully.

Effect of Limitations of the Business Enterprise upon the Sales Budget.—In the preceding discussion, we have considered analyses of past sales, surveys of the market, analyses of future sales probabilities, and the relation of general and specific economic conditions to the sales forecast. Another factor which enters into sales budgeting is the internal condition of the business for which the sales are being planned.

Four types of conditions which are significant in preparing the sales plans are: (1) the adequacy of the plant for the most economical production; (2) the control over the sources of raw materials and supplies, and the ability to acquire the quantities when desired; (3) labor conditions; and (4) the financial strength or weakness of the business. Unless the business has plant facilities for economical production, unless raw materials and labor can be secured as needed, and unless the amount of business anticipated in the sales program can be financed, the sales budget will not be effective.

Production budgets, expense budgets, plant and equipment budgets, and financial budgets will now be considered.

The Production Budget.—After the sales budget has been determined for the future weeks, months, quarters, or years, production plans can be prepared for these periods on the basis of the sales plans. In preparing the production plans, two main types of problems are present. First, the total amount of each product to be produced during the budget period must be determined, and this total production must be scheduled or prorated over the budget period to meet sales orders when received. Second, the factors of production cost—namely, raw material, labor, and overhead—must be budgeted in relation to the expected sales.

It is essential to obtain coordination between sales plans and production plans. Production must provide the necessary stocks to meet the expected sales. And yet, the sales quotas must not exceed production capacity. Obviously, the ideal relationship exists when sales quotas require the maximum production which can be obtained efficiently with the available plant, materials, and men.

The problem of scheduling the quantity of products to be made and the time when they are to be produced involves two statistical considerations. First, the sales and the production should be expressed in the same units of measurement. If sales are in dollars, then production can first be scheduled in dollars at selling prices. If physical volume is the basis upon which sales quotas are made, then physical volume should be used as the basis for the estimate of the total quantity of production and for the quantities to be produced in each part of the entire period. However, in planning production costs, it will be necessary to convert the physical volume of production, or the volume in sales dollars, into dollars of cost so that the elements of cost (raw materials, labor, and overhead) can be budgeted in relation to the total cost of each product. Therefore, usually the entire production budget can best be prepared on the basis of cost, which necessitates a statement of sales quotas on the basis of estimated cost of production. Thus, the most consistent plan for budgeting production is to convert the sales quotas to a cost of sales basis, and then plan the production upon a cost basis to meet the expected sales requirements.

Second, in scheduling production, the expected sales do not always determine the quantities to be produced in each period. Finished stock on hand and the period required for production are important factors in plants which manufacture for stock. This problem is largely one of inventory control of finished goods. The amount to be produced each month (quarter, week, or day) is determined by estimating the inventory of finished stock desired at the end of the month (or other period), to which are added the estimated sales of the period, and from this total the inventory of the beginning of the period is deducted. Then the production schedule for the subsequent months is similarly prepared by estimating the inventory to be carried over at the end of each period, adding the expected sales of the month, and deducting the estimated inventory at the beginning of the period.

It will be noted that the preceding discussion refers only to the budgeting of production in plants which produce for stock. In plants which produce for special orders only, and do not carry finished goods inventories, detailed budgeting of production in the above manner is not practical. In such plants, the unfilled sales orders, with perhaps certain allowances for the normal expectancy of sales orders, form the basis for

production volume. Also, the budget of materials, labor, and overhead costs in such companies cannot be prepared for a period far in advance of current orders. In the discussion that follows, we shall consider the budgeting of manufacturing costs principally in companies producing for stock.

Budget of Materials.—After the volume and time of production have been determined, detailed plans for the production costs must be made. The purchasing department then has a basis for scheduling the raw-material requirements, the personnel department has a basis for estimating direct- and indirect-labor requirements, and the cost-accounting department can determine in advance the proper standard unit costs of materials, labor, and overhead expense, to be used in estimating profits.

In estimating the inventory of raw material and supplies to be carried over from one period to another, the budget committee has an important task. Sufficient supply of each item of material to fill orders is necessary, but oversupply is costly. For example, in an article on "The Prevention of Failures in Business," one writer refers to a company that was refused loans by banks and bond houses. The company needed more money and finally a constructive accountant was employed to investigate the situation. "He examined into their inventory, and found that they had in the inventory seven years' supply of some of the parts that go into their assembly, and seven days' supply of other parts. And he found that if the inventory was properly proportioned as to time needs, they would have three times the amount of capital available that they were trying to get. In other words, three times the capital that they needed was tied up in an inventory, part of which would not be called for within seven years."[1]

Planned purchasing offers opportunities for savings which can be obtained in no other way. When a definite volume of each product is to be produced, quantity purchases of raw materials can safely be made. Prices of materials often vary considerably over a period of six months. If the purchasing department analyzes the price conditions of the chief materials required and takes advantage of low prices and quantity discounts, considerable savings will be realized. Obviously, storage facilities will have to be considered, and capital requirements also, but usually if the company is in a position to buy at the opportune moment and knows the right quantities for future production, it can effect distinct savings in raw-material costs. If the right quantities are not known, as in cases where budget plans are not made, a large purchase at low prices may prove costly because of carrying charges and deterioration of the

[1] PINKERTON, PAUL W., C.P.A., "The Prevention of Failures in Business," in *The Certified Public Accountant*, September 1928.

material before it can be used. Under proper buying methods, adequate stores records are maintained showing exactly the quantities on hand, detailed requirements of the future are obtained from well-planned budgets of production, and price studies are made from information on crop reports, warehouse storage reports, export and import forecasts, current forecasts of domestic demand, and similar bases.

The Labor Budget.—Planned production encourages the training of employees. Under the procedure of planned production, work schedules often can be arranged which substitute permanent workmen for transients. As in the case of the California and Hawaiian Sugar Refining Company, previously described in this chapter, production may be planned to provide almost uniform employment, even though the sales demand is seasonal. In some types of work, this is very important in obtaining an efficient and contented personnel.

Piece-rate methods of wage payments are excellent in budgeting labor costs. Furthermore, bonus plans and premium rates not only encourage labor efficiency but also assist in obtaining uniform labor costs, for the workers are paid on the basis of performance rather than time. The development of such payment plans is not necessarily part of the budget procedure, but such plans often grow out of the adoption of budget methods and they assist materially in obtaining accurate budgets.

Budgets of Manufacturing Expenses, or Overhead.—In budgeting production, all costs must be scheduled. The cost of manufactured goods includes not only materials and direct labor, but also dozens or even hundreds of miscellaneous indirect costs of maintaining and operating the plant. Some of these indirect costs exist regardless of the quantities produced, but other costs increase in total amount with the increase of production. Unless the total of these costs is computed in advance, the profits of the budget period cannot be estimated.

Effective budgeting of manufacturing expenses can best be secured by working out "standard" overhead rates of cost. These standard rates, however, must be based upon estimated total *future* costs and *future* volume of production. Formerly, standard rates usually were determined entirely from the records of the past. Under budgeting methods, the standard overhead rate is a rate for the future period, to be determined on the basis of a planned future. Therefore, this standard is one with which actual costs can properly be compared. It is determined by compiling the total overhead costs which definitely can be expected in producing the budget volume of each product, and then dividing this total by the production planned for each respective product. In compiling many of the expected overhead or manufacturing expenses of each department, the department foremen and the factory superintendent can be of great assistance, and it is important in the control of these

costs that they be made to feel their responsibility for keeping controllable costs at a minimum. However, the actual allocation of general overhead costs to departments and the determination of the standard cost rates per unit of each product is a technical problem of cost accounting which usually requires the assistance of the cost-accounting department.

Summary of the Results of the Production Budget.—To summarize the value of budgeting production operations in a manufacturing plant, reference is made to a statement of the industrial engineer of the Monroe Calculating Machine Company. "The value of budget control cannot be too strongly emphasized. It gives definite control over three important manufacturing elements—quality, amount of output, and unit cost. It means:

1. Better quality.
2. Greater quantity.
3. Lower cost.
4. Satisfied personnel."[1]

Budget of Purchases in Merchandising Companies.—Analogous to the purchasing of materials for production is the purchasing of merchandise for sale. However, the budget procedure is more directly related to sales in planning purchases of merchandise than in planning raw-material purchases. In manufacturing plants, the material purchases are budgeted to meet production requirements which furnish finished goods inventories in preparation for expected sales demands, whereas in budgeting merchandise purchases the production plans are eliminated.

Modern department stores plan the sales of each department for the year in advance, for each month of the year, for each week of the current month, and even for each day of the week. On the basis of the sales plan for each month, "open-to-buy"[2] allotments are made to each department buyer for several months in advance. These allotments are subject to revision, and even the current month open-to-buy total may be cut down or increased as the sales of each week or day fall short or exceed the budgeted amounts. In other words, the open-to-buy budget controls the entire merchandising operations in a very sensitive manner. But in order to use the open-to-buy effectively, statistical records must be

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[1] McGrath, Kenneth, Industrial Engineer, Monroe Calculating Machine Company, "Cost Reduction through Budget Control," *Manufacturing Industries*, Vol. XIV, No. 4, p. 285, October 1927.

[2] The term "open-to-buy" means the balance of purchases budgeted for the period; it is the amount that is "open" to be bought within the period, or the remaining portion of the period, according to the budget plan.

prepared daily in the accounting department. These records show for each department the open-to-buy for the month, the balance unused in each open-to-buy budget each day, the actual and the planned sales of the month to date, and a comparison of these amounts with corresponding periods of the previous year.

Budget of Expenses.—Planning the operations of a business does not end with the sales budget and the production budget or with the merchandise-purchases budget. All the expenses must be budgeted if profits are to be controlled. Whether the enterprise is a manufacturing or a merchandising concern, the selling, general, and administrative expenses must be controlled in accordance with the sales plans.

For example, selling expenses should not be allowed to exceed profitable limits in securing the estimated sales. In order to attain profits, standard costs or ratios of advertising, of sales salaries, of delivery costs, and of other selling expenses can be established in each organization. These are not only goals set for attainment; they often are the total allotments which will be allowed and paid for by the comptroller. Thus, upon the basis of the amount of sales budgeted for any one product, or for a given sales department, advertising expenditures are usually allowed in a predetermined percentage of these sales. The department or the product must show greater sales than were planned to justify increased advertising expenditures; that is, the ratio of advertising expenses to sales which is known to be profitable will be allowed, and the value of more advertising expenditures will have to be proved by actually obtaining more sales per dollar of advertising allowed than was expected. Then, as the actual sales exceed the budgeted sales, larger advertising appropriations may be made up to the point where it appears that sales will be secured at a cost not exceeding the standard ratio of advertising to sales.

The determination of standard ratios of all selling and general expenses to sales is essential in preparing the budget of these expenses, but the standard ratios require keen judgment, both in computation and in application. Thus, standard ratios must be determined chiefly from past experiences of the company and of other companies,[1] and yet corrections must be made for costs which will be changed by new methods or changed conditions of the future. Also, in applying the ratios to determine budget allotments in dollars, temporary differences in conditions, as compared with standard conditions, must be recognized.

Budget of Plant Requirements.—It has been previously mentioned that the master budget includes both an estimated profit and loss state-

[1] The Harvard University Bureau of Business Research, and other business bureaus have collected and published information on expense ratios from many types of business organizations.

ment and an estimated balance sheet. The budgets considered in the preceding pages are budgets of operating income and expenses, from which the estimated profit and loss statement is prepared for the budget period. To assist in obtaining the balance sheet expected at the end of the budget period, a budget of plant requirements and a financial budget must be prepared. Both of these budgets, however, must be considered in connection with the budgets previously described, for the sales, production, and departmental expense plans are predicated upon definite plant and cash requirements. In short, the plant must be provided to meet the operating plans, and the cash must be available when needed. If plant and financial plans cannot provide for the planned operations, the operating plans will require revision.

In a plant having a large amount of fixed assets, replacements and additions are continually necessary in some part of the plant. And yet it is important not only that the new equipment or buildings be ready for use when needed, but it is also important that equipment is not wasted by unwise selections or inopportune purchases. Under a planned budget of plant requirements, the needs of the plant in the way of additions and renewals are carefully surveyed, and not only is the plant maintained in a better condition for operation, but also less funds are wasted in choosing and installing the equipment.

The plant requirements necessarily are compiled from within the plant. Department foremen in the factory, the delivery department manager, the sales manager if he is in charge of the warehouse and the store facilities, the office manager—in fact, all department heads who use equipment in their department operations—must survey their equipment needs for the approaching budget year, and report upon them to the budget committee. From the reports of these junior executives. the budget plans can be prepared after considering the relative importance of the various requests for equipment.

In some companies, the magnitude of the plant or equipment requirements necessitates a central planning department with trained engineers who work continuously on the details of plans for additions to the plant. In railroads and other public utilities, it is the common practice to require these detailed plans and estimates of costs before the general plant budget will be adopted. When there is no central planning department, the individual department heads prepare the plans and estimates for the new equipment, often obtaining the assistance of the purchasing department in securing prices.

The Financial Budget.—The last budget, but by no means the least important under budgetary methods, is the budget of cash—the financial budget. This budget is a careful estimate of the cash which will be available during the budget period, and the cash expenditures.

In order to determine the cash which will be available from customers, the accounts and notes receivable at the beginning of the period should be added to the amount of sales budgeted for the period, and from this the estimated accounts and notes which will remain uncollected at the end of the budget period must be deducted, together with the estimated cash discounts. Consideration of collections and general money conditions in the various sales areas is necessary in estimating the accounts which will remain uncollected at the end of the budget period. Many other factors will have to be studied also, such as the effectiveness of the credit and collection department, the possibility and the advisability of changes being made in the terms of sales, and tendencies toward deferred datings on invoices.

All sources of cash must be considered. Investments may be held which it will be advisable to turn during the budget period. Additional issues of stock may appear advisable. Bank loans or bond issues may have to be planned to provide the funds needed. However, these latter sources will be considered only after determining the funds required for the planned operations.

In computing the cash needed, the requirements for purchases is one of the first considerations. To the accounts and trade notes payable at the beginning of the period must be added the planned purchases of materials or merchandise for the period, less expected cash discounts. Then the accounts and trade notes which will remain unpaid at the end of the budget period must be deducted. Similarly, purchases of equipment planned in the equipment or plant budget will have to be considered as to the cash payments which will be made thereon during the period, and as to the portion which will be unpaid at the end of the period. Furthermore, other cash requirements, as for bond sinking-fund payments, for maturing bonds and other loans, and for current expenses, will enter into the computation of the financial budget.

In estimating the cash required for current expenses, it must be recognized that not all expenses require cash in the period for which they are recorded, and some expenses may be paid for in advance. Particularly, depreciation and depletion do not require cash, nor do the bond discount and expenses written off from deferred charges in a given period. But if interest is prepaid for periods in advance, or if rent, insurance, and other items are paid for in advance, cash is required in excess of current expenses.

The advantages of the financial budget in controlling the finances are too obvious to enumerate. But one particular advantage should be mentioned. The amount of cash on hand and the amount of liabilities of a company mean little, except in relation to the other assets and the volume of operations. To one trained in accounting principles, it will

be evident that the procedure of working out the financial budget results in estimates of many assets other than cash and in predictions of the amounts of all liabilities. By using these estimates and predictions, and by taking the estimated inventories from the production budget and the estimated plant additions from the plant budget, a balance sheet can be prepared showing the condition which the budgeted operations will produce. To the management, this balance sheet is valuable because all assets and liabilities are given, and the executives can, therefore, anticipate the strength or weakness of the condition toward which the business operations are planned.

The Relation of the Statistical Department to the Budget.—The purpose of this chapter has been to describe the nature and the value of budgeting methods, with the emphasis placed upon the statistical methods and analyses that are necessary in preparing the budget plans.

Some of the statistical information used in budgeting will be collected from sources outside of the factory, the sales department, or the accounting offices. Other information will be obtained from records or from employees' estimates within these departments. Much of the work of collecting, tabulating, and analyzing this information is the work of the statistical department. However, it is not the duty of this department actually to prepare the budget. In other words, it should prepare information and analyses for the use of the budget committee, but the actual budget adoption is a function of the management.

Usually the statistical department is separated from the accounting department. Yet the accounting department should assist materially in collecting and in applying much of the information to the problems of budgeting. Many of the procedures in preparing estimates to submit to the budget committee require accounting knowledge and technique.

The Budget as an Aid to Executive Control.—Again it should be emphasized that the budget system is more than a series of estimates. Although many estimates are made in preparing the budget plans, these plans, when adopted, become the basis of administrative policies and executive decisions. They are indeed the "blue prints" and the architect's specifications for the building of the business enterprise during the period for which they are prepared.

In a later chapter dealing with statistics in executive control, the aid of budgeting methods in management will be explained further.

Questions and Problems

1. Explain the difference between forecasting and budgeting.
2. Discuss the preparation and operation of a budget.
3. What part should operating executives have in budget making and by whom should budget plans finally be prepared?

4. Name the five parts of the master budget.

5. What is the relationship of the master budget to the main accounting statements or exhibits prepared by the accounting department?

6. Which part of the master budget should be prepared first?

7. Discuss the period for which the sales budget should be prepared. What determines the period of preliminary and of definite plans? Give three examples of period requirements.

8. How should data on past sales be classified or analyzed for the sales budget? Name three bases of classification.

9. What correction may be necessary in applying data on past sales to future expectations of sales?

10. What other steps in preparing the sales budget are necessary in addition to the study of past data?

11. Discuss the use of market surveys and analyses in budgeting sales.

12. How do economic conditions affect demand, and what data should be obtained to indicate the relation of supply to demand? From what points of view should economic conditions be studied in budgeting?

13. What internal conditions within the organization influence the budget?

14. Describe the procedure in coordinating production plans with the sales budget. What parts or elements of production must be budgeted?

15. Describe how savings may be effected by careful budgeting of material purchases and labor costs.

16. To what type of rates should manufacturing or overhead costs be reduced? Explain.

17. Summarize the results of production budgets.

18. What is meant by the "open-to-buy" in department stores?

19. How do ratios assist in preparing the budget of selling and administrative expenses?

20. Why are budgets of plant requirements and of cash necessary and how are they obtained? How can complete balance sheets for future dates be obtained? Are they necessary? Why?

21. What is the relation of the statistics department to budget preparation?

22. Prepare a report upon the budget plan of a concern in your own city which will cooperate with you in furnishing information. Include a careful analysis of the statistical operations involved.

CHAPTER XIX

POPULATION AND PURCHASING POWER

Population data are of great importance in analyzing commodity markets, and in determining real estate situations, while in manufacturing and other types of production, they are especially significant in connection with the labor supply. Because of this importance, such data are, in turn, of great value to investors, to bankers, and to management in general. If one is to appreciate the significance of population data in marketing commodities, real estate, or securities, he must also know something about the purchasing power.

The general use of data on population and purchasing power and practical methods of making estimates will be explained in the present chapter. Discussions of the more specific applications of these data will be included in later chapters on the use of statistics in production, marketing, real estate, investing, banking, and management in connection with the practical problems that are being analyzed.[1]

Detailed descriptions of methods of estimating population for periods when census enumerations are not available are included in this chapter for two reasons. The *first* is to provide data for the purposes indicated above. The *second* is to furnish examples of methods which illustrate the practical and systematic development of simple estimates. If the student fully appreciates the procedure involved in making these estimates, he will be able to apply similar methods in solving other business problems.

Uses of Population Data.—As previously indicated, population data are used to a great extent in some types of business and to some extent in practically all types. As a matter of fact, the importance of watching population changes as an index of future business in many areas cannot be too strongly emphasized. This is especially important in the marketing of commodities and real estate. For many products, the market will vary reasonably directly with the population. Consequently, accurate population data are necessary in determining the relative importance of the different parts of the market area, and in determining what changes are taking place in the different communities.

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[1] Chapters XX, Production and Labor Statistics; XXI, Marketing Analysis; XXII, Real Estate Analysis; XXIII, Investment Analysis; XXIV, Statistics in Banking; and XXV, Executive Control and Management Statistics.

If a community is growing rapidly, the condition is favorable to a high rate of building activity, and to the expansion of retail stores, banks, and other business enterprises. If such a community ceases to grow, the continuance of the high rate of building construction, with the accompanying financing, subdividing, street work, house furnishing, etc., is not necessary. It usually happens, however, that many poorly informed builders, subdividers, stores, banks, and others continue to prepare for an increasing population for some time after the growth has ceased or a decline has begun. The result is overbuilding, oversubdividing, and overexpansion of businesses, which, in turn, cause a great amount of financial loss and a large number of failures. This not only shows the value of population data, but emphasizes the need for *current* population indexes.

Salesmen's territories, branch banks, wholesale agencies, retail stores, distributive systems, etc., should be apportioned in part according to the distribution of the population. In such work, it is again necessary to observe any important changes in population growth. The importance of this is illustrated by the experience of a certain concern in relation to its California sales territory. The state was divided into 10 districts in such a manner that they were approximately equal on the basis of the 1920 census of population. Five years later, three of the salesmen were having great difficulty in selling the required amounts, while four of them were substantially overselling their quotas. At first the sales manager thought that the differences were due largely to differences in the ability and energy of the salesmen. But when an analysis was made of sales-district populations, it was found that six of the smaller counties had experienced decreases, one important county had an increase of over 100 per cent, and three other important counties had increases of over 60 per cent. When allowances were made for the changes in population, there were no significant differences in the work of the 10 salesmen.

In many cases, it is desirable to study the distribution of population within a city. Sometimes the purpose can be served by using the figures as reported by wards or enumeration districts. These areas may not be satisfactory, however, if the study covers a long period, as their boundaries are changed from time to time. In a number of cities, the total area is now divided into small areas, called census tracts,[1] which have permanent boundaries. The recording not only of population but of other kinds of data by these permanent basic unit areas is a most important aid in making local statistical analyses.[2]

[1] Census tracts, which have permanent boundaries, should not be confused with census enumeration districts, which do not have permanent boundaries.

[2] The cities which had population recorded by census tracts in 1930 were: New York, Chicago, Philadelphia, Baltimore, Los Angeles, Boston, Cincinnati, St. Louis,

In connection with the use of population data, it should be pointed out that erratic changes are not likely to occur under present conditions in large figures, such as the total for the United States. Similarly, erratic changes may not occur in some of the larger cities. But in many suburban areas, small communities, and even small sections of the country, immigration or emigration is often taking place so rapidly that the short-time changes in the population are relatively large. This condition has not only characterized the growth of Florida and southern California, where the changes have been spectacular, but it is characteristic of practically all active communities throughout the country.

Importance of Population Classes and Types.—It is not only important to know what the total population is for a given area but it is also important and sometimes necessary to have the data by classes and types. In analyzing the market in Peru, for instance, it is not enough to know that the population of Peru is about 5,000,000, which is roughly equal to the present population of Michigan. It is also necessary to know that about 50 per cent are Indians; 32 per cent are half-breeds; 13 per cent are whites or slightly mixed whites; and 5 per cent are Negroes, Chinese, Japanese, and others. It is obvious that the wants of these different classes will vary widely and that they must be considered separately. And if one is to appreciate the true significance of the numbers in these classes, he must have reliable information on the purchasing power.

Permanent and Non-permanent Population.—When one is using population data in making a business analysis, it is very necessary that he know who have been included in the enumeration or estimate. As obvious as this point may seem to be, it is often overlooked by those who are using population data, and this results in much confusion and inaccurate application.

As pointed out previously (footnote, page 15), the United States Census includes in the population of a city only permanent residents or those whose actual homes are not elsewhere. Those whose actual homes are in New York, for instance, but who are in other parts of the country when the census is taken, are counted as of New York.

When the transients or temporary residents in a city constitute a large number as compared with the permanent residents, it often happens, obviously, that the Federal Census population data are not satisfactory in making various kinds of market analyses, as they represent only a part of the market. Not only must the permanent population be provided with housing facilities, but also the persons who stay in a city a few weeks or months must be provided with temporary homes or hotel

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Buffalo, Washington, Cleveland, Pittsburgh, Nashville, Columbus, Indianapolis, Berkeley, Yonkers, and Syracuse.

accommodations. And the transients who remain overnight must have lodging, while those who come in and leave the city the same day must be provided with working space, educational facilities, meals, shopping facilities, amusement, street space, etc. In other words, various population figures may be required for a certain city depending upon the uses that are made of the data.

Sources of Population Data.—The Federal Government took the first population census of the United States in 1790. Since that time a census has been taken every 10 years. In recent enumerations, the information that has been gathered has been very complete. The published reports give the numbers of persons living in the various states, counties, townships, cities, wards, towns, and other civil divisions. These data are classified in many different ways, such as by sex, age, urban or rural, marital condition, race, nativity, illiteracy, and occupation.

In some instances, a state or a city will take a census between the Federal Census dates. Such enumerations are especially valuable when there is a rapid rate of growth or when there is a change in the rate of growth.[1]

For most practical purposes, it is necessary to have population figures at short intervals, say, of not greater than one year, and ordinarily they must be obtained by making *estimates* for the years between census dates, because the expense of making an actual enumeration is prohibitive. And since the census does not include the non-permanent population, estimates have to be made, in making most practical studies, to determine the *total number of persons living in an area.*

Population Estimates.—In making a population estimate, it is first necessary to determine exactly who shall be and who shall not be included in the figure. As previously indicated, this requirement, though it may seem to be a very obvious one, is frequently overlooked. Unless this requirement is clearly appreciated and carefully observed the estimator may include substantial numbers of non-permanent population in an estimate of permanent population. Or he may make an estimate of the permanent population and unintentionally overlook the non-permanent population.[2]

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[1] Special State censuses were taken in 1935 in Florida, Kansas, Massachusetts, South Dakota, and Michigan, and in 1936 in Rhode Island. Special city censuses were taken in Chicago in 1934 and in Cincinnati in 1935.

[2] Population estimates often lead to important controversies in connection with the rivalry between cities. And such controversies are not of recent origin. When Sir James Petty, for instance, estimated the population of London back in 1686 and asserted that this city was the largest in the world, M. Auzout of Rome objected and asserted that the city of Rey in Persia was "far bigger than London." The population estimates published by booster organizations at the present time are often inflated and must be carefully checked before being used in any important analysis.

Most population estimates in the United States are based upon determining the increase that has occurred since the last Federal Census. The methods used will vary all the way from those which are based entirely upon mathematical projections of Federal Census data to those which determine the increase on some independent basis. The methods of making population estimates based on census projections, water services, building permits, city directories, enrollment and average daily attendance in elementary schools, telephones, and a number of other series will now be discussed.

Estimates Based on Projection of Rate of Growth.—Probably the simplest method commonly used is that of projecting the rate of growth which has been found to characterize the decade previous to the last census date. That is, the rate of growth that is found to exist from 1920 to 1930 is projected until the 1940 census data are available. Graphically, this would be plotting 1920 and 1930 data on a semilogarithmic scale, fitting a straight line to these points, and projecting it to 1940. Obviously, this method is not reliable, except in communities where the rate of growth is known to be stable and free from erratic influences. In periods of rapid growth, such as those which characterized Detroit, Florida, and southern California in the 1920's, it is obvious that this method should not be used at all for most practical purposes and that the estimator will have to use other methods.

The Bureau of the Census, in making its estimates for cities and states before July 1, 1928, used the straight line projection method, which is fairly satisfactory for the nation as a whole and for communities with a reasonably stable rate of growth, but it is unsatisfactory where current data are most needed; that is, where changes are large or erratic. The Bureau now bases its estimates of the total population of the United States on the records of births, deaths, and net immigration, so that the result can be accepted with a good degree of confidence notwithstanding the lapse of time since the last census. But this is a method which is not applicable to the individual states or to smaller political units, since no record of the migration to or from states and cities is available. In estimating the population of the states, therefore, the Bureau gives consideration to trends indicated by school enrollment, birth and death data, results of any state censuses that have been taken, and any other significant items that may be available.

In estimating the population of cities and counties, the Bureau formerly used the method of projecting the increase shown by the last two censuses. This method, however, yielded very uncertain results when applied a few years after a census had been taken, and the Bureau discontinued its use. The estimates made for July 1, 1933 were based on the assumption that the increase in any area between 1930 and 1933 bore the same relation to the increase in that area between 1920 and

1930 that the increase in the United States as a whole between 1930 and 1933 bore to the increase in the United States as a whole between 1920 and 1930. Since that time, however, city estimates have not been made because it became evident that because of the unusual movements of population that had taken place after 1930, estimates made by any mathematical formula were far from satisfactory.

Estimate Based on Water Services.—Although the method of projection just described is commonly used for general purposes, it is ordinarily necessary to use other methods when making careful studies. Most of the methods of estimating population are based on the use of readily available series which have a high degree of correlation with population. One of these series is domestic water services. It is obvious that domestic water consumption, or domestic water services, should have a relatively high relation to the number of persons in the area, because water for domestic use is a common necessity, with a relatively stable rate of consumption per capita.

In general, the method of estimating population on the basis of water services is as follows: If there was a population of 500,000 in a certain city in 1930 and this city had 100,000 domestic water services at that time, it follows that there were 5 persons per service. Suppose that we find that practically the same situation existed in 1910 and 1920, and that apparently nothing has happened to change this relation materially. Then if there are 200,000 domestic water services in the city on January 1, 1938, our estimate of the population of that date would be 200,000 × 5, or 1,000,000 persons.

The method of estimating population on the basis of water services, as mentioned above, may seem to be simple. In general, it is merely that of dividing the Federal Census population figure by the number of water services for the census year, and then applying the quotient as a factor to the service data for any other year. Ordinarily, however, in computing the basic factor and in making the final estimate, many adjustments and corrections must be made. The more important difficulties that were met in a practical problem of estimating the population of Los Angeles for January 1, 1927, illustrate the types of difficulties that may be encountered. These will now be discussed.

It was found that the installation of water services had a distinct seasonal variation and that it was influenced by the business cycle in building activity. This seasonal variation and cycle did not correspond to the population seasonal variation and cycle. Care had to be taken, therefore, that the estimate was not influenced by seasonal changes, or by the upward or downward slope of the cycle.

It often happens that the boundaries of an area served by a city water department are not the same as the city's corporate limits. In this city under discussion, the city water department served an area which

extended beyond the corporate limits of the city itself, but the number of services was small. The exact number outside was not known, but in 1923 it was estimated that about 3,800 services of the department were outside the city limits. To counterbalance this, however, an investigation at that time showed that five private water companies operated within the city limits and provided approximately 4,000 services to city dwellings.

The annual increase of services of the city water department included new services installed, and also the services acquired by purchase from private companies operating in the city and in annexed territory. Services acquired in the older parts of the city may represent no new increases in population, but those acquired when an area is annexed to the city may be good indicators of the increases due to annexations.

It was desirable to know the number of water services used for domestic purposes only, but the data were not available. Many services furnished water to buildings where shops and stores existed jointly with living quarters. The commercial, industrial, and domestic services were, therefore, in one amount. As long as the percentage relation between the domestic and business services remains the same, no error develops, but if one grows faster than the other, the resulting estimate may be faulty.

In addition to the foregoing difficulties, the data included (under varying designations) free services for parks, city buildings, and sewer flushing; services for private fire protection; irrigation services; services with meters out; services disconnected entirely; and services shut off due to vacancy, delinquency in paying bills, and other causes. In the case of disconnected services, the serial number of the service was resold as a new installation. Services are sometimes ordered but never put in, or are ordered one year and installed the next. All of these conditions must be observed and corrections must be made to offset the difficulties when possible. When it is impossible to make corrections, allowances must be made when using such an estimate. When actual data for making the adjustments and corrections cannot be secured, the expert opinions of the water department officials are often very valuable in improving the basis for the estimate.

In this case, analysis brought out the fact that although free services were uniformly about 3 or 4 per cent of the total, disconnected services had decreased from 7 per cent to 1.3 per cent over a 15-year period. An attempt was made to deduct all items not related to domestic services. It was possible, however, to deduct only the free services and disconnects.

Since November 9, 1926, the water department of this city has installed water services for vacant lots for the purpose of avoiding as

much as possible the necessity for future excavation in the pavement. These services are easily deducted as the number is reported, but care must be taken to see that such services are not included in the figure used in making the population estimate.

On the basis of various estimates, it was determined that, under average conditions, water services installed for residence use would not be used by tenants until approximately three months after installation, because a service usually was installed to provide water for construction as soon as the building permit was secured. Even after the house is completed it often happens that it is not occupied for some time. In order to allow for this lag between the installation of the service and its domestic use, therefore, the services which were related to the people counted in the census of January 1, 1920[1] were those inventoried three months earlier—about October 1, 1919. For the 1910 factor, the services were adjusted in conformity with this lag to January 15, 1910, to apply to the census date April 15, 1910.[1] The same adjustment was made in the data used for the 1927 estimate.

If the proportion of vacancies in the various types of dwellings changes, then the relation between water services and population will also change. It was estimated that in January 1920 only 3.2 per cent of the residential accommodations in the city were vacant. A survey made on December 15, 1926, showed that the proportion vacant was 8.2 per cent. It is possible to make a reasonably accurate correction for vacancies in this city from the quarterly vacancy figures that are available. In this instance, the correction amounted to a deduction of 18,400 meters from the 1927 figures.

After the necessary corrections and adjustments were made in the water-service figures, the persons-per-service factors found for 1910 and 1920 were 5.142 and 5.325, respectively. This seemed to indicate a gradual increase of persons per service. And there were a number of other conditions which indicated that a progressively increasing factor should be used. Undoubtedly there was a change in relation between water services and population, due to the increasing proportion of multiple family dwellings, since the city water department installs but one service for a duplex, a flat building, or for an apartment house in this particular city. Consequently, from one to a considerable number of families may be represented by one service. While such conditions indicated the necessity for a progressively increasing factor of persons

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[1] The Federal Census of 1920 was taken as of January 1, 1920, while the Census of 1910 was taken as of April 15, 1910. In both years the water services were inventoried as of June 30 or July 1. The lag mentioned must be checked every time an estimate is made as it is probable that it is subject to considerable change. (The Census of 1930 was taken as of April 1.)

per service, there were two offsetting factors which had to be considered. In the first place, there is a strong tendency for the average-sized family in the city to become smaller as the proportion of apartment dwellers increases.[1] In the second place, the number of vacancies in apartment houses and other multiple-family structures was relatively high, and consequently the increase in the factor of persons per service did not bear a direct relation to the increasing proportion of apartment house units furnished. This presents a real difficulty in using this method when there is a large increase or decrease of the population dwelling in apartment houses. The change would not be reflected in water services because as long as there is any occupancy at all in a multiple-family structure the service will be operated.

Obviously, it was desirable to allow for some increase in the factor of persons per service. Just what this increase should be was a question that could not be decided on a statistical basis without making more elaborate calculations than were possible under the circumstances. Finally, it was decided to use an approximate projection of the rate of change indicated from 1910 to 1920, which gave 5.46 as the 1927 factor of persons per service.

On the basis of the foregoing methods, qualifications, and adjustments, the final number of services to be used with the factors computed in this investigation was 205,222 for January 1, 1927. This number multiplied by the persons-per-service factor of 5.46 gave 1,120,512 as the estimated permanent population of Los Angeles for January 1, 1927.

There are many methods, other than those discussed, of making adjustments and corrections in water-service data when estimating population. The methods that are given were actually used in a practical problem of business analysis and are submitted as examples which indicate the common types of problems that arise, and show the solutions that were believed to be practical for the purpose at hand. These suggestions should be helpful in developing a procedure when estimating population for this or some other city.

Estimate Based on Building Permit Data.—When making a population estimate on any basis, it is important to check it with other independent bases. Before accepting the preceding estimate of population for Los Angeles for January 1, 1927, estimates were also made on several other independent bases. One of the best of these was the number of family accommodations in residential building permits. If the building department of a city keeps records showing the number of family accommodations for which permits to build have been issued, one may have a

[1] The average number of persons per family in Los Angeles decreased from 4.057 in 1910 to 3.616 in 1920 and 3.342 in 1930.

good basis for estimating population. Such an estimate is based principally upon adding the increases in family capacities to the total capacities that existed on the census date, subtracting the vacancies, multiplying by the number of persons per family, and correcting for non-permanent residents and annexations.

The process of making the estimate will be explained in detail in connection with Exhibit 186. The first item (number of permanent and non-permanent families for January 1, 1920) is based upon the census of that date. The Federal Census, however, gives only the permanent residents. Obviously, it is necessary to know the proportion of non-permanent residents in order to get the total of all residential capacities. This figure was not readily available, but upon investigation it was found

Population Estimate Based upon Family Capacities of Building Permits, Los Angeles, January 1, 1927

1. Families, permanent and non-permanent, January 1, 1920	181,223
2. Estimated family capacities vacant January 1, 1920	5,763
3. Estimated family capacities under construction January 1, 1920	1,930
4. Family capacities, permanent and non-permanent, added to January 1, 1927	174,059
5. Total family capacities, permanent and non-permanent, January 1, 1927	362,975
6. Deduct for vacant and under construction	−34,483
7. Total family capacities, permanent and non-permanent, occupied	328,492
8. Deduct non-permanent families	−44,346
9. Total permanent families	284,146
10. Total permanent persons (284,146 × 3.616)	1,027,472
11. Add population annexed	86,890
12. Total permanent population	1,114,362

Exhibit 186.—Illustration of procedure in making population estimate on the basis of family accomodations added according to building permits.

from a private source, which had a close check upon these conditions in 1920, that the proportion of non-permanent residents at that time was approximately 12 per cent. The number of permanent families enumerated on January 1, 1920, by the Federal Census was 159,476. Considering this figure as 88 per cent of the total, we have 181,223 as the total permanent and non-permanent families, as shown by the first figure in the table in Exhibit 186. This figure was taken to represent the total occupied dwelling capacities. The next step in building up the total of all dwelling capacities, was to add the number vacant. This was determined by an estimate and is shown as the second figure (5,763) in Exhibit 186. As the increase since the last Federal Census is based upon the total increase of dwelling accommodations for which permits to build have been issued, it was necessary to add also the family capacities under construction in January 1920. No information or definite esti-

mate was available on this point, but an investigation indicated that the average length of time required for construction of residential buildings at that time was about three months and, consequently, the number of family capacities for which permits had been issued during the three months prior to January 1 was taken as the number under construction, and this number (1,930) is shown as the third figure in Exhibit 186. To the figures previously obtained, the total number of family capacities for which permits were issued during the period from January 1, 1920, to January 1, 1927 (174,059), was added as shown in Exhibit 186, and this gave the total family capacities for the desired date. This number is 362,975 (see Exhibit 186).

With the total family capacities determined for both permanent and non-permanent population, including those vacant and under construction, the next step was to deduct the vacancies and the accommodations under construction. Fortunately, in this city, quarterly surveys were available which furnished data on the proportions vacant and under construction. The total number of family capacities was multiplied by this percentage (9.5)[1] and the result (34,483) was subtracted as shown in Exhibit 186. In order to arrive at a figure which is comparable with the Federal Census figures, the estimate must include only the *permanent* population, and, consequently, the non-permanent must be deducted. From time to time, surveys have been made on a sampling basis in this city in order to arrive at a reasonable estimate of this proportion, which was deducted as shown by item 8 in Exhibit 186, leaving a total of 284,146 permanent families.

Having determined the total number of permanent families, the next step was to multiply this figure by the number of persons per family as indicated by the last Federal Census. To this figure, estimates of the population of annexations, as indicated by the city records, were added, which gave a final figure of 1,114,362 for the total permanent population of the city on January 1, 1927 (last figure in Exhibit 186). This figure, it will be noted, corresponds closely to the estimate based on water services.

No direct correction was made in Exhibit 186 for family capacities added by alterations and additions, since no definite figures were available. After a study of the problem, however, it was believed that these additions were very closely offset by the deductions due to family capacities demolished, destroyed by fire, or converted into business buildings, and permits issued but not acted upon.

The preceding discussion indicates the steps that were taken, but it does not indicate fully the care that must be used in taking each step.

[1] Strictly speaking, this is a factor for December 15, 1926 (published in the Eberle & Riggleman Economic Service for December 27, 1926), adjusted to January 1, 1927.

As an indication of the analysis that should ordinarily be applied, the factor of persons per family, used in item 10 in Exhibit 186, will be examined. The Federal Census enumerates the families as well as the individuals in each civil division. By dividing the number of persons by the number of families, the number of persons per family was obtained. On January 1, 1920, Los Angeles had 576,673 persons and 159,476 families, which places the average size of its census family at 3.616 persons. The census family differs from the natural family. The following quotation from the 1920 Census explains this difference:

> The term "family," as used in the census, signifies a group of persons, whether related by blood or not, who live together as one household, usually sharing the same table. One person living alone is counted as a family, and, on the other hand, all the occupants and employees of a hotel, boarding house, or lodging house, if that is their usual place of abode, and all the inmates of an institution, however numerous, are treated as constituting a single family. Thus the census family may be either a private family or an "economic family."

In the Census of 1900, an investigation was conducted to determine to what extent the census family differed from the private family in numbers of persons. It was found that there was practically no difference for the nation as a whole or for many of the states. However, in communities where unusual conditions prevailed, such as the presence of a large floating population, large schools, hospitals, or construction camps, the difference was found to be marked. The number of persons per family has been decreasing steadily in the United States. In 1850, there were 5.6 persons per family as compared with 4.3 in 1920. It was of greater significance in this survey, however, to consider the situation in California and Los Angeles. In California, the number of persons per family increased from 3.769 in 1840 to 4.917 in 1890, and then decreased to 3.807 in 1920. The early increase was caused by a large floating population entering the state at the time of the mining and railroad booms. Many persons at that time lived in public places. The decrease has probably been due to a change from living in hotels, boarding houses, and camps to living in private homes; a decrease in the birth rate; and an influx into southern California of many persons who do not bring their families with them. Data on sizes of families were not available for Los Angeles for dates earlier than 1890, but since that time the number of persons per family had decreased from 4.558 to 3.616 in 1920.

Obviously, in an estimate of population which is based upon a computation of families multiplied by the number of persons per family, considerable importance must be attached to the accuracy of this factor, because an increase or decrease of a few tenths will materially raise or lower the estimate. Accordingly, the question arose as to whether the

factor of 3.616 persons per family for Los Angeles should be accepted or changed. On the one hand, strong claims were made that the increase in industrialization during the preceding seven years had caused the number of persons per family to increase, while, on the other hand, it was pointed out that the trend of the last four Federal Census factors indicated further decrease. It was difficult to see how the factor could increase, because it was evident that a large part of the incoming population was of the retired type, and apparently those who came to work in the industries were composed largely of those with no families or with small families, because they could move more readily than those with large families. However, it did not seem probable that the factor had decreased further, as it was lower in 1920 than the factor for any other city of over 100,000 in population. In view of these considerations, it was decided to use the 1920 size of family in making the estimate under consideration. (It may be of interest to note that when the Federal Census was taken three years later, the size of family in Los Angeles was found to be 3.359 persons.)

Estimate Based on City Directory.—Though the proportion of a city's population that is listed in city directories sometimes fluctuates widely over a period of time, it is ordinarily a good basis for estimating population, providing the estimates are made by those who are familiar with current population characteristics. As in the case of water services, this estimate depends upon finding the relation of the number of names in the directory to the Federal Census, and then applying the factor that is found to the number of directory names for succeeding years. That is, if the city has a population of 500,000 on January 1, 1930, and there are 100,000 names in the city directory as of this date, then the number of names in the directory multiplied by 5 would give the population for the following years. It is obvious that the accuracy of this estimate depends upon a constant relation between city directory names and population, or, in other words, it depends upon securing the same proportion of names year after year. One check which is of value in this connection is to see whether the factor is the same for several preceding census dates. Other checks, however, which fit the particular case must be applied for the purpose of finding out as much as possible about conditions since the last census date. This basis will not be discussed further at this point. From the previous detailed discussion of water services and building permits, the student will appreciate that the same type of procedure and care characterizes the making of population estimates on other bases.

Estimate Based on School Data.—One of the most readily available series upon which a population estimate may be based is that of elementary school enrollment. This is ordinarily available by cities, counties,

and states. Average daily attendance figures are also valuable either for use as a direct basis or as a check upon the enrollment figures. In communities where there is a rapid turnover of population, the average daily attendance figures sometimes constitute a more representative basis than the enrollment figures. Elementary school figures are ordinarily the best, as they have a more constant relation to the population than high-school or evening-school enrollment. As is the case with water services and building permits, there are often many corrections and adjustments which must be made. The exact nature of these will be indicated by a study of the individual case. Ordinarily, allowances must be made for seasonal variation, schools outside the city proper but in the city system, schools recently added to the city system, and changes in the proportion of private schools. All that was said in connection with the building permit basis regarding changes in the size of the family applies with equal importance to the school enrollment basis.

Telephones, Gas and Electric Services.—Three other bases that can be used in a manner similar to water services are telephones, gas services, and electric services. As is the case with the preceding bases, these series must be corrected and adjusted to make the correlation between them and the population as high as possible. If, for instance, the proportion of business telephones is increasing, the proper allowance must be made. Adjustments usually must be made when using telephones and gas or electric services because of the fact that the serving areas of these public utilities are seldom the same as the areas within the municipal boundaries of the city. Such figures as electric services must be carefully analyzed to see that the correlation with population is high. In one city, for instance, there were large figures for new services. It was found, however, that a large portion of these did not represent increased population but were simply replacement installations required because of the increased use of electric appliances. Similarly, domestic electric power consumption is not a good index of population growth because the domestic use of electric power is increasing at a more rapid rate than population, owing to the increased use of new electrical household equipment.

Estimate Based on a House Count.—In certain instances, none of the preceding bases can be used. In suburban cities and towns, it often happens that conditions change so rapidly that the relations between population and the various series previously mentioned cannot be determined definitely enough to be dependable. In such instances the house-count method has been found to be a practical one. In this method, enumerators simply count the occupied dwellings and the result is multiplied by the average number of persons per dwelling. It is usually advisable to make a sample check to determine a reasonable factor of persons per dwelling.

This method is a reasonably simple one when the dwelling places counted consist of single-family dwellings. When apartments, hotels, and rooming houses are present, the problem of determining the population becomes a more difficult one.

In some instances, the Federal Post Office, public utilities, or other organizations have made such house counts for their own purposes, and the data may be obtained from them for use in making population estimates.

The house-count method is the only method included in this discussion which is independent of the Federal Census. In all of the other methods, the census figures are used as a basis for determining the factors to be applied to the data, or the census enumeration is used as a base to which subsequent increases are added. It should also be noted that the house-count method, as above outlined, includes both permanent and non-permanent population.

Semiannual, Quarterly, and Monthly Estimates.—Annual estimates of population are sufficient for most purposes, but in some instances it is desirable to have estimates at semiannual, quarterly, or even monthly intervals for areas where it is known that the population is rapidly changing. Any of the bases previously discussed can be used in making semiannual, quarterly, or monthly estimates if the data can be secured for corresponding intervals. Ordinarily, however, this is possible for only a few of these series. But this is not a serious difficulty, as an annual estimate can be made by using several bases and the, say, monthly estimates can be carried forward through the year by using a few available series. Then, at the end of the year, another annual estimate can be made by using a larger number of bases, and so on.

Estimating Non-permanent Population.—As has been pointed out from time to time, the practical use of population figures usually depends upon having the total number of all persons in the area under consideration. Census data (or comparable estimates) can be made to include the non-permanent population by making an investigation on a sampling basis to determine the percentage whose permanent residences are in other parts of the country, and then raising the figures in the proper proportion. This investigation ordinarily must be made by having a field enumerator make personal calls upon a representative number of families. On his schedule he can record the number of persons in each family and indicate whether the family resides in the city permanently or whether some other city is considered as its permanent address.

The house-count method, as previously described, includes both permanent and non-permanent population. In other words, it includes the total of all persons in the area covered, and no correction need be made if the total figure is the one desired. If, however, a figure including

only the permanent population is wanted, the non-permanent will have to be estimated and deducted.

Importance of Purchasing Power.—As previously stated, it is difficult to base any conclusions on population unless one knows something about the purchasing power. This not only is important to those who are selling goods directly, but it is also important to bankers, realtors, and other business men whose operations directly or indirectly depend upon the purchasing power of the districts into which they are extending their operations. One of the reasons why one sees so many suburban stores going out of business is that there is not enough purchasing power in their immediate communities to justify their existence. This is especially true during a decline in business activity. It is understood, of course, that purchasing power is only *one* of the factors that should be considered in the preceding instances. Competition should be carefully analyzed while the types of buyers, their customs, nationalities, occupations, standards of living, and the like must be studied.

The importance of population to different kinds of commodity markets varies with the size of the individual incomes. When using purchasing-power statistics the following tendencies should be observed: (*a*) the greater the individual's income, the smaller is the percentage spent for food; (*b*) as the income increases, the percentage spent for clothing is approximately the same; (*c*) as the income increases, the percentage spent for rent, fuel, and light is approximately the same; and (*d*) as the income increases, the percentage spent for luxuries becomes greater.[1] The relative position of the buyer in the business cycle should be carefully determined. In a period of increasing or high prosperity, the buyer tends to spend relatively more of his income than when conditions are poor. In a period of depression, purchasing power will ordinarily decrease, but even in cases where one can maintain the same income it will not be spent as freely.

In analyzing long-time trends in the United States market, the seller must appreciate the basic changes that have taken place. Not only has there been a tremendous increase in the national income, but the relative importance of the sources has changed. One of the curves in Exhibit 187 has been adjusted for changes in price level. Such an adjustment should, of course, be made in problems where the data are considered as representative of real purchasing power.

Purchasing Power Compared with Population.—Obviously, the distribution of purchasing power will often be quite different from the distribution of population. That is, an area which has a small population may have a high purchasing power, while an area that has a large popula-

[1] These tendencies are sometimes referred to as "Engel's law."

tion may have a low purchasing power. In Los Angeles in 1926, for instance, about one-twelfth of the families received an income of over $5,000 per year, but the total income received by this one-twelfth constituted about one-third of the income of the entire city. Over 55 per cent of the families in the city received incomes between $1,000 and $2,500, but the total income of this group aggregated less than one-third of the total purchasing power of the city. In such problems as

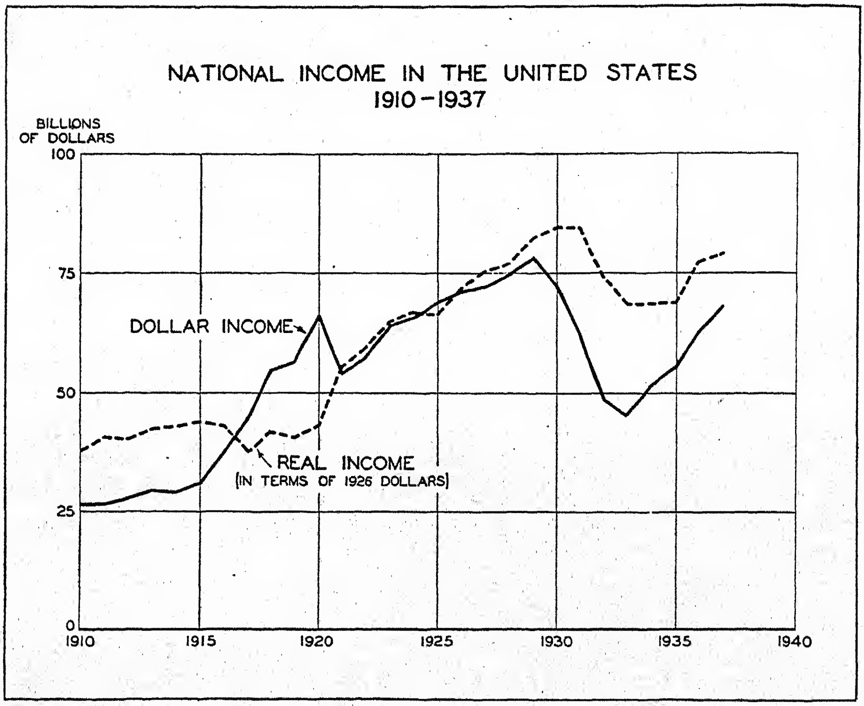

EXHIBIT 187.—In such problems as analyzing the United States market, the changes in both money income and real income must be carefully considered.

locating neighborhood stores or branch banks, the purchasing power per capita of the different sections of a city must be carefully analyzed.

Recent Developments in Measuring Purchasing Power and in Using Purchasing-power Data.[1]—"Until within a few years the buying capacity of the United States market, as applied to the sale of the products of any one concern, could be measured with sufficient accuracy by any good

[1] The first four paragraphs of this discussion of recent developments in measuring purchasing power and in using purchasing-power data are abstracted from "Some Recent Developments in Market Analysis," an address delivered before the Taylor Society by Paul T. Cherington, Director of Research, J. Walter Thompson Company.

thumb-rule. The National Cash Register Company, for example, is said to have based its sales plans quite successfully for many years on an estimated annual consumption of one cash register for each 400 people. Other concerns for years did a satisfactory business on similar rough general approximations of consuming capacity. But in recent years, the shortcomings of such general averages have been increasingly apparent. They have required so many local adjustments that they have long ago ceased to be even roughly approximate bases for the construction of sales quotas. Within the past ten years many new suggestions have been tried out, and within the past five years substantial progress toward more accurate methods has been made. In this connection, one of the principal efforts has consisted of weighting the counties according to some measure of purchasing power.

"One of the earliest of the widely accepted suggestions for indicating buying power by counties, was that developed by the Curtis Publishing Company in which it based the buying capacity, first of states and then of cities, and finally of counties, on the combined circulation figures of their three publications—a woman's magazine, a popular weekly, and a farm magazine. These three publications operating in three distinct fields had, for several years, been consistently at work building up circulation of these three types. These circulation efforts constituted a process of selection of three well-defined types of people which had been carried on consistently on a large enough scale and over a long enough period to result in characteristic selections from the total population. These figures have been widely used and have proved to be of real value in sales quota construction.

"A similar selection has been made use of by the Crowell Publishing Company, which pushed the use of such figures a step farther by combining them with other figures indicating the accessibility and purchasing power of the country by counties. Much use has been made of the figures on income prepared by the National Bureau of Economic Research, and the estimates of purchasing power by counties prepared by H. G. Weaver of the General Motors Corporation. With these in combination a set of figures has been developed covering effective incomes by counties which have proved to be of great value in sales quota construction.

"Another line of effort which paralleled these in a measure, in point of time, has been working out of trading centers. In this work, the two most conspicuous contributions have been the list of 657 trading centers worked out by the International Magazine Company, and the figures covering the 81 cities of 100,000 population, or over, combined with the population and purchasing-power figures for the surrounding territory covered by a leading newspaper in each of the cities."

A compilation of statistical data, which no one should be without who is concerned with the determination of purchasing power by trading areas, is the "Consumer Market Data Handbook of the United States" (published by the United States Bureau of Foreign and Domestic Commerce). This handbook presents for the counties of the United States data of many types, which can be used as indexes of purchasing power, both as to quantities and types. Another valuable statistical aid is the quarterly *Brookmire Income Map*, published by the Brookmire Economic Service, which forecasts consumer income by cities, states, and industries, see page 380. In problems involving national income data, the estimates of the United States Bureau of Foreign and Domestic Commerce, the National Bureau of Economic Research, and the National Industrial Conference Board are generally used.

More will be said about the application of purchasing-power data in Chapter XXI on Marketing Analysis.

General Motors Purchasing-power Index.[1]—As an example of a practical method of developing a purchasing-power index on a county basis, the General Motors Purchasing Power Index will be described briefly. This explanation will also serve as a suggestion of how similar statistical problems may be approached and worked out on a practical basis.

"Many data exist which reflect directly or indirectly the relative economic importance of the various counties in the United States, and extensive use has been made of them in connection with market analysis and advertising. The market for the average product depends upon purchasing power and this, in turn, depends primarily upon the income enjoyed by a given territory.

"The county data generally available are:

Value of manufactured products.
Value added by manufacture.
Value of mineral products.
Value of fishery products.
Value of farm crops.
Population data.
Income-tax data.
Magazine circulation.
Number of retail outlets.
Automobile registrations.
Number of telephones.
Bank deposits.
Number of banks.
Industrial wage earners.

"Incomes by counties are not available, but for certain past years reliable estimates of incomes by states have been developed by the National Bureau of Economic Research. On the basis of tax returns, crop reports,

[1] This description of the General Motors Purchasing Power Index is quoted from an article on the subject by H. G. Weaver of the General Motors Corporation, in *Manufacturing Industries*, Vol. XI, No. 4. The index has been used extensively by the General Motors Corporation. Mr. Weaver was awarded the Edward Bok prize of $2,000 for this development as the most outstanding research in the selling and advertising field in 1925.

debits to individual accounts and other available data, it is possible to develop similar estimates for later years. By this method the income of each state in the union was estimated and these estimates, coupled with the estimates of the National Bureau of Economic Research, provided us with data whereby the average income of each state might be estimated for the past five years.

"It was evident that if some kind of formula could be developed which would enable us approximately to calculate this known quantity of income for each state, and if in the development of this formula we would use only such data as were also available by counties, we would then have a formula which, in all probability, would be approximately true for the calculation of county income.

"By reducing the state data to a percentage of the United States basis it was possible to appraise each factor as regards its correlation with the per cent of effective income in each state. Over 1,000 tests were then conducted involving various combinations of the percentage data to determine just which elements provided the best indices to effective income. The following were chosen as the most pertinent or 'appropriate' material to use for this particular purpose:

A = Per cent of value of products including:
  Added by manufacture (United States Census, 1919).
  Farm crops (United States Census, 1920).
  Live-stock products (estimated by Crowell).
  Mineral products (United States Geological Survey, 1922–1923).
  Fishery products (United States Bureau of Fisheries, 1920–1923).
B = Per cent of retail outlets (R. L. Polk Census, 1922).
C = Per cent of total population (United States Census, 1920).
D = Per cent of income-tax returns (United States Bureau of Internal Revenue, 1922).

"It was found that the proper combination of these percentages lay somewhere between a simple average and a weighting of the income-tax data twice, three, four, and five times. The formula reads:

$$\frac{A + B + C + M \times D}{N},$$

where M is a multiple (1, 2, 3, etc., of the factor D) and N is an arbitrary divisor.

"Thus, the State of Alabama represents:

A = 1.22 per cent United States total value of products.
B = 1.26 per cent United States retail outlets.
C = 2.22 per cent United States total population.
D = 0.64 per cent United States income tax returns.

"The approximate ratio to be arrived at is:

$$K = 1.09 \text{ per cent United States total effective income.}$$

"The four pertinent factors were worked into formulas as follows:

$$\textit{Formula } 1.\quad \frac{1.22 + 1.26 + 2.22 + 0.64}{4} = 1.33 \text{ per cent.}$$

$$\textit{Formula } 2.\quad \frac{1.22 + 1.26 + 2.22 + 2 \times 0.64}{5} = 1.20 \text{ per cent.}$$

$$\textit{Formula } 3.\quad \frac{1.22 + 1.26 + 2.22 + 3 \times 0.64}{6} = 1.10 \text{ per cent.}$$

$$\textit{Formula } 4.\quad \frac{1.22 + 1.26 + 2.22 + 4 \times 0.64}{7} = 1.04 \text{ per cent.}$$

$$\textit{Formula } 5.\quad \frac{1.22 + 1.26 + 2.22 + 5 \times 0.64}{8} = 0.99 \text{ per cent.}$$

"By applying these five formulas to all the states it was found that the following number of calculations fell within 10 per cent of the known quantity K, the estimated state income: Formula (1) 26; (2) 35; (3) 46; (4) 44; (5) 41.

"While the foregoing results are predicated upon a 'cut-and-try' process, there is ample scientific justification for the use of these combined factors as an index to income. However, it was evident that no one formula would answer for all states, but that a final choice would have to be made on the basis of individual state groups. Hence a 'similarity table' of states was worked up in seven columns headed:

1. Average effective per capita income.
2. Ratio $\dfrac{\text{effective income}}{\text{number of tax returns}}$.
3. Per cent population in cities over 25,000 (1920 census).
4. Retail outlets per 100 square miles farm area (Polk and United States Census).
5. Per cent native white population (1920 census).
6. Ratio $\dfrac{\text{value added by manufacture}}{\text{value crops and live-stock products}}$ (Crowell Publishing Company).
7. Value of mineral and fishery products as a per cent of total value of products (Crowell Publishing Company).

"An exhaustive analysis of this table led to the conclusion that states could not be successfully grouped solely on geographical location. The only practical grouping for measuring purchasing power seemed to be by characteristic factors as developed in the similarity table.

"The basic classifications determined upon through the use of this table were as follows:

1. *Southern Group.*—Characterized by low per capita income and extensive agricultural operations. (South Carolina, Georgia, Alabama, Mississippi, etc.)

2. *Sparsely Settled Western States.*—Characterized by high incomes, range-cattle and mining operations. (Arizona, Nevada, Montana, etc.)

3. *Urban Group.*—Thickly settled industrial states having a high percentage of population in large cities. (New York, Rhode Island, Massachusetts, Illinois, Michigan, etc.)

4. *Miscellaneous and Hybrid Groups.*—Some states do not fall under the distinct classification and must therefore be tested with groups especially designed for the purpose. (West Virginia, for example, appears to be a cross between Pennsylvania and Oklahoma. Maine, New Hampshire, and Vermont have the characteristics of Massachusetts, on the one hand, and Montana on the other.)

FORMULAS APPLIED TO INDIVIDUAL STATES

State	Formula	State	Formula	State	Formula	State	Formula
Alabama	2	Iowa	3	Nevada	2	South Dakota	2
Arizona	2	Kansas	3	New Hampshire	3	Tennessee	3
Arkansas	2	Kentucky	3	New Jersey	4	Texas	2
California	4	Louisiana	2	New Mexico	2	Utah	3
Colorado	3	Maine	3	New York	4	Vermont	3
Connecticut	4	Maryland	3	North Carolina	2	Virginia	2
Delaware	3	Massachusetts	4	North Dakota	2	Washington	3
Dist. of Columbia	..	Michigan	3	Ohio	4	West Virginia	3
Florida	2	Minnesota	3	Oklahoma	3	Wisconsin	3
Georgia	2	Mississippi	2	Oregon	3	Wyoming	2
Idaho	2	Missouri	3	Pennsylvania	4		
Illinois	3	Montana	2	Rhode Island	4		
Indiana	3	Nebraska	3	South Carolina	2		

EXHIBIT 188.—The General Motors Corporation found it necessary to use different formulas for different states in its purchasing-power index. This illustrates a common problem in business statistics. Uniform formulas or uniform methods may not produce the most uniform results.

"We are now in a position to apply our formulas to selected groups corresponding more closely to the conditions represented by the relations between the different counties within an individual state.

"On the basis of the various groupings and combinations of groupings, 18 groups were tested in addition to the preliminary tests on the United States as a whole. These tests not only covered states that were economically similar but also a number of groups that dovetail into one another with the idea of getting as many different kinds of combinations as appear likely to occur as between the counties in any single state.

"Through such group testing, the formula best suited to each state was determined, as shown in Exhibit 188. Incidentally, the formula finally chosen for a given state was not necessarily the same as the formula which showed the best agreement for that particular state according to the calculations above described.

"In the case of such states as Alabama, Mississippi, Georgia, etc., it was established that Formula 2 would give the most accurate results. Hence, the conclusion is drawn that Formula 2 will give the best results for the individual counties composing Alabama.

"By applying the percentages developed in this way to the effective income for the state as a whole, we arrive at an estimate of the total effective income received by each county, and then by dividing this effective income by the population of the county we can determine the effective per capita income for each county. Our final product, therefore, provides us with income data for each county expressed in three ways:

1. The effective income of each county as a per cent of state total.
2. The amount of effective income received by each county expressed in dollars.
3. The effective per capita income of each county expressed in dollars.

"The following example shows the derivation of this data for Autauga County, Alabama:

Autauga County as a per cent of the Alabama state total has

.91 per cent of total value of products in the state.
.59 per cent of retail outlets in the state.
.80 per cent of total population in the state.
.23 per cent of number of income tax returns in the state.

$$\text{Income ratio} = \frac{.91 + .59 + .80 + 2 \times .23}{5} = .552 \text{ per cent.}$$

"In other words, Autauga County receives .552 per cent of Alabama's income. The average effective income of Alabama for the past five years is estimated at $706,600,000. Therefore, .552 per cent of $706,600,000 gives an estimate of $3,900,432, representing the annual effective income received by Autauga County. The population of Autauga County is 18,908. And $3,900,432 divided by 18,908 gives us an average annual effective per capita income of $206.

"The adaptation of such basic data to the practical problems of marketing must obviously vary according to the individual nature of each product."

Estimating Local Purchasing Power.—When it is desired to have a definite representation of the local geographical distribution of purchasing power, it is practically always necessary to resort to some method of estimating, as no census of purchasing power is ordinarily available. If a careful search is made, it is usually possible to find information which can be used to make a fairly detailed estimate of the purchasing-power situation. This can be illustrated by referring to a survey which was made in connection with locating a large store in Los Angeles. The question had arisen as to whether, by moving from the old shopping

center, the store could be more conveniently located in relation to the purchasing-power class from which it drew its trade. Consequently, it was desirable to know how the purchasing power was distributed throughout the city by four different groups—namely, high-class purchasing power, high middle-class purchasing power, low middle-class purchasing power, and low-class purchasing power.[1] It is obvious that in such an instance, a house-to-house canvass could not be made for two reasons. In the first place, the cost would be prohibitive; and in the second place, the chances are that, as a rule, the person interviewed would either refuse to give the information or would give inaccurate or unreliable information. Therefore, the problem was to find some reliable index which would indicate the amount of purchasing power for the city and how it was distributed.

It was found that about two years previously one of the public utilities had made a survey which showed estimated rental figures for every family capacity in the city. These were classified by different types of dwellings and were recorded by about 500 small sections of the city. While these data were confidential, it was possible to make arrangements to use them for the purpose at hand. An investigation was then made on a sampling basis which determined the relation of rents to income. The next step was to convert the rental figures for each of the 500 small sections to income figures on the basis of this factor. In this manner, the amounts of purchasing power for 500 different sections of the city were secured, but there still remained the problem of bringing them up to date. In this case, this was accomplished by compiling building permit data which showed the value of new dwellings by classes that corresponded to the four income groups for the 500 sections of the city. In this process, of course, a factor was determined which would show the relation of the cost of the new dwelling to the average income. It was necessary also to correct the data for vacancies. Then these increases in purchasing power were added to the amounts that had already been determined.

General Application of Preceding Methods.—The examples described in the preceding discussion illustrate the problems that actually arose in making practical estimates of population and purchasing power. The methods as specifically described may not be entirely satisfactory for every project, as different problems will arise and other data may have to be used. They should, however, provide a working basis and indicate an approach that can be followed in solving such problems. An appreciation or understanding of the problems described will provide a back-

[1] Classification of purchasing-power data by income groups, whether on the basis of large or small geographic areas, is often of great importance in making practical applications. The sales of Cadillac or Lincoln cars, for instance, could hardly be estimated accurately on the basis of the total purchasing power in an area.

ground which will enable these or similar methods to be fitted, not only to other population and purchasing-power problems, but to problems of making other kinds of estimates. Alterations and improvements can be made that will be practical for the particular purpose at hand.

Questions and Problems

1. Discuss the importance of current population data in establishing business enterprises, and in organizing sales forces.

2. Suggest several bases on which the population of a given locality may be classified to assist in formulating business policies.

3. Why is it important to distinguish between permanent and non-permanent population?

4. Is the Federal Census of population the only one taken? How often is it taken?

5. List several common bases for making population estimates.

6. What are the limitations of the projection-of-rate-of-growth method? What modifications have been applied in Federal Census Bureau estimates?

7. Explain the method of estimating population on the basis of water services, and point out some of the difficulties in using this method.

8. How may building permit data be used for estimating population and what collateral data must ordinarily be used?

9. How are city directory data and school enrollment used in estimating population, and what difficulties are likely to be found in using these data?

10. What public utility services (other than water) are useful in estimating population? Discuss.

11. What are the difficulties encountered in the house-count method of estimating population, and in what ways does this method differ from those previously considered?

12. When are semiannual, quarterly, and monthly population estimates necessary?

13. How can the non-permanent population of a city be estimated?

14. Of what importance is an estimate of the purchasing power of the population?

15. As incomes increase, does the percentage of income spent for food tend to increase, decrease, or remain stationary? The percentage for clothing? For rent, fuel, etc.? For luxuries?

16. How has the national income increased during the last 20 years? Of what significance is this in analyzing purchasing power?

17. Is the place of greatest density in population always the place of greatest purchasing power? Discuss.

18. How did the Curtis Publishing Company indicate buying power by counties?

19. How does the Crowell Publishing Company indicate purchasing power by counties?

20. Describe the development of the General Motors Purchasing Power Index.

21. How can the distribution of purchasing power within a city be determined?

22. Can purchasing power be estimated accurately by house-to-house enumeration? Why?

23. On the basis of methods similar to those described in this chapter, how would you estimate the number of prospects for electric irons? For screen door silencers? For a new breakfast food? For a new theatre?

24. On the basis of available series, estimate the population of your city, or of a certain area, as of January 1 of the current year.

25. On the basis of available series, estimate the purchasing power of your city, or of a certain area, as of January 1 of the current year.

CHAPTER XX

PRODUCTION AND LABOR STATISTICS

It is the purpose of this chapter to point out some of the more important ways in which statistics are used in increasing efficiency in production.[1] Since the limited scope of the present chapter does not permit detailed consideration of the different kinds of production (farming, mining, manufacturing, etc.) the discussion is confined to the more important uses of statistics in manufacturing. The principles considered, however, are fundamental, and can usually be applied to the other kinds of production.

Approach to the Problem.—As previously indicated, the business investigator must have a definite and orderly way of approaching a production, labor, or other problem. This is a practical necessity if the final report is to be satisfactory in scope, cost, and results.

The first step in approaching the problem is to define the *real* objective. For example, the real objective in wage fixing is not to pay according to productivity; it is to adopt such a combination of wage payments, sick benefits, pensions, unemployment reserves, etc., as will result in the most effective long-time combination of low total unit costs and contentment of working forces. Moderate increases in wages and benefits with length of service, regardless of individual efficiency, may produce the best results, and it may be of utmost importance to collect information on this question, if the real objective of a wage-fixing investigation is clearly and accurately defined.

The next step in approaching the problem is to eliminate, or at least to recognize, the personal equation. Statistical investigations are not rigid. They may be greatly influenced by personal opinions or bias on the part of the investigator. Many factory managers, as well as other business men, do not realize the extent to which their own personal interests interfere with arriving at a scientific solution.

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[1] In this text the term "production" is used as it is broadly understood in business. It includes such industries as farming, mining, lumbering, and manufacturing or, in other words, those activities which are principally involved in the creation of form utilities. Attention is called to the meaning of the term production because of the fact that the business man does not use the term with the same meaning as the economist. To the business man, the term means the creation of form utilities, while to the economist, the term includes also the creation of time and place utilities.

Another step in approaching the problem is to break up the major problem into its several component parts. The solution of these minor units is much more simple than the solution of the problem as a whole. The importance of this procedure will be readily appreciated in the discussion which follows, especially in connection with production control and motion studies.

After the problem has been separated into its parts, the factors entering into the solution of each part should be listed and arranged in a logical order. Sometimes these factors can be weighted in accordance with their relative importance. The advantage of having the problem laid out in this way is obvious.

Finally, the investigator should take a fresh point of view. And the business man should make a conscious effort to take a fresh point of view from time to time throughout the investigation. The value of this open-mindedness is readily appreciated when it is realized how easily bias and prejudice creep into an investigation, and how greatly business is hampered by tradition.[1]

Factory Location.—Among the more important factors which should be taken into consideration in locating a plant are: access to markets, sources of material, transportation and storage facilities, supplies of labor, supplies of capital, site land conditions, climatic conditions, facilities for repairs, and such indirect factors as living conditions. Systematic study of these factors should provide a basis for locating most factories in such a way that they will have a distinct advantage over industries which are not well located. Most factories, however, have not been located in this way, and probably most industries are still located on the basis of convenience and sentiment. This is usually the basis of selection when a man starts an industry in his home town.

Suggestions for analyzing markets from the point of view of both selling and buying will be given in a later chapter on market analysis, and site land problems will be discussed in a later chapter on real estate. The other factors in determining plant location will be discussed indirectly from time to time.

The uses of statistics in manufacturing will now be considered from the point of view of an operating factory.

Production Control.—Production control (in manufacturing) may be defined as "that system which, extending over a long period of time, controls the order of movement of the elements of a productive program in relation to each other and to the whole."[2] The more numerous the

[1] See A. W. Shaw, "An Approach to Business Problems," pp. 13–24, Harvard University Press, Cambridge, 1916.

[2] Babcock, George D., in "Management's Handbook," p. 599, edited by L. T. Alford, The Ronald Press Company, New York, 1924.

elements involved, the greater will be the difficulty in systematizing and controlling them, and the greater will be the value of statistical aids.

The greatest precision of operation is obtained by accurately planning in advance every operation and every movement necessary to the manufacture of the final product. This involves a specific knowledge of: (1) kind and quantity of material required; (2) time required to obtain raw material; (3) methods of performing required manufacturing operations; (4) productive capacity of equipment; (5) time required for each operation on the product; and (6) limits of quality necessary to manufacture.[1]

While not all of the information required in the preceding list is statistical, statistics are necessary throughout the list in order that the production manager may plan a logical and definite sequence of operations, and so that he can accomplish a definite objective at a definite time.

The Production Schedule.—Products may be manufactured for stock or they may be manufactured on customers' orders. The product of a manufacturing plant may fall in both or either of these classes. In manufacturing either type of product, a schedule of production is ordinarily essential. The preparation and operation of the production schedule is an integral part of the budget system described in Chapter XVIII, although it may be used where the budget plan has not been adopted.

In formulating a production schedule, the first step is to make an estimate of the probable demands for the product.[2] A sales analysis to be used in establishing a production schedule should include an estimate of the probable demand for as long a period as possible, and in most instances this should not be less than three months, while in many instances the period should be not less than six months or even a year. In addition to this, a sales analysis should show the seasonal, cyclical, and other variations. From these data, the manufacturing schedule may be determined. Such a production schedule is shown graphically in Exhibit 189.

In Exhibit 189, which was prepared before the beginning of the year, curve *A* represents expected monthly sales for the year. Curve *B* represents the same data as *A* but expressed as cumulative sales, or sales to date, from the beginning of the year. Sales, it will be noted, are subject to a heavy seasonal variation. And it would not be economical to have a similar seasonal variation in production. However, if the monthly production shown by curve *C* is carried out, the net monthly inventory will

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[1] See second footnote, p. 454.

[2] For methods of making market analyses, see Chap. XXI.

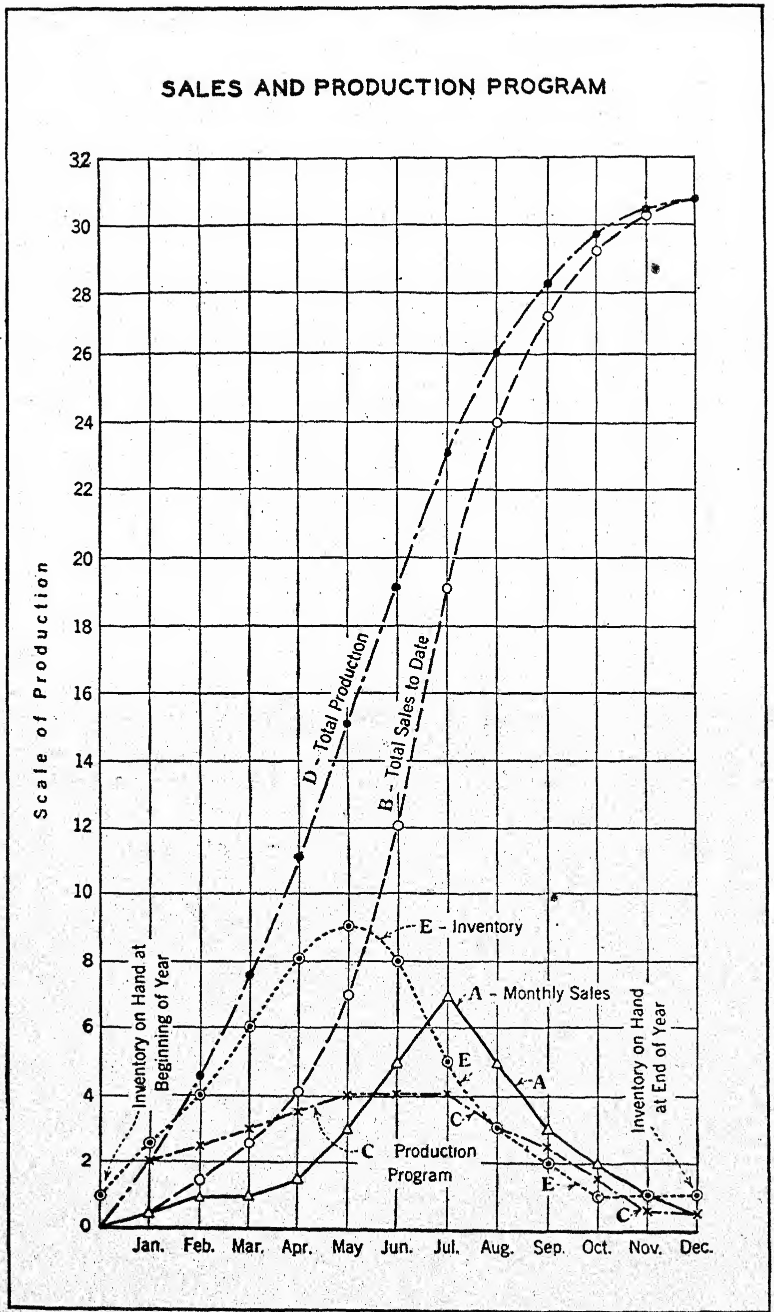


EXHIBIT 189.—Scheduling future production in relation to expected sales. (*Adapted from "Management's Handbook."*)

be as indicated by curve *E*. This furnishes the necessary margin of safety between production and sales, as shown by comparing the cumulative curves *B* and *D*.

Purchasing and Stock Records.—Before actual manufacturing begins, materials must be purchased, received, stored, and issued. In the larger concerns, the purchasing department usually is separate from the stores department, while in the smaller concerns they may be combined. In the present discussion. the two types of work will be considered together.

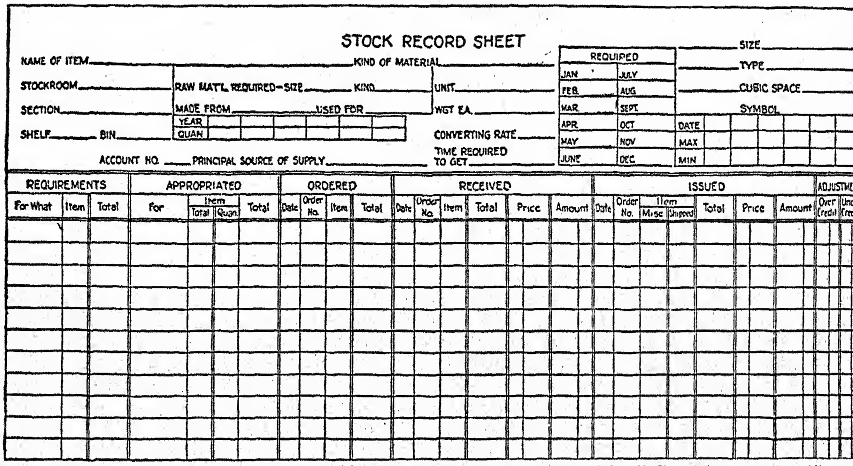
The purchasing department should have complete records of sources of materials together with an accurate description of the services rendered and time required for delivery. Definite grades of the quality and figures which will indicate the dependability of the sources should be included, and sources of supply other than those commonly used should also be recorded. Some of the figures may duplicate information elsewhere in the accounting records, but it is well for the purchasing department to have such information readily available in its own files, especially in large organizations.

The purchasing procedure includes: (1) the receipt of a request for purchase from the department for which purchases are to be made; (2) the issuing of a purchase order; (3) the recording of the material received; (4) the preparation of (*a*) orders to return materials received and (*b*) debit or credit memoranda to the accounting department to adjust invoices for materials returned and for price or quantity errors on these invoices; and (5) entries upon the invoice of the account names or numbers to be charged, and other information which will assist in obtaining authorization for payment. Where the duties of the stores department are performed by the purchasing department, this department will also maintain the stock records, and will control the physical location and the issuing of materials.

Each part of the mechanism involved in purchasing, storing, and issuing materials requires carefully prepared forms, upon which a complete record of the procedure should be made. The "request for purchase" form (also called "purchase requisition"), for example, should show the quantity, description, grade, and price of the articles desired, the data for delivery, the firm from which the articles may be obtained, and the signature of the department foreman and plant superintendent. It is to be noted that many purchases will be made which will not originate with the department that will use the materials, because certain standard materials are maintained in stock for recurring requirements. For such materials, the "request for purchase" will come directly from the stores department when the quantity on hand has been reduced to a minimum or "margin of safety" point.

Forms of purchase orders, of records of materials received, of debit and credit memoranda, of return shipment orders, of invoice authorization, and of distribution stamps will not be described in this discussion. The nature of the information required is suggested in the name of each form. The stock records to be maintained in the stores department, or in the stores section of the purchasing department, will now be considered.

"Stock" records (also termed "stores" records, "balance of stores" records, etc.) are maintained to show the exact situation as regards materials on hand, received into stock, ordered but not received, issued upon material requisitions, and apportioned for manufacturing requirements but not yet issued. Information that will assist the purchasing department in connection with ordering materials may also be included


STOCK RECORD SHEET

NAME OF ITEM ____ KIND OF MATERIAL ____

STOCKROOM ____ RAW MAT'L REQUIRED—SIZE ____ KIND ____ UNIT ____

SECTION ____ MADE FROM ____ USED FOR ____ WGT EA. ____

SHELF ____ BIN ____

YEAR						
QUAN						

CONVERTING RATE ____

ACCOUNT NO. ____ PRINCIPAL SOURCE OF SUPPLY ____ TIME REQUIRED TO GET ____

REQUIRED	
JAN	JULY
FEB.	AUG
MAR.	SEPT.
APR	OCT
MAY	NOV
JUNE	DEC

____ SIZE ____

____ TYPE ____

____ CUBIC SPACE ____

SYMBOL

DATE						
MAX						
MIN						

REQUIREMENTS			APPROPRIATED				ORDERED				RECEIVED						ISSUED							ADJUSTMENT	
For What	Item	Total	For	Item Total	Item Quan.	Total	Date	Order No.	Item	Total	Date	Order No.	Item	Total	Price	Amount	Date	Order No.	Item Misc	Item Shipped	Total	Price	Amount	Over Credit	Under Credit

EXHIBIT 190.—Stock record sheet. (*From Jordan and Harris, "Cost Accounting."*)

in the stores records. In Exhibit 190, a stock record sheet is illustrated upon which there can be recorded all of the information suggested above. The forms used in different manufacturing companies vary greatly in arrangement and appearance, and the number of forms used to record the essential information also is not uniform. The illustration presented here provides in one place a large portion of the information needed in purchasing for stock requirements and for the control of stock.

At the top of the stock record sheet, shown in Exhibit 190, space is provided to state for the particular item of material such facts for the purchasing department as: (1) the principal source of supply; (2) the time required to get delivery; (3) the specifications of size, ingredients, etc.; (4) the usual monthly requirements, or budgeted requirements, of the manufacturing plant; and (5) the maximum and minimum quantities set for normal stock conditions. To control the location of the physical

stock, the position of the materials in the storeroom is entered on the form. Six groups of columns are provided on the form illustrated in Exhibit 190. As requirements are ascertained from the production planning department, the quantities are noted in the first group of columns. Quantities appropriated from stock on hand for the known requirements are entered in the second series of columns. From the purchase orders issued, the quantities that have been ordered are entered in the "Ordered" columns. When the goods are received, the quantities, prices, and amounts from the invoices are entered in the columns headed "Received." Likewise, when the material requisitions are filled, the quantities, unit prices, and total amounts or values are entered in the "Issued" columns. At the same time that these issues are entered, the quantities are deducted from the "Appropriated" columns, as it is no longer necessary to reserve stock ordered or on hand for these requisitions.

In order to determine the material on hand at any date, the quantities in the "Received" column are totalled, and those in the "Issued" column are also totalled. Periodically (monthly, quarterly, etc.) these columns are closed by ruling, and the balance on hand is brought down below the ruling in the "Received" column. In valuing the inventory balance on hand, the price to be used may not correspond with the price at which the balance on the record is valued. Also, when the materials are physically counted, shortages may appear. Therefore, columns are provided for adjustments of the amounts (monetary values), which are made at the time the balances are entered.

Some stock record cards provide columns for the balances, in quantities and amounts, that remain on hand after each receipt and issue. The use of such forms requires the computation of a balance after each entry. However, the form illustrated in Exhibit 190 is usually satisfactory, as the balance of material on hand can be quickly determined from the running totals, when needed.

Other record cards that are used in connection with the control and issue of materials include the bin tag (used to indicate quantities in each bin or stores compartment), the requisition for material that is in stock, and the material credit slip (for material returned to stock). The nature of each of these records is indicated by its title.

The purchasing department also needs information on markets and market-price conditions in order that it may purchase in the best markets at the best prices. Since the work of analyzing market conditions in buying materials for manufacturing is the same as that in buying materials for resale, the discussion of the technique involved in studying market conditions is included in Chapter XXI on Market Analysis.

Control of Material.—Material control covers the following: checking quality in accordance with definite specifications or standards; requisi-

tioning and purchasing in accordance with a predetermined manufacturing program; receipt in stores; issue to the manufacturing department; and movement through the plant until the finished product is delivered to the warehouse or shipping department.

In the first place, the raw-material requirements must be determined. From a study of the product to be made, the total quantity of each class of material needed for a unit of the finished product is estimated. This quantity, multiplied by the number of units to be made, plus the proper allowance for waste or spoilage, determines the minimum quantity of materials for which orders should be placed. If the material is of the kind that is used continuously, it is customary to keep on hand a minimum quantity sufficient to provide for manufacturing requirements while the supply is being replenished. Whenever the quantity reaches this minimum, orders should be placed for additional material.

The time required to secure raw materials is an important factor in production control. The time that must be allowed depends upon such factors as distance from source of supply, the competitive condition in the market, and whether the materials are kept in stock or have to be manufactured on order. The average and maximum lengths of time that must be allowed for purchases of materials should be tabulated for the use of those who determine when orders should be placed. Exhibit 191

NUMBER OF WORKING DAYS TO BE ALLOWED FOR RECEIVING STOCK AFTER PURCHASE ORDER IS PLACED

Commodity	Time from warehouse	Time from mill	
		Average	Maximum
Black sheet steel	10	60	[illegible]
Cold-finished steel—round	10	60	[illegible]
Cold-finished steel—square	10	60	[illegible]
Drop forgings	..	40	60
Malleable-iron castings	..	45	60
Mild-steel plate	10	60	90
Seamless steel tubing	10	45	60
Yellow bar brass	20	90	120

EXHIBIT 191.—Delivery time table used in production control.

shows such a tabulation taken from the records of an Illinois manufacturing concern using various kinds of metal products.[1] Such figures, of course, should be carefully revised from time to time in accordance with the changes in the market situation and other conditions.

[1] The table was compiled from a more extensive table in "Management's Handbook," p. 608.

Routing.—Efficient routing of products through a manufacturing plant requires the following:[1]

1. Determination of possible methods of manufacture and a selection of the best method.
2. Determination of needs for special equipment.
3. Determination of capacity of available equipment.
4. Determination of sequence of operations.
5. Decision as to speed of operations.
6. Determination of time required for each operation.
7. Preparation of route sheets.
8. Grouping of route sheets into assemblies so that the components of the final product will be started in process at such time as will insure their being completed together.
9. Preparation of work orders for carrying the routing details into effect.

It is obvious that statistical records are necessary in connection with the preceding list. The first requirement involves accurate records of time, materials, and waste, as well as some definite determination of results. The second item requires the same treatment in connection with special equipment. The third requires similar records regarding available equipment and its capacity. In the fourth, statistics are not, ordinarily, so important. In the fifth, accurate records regarding the most economical speed are necessary; and in the sixth, detailed records must be compiled in order to determine the time required for each operation. In the seventh, the value of statistics is largely indirect. The eighth item involves the accurate determination of time required in process and a careful coordination of the statistics. The ninth is a process of a routine nature in which the importance of statistics depends upon the previous items.

[redacted] Gantt Chart.—Up to the present time, the principle of the Gantt [redacted] applied most extensively to industrial production. There [redacted]any possibilities of extending its use in this field as well as in [redacted]ers. Most of the Gantt charts used at the present time may be considered as falling into three general classes: (1) man and machine record charts; (2) layout and load charts; and (3) progress charts. The man record chart indicates whether or not the worker makes the proper use of his time, and if not, it indicates the reason why. The machine record chart shows when a machine is not in use and the reason why. The purpose of the layout chart is to plan work so as to avoid idleness of men and equipment and to get work done in the order of its importance. The load chart shows, in hours or days, the amount of work ahead of a plant or any part of a plant. Since the executing of a plan is of equal importance with the making of that plan, it is the purpose of the progress

[1] List from "Management's Handbook," p. 622.

chart to get work done by showing a comparison of the accomplishments with the plan and the reason for any failure to live up to that plan. For the purpose of illustrating the basic principles of Gantt charts, two layout charts (Exhibits 192 and 193) and one machine burden record (Exhibit 194) will be described briefly.[1]

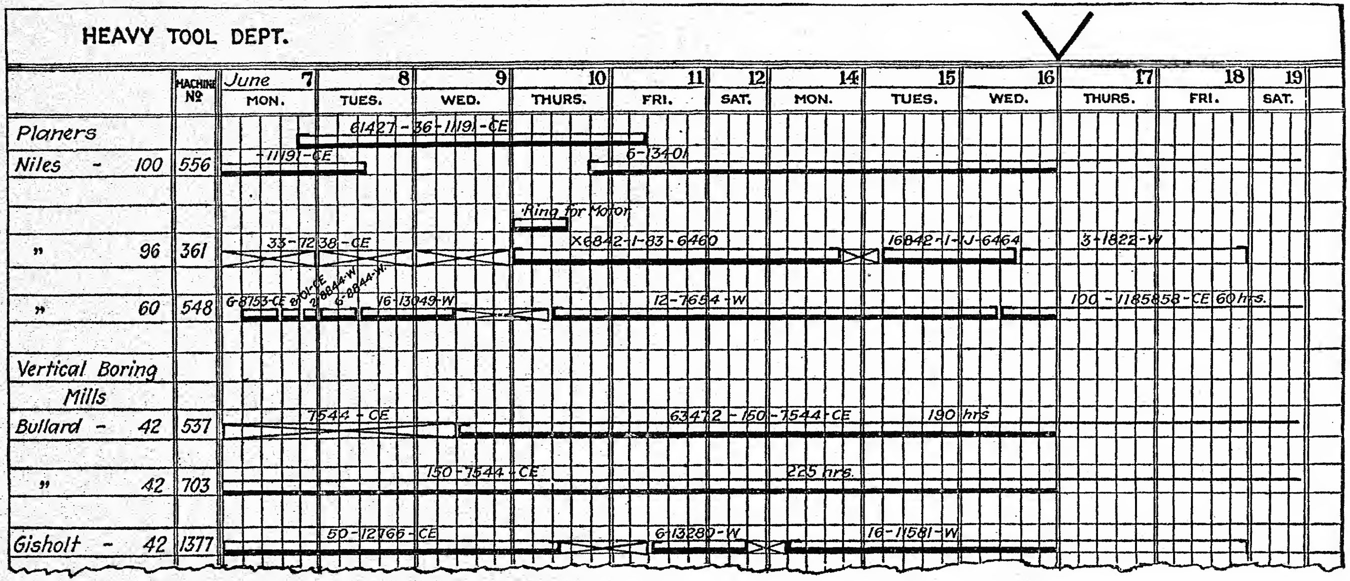
Exhibit 192 illustrates the form of Gantt chart which is used in scheduling production. The columns represent work days; these are subdivided into convenient fractional parts. To each piece of productive equipment two lines are allotted on the chart. When work is scheduled, the time at which an operation should start on any particular machine is indicated by placing a ⌈ mark on the lower line allotted to the machine. This mark is made in the column which represents the exact hour at which the operation is to start. The time at which each operation is to be completed is indicated by a ⌉ mark. A light line joining the two marks at the top indicates that the machine is not available for work during the period covered. Order numbers or any other desired information may be entered above this connecting line.

The performance of an operation is shown by drawing a heavy line between the bottoms of the two ⌈ ⌉ marks (Exhibit 192). If the time required for the operation is greater than that scheduled, the period of excess time is indicated by two intersecting inclined lines under a light line, as indicated in the legend of Exhibit 192. If an operation is completed in less than the scheduled time, this is shown by entering the next operation on the upper line allotted to the particular equipment, and if this job is completed ahead of time, the fact would be shown by its overlapping the next job, which would be entered on the lower line.

The machine-shop work layout chart in Exhibit 192 shows how the work was laid out in a department equipped with large machine tools on which but one job can be done at a time. On the first machine, the making of part 11191-CE, according to the schedule, was to have been finished Tuesday noon, but it was completed on Monday and Order 61427 was begun. This job was also completed ahead of the foreman's estimate (11 o'clock Friday) and the third order was begun Thursday afternoon. When the chart was copied on Wednesday the 16th, the work on this machine was just on schedule. (The check mark at the end of the day, Wednesday the 16th, indicates that the chart was reproduced Wednesday night and shows how the work stood at that time.)

On the second machine on the Gantt chart of Exhibit 192, the work was already three days behind schedule, when it was carried over from the

[1] For a detailed description of the making and using of Gantt charts, see Wallace Clark, "The Gantt Chart," The Ronald Press Company, New York, 1922. The present discussion of the Gantt chart is abstracted from Mr. Clark's book. See also "Management's Handbook," pp. 646–649.



Date job is scheduled to start.

Date job is scheduled to be completed.

Total time scheduled for order.

Work done.

Time required to make up for past delays.

Figures above lines indicate order numbers.

Indicates that chart was reproduced Wednesday night and shows how the work stood at that time.

REASONS FOR STOPPING WORK

B—Break-up.
H—Lack of help.
M—Lack of material.
P—Lack of power.
R—Repairs.
T—Lack of tools.

EXHIBIT 192.—Gantt work layout chart for a machine shop. (*From "The Gantt Chart" by Wallace Clark.*)

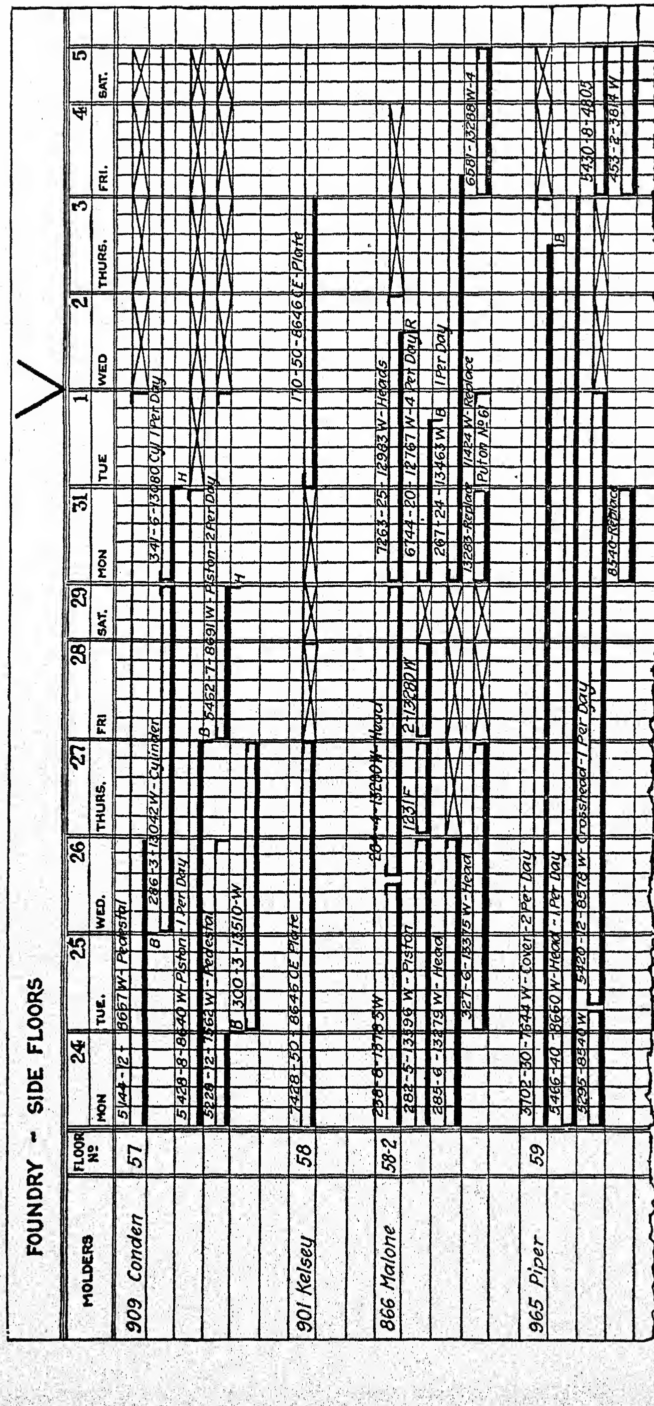


EXHIBIT 193.—Gantt layout chart for a foundry. (For explanation of symbols, etc., see Exhibit 192. (From "The Gantt Chart" by Wallace Clark.)

previous sheet. At that time, order X 6842 was scheduled to be begun Thursday morning and to be completed Monday afternoon, but it was necessary to run in an emergency job, a ring for a motor, which caused a four-hour delay (indicated by crossed lines) before Order 16842 could be started. When the chart was copied Wednesday night the work on this machine was four hours behind schedule.

On the molding floors in a foundry it is possible to do more than one order at a time. Consequently the laying out of work in a foundry differs from that for machine tools as discussed in connection with Exhibit 192.

In Exhibit 193, a work layout chart for a foundry is presented. According to this chart the first molder (Conden, on floor 57) could put up molds for three orders each day. This chart shows that he worked on Order 5144 until Wednesday morning when he had to "break up" (indicated by *B*, see legend in Exhibit 192) and start Order 286. He had finished this when the factory closed Saturday and on Monday morning he began work on Order 341, but was stopped Tuesday morning because of lack of help (see *H*). Then he went back to the first order, 5144, and did a day's work. Conden worked on his second order (5428) through the week until Friday morning when he had to break up (*B*) and begin work on Order 5462, on which work was stopped Monday morning because of absence of help (*H*). After one day's work on his third order, 5228, he broke up and worked on Order 300 until Thursday night. When the Gantt chart was copied on the first of the month (Tuesday), Conden was so far behind with the work already scheduled for him that crossed lines had to be drawn through the remainder of the week to indicate that this amount of time would be required before any further work could be assigned to him.

Machine Burden Records.—In order to schedule work properly in a factory, it is necessary to have an exact knowledge of the available machine capacity and the extent to which this capacity is being used. Such information is given by a machine burden record. This record shows all the equipment in the factory and the time required for performing each operation at each machine.

Forms of machine burden records vary with different classes of manufacturing. Exhibit 194 shows an example of a machine burden record where the manufacturing is largely repetitive in character. In using this form, a sheet is assigned to each machine in the factory. As each operation sheet is made out, the time required for each operation is entered on the burden record. If the total of the time so entered multiplied by the number of units of product scheduled for production each day exceeds the total working hours of the day the machine is overburdened, and the situation must be corrected if the production schedule

MACHINE BURDEN RECORD

Machine Symbol	B4V4
Serial No.	M-1078
Location	K1H2
	K1H2

Group BORING MILL VERTICAL

Machine Name BULLARD 1 SH 5 T 42"

LOT SIZE	LOT TIME Hrs.	LOT TIME Dec'l	SYMBOL	OPER. NO.	PIEC'S PER MODEL	IDEAL TIME Hrs.	IDEAL TIME Dec'l	Total Time Per Model Hrs.	Total Time Per Model Dec'l
50			5-11 61059	16-17 18-19	1		.241		.241
50			17604	2-3	2		.410		.820
50			17604	4-5	2		.177		.354
25			60281	8-9	1		.120		.120
24			60002	1-2	2		.623	1	.246
24			60002	3-4	2		.437		.874
50			61059	19A-19B	1		.210		.210
25			6925	2-3	2		.197		.394
25			6925	4-5	2		.354		.708
24			17678	2-3	2		.770	1	.540
						Total		6	.507

LOT SIZE	LOT TIME Hrs.	LOT TIME Dec'l	SYMBOL	OPER. NO.	PIEC'S PER MODEL	IDEAL TIME Hrs.	IDEAL TIME Dec'l	Total Time Per Model Hrs.	Total Time Per Model Dec'l
						Total			

Front

MACHINE DATA

Inv. P7775.00
101.77

Date Purchased 11/25/	Purchased From	Cost

EQUIPMENT 42" Bullard Turret Lathe

REPAIRS

DATE	AMOUNT		TOTAL		DATE	AMOUNT		TOTAL
Jan. 19					July 19			
Feb.					Aug.	Labor	.35	
Mar.					Sept.			
Apr.					Oct.	Labor	9.98	
May					Nov.			
June					Dec.			

NOTE:

DATE SOLD SCRAPPED	SOLD TO	VALUE

Back

Exhibit 194.—Machine burden record used in connection with scheduling production. (*From "Management's Handbook."*)

is to be followed. If the time assigned to the machine multiplied by the units of product scheduled per day is less than the working hours per day, the difference represents the time that can be assigned to other work.[1]

[1] The discussion of Exhibit 194 is abstracted from "Management's Handbook," p. 650.

Exhibit 195 shows a form of Gantt chart used as a machine shop burden record. On such burden charts, the classes or groups of operators, machines, workbenches, or floors are shown in the stub at the left of the sheet, and in the next column the numbers in each group are listed. In the month columns, the light lines show the hours of work which have

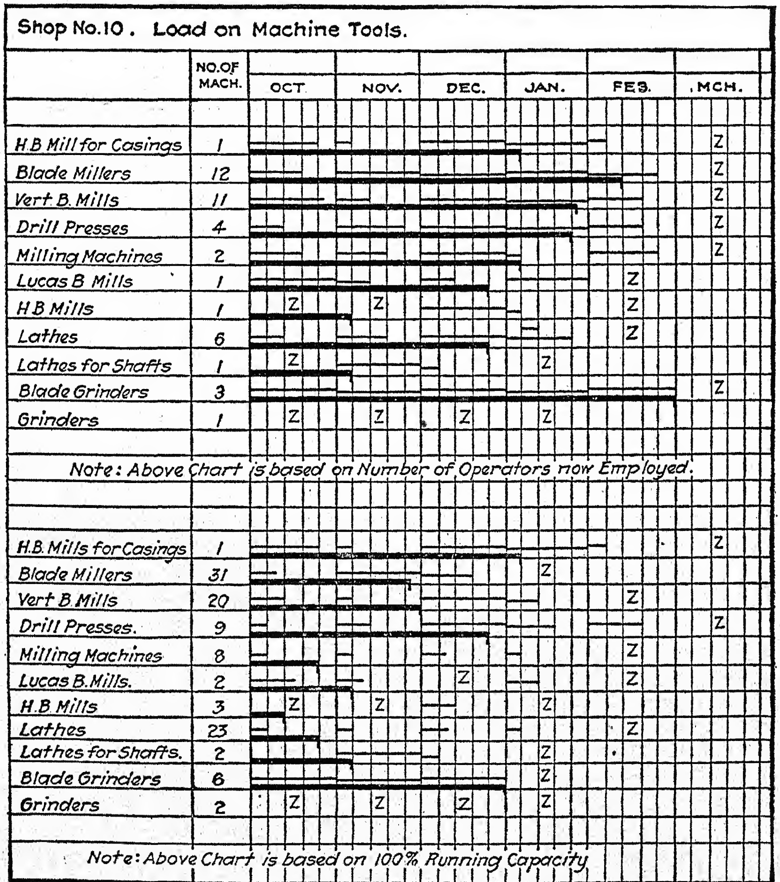


Exhibit 195.—Gantt chart used as a machine-shop load chart or burden record. (*From "The Gantt Chart" by Wallace Clark.*)

been assigned to that group during each month, and the heavy lines indicate the total hours of work to be done. The letter *Z* (zero) indicates no work ahead during the month in which it appears. The chart "shows how far in the future the machine tools will be kept busy by orders in the plant when it is drawn up. On the first line, for instance, it is clear that the one horizontal boring mill will be busy 80 per cent of the time during October, 20 per cent during November, and all of December and January." The information for this chart is secured from the layout

charts which show what orders are ahead of each machine; from these it is easy to add up the hours of work scheduled for each week or month. The Gantt load chart or burden record shows the manager or superintendent very clearly which machines are going to be kept busy in the near future, which ones are overloaded, and which ones have little or no work ahead.[1]

Dispatching.—A number of statistical records are necessary in dispatching. In production, dispatching consists of: assignment of work to machines or workmen in accordance with the schedule and according to importance or precedence; the procurement of tools, etc.; the initiation of inspection; movement of work from one operation to the next; recording of time of starting and completing each operation; and keeping records of completion of jobs, spoils or defective work, and idle time of workmen and equipment. The forms and apparatus used in dispatching include the work order or job ticket, the move order, the instruction card, the idle man card, the idle machine record, the route sheet, the work order file, dispatch racks, and time-stamps or clocks. Other forms which are sometimes valuable are the absence report, the tardy report, the behind-schedule tag, and the lot-urgent tag.[2]

The work order is particularly important in dispatching. This form should give the workman the necessary information concerning his job as regards the material and quantity upon which he is to work, the order number to which it is to be charged, the name or class of operation, and the time that is allowed for the completion of the job. It is desirable to have a record on each work order of the time it was issued and the time it was returned upon completion of the job. This record serves several useful purposes: it furnishes data on an individual's progress; it furnishes a basis for the comparison of different workmen; it furnishes a check upon the rate-setting department; it indicates the need of inspection of workmen's methods; and it may serve as a basis of wage determination.[3]

Control of Quality—Working Standards.—It is obvious that if the quality of a product is to be definite and uniform, some method of controlling the quality within certain specified limits is necessary. To state a quality at all accurately it must be compared with some arbitrarily selected standard. The determination of whether or not products are up to standard is accomplished by what is known as "inspection." The details of inspection vary widely with different products. Some prod-

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[1] This discussion of the Gantt load chart is abstracted from Wallace Clark, "The Gantt Chart," pp. 72–74, The Ronald Press Company, New York, 1922.

[2] For further details regarding dispatching and dispatching records, see "Management's Handbook," pp. 656–676.

[3] For additional information on time-keeping, see "Management's Handbook," pp. 675–676.

ucts can be inspected throughout the process of manufacture, while others can be inspected only after completion. Since inspection is often a costly process, it is important to determine just how much is necessary in order that there will not be a waste due to unnecessary inspection.

In manufacturing some products, it is necessary to use 100 per cent inspection, while in other instances a sample is sufficient. If less than 100 per cent inspection is sufficient, the method of sampling is employed. This involves the assumption that a piece selected at random probably is representative of the rest of the lot, or that a portion of a quantity of some substance is probably like the remainder.

Sampling can also be used in checking inspectors by occasionally sending the same part to the same inspector twice without notifying him. This procedure furnishes a good check upon how carefully the inspector is doing his work.

Operation Study and Rate Setting.—Operation studies are made in order that statistical facts may be substituted for guesswork in evaluating the work which the employer is buying and the employee is selling. These studies furnish a basis for raising the task to the highest standard that can be maintained satisfactorily. They make possible an accurate recording and analysis of the elements of a task, and this provides information which enables the workers to improve their skill. Operation studies ordinarily reveal sources of wasted energy and materials, and they indicate ways in which methods can be improved and unnecessary work eliminated. These studies also furnish a basis for establishing wage rates; they help the employer to make schedules of production and promises of delivery which he can maintain; and, by increasing efficiency, they lower the costs of production.

Analytical time and motion study is the basis of job standardization. When Frederick W. Taylor was confronted with the problem of systematic soldiering, he decided to determine what constituted a reasonable day's work. He discarded previous practices, and by breaking this complex problem down into its elements, studying these separately, and combining results, he developed methods for determining not the maximum work that a man could do on a short spurt for a few days, but "what really constituted a full day's work for a first-class man; the best day's work that a man could properly do, year in and year out, and still thrive under."[1]

Steps in Job Standardization.—Job standardization, or motion and time study consists of the following:[2]

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[1] TAYLOR, FREDERICK W., "The Principles of Scientific Management," p. 55, Harper & Brothers, New York, 1919.

[2] The outline of steps is abstracted from "Management's Handbook," pp. 799–800.

Preliminary

1. Recording and analyzing existing conditions as to methods and equipment to determine whether the best obtainable are being used.

Time and Motion Study

2. Breaking down an operation into its smallest motion elements.
3. Critical study of these elements, eliminating unnecessary delays, and discarding useless and inefficient elements. Improving the remaining ones until they are the best obtainable.
4. Putting together these improved elements into the best combination.
5. Determining allowances for necessary delay and for fatigue.[1]

Rate Setting

6. Determining the proper wage rate and incentives.[1]

Instruction and Training

7. Drawing up detailed instructions covering tools necessary, elemental operations involved in detail and in proper sequence with time allowed for each, and rate of pay.
8. Training the workman to perform the task in the manner and the time set until habit develops and motions are automatic.

Control of Conditions

9. Maintaining requisite working conditions, equipment, and supply of material.

"Mere statistics as to time which a man takes to do a given piece of work do not constitute time study. Time study involves careful study of time in which work *ought* to be done."[2] Though time study usually results in a shorter time, it is not a mere speeding-up process. It seeks to shorten the time by use of improved methods and elimination of wasted energy but not by over-speeding the worker. A task which results in immediate or gradual exhaustion of the worker has been wrongly set. Proper job standardization always limits the task to "that rate at which the worker can work indefinitely, year after year, thrive and be happy."

Methods of Operation Study.—Motion studies of production jobs are ordinarily carried on by two methods: (1) the stop-watch method; and (2) the micromotion method. The first method is based on stop-watch time measurements in the shop. The second method involves the use of the motion picture camera, the microchronometer, and other special devices. The breaking down of work into its elements is similar

[1] It is often well to defer Steps 5 and 6 until the new manner and time of performance have been thoroughly tested.

[2] TAYLOR, FREDERICK W., "The Present State of the Art of Industrial Management," *Transactions, American Society of Mechanical Engineers*, 1912.

in both methods, although the study can be worked out in greater detail under the micromotion method.

The stop-watch method, as compared with the micromotion method, requires less elaborate equipment and less specialized experience, it is easier to use, it creates less disturbance, and it is sufficiently accurate, adequate, and reliable for many practical simple motion studies. The advantages of the micromotion method over the stop-watch method are that it records many more conditions and the records are of more permanent value, the emphasis is placed on the method of performing the task, time and motions are recorded automatically and accurately, visual records of best practice may be used in teaching, and human judgment is reduced to a minimum.

Stop-watch Time and Motion Studies.—The value of stop-watch time and motion studies depends largely upon how thoroughly the governing conditions are noted and recorded. The first step in this connection is to prepare observation blanks which will adequately cover all requirements. It is imperative that all important items be entered and that all studies agree in arrangement, if the results are to be satisfactorily compared. Exhibit 196 shows such a blank for a short sewing-machine operation. In this instance, the length of the operation is determined by the quantity in a "bundle." The observation blank is divided vertically so that one side can be used for motions of the left hand and the other for those of the right hand. When both hands are used, the entry extends across the dividing line. It will be noted in Exhibit 196 that the items are divided into three groups. The first one, or "preparation" group (items 1 to 11) includes motion items which occur only once during the operation. It is separated from the "machine running cycle" (motion items 12 to 26 which occur with each garment). The "completion" group (items 27 to 31), also composed of motions which occur only once in this operation, is similarly separated.

In making a time study, the ordinary athletic stop-watch is not suitable, but stop-watches which are specially adapted to time-study work are readily obtainable. The athletic stop-watch is divided into tenths of a second, while the large dial on a time-study stop-watch is divided into one-hundredths of a minute. This decimal division facilitates the addition and computation of the readings.

If the elemental operations occur in a fixed order, it is best to enter them on each blank before beginning to record other information. When the order cannot be determined, they may be listed at the bottom of the form and given letters or other symbols, which can then be used in any sequence.

In making time studies, nothing should be left to the memory of the observer. Even those items which appear self-evident should be carefully and accurately recorded.

In making these studies it is important that an appropriate method of timing be used. The over-all timing method, the repetition method, the discontinuous method, the continuous method, and the cycle method will now be discussed briefly.

The *over-all timing method* is not satisfactory as a rule in this work, as it does not permit the studying of small divisions of the operation.

THE BLANK UNDERWEAR CO. No. 1. PLANT

Motion Study—Observation Sheet

OPERATION—Collar Seaming
OPERATOR—W. P.
DEPARTMENT—Finishing
OBSERVER—C. W. L.

DATE—
TIME—9:00 to 11:00 A.M.
GARMENT—Reg. Light Weight Union Suit
MACH.—Union Special

No.	Left Hand	Both Hands	Right Hand
1.	Turn bundle right side up		....
2.		Lift bundle to lap with necks at left and straighten	
3.		Untie cord	
4.	Pull out cord		Hold back garments
5.	Hold ticket		Sign ticket
6.	Hold ticket		Clip ticket
7.		Open bag	
8.	Hold bag open		Take out collar and lay on table at right
9.	Hold bag open		Take out back pieces and lay on table at right
10.		Shirr up bag	
11.		Lay bag on table at left: cord ready for bundle	
12.	....		Pick up garment
13.		Open top part	
14.		Place on machine left front corner ahead	
15.	Hold garment		Pick up one back-piece
16.	Hold garment		Pick up one collar
17.	—first one only—hold end of collar.		Fold other end over to test marks
18.	Place collar on garment and hold		Hold garment
19.		Feed and guide to second mark, first mark at seam	
20.	Hold garment		Put in back piece from under side
21.	Bring collar even		Fold back piece
22.		Feed and guide to come out even	
23.	Break thread		Hold garment
24.	Lift garment		Take it by middle
25.	Fold in the middle		....
26.		Lay on table over cord	
27.	Pick up ring end of cord		Pick up free end
28.	Hold ring		Put free end through ring
29.	Hold bundle		Pull cord through ring
30.	Hold ring and cord		Loop and tie
31.		Lay bundle aside	

EXHIBIT 196.—Simple motion study observation sheet. (*From "Management's Handbook."*)

If the data are collected in greater detail than necessary, they can always be combined, but if the divisions are not small enough, they cannot be subdivided without repeating the work. Consequently, it is ordinarily better to break down the operations into smaller divisions than are later found to be necessary, than it is to have the divisions too large.

In *the repetition method*, each successive step is timed separately, starting and stopping the watch for each observation. In this method.

however, a part of an operation may be missed and the worker may be disturbed. This method should be used only when making detailed observations of one elemental operation.

The *discontinuous method* consists of timing every alternate elemental operation in a complete operation, and then timing the remaining ones. This method has a serious disadvantage in that the complete actual sequence is lost.

The *continuous method* as a general rule gives the best results in most instances. Under this method, the elements are recorded in sequence without stopping the watch. During the period of observation, the time as shown on the watch is noted at the instant each elemental operation is completed, and this time is recorded opposite the proper symbol on a special sheet prepared for the purpose. The continuous method not only gives the exact time taken for each elemental operation, but it gives the time in the order of performance. Further advantages of this method are that it charges up every minute of time to some element, and it eliminates any danger of omitting delays.

In instances when the elemental operations are shorter than .04 or .05 minute each, it becomes difficult to time them with a stop-watch. Noting the transition point, reading the watch, entering the reading, and looking back at the operation cannot be done accurately in less than this time. It is possible, however, to determine times by taking them in groups as in the "cycle method."[1] But if it is necessary to make extended studies involving close measurements, the micromotion method can be used.

After the data have been enumerated and tabulated, the next step is the *determination of the base time.* Early industrial analysts were content to use the arithmetic average of the observed times as the base time. This average, however, is not reliable when the errors are cumulative in one direction, and this is often the case as the operations requiring longer times are retained. In such instances, the median is untrustworthy for the same reason. The mode is often the most satisfactory since it shows the most common occurrences and indicates the prevailing times required for the operations. The time which occurs most frequently (or the time slightly less) is quite widely used as the basis for setting the task. A still more satisfactory basis for many purposes, however, is minimum time. Maximum times may vary widely, but minimum times approach a limit below which they do not fall. It should be appreciated, of course, that a single observation below all the others may be an error. In many operation studies, it is also desirable to throw out large figures if they exceed the adjacent items by as much

[1] For an explanation of the details of the cycle method of stop-watch timing, see "Managenent's Handbook," pp. 813 and 818.

as 30 per cent. After the unreliable items have been eliminated, a mode, or sometimes a minimum, can be determined and used as the base time.[1]

OBSERVATION SHEET . **A**

OBSERVER'S NAME ....F MACHINE NO. DATE 8-3 19

WORKMAN'S NAME AND QUALIFICATIONS............C. K L-421

PART AND SIZE..........30 x 3½ Clincher Beads

OPERATIONS..........Hand Built AVERAGE DEVIATION 1. 26

	Detailed Operation	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total Average	% Deviation	Selected Minimum
1	BEAD COVER:																							
2	Get Chafer and lay on Table (10 ft.).	.09 .09	.10 1.05	(.22) 2.12	— —	.12 4.58	.05 5.50	.11 6.27	.15 7.39	.11 8.48	.13 9.39	.16 11.00	.14 12.13	— —	.06 14.00	— —	.12 16.04	.12 17.55	— 18.37	.18 19.42	.15 20.32	1.70 .113	1.25	.09 .09
3	Splice to Extra Section of Chafer	.04 .13	.05 .10	.07 .19	— —	(.15) .13	.03 .53	.03 .30	.08 .47	.04 .52	.06 .45	.09 .09	.05 .18	— .08	(.12) .26	.06 15.21	.03 .07	.04 .59	.10 .47	.03 .45	(.14) .46	.76 .0543	1.35	.04 .045
4																								
5	Spot on Wheel	.03 .16	.02 .12	.03 .22	— 3.21	.02 .75	.04 .57	.03 .33	.03 .50	.03 .55	.03 .48	.04 .13	.02 .20	.03 .11	.04 .30	.05 .26	.03 .10	.06 .65	.05 .52	.03 .48	.03 .49	.64 .0326	1.63	.02 .026
6	Run on	.02 .18	.03 .15	(.13) .35	.07 .28	.04 .79	.03 .60	.03 .36	.03 .53	.04 .59	.03 .51	.04 .17	.03 .23	(.20) .31	(.16) .46	.06 .32	.03 .13	.05 .70	.04 .56	.04 .52	.03 .52	.57 .0356	1.18	.03 .028
7	Cut and Splice	.07 .25	.06 .21	.04 .39	.04 .32	.10 .89	.06 .66	.06 .42	.05 .58	.05 .64	.06 .57	.04 .21	(.15) .38	.06 .37	.07 .53	.05 .37	.06 .19	.07 .77	(.13) .69	.09 .61	.07 .59	1.10 .0611	1.22	.05 .05
8	Adjust	.05 .30	.05 .26	(.16) .55	.06 .38	.04 .93	.04 .70	.04 .46	.04 .62	.04 .68	.04 .61	.06 .27	.04 .42	(.10) .47	.07 .60	— —	.03 .22	.04 .81	.04 .73	.03 .64	.04 .63	.75 .0441	1.47	.03 .035
9																								
10	Get section of (6) Beads and place on table	— —	— —	— —	.27 .65	— —	— —	— —	— —	— —	.29 .90	— —	— —	— —	— —	— —	.33 .55	— —	— —	— —	— —	.89 .297)	1.10	.27 .24
11	Measure and cut to Length	— —	— —	— —	.15 .80	— —	— —	— —	— —	— —	.15 10.05	— —	— —	— —	— —	— —	.13 .68	— —	— —	— —	— —	.43 .143)	1.10	(.13) .115
12																								
13	BEAD:																							
14	Pick up and spot Bead on Wheel	.12 .42	.18 .44	(.08) .63	.10 .90	.10 5.03	.08 .78	.10 .56	.15 .77	(.04) .72	.10 .15	.12 .39	(.18) .60	.11 .58	.15 .75	— —	.12 .60	.14 .95	.08 .81	.11 .75	.17 .80	1.93 1206	1.20	.10 .10
15	Run On		.06 .50	.09 .72	.06 .96	.07 .10	.06 .84	(.15) .71	.05 .82	.10 .82	.08 .23	.09 .48	.08 .68	.10 .68	.07 .82	.11 .48	.07 .87	.07 18.02	.06 .87	.10 .85	.06 .86	1.38 .0767	1.28	.06 .06
16	Cut and Splice	.57	.06 .56	(.13) .82	.04 4.00	.04 .14	.04 .88	.04 .75	.04 .86	(.10) .92	(.25) 48	.09 .57	.06 .74	.06 .74	.04 .85	.06 .54	.07 .04	.07 .09	.06 .08	.04 .89	.04 .90	.85 .0531	1.33	04 .045
17																								
18	BEAD COVER Cont'd:																							
19	Turn Chafer one Side	.12 .69	.10 .66	13 .95	.11 .14	.10 .21	.10 .98	.12 .87	.11 .97	.12 9.04	.14 .62	.11 .68	.12 .86	.14 .88	.10 .96	.16 .70	.11 17.05	.10 .19	.10 19.03	.08 .97	.10 21.00	2.30 .115	1.27	.09 .09
20	Turn Chafer other Side	.15 .84	.11 .77	.09 3.04	(.22) .36	.10 .34	.08 6.06	(.26) 7.13	(.29) 8.26	.10 .14	.12 .74	(.22) .90	.09 .95	.10 .98	.09 15.05	.12 .82	(.28) .33	.09 .28	.10 .13	.11 20.08	(.27) .27	1.36 .0972	1.21	.08 .075
21	Stitch with flat Roller	.06 .90	.06 .83	.07 .11	.05 .41	.05 .39	.06 .12	.06 .19	.06 .32	.06 .20	.05 .79	.05 95	.05 13.00	.06 14.04	.06 .11	.05 .87	.05 .38	.05 .33	.05 .18	.05 .13	.06 .33	1.11 .0556	1.11	.05 .045
22	Remove to Rack (10 ft.)	.05 95	.07 .90	.04 .15	.05 .46	.06 .45	.04 .16	.05 .24	.05 .37	.06 .26	.05 .84	.04 .99	.04 .04	.04 .08	.04 .13	.05 .92	.05 .43	.04 .37	.06 .24	.04 .17	.04 21.37	.96 .048	1.20	.04 .038
23																							18.90	
24										(0 9069 ÷ 0.72 = 1.26 Deviation)													1.26	

Boldface figures represent differences. (Courtesy of D. V. Merrick.)

EXHIBIT 197.—Time-study observation sheet with data filled in. (*From "Management's Handbook."*)

In the stop-watch study of Exhibit 197, the watch was not stopped until the end of the twentieth cycle. (In this table, the cycles are shown

[1] See D. V. Merrick, "Time Studies as a Basis for Rate Setting," pp. 13–19, McGraw-Hill Book Company, Inc., New York, 1919, for statistical procedure for determining a selected minimum.

in the captions and the elemental operations are shown as stub items.) Readings were entered lightly at the bottom of each space. Later, the time periods were computed and entered heavily above their respective readings. That is, when the first elemental operation ("get chafer and lay on table") had been performed, the stop-watch reading of .09 minute was recorded in row 2, column 1 (bottom figure). The watch ran continuously, however, and when the second elemental operation ("splice to extra section of chafer") had been completed, the reading .13 minute was entered in row 3, column 1, and so on, until the last elemental operation ("remove to rack") was performed and entered (see row 22, column 1), which completed the entire operation, and the workman was ready to begin the second cycle. After the first elemental operation in the second cycle was completed, the reading 1.05 was entered (see row 2, column 2) and so on. The top figure in each space in the cycle columns represents the difference between the reading in that space and the preceding reading, or, in other words, it represents the time taken by the particular elemental operation. In the "Total" and "Average" column, the upper figure in each space is the total of the time periods and the lower figure is the average time required. Time figures marked ( ) were thrown out as worthless. It should be noted that the elemental operations "get section of six beads" and "measure" are intermittent and occur in every sixth cycle. Their averages, therefore, are figured on a basis of 3 readings, while all others are figured on a basis of 20, unless missed by the observer. Items missed by the observer are, of course, not included in computing the average. The first average, therefore, was based on 15 items as one was excluded and four were missed.

After the untrustworthy readings have been discarded, and the average time for each elemental operation has been determined, this average is divided by the lowest time accepted, giving the ratio of deviation. If, for instance, the average is .113 minute and the lowest accepted time is .09 minute, the ratio 1.25 will show how far the average time of the worker exceeds his best time (see row 2 in Exhibit 197). A similar ratio is found for all other elemental operations and these ratios are averaged. (Although care should be used in averaging ratios, the method is ordinarily satisfactory for this purpose.) The average ratio (1.26, see last figure in last row, Exhibit 197) may be considered as the worker's characteristic for that study. The actual minimum and the actual average may be compared in the light of this characteristic in selecting the base time. Minima derived in this way and used in place of observed minima, are called "selected minima." Selected minima usually differ but little from observed times. Since the average ratio of deviation is a dependable characteristic, a high or low figure will always be significant to the experienced analyst. Exhibit 198 is an instruction card made up from Exhibit

197. This card gives the time allowed for each elemental operation in the task, and when work cannot be performed within the limits set, the supervisor must help the workman solve his difficulties if possible.

"C" INSTRUCTION CARD 20

NAME OF PART AND SIZE 30 x 3½ Clincher Beads	SHEETS, SHEET NO.
NUMBER AND NAME OF OPERATION B C-1 Building	TIME PER ONE PIECE 1.20 MINUTES
MACHINE NO. Cast Iron Hand Revolved Forming	Wheel

ITEMS DETAILED INSTRUCTIONS	ELEMENTARY TIME Prep.	3½"
1. Weighing 0.36 × 1/18	.02	
BEAD COVER:		
2. Get cover and lay on table		.09
3. Splice to extra section		.045
4. Spot on "V" forming wheel & run on		.055
5. Cut & Splice		.060
6. Adjust in to "V"		.040
7. Get section of 6 bead stock & place on table		.040
8. Measure & cut to length		.025
BEAD:		
9. Pick up bead, spot & run on wheel		.16
10. Splice		.045
BEAD COVER (Cont'd):		
11. Turn cover over bead one side		.090
12. " " " " other side		.080
13. Stitch with flat roller		.060
14. Remove bead from wheel & hang on rack		.050
15. Roll up line 0.12 × 1/6	.02	
Total Preparation	.04	
Total Selected Time		0.840
Allowance		.310
Working Cycle		1.15
0.04 Prep. × 1.25		.05
Task Time		1.20

WHEN WORK CANNOT BE DONE AS SHOWN, REPORT MUST AT ONCE BE MADE TO MAN WHO SIGNED THIS CARD.	DATE, 8–6–	SIGNED D. V. M.

EXHIBIT 198.—Instruction card for time study referred to in Exhibit 197. (*From "Management's Handbook."*)

There is no fixed rule for determining the number of readings that should be taken. Usually from 10 to 30 readings will be sufficient. However, the number depends in part upon the cost of the operation for the period, the number of employees engaged, and the amount of uncontrolled variation present. The more important these factors are, the

greater is the number of observations that should be taken before a standard is set. Usually in such cases the job is studied, then improved, and then studied again. Final studies mark the stage for standardization.

When timing operations where there is a range of sizes of the same product, it is usually possible to determine the base times of a few sizes at selected and suitable intervals and to determine the intervening sizes by interpolation. This can be done by plotting the base times and drawing a smoothed curve through the points, which will indicate the base times for the intervening sizes. By extrapolation, the principle can also be extended to sizes larger than those actually observed, although the risk of error is greater. With both interpolation and extrapolation there is considerable risk that somewhere within the range of sizes there may be an abrupt change in conditions, such as the substitution of cranes for labor when handling sizes exceeding a certain weight. It is imperative that such changes in conditions be recognized, and studies should be made of the sizes just above and just below to determine the discontinuity.

In the preceding discussion of stop-watch methods, the emphasis was placed on selecting the methods of performing elementary motions on the basis of the times of performance. It should be appreciated, however, that motion study is much more than this. In fact, it is principally concerned with the motion paths by which the work is accomplished, considering time as one method of measuring the motion path along with the three space dimensions. In making such studies, the real objective is to find the right paths and speeds of the motions. If these are right, the times will be right also.[1] Except in the simplest cases, the making of motion studies is greatly facilitated by the use of the micromotion method.

Micromotion Method.—The micromotion method of motion study is based on recording motion in relation to a cross-section background by means of a motion picture camera, and recording time on the film by having a rapidly moving clock (microchronometer) within the range of the camera. This method records performances of the most intricate and skillful nature. The studies are usually made in a laboratory where conditions as they exist in a shop are duplicated as nearly as possible, or in a part of the shop which is set aside for the purpose. The micromotion method includes a number of special devices and practices which require a very careful study before they can be used. However, they are readily mastered by those who will give them the proper attention. It is not the purpose to explain this method in detail in this text, but rather to call attention to it as a method of carrying on motion studies

[1] See L. M. Gilbreth, "Handbook of Business Administration," Sec. III. Chap. VIII, McGraw-Hill Book Company, Inc., New York, 1931.

which are much more complete and in much greater detail than is possible in the stop-watch method previously discussed.[1]

One of the most important steps in making a motion study is to break down the operation into small elements which can be considered separately. All operations can be divided into 17 elements of a cycle of decisions and motions (occurring in varying sequences) as follows: (1) search; (2) find; (3) select; (4) grasp; (5) transport, loaded; (6) position; (7) assemble; (8) use; (9) "dis-assemble" or take apart; (10) inspect; (11) pre-position for next operation; (12) release load; (13) transport, empty; (14) rest (for overcoming fatigue); (15) wait (unavoidable delay); (16) wait (avoidable delay); and (17) plan. These elements are called "therbligs."

When the micromotion film is ready for analysis, every frame that contains data that might be valuable in determining the standard method of doing the work is studied, and the data are transferred to a record called a "simo-chart," which is a record of the motions and time as revealed by the film. It is made on standard cross-section paper, the therblig data being recorded in colors by means of symbols. A simo-chart is prepared from the film of each worker recorded. "When these are completed, they are hung side by side so that one can compare the therbligs. Every slightest deviation in the therblig and in the time of the therblig is noted, and a tentative standard method is built up by synthesizing the best therbligs from all the charts. It is very seldom the case that the derived method lies in the consecutive motion cycles of any one worker. Usually the synthesis consists of a therblig from one chart, then one from another, or a complete motion cycle may be used, if it is that of an especially efficient worker. The completed chart contains the best practice of each of the group of workers studied, combined into a work method."[2]

Additional motion factors not covered in the above method may be studied by means of the cyclograph. With this method, small electric lights are attached to the worker's fingers, elbows, head, etc., as may be desired, the wires which furnish the current being fastened to the clothing in such a way that they will not interfere with the worker's motions. When an ordinary photographic plate or film is exposed for a short time, the path of each motion of the cycle of work will appear on it as a streak of light. If a stereoscopic camera is used, the record is made in three

[1] For details on the micromotion method, see "Management's Handbook," pp. 868–873; Frank B. and L. M. Gilbreth, "Applied Motion Study," Sturgis and Walton Company, 1917; and "Handbook of Business Administration," Sec. III, Chap. VIII, McGraw-Hill Book Company, Inc., New York, 1931.

[2] Gilbreth, L. M., "Handbook of Business Administration," Sec. III, Chap. VIII, McGraw-Hill Book Company, Inc., New York, 1931.

dimensions. If an interrupter is placed in the light circuit which flashes lights on and off at a uniform rate per second, the photograph will then show a series of dashes instead of a continuous line. Then, by counting these dashes, the time of a motion or part of a motion can be determined very accurately. It is also possible to control the operation of the lights so that the direction of the motion is indicated, and so that the different motion paths are differentiated. These records, therefore, may be made to show the paths of the motion, the time, the direction, the speed, and, if several moving parts are to be recorded, the relative speed.

The objective of motion study has not been achieved until the best worker has been taught the motions of the standard method, and is able to do the work by the prescribed method in the prescribed time. After the method has been checked in this manner, the data are turned over to those in charge of training and task and rate setting, to be incorporated in the standards which are to be taught and used.

Motion study is confined to determining the best methods by which the work can be done, and teaching the method. It does not determine certain allowances that must be made and added to the observed time for the best interests of all concerned.

Allowances.—In determining the standard task, certain allowances must be added to the observed times. The difference between the rate at which a man can work for short times and the rate that he can maintain for a long period is often greater than realized. The average daily production of a good worker may be 50 per cent less than that which would result from maintaining his highest rates. This variation is even greater for poor workers. The standard task must be such that the worker can maintain it indefinitely without injury to health. Allowances have been standardized under five heads: fatigue, variation in rhythm, unavoidable delays, personal delays, and machine delays. Allowance curves for fatigue and variation have been plotted for cycles of short duration.[1] Exhibit 199 gives 10 such allowance curves varying from 10 per cent handling time and 90 per cent machine to 100 per cent

[1] Allowances have been standardized and allowance curves have been plotted by D. V. Merrick on the basis of thousands of tests, which were made largely in the metal trades but which are applicable to many others. The general formula derived by Barth from such series of curves is:

$$P = 20 + \frac{49.5 - 0.325C}{\sqrt{0.376 - 0.0000216C^2 + T}}.$$

In this formula P is percentage allowances, C is percentage of handling time, and T is the minimum selected time for the cycle in minutes. (See D. V. Merrick, "Time Studies as a Basis for Rate Setting," pp. 53–65, McGraw-Hill Book Company, Inc., New York, 1919.)

handling time and no machine for the cycle on which the allowance is to be made. Abscissas of the larger chart cover time periods of from 0 to 20 minutes. The curves on the smaller chart have abscissas up to

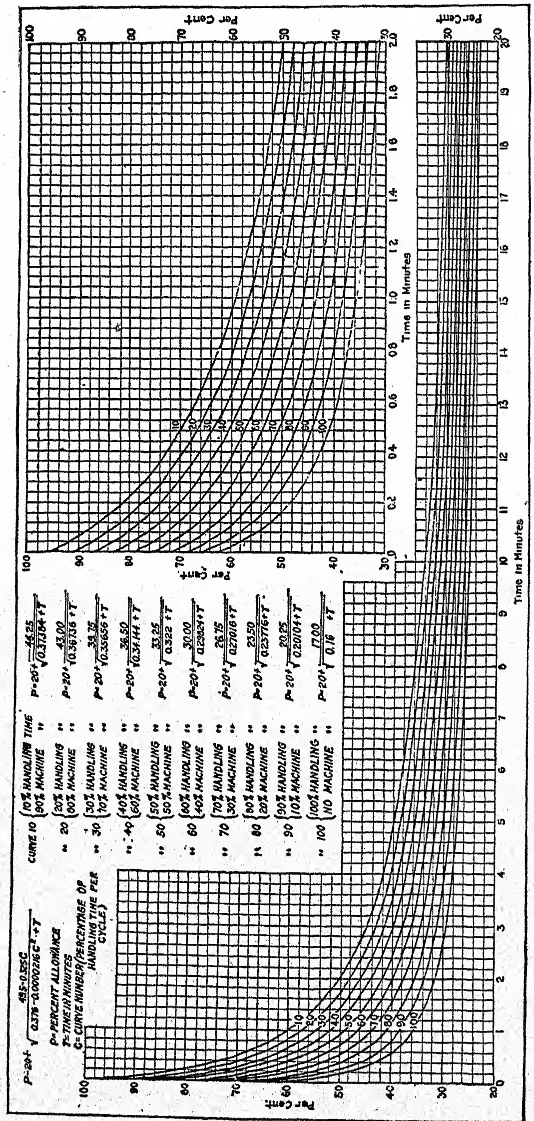

EXHIBIT 199.—Allowance curves. (*From "Management's Handbook."*)

2 minutes, and are useful for short time cycles. According to the general formula, the allowance time is never less than 20 per cent. The percentage allowance is based upon the total of the handling time. If, for

instance, a job requires 2 minutes machine time and 2 minutes handling time, the 50 per cent curve is used, and the intersection of the 2-minute ordinate with the curve (see Exhibit 199) determines the allowance percentage (42 per cent) to be added to the selected handling time in making up the task time. If there is no machine time, the curve representing 100 per cent handling time is used.

Delays should be classified as avoidable and unavoidable. The former should be eliminated and the latter should be provided for by percentage allowance in setting the task. An allowance of 25 per cent of preparation time is sometimes used. Good judgment must be used in applying these allowances as they are often but a very small proportion of the time for the whole job, and sometimes they are of but little practical value, excepting for the fact that including these allowances may have a desirable effect upon the worker.

Allowance must also be made for such personal needs as rest and washing up. In some instances, such as intensive automatic work, time studies extending over relatively long periods must be made in order to determine a reasonable allowance. Delay and fatigue allowances should ordinarily be kept separate, although allowances for necessities of life are commonly combined with those for fatigue. Allowances for necessities of life which should be added to observed time should range from about 2.5 per cent under easy conditions to about $33\frac{1}{3}$ per cent under severe conditions, such as work in the sun or in an overheated room. The allowance for fatigue is even more variable and the addition to observed time ranges from 5 to 100 per cent.

The allowance for machine delays varies according to the type of machine. For operation on ordinary machines like printing presses or machine tools the allowance will ordinarily vary from 5 to 25 per cent.

In many instances, hand and machine work may be simultaneous and not separate. It is not safe in such instances to use allowance curves derived from other industries without a careful check.[1]

Wage Rates.—The influences which operate to change the labor cost of production are often more intangible than many of the other conditions subjected to analysis. Conditions under which men work vary widely from place to place, from plant to plant, and from one type of work to another. It has been noted that workmen work at different speeds in different seasons of the year and at different periods in the business cycle. Furthermore, a rate suited to one nationality will not be suited to another. The situation relative to the labor supply, conditions in the plant, attitudes of labor unions, and the like, all have important effects in setting

[1] For further details on allowances, see "Management's Handbook," pp. 820–829, and D. V. Merrick, "Time Studies as a Basis for Rate Setting," pp. 16–19 and 49–75, McGraw-Hill Book Company, Inc., New York, 1919.

the day's work and the wage rate. To be effective, a wage standard must be reasonable, capable of attainment, consistent with the work, and should be guaranteed as long as methods and manufacturing conditions continue unchanged.

There are many wage plans in use at the present time. Usually several of the best for any particular case must be carefully studied before one can be selected. It may even be advisable to combine part of one plan with part of another. Through the use of various computations and graphic methods, it is possible to examine a plan and determine what may be expected from it before it is put into operation. It is impossible, however, as a practical matter, to present a complete description of methods of analyzing wage plans within the limits of the present general text. It should be clearly understood, therefore, in connection with the discussion which follows, that such is not the object, but rather that the purpose is to indicate the possibilities of making definite analyses of different wage plans. With this purpose in mind, six well-known wage plans will be analyzed briefly.[1]

Wage Payment Plans.[2]—The effectiveness of wage plans varies widely with the special conditions under which they operate. It is a general principle, however, in determining a wage payment plan, that ordinarily it should include an incentive for increasing the value of an employee's service, and that this gain should be shared by the employee and the employer. In other words, the employee should realize more than he otherwise would, and the employer should realize a substantial gain due to the increase in productiveness of the worker. Often it is desirable to provide an incentive to improve quality as well as quantity.

Time or day rate is commonest and simplest of all methods of wage payment. By itself it has no relation to the task, as it offers the workman a given sum for a fixed period of his time. The rate is ordinarily quoted as a certain amount per hour, per day, or per week. The limits of a given rate are as follows: The lower limit is the point in efficiency which brings discharge, and the upper limit is the degree of excellence which ordinarily wins promotion. Within these limits, earnings remain constant as shown by the "wages earned" curve in Part *A*, Exhibit 200.

[1] In discussing the six selected wage plans it is convenient to assume a definite financial distribution for purposes of illustration. This assumed distribution of material, labor, and overhead, with the range of variability, is described on p. 925 of "Management's Handbook." Formulas for computing earnings for these and other plans are also given and described on pp. 924–955, inclusive, of this handbook.

[2] This discussion of day rate, straight piece rate, Halsey premium, Barth premium, Taylor differential piece rate, and Gantt task and bonus wage plans, with the accompanying illustrative data, is abstracted from pp. 925–941 of "Management's Handbook." For more detailed discussions of these plans and for information on other well-known plans, see "Management's Handbook," pp. 909–974.

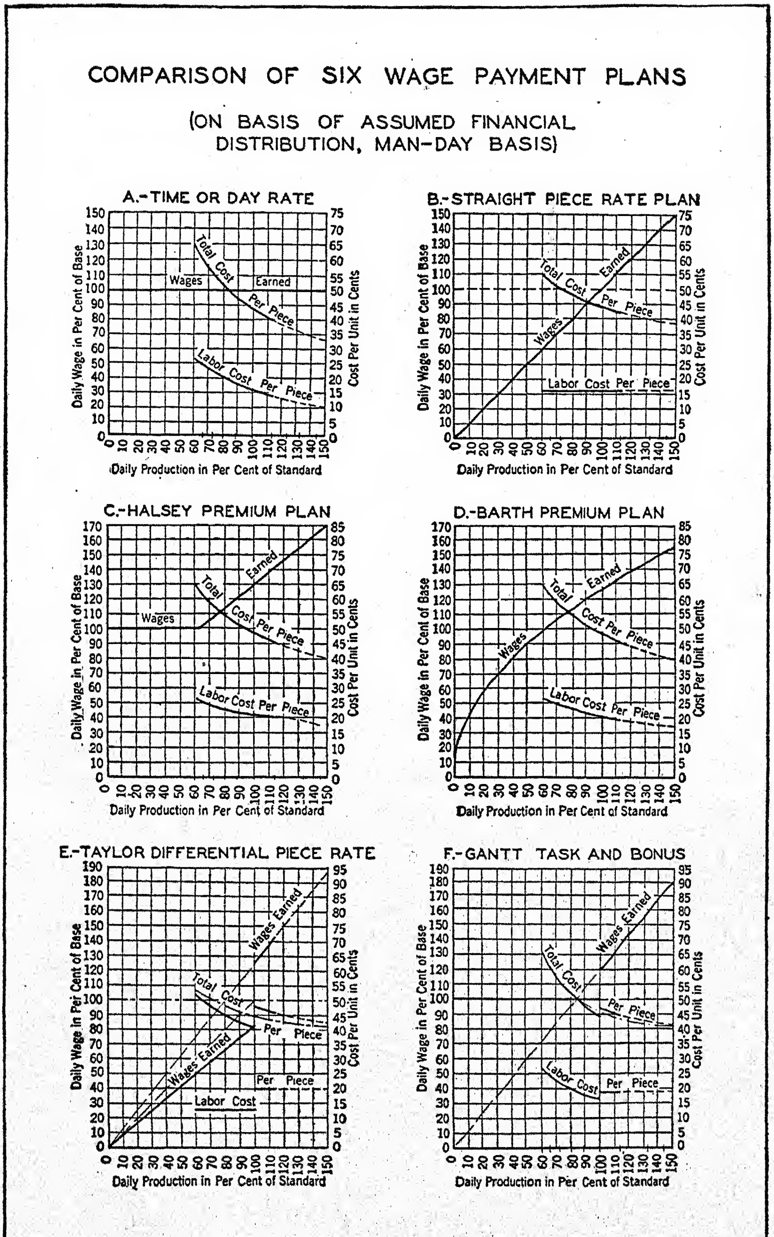

EXHIBIT 200.—Six well-known wage plans compared. Usually several of the best plans should be carefully analyzed for a particular plant before any one is selected. (*Adapted from "Management's Handbook."*)

Within these limits, the day rate pays exclusively for the workman's time, taking no account of quantity or quality of work done. So long as the workman remains safely within these limits he is in a passive state, and consequently this method of time payment may permit the grossest kinds of inefficiency. From numerous experiences it has been discovered that the average day worker takes about twice as long to do his work as is really necessary without overexertion.

There are several statistical measures which can be used in connecting the time wage with the task. The time which elapses on a good job can be recorded carefully and checked up together with the conditions of performance. The production records can be kept and followed by individuals and foremen. If these two steps are taken, and if adjustments in the rate are made according to performances, the time plan can be fully as successful as many of the more complex wage plans. While it is therefore highly undesirable by itself, it can be very successful when properly supported. Cost per piece is high at lower productions and apparently low at higher productions (see "labor cost per piece" curve in Part *A*, Exhibit 200) but as a matter of fact higher productions are seldom obtainable under this system, so that the total cost per piece should be considered high (compare "total cost per piece" curve in Part *A*, Exhibit 200 with the corresponding curves on the other graphs on this exhibit). That is, this is true unless the measures just mentioned are sufficiently successful to obtain higher efficiency.

The straight piece-rate plan is that method of payment where the worker is paid a constant rate per unit produced. Piece rates for many purposes are sounder in principle than day rates, but they may be very inefficient and unsatisfactory unless the rate is carefully set. Under the piece-rate plan, the worker's earnings depend upon his own effort. The piece rate is discouraging to a beginner, but it furnishes an opportunity for the ambitious and capable worker to raise himself to high earnings per day if the rate is properly determined. Under the piece-rate system, the employee takes all of the gains or losses from his own labor. As in time work, it is necessary to eliminate the poor workers after giving them a chance, and it is also necessary to limit the overambitious workers to a rate that is consistent with good health. It is obvious that the piece rate would not be satisfactory when workers have to change frequently from one job to another. A survey of factories employing about 250,000 workers showed that 75 per cent of the plants used the straight piece-rate plan in paying about 40 per cent of their employees. A large proportion of these plants paid from 60 to 85 per cent of their employees on a piece basis.[1] Cost per piece is constant so far as direct labor is con-

[1] Data from "Management's Handbook," p. 928.

cerned (see "labor cost per piece" curve in Part *B*, Exhibit 200), and a total cost is more nearly constant than for any other plan (compare total cost per piece curves). Exhibit 200 also shows the straight line for wages earned which always points to origin.

One of the earliest of the modern plans is the one known as the *Halsey premium or gain-sharing plan.* Under this plan, the time wage is guaranteed to the workman, and he is allowed to choose whether or not he should be paid the premium. A standard task time is determined on the basis of previous experience rather than by modern job standardization. The amount of time the workman can save relative to this task multiplied by his hourly rate is divided between the employer and the employee, the latter receiving from 25 to 50 per cent according to agreement. Because of the guaranteed rate and the right to choose, this plan is easy to introduce. It calls attention to the value of the time saved, which has an excellent effect upon the workman. Since the task is derived from the best practice without time and motion study, it is usually much lower than it is when determined by scientific methods. Exhibit 200, Part *C*, shows the operation of the Halsey premium plan. It will be noted that after work is brought up to task, the cost per piece decreases as higher productions are reached. The cost per piece is high, however, for the lower rates of production because of the guaranteed time wage.

The Barth premium plan was designed to substitute the geometric mean for the arithmetic mean used in the Halsey plan—that is, instead of half the sum of the standard time and the actual time, the square root of the product of these values is used. The resulting curve rises rapidly during low productions and approaches a straight line having a lesser slope than the Halsey plan. In this way, it avoids the guaranteed day wage and yet gives a higher earning for relatively low production than the straight piece-rate plan (see Parts *B*, *C*, and *D*, Exhibit 200). This feature makes it exceptionally appropriate for beginners. Costs fall off rapidly for high productions (see Part *D*, Exhibit 200). As a whole, however, costs are higher than under ordinary piece rates (compare total cost curves in Parts *B* and *D*).

The Taylor differential piece-rate plan was developed by F. W. Taylor in 1884. Taylor had found that low wages did not necessarily mean low cost. He believed that to secure lower costs, higher wages were necessary, and he was the first to point out the necessity of a scientifically derived task for a day's work. He believed also that low producers should be penalized and that high producers should be rewarded generously. Two differential piece rates, one high and one low, resulted from his experiences. He put the step from the low to the high rate at the task point, and he insisted that this step be of an appreciable amount. No

other wage plan has offered so strong an incentive to the ambitious and skillful worker. The operation of the Taylor differential piece-rate plan is shown graphically in Part *E*, Exhibit 200. Under this plan, the employees push the management to keep everything in perfect condition, and the management recognizes this obligation to comply, in order not to make the worker suffer for matters outside his control. This plan, however, has been difficult to use in certain instances because the low piece rates seem to penalize the workers too severely.

The Taylor differential piece-work plan like the straight piece-work plan gives direct labor costs which are constant, excepting for a step at task (see Part *E*, Exhibit 200). The total cost per piece when plotted shows the gradual slope curve characteristic of the piece rate, but on account of the change in the rate, there is a step at task which makes the cost much greater at 100 per cent production than at 99 per cent production. This, of course, refers to the individual case. When the whole shop is considered, as it must be in judging the merit of any plan, it will be seen that the cost due to those working above 100 per cent efficiency will be merged with the cost due to those working below 100 per cent efficiency. In other words, when the average production of the shop is 99 per cent standard, there will be some individual performances above 100 per cent; and again when the average production for the shop is 101 per cent, some of the cost will be due to those producing below 99 per cent efficiency. In changing from the 99 per cent average of one day to the 100 per cent efficiency of the succeeding day, there will be no step or pronounced increase in the total cost per piece. (These individual and total cost curves are presented in Exhibit 200, Part *E*.)

The Gantt task and bonus plan was devised to utilize the strong incentive of the Taylor differential plan and to eliminate the drastic effect of the low piece rate from low production. The high task standard is set by time study, and time wages are guaranteed until this amount of production is accomplished. At the point of task performance, a step in wages is made which puts the emphasis upon the amount of work desired. This step may be made anywhere from 20 to 50 per cent of standard time. When Gantt first tried out this plan he used the step of 35 per cent of standard time, and in addition gave only time wages for performances above the task. The plan has three outstanding merits. It avoids a low wage for those who fail to make the task, it puts a high incentive at task desired, and it gives full earnings saved for all performances above task. Gantt offset the disadvantages that might arise from a time guaranty by encouraging records of production by individuals. These individual records may be shown to the workers themselves, and will often add as much stimulus as a financial incentive. Direct labor cost per piece combines the figures given for time wage up to task and those

given for piece work beyond task. The total cost per piece is, therefore, high for low production, but quite low for higher production. The operation of the Gantt task and bonus plan is shown graphically by Exhibit 200, Part *F*.

Selection of a Wage Plan.—From the preceding discussion of a few of the well-known wage plans, it will be appreciated that the selection of the best wage plan for a particular plant can be facilitated by systematic statistical and mathematical studies. Before selecting any one of the many wage plans that may be under consideration, an estimate can be made of the probable effects of the various plans. Some plans will be obviously unsatisfactory and can be eliminated at the start. If, for instance, the labor situation and the nature of the work are such that a day guaranty is essential, the regular piece-rate plan can be eliminated at the outset. If the plan already in use is proving satisfactory, it should not, of course, be changed unless there is some distinct reason for doing so.

In selecting a wage plan, it often happens that actual experience data cannot be obtained, and when they can be obtained they seldom fit other cases exactly. Consequently, the final decision as to what plan is best will always depend upon good judgment. The final test of a plan is twofold: (1) How will it affect wages and relations between the employer and the employee? (2) How will it affect total production and total cost? It is impossible to answer these questions completely in advance, but one should make an earnest attempt to get a good answer based upon reliable statistics.

Incentives for Piece-rate Beginners.—Since a piece rate is discouraging to a beginner and a day guaranty is lacking in incentive, it is often desirable to pay apprentices according to a wage plan based upon length of employment and rate of improvement. Under this plan, a beginner may be put at once on piece work and during the period of training he receives, in addition, auxiliary amounts which decrease over the period from nearly a full wage to zero in arbitrary amounts or according to the more preferable auxiliary percentages. If, for example, a certain job can ordinarily be learned in six weeks, and a beginner can do a third of the normal week's task during the first week, the wages for that week might be piece work plus 200 per cent of piece work (which is equal to 300 per cent of piece work). If he can do one-half of the required amount the second week, the auxiliary percentage would be 100 per cent and so on, until the seventh week when he *should be* up to the standard rate of production and would receive only the piece-rate wages.

In working out plans for remunerating new employees, it is often assumed that a steady increase in their efficiency should be expected. It has been found, however, that this rate is not constant. One investi-

gator,[1] after analyzing thousands of cases, has formulated the curve presented in Exhibit 201, which shows the average reaction of new workers to a task with which they are unfamiliar. In his analysis he states the belief that when the average worker has covered about one-half the

APPRENTICE RECORD

Name *Mary Jones* No. *161* Operation *Grind* Class *II* Training Period *6* Date Started *1/3*
Address ______ Age ______
Experience ______
Education ______
Married ______ People dependent upon support ______
Born ______
Father Born ______
Where last employed ______
Position held ______ Wages ______
Where before that ______
Position held ______ Wages ______
Reason for leaving last job ______
Reason for leaving next to last job ______
Employed before by this Company ?
Under whom ______
Relatives employed here ______

Hours Worked	Wk.	% Efficiency Theo.	% Efficiency Actual	Pay Theo.	Pay Actual	Paid
48	1	15	18	11.50	11.50	✓
48	2	27	31	11.85	11.85	✓
48	3	34	38	12.20	12.22	✓
48	4	39	40	12.55	12.55	✓
48	5	42	44	12.90	12.90	✓
48	6	45	40	13.25	12.90	✓
48	7	47	50	13.60	13.60	✓
48	8	48	48	13.95	13.95	✓
48	9	49	54	14.30	14.30	✓
48	10	52	57	14.65	14.65	✓
48	11	58	62	15.00	15.00	✓
48	12	70	74	15.36	15.36	✓

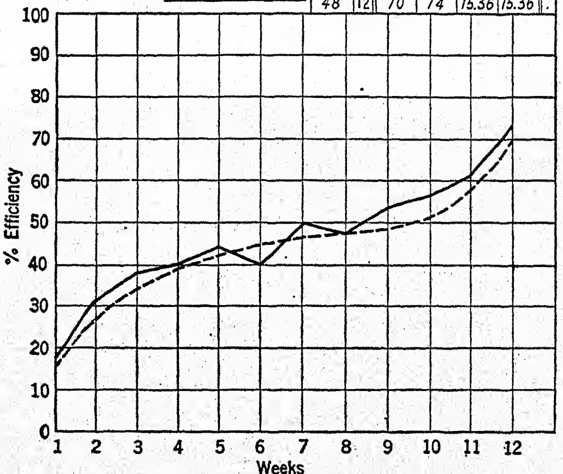

Exhibit 201.—Apprentice record and progress curve. (*From "Management's Handbook."*)

normal training period, there must be expected a period of time, depending upon the nature of work, where progress or increase in skill practically ceases. Then, after a short while, the worker rapidly attains a balance of average proficiency. Especially in plants employing female labor, the loss of operatives before they have become fairly efficient workers is a

[1] Carle M. Bigelow.

serious problem largely due to the fact that remuneration is based upon a required constant increase in proficiency which is practically impossible for the average worker to maintain.[1] Accordingly, the plan of Exhibit 201 remunerates apprentices upon a basis which will enable them to receive a constantly increasing wage during the time of apprenticeship based upon a rational demand of average performance.

Labor Turnover.—It is often desirable that the management should have data which will indicate changes in the rate at which employees separate from the particular establishment. In most plants, hiring and training new workmen are costly and efforts should be made to make conditions such that this replacing of workers is reduced as much as is practical, and so that the concern may be said to have a low "labor turnover."

"Labor turnover is the ratio of separations—or of replacements—to the total number of employees." There are wide variations, however, in the methods of computing this ratio. One widely used method is to divide the total number of separations, for the period under consideration, by the average number of employees on the pay roll during the same period. Thus, if in a year there are 1,000 terminations of employment, out of an average working force of 2,000, the labor turnover is represented by the following formula:

$$T \text{ (turnover)} = \frac{S \text{ (separations)}}{F \text{ (average working force)}}$$

or

$$T = \frac{1{,}000}{2{,}000} \text{ or } 0.5 \text{ or } 50 \text{ per cent.}^{2}$$

When using turnover figures, it is always important to have other statistics which will assist in their interpretation. For instance, one should have such information as whether changes are permanent or temporary, or whether men were laid off because of lack of work, or whether they quit or were discharged because of grievances. Many concerns compute labor turnover by groups, according to causes of separation; others make distinction between preventable and unpreventable turnover; and others record turnover separately for old and new employees, for men and women, for skilled and unskilled labor, and for other significant classifications.

Since methods of computing turnover are not uniform, care must be used in any attempt to compare turnover figures of different concerns. However, this does not detract from the value of analyzing turnover

[1] "Management's Handbook," p. 962.

[2] See Gordon S. Watkins, "Labor Management," pp. 260–270, McGraw-Hill Book Company, Inc., New York, 1928.

statistics in connection with efforts to minimize the loss of desirable workers in an individual establishment.[1]

Attendance and Absenteeism.—Regularity and punctuality of attendance constitute a necessary factor in careful planning of manufacturing operations. To secure this regularity and punctuality, however, requires careful study and management. Some companies keep detailed records of absenteeism and make analyses based on percentages and other mathematical methods which resemble those used in computing labor turnover.[2]

Accident Statistics.—Management, in connection with its efforts to increase the safety of workmen, may profitably spend much time in analyzing such statistics as those showing frequency of accidents, their severity, or times of occurrence—all by classes of workmen. Based on such studies, safety activities may be emphasized where there is the greatest need. For instance, if it is found that most accidents are among new employees, it may be possible to increase the effectiveness of their training in this respect. If most accidents occur late in the day, it may be possible to make workmen conscious of the tendency so that they will be on their guard, or it may be an indication that hours are too long, or that heating or ventilation is insufficient.

Current Wage and Cost-of-living Data.—As has been pointed out previously, management must carefully watch changes in the cost of living, and changes in the current rates of wages paid in other concerns and industries from which the different classes of workers are ordinarily drawn or to which they go upon separation, if it is to set its own wage rates justly and intelligently. One of the most important sources of statistical information for use in this connection is the Bureau of Labor Statistics of the United States Department of Labor. The bureau publishes statistical bulletins on wholesale prices .etail prices and cost of living, wages and hours of labor, employment and unemployment, women in industry, workmen's insurance and compensation, industrial accidents, productivity of labor, industrial labor conditions, and other subjects of interest in connection with labor problems. In July 1915, the publication of what is now the *Monthly Labor Review* was begun. The special purpose of the *Monthly Labor Review* is to make available, regularly and promptly, information concerning various phases of the labor question, both in this country and abroad, which is not available in collective form through any other source.[3]

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[1] See "Management's Handbook," p. 1471.

[2] See Gordon S. Watkins, "Labor Management," pp. 270–271, McGraw-Hill Book Company, Inc., New York, 1928.

[3] The reports and bulletins of the Bureau of Labor Statistics are furnished gratis as long as the bureau has a supply available. The subscription rate to the *Monthly Labor Review* is $1.50 per year.

If adequate cost-of-living data cannot be secured from the usual published sources, it may be necessary to make special investigations to determine the situation. Large companies often carry on these investigations locally, and small companies can do the same on a cooperative basis.

Questions and Problems

1. Discuss the steps necessary in approaching a statistical problem.

2. In considering the location of a factory, data upon what factors should be collected?

3. What operations should be planned definitely to secure effective production control?

4. Upon what data is the production schedule based?

5. Discuss the records of the purchasing and stores departments used in controlling purchases and materials on hand.

6. What factors must be studied for the control of raw materials?

7. In routing products through a manufacturing plant, what are the requirements?

8. What is the purpose and nature of the Gantt chart for scheduling production?

9. What is the purpose of machine burden records?

10. Discuss "dispatching."

11. How is the "work order" record used in dispatching?

12. How can quality be controlled in relation to working standards?

13. Discuss the importance of sampling in inspection.

14. What is the purpose of time and motion study?

15. What are the principal steps in job standardization?

16. Describe the stop-watch method of operation study.

17. What is the micromotion method of operation study?

18. How should one proceed in establishing the "base" time for an operation? How many time readings are necessary?

19. What allowances require additions to observed time, and how are allowances determined?

20. How can wage rates be compared?

21. Discuss the relative merits of the time or day-rate and the straight piece-rate wage plans.

22. What is the Halsey wage plan? The Barth premium plan?

23. What are the characteristics of the Taylor differential piece-rate plan? What are those of the Gantt task and bonus plan?

24. What is the final test of a wage plan?

25. How may incentives for beginners be provided?

26. What is "labor turnover"? How is it computed? What important uses of such data are made?

27. How may attendance and accident statistics be used in industry?

28. Discuss the use of current wage and cost-of-living data in determining fair wages.

29. Prepare a set of forms to be used in the statistical control of production in a specific concern which will cooperate with you in furnishing information.

30. Make a stop-watch time and motion study in a small concern which will cooperate with you in making such an investigation.

31. Draw up a detailed plan for making a micromotion study in a certain plant.

32. In a certain plant, which will give you information on the wage plan now used, analyze the plan as compared with other plans, using the methods suggested in this chapter.

CHAPTER XXI

MARKETING ANALYSIS

The degree of success that a manufacturer or trader has attained ordinarily depends in a large measure upon how effectively he has built up and maintained a market for his products. Production capacity in general, at the present time, is greater than the market capacity for absorbing the products at a profit to the producer. The market is normally a buyer's market, which means that the seller must aggressively push his goods if they are to be sold under the present conditions of keen competition. An intensely competitive situation in a market does not mean, however, that one should stay out of it. In fact, extensive markets are very likely to be highly competitive, but the opportunities may be very good to sell an improved product through superior sales efforts based upon definite and accurate information. During recent years, the statistical analysis, often based upon specially collected data, has come to be an important factor in determining what, where, when, and how goods should be marketed.

Analyzing the Market.—Before attempting to build up a market, one should find out what can be sold and determine as far as possible the extent and nature of the market. *What* can be sold depends upon the consumer's conception of the satisfying powers of the various commodities, and *how much* can be sold depends largely upon the number of people and their purchasing power. A skillful analysis of these factors should precede any attempt to establish a new market.

The value of statistics in solving the above problems is now widely recognized. And it is just as important that the difficulties, dangers, and limitations attending their use be thoroughly appreciated. As pointed out previously, statistics cannot take the place of good judgment, mental alertness, or common sense. But when the business man is attempting to decide upon a marketing policy, he finds it of great value to have, among other things, an unbiased and true report upon the statistical facts regarding the situation.

It is particularly important that the scope of a market investigation be defined as definitely as possible before beginning the work of collecting data. After the problem has been defined, one should ordinarily use secondary information as far as possible, but most investigations will require the collection of at least some primary data. A complete market analysis not only includes a careful measurement of existing and potential

demand, but it also includes a careful checking up on the selling points or satisfying powers of the product itself.

Preliminary Study of the Market.—Before making a complete study of the market, it is often advisable to make a preliminary survey for the purpose of finding out whether or not a more detailed survey should be made. It often happens that a preliminary analysis will indicate quite definitely whether or not the proposition is favorable. If it is shown to be decidedly unfavorable, much of the expense which would be necessary in making an extensive survey will be saved. If it is decidedly favorable, only the details for carrying out the plan will remain to be worked out. If the preliminary results do not indicate a definite answer, they will practically always at least indicate how a much better analysis can be made than would have been made if a complete survey had been attempted at the outset.

Preliminary surveys are ordinarily based to a greater extent upon secondary data than are final surveys. By using available data that are already compiled, much can be learned about a marketing problem at a relatively small cost. The gathering of primary data is an expensive process and should ordinarily be done only when it is obvious that valuable results will be obtained which cannot be secured from secondary data alone.

Demand and Supply Measurement.—Questions of supply and demand are usually involved in marketing analyses. The terms, however, are often used loosely in business and sometimes quite inaccurately. Strictly speaking, the *supply* of goods on the market does not mean the total stock in existence, but that quantity which the owners are willing to sell at a certain price. The goods which are in existence, but which are not for sale, constitute only a potential supply. *Demand* in a market means the quantity of goods that will be taken at a given price. It must not be confused with desire, to which there must be added ability to pay and inclination to buy before there is any demand. Both supply and demand often fluctuate widely over a period of time and as much definite information as possible should be collected concerning potential supply and demand in order that one may have some indication of the probable future condition.

For certain staple commodities, with a relatively stable rate of consumption, it is possible to determine the demand with reasonable accuracy. Likewise, it is possible to determine the supply of many staple commodities, as extensive records are compiled by government and other agencies. In many instances, however, direct measurement cannot be made of demand and supply, and it is necessary to estimate the situation on the basis of indexes or indicators developed from related series of available data.

Demand Conditions—Indexes of Demand.—In determining the extent of demand, a high degree of accuracy is seldom practical because a certain margin must be allowed to take care of the changes which will undoubtedly occur in a short time. It is important, however, to obtain indications as to whether there is strong, good, fair, poor, or no demand, and sometimes it can be gaged much more closely than this. Instead of trying to make absolute measurements of the demand for a specific product, it is usually satisfactory to obtain some kind of an index or indicator of the situation.

There are many forms and variations of indexes of demand and they have varying degrees of value. Often there is an obviously direct relation which makes one series an index of another. For instance, permits to build, that are issued by a city building department, indicate the demand for certain building materials, and sales of gasoline may be indicative of the demand for replacement tires and automobile accessories.

Since demand varies with differences in people, commodities, seasons of the year, customs, religions, and ages, as well as with the population and purchasing power, it is obvious that it is necessary to use many different kinds of indexes. Purchasing power varies widely with the business cycle; consequently, it is necessary to have indexes of general business conditions in determining the general demand situation.

National distributors find it of great value to have indexes of demand for many different geographic sections of the country. This enables them to know which districts are the best in which to push sales. When demand in some districts is low or declining, they will concentrate their sales efforts in the districts where the demand is good or improving.

In order to determine the extent and distribution of demand, data on population and purchasing power are of utmost importance in making practically all marketing analyses. For many products, the market will vary reasonably directly with the population. This is important in determining the relative importance of different parts of the country and in determining changes which are taking place in the market in rapidly growing communities. And, of course, if the population data are to be of greatest significance, it is usually necessary, and sometimes imperative, to have classified detailed data on purchasing power.

The methods of making and applying population and purchasing-power estimates have been discussed previously.[1] While these methods are of great importance in making other types of investigations, it is in making analyses of the market that they ordinarily find their greatest value.

Supply Conditions—Nature and Extent of Competition.—Most of the preceding discussion applies to the demand side of the market. This

[1] See Chap. XIX on Population and Purchasing Power.

is, however, only one side of the problem and it is also important to find out what the situation is on the supply side, or, in other words, to analyze competition. Among the more important factors to be determined are the source of competing products, the strength of those furnishing competing products, and the condition of present stocks.

It is necessary to know the source of competing products in order to know something about the conditions of production, cost of transportation, tariffs (if the goods are foreign), and other similar factors. It is obvious that the strength of competitors due to financial backing, good will, and other factors, which would make it difficult to introduce a new product, should be very carefully considered. If large stocks of a product are already in the existing market, prices may be unsatisfactory for some time, and it will be better to concentrate upon developing a market somewhere else.

Statistics on actual production, shipments, importations, stocks, etc., of many commodities may be used in determining the competitive situation of other commodities in the various markets under consideration. Large importations of bananas in New York City, for instance, may indicate poor conditions for oranges and other fruit in this market.

Statistical information on the stocks of many of the principal commodities in the country are compiled and published by governmental departments, trade associations, and other organizations. The *Survey of Current Business* publishes data on stocks on hand of beef, pork, mutton, sugar, cottonseed products, flour, wheat, rice, corn, oats, rye, barley, apples, butter, cheese, eggs, poultry, fish, beverages, coffee, tea, tobacco, wool, cotton, silk, rayon, hosiery, hides and skins, coal, coke, petroleum, gasoline, iron ore, steel, copper, lead, zinc, tin, lumber, roofing, cement, brick, turpentine, rosin, alcohol, rubber, tires, paper, wood pulp, and other products. When conditions in the market for one or more of these commodities are being taken as indicators of conditions in markets for commodities on which information is not available, care should be taken to see that changes in styles, customs, prices, etc., have not destroyed their significance.

As previously pointed out, finding a keen condition of competition does not mean that one should stay out of a market. If a market is a good one it is to be expected that it will attract a considerable amount of trade. A highly competitive market, if it is an extensive one, is often a good one in which to sell goods through superior marketing methods, and it is one of the objects of the market analysis to determine the best selling points and best selling methods.

Determination of Market Areas and Sales Territories.—If a manufacturer, wholesaler, or other distributor is to have an adequate check upon his salesmen and what may be expected of them, it is ordinarily

necessary to divide the total market area into sales territories. This involves first a determination of the limits of the total market area, and in doing this it is ordinarily necessary to make a study of the distribution of the population and purchasing power, the extent of competition, the time and cost of deliveries, and the probable future development. Then the total area can be divided into territories of approximately equal importance, or if they are not equal, the management will know about what is to be expected from each area. In problems of national scope, it is often desirable to lay out the sales districts on the basis of the large population centers with their tributary territories.

In laying out and analyzing trading areas on the basis of important centers and tributary territories, the problem is to group properly the county population and purchasing-power figures (previously discussed in Chapter XIX) around the centers to which they are tributary. The principle of this plan is simple and obvious, but its practical working out involves many difficulties. In explaining this situation the director of research of the J. Walter Thompson Company pointed out that "perhaps the most serious difficulty in this work is that growing out of the vagueness of the term 'tributary territory.' There are for example about 180 cities in the country which are primary grocery jobbing centers with large grocery jobbing houses; but taking large centers and small ones together, there are 567 centers with grocery jobbing houses of substantial size. One of our first efforts was to group about these 567 centers the territory more accessible from those centers, respectively, than from any others. In revising this work for publication, the Bureau of Domestic Commerce of the Department of Commerce regrouped these territories about the 180 primary centers alone.[1]

"Either plan is effective for groceries, but neither grouping works wholly satisfactorily for drug products. There are 205 centers with substantial drug jobbing houses and when the county figures are grouped around these with proper allowances for the large center jobbers operating on an interstate basis, the result is quite a different grouping from that for groceries.[2]

[1] As part of the 1929 edition of the "Market Data Handbook," the Bureau of Foreign and Domestic Commerce published a wholesale grocery map with 183 territories (see Exhibit 202, p. 499). In the 1938 edition of the "Atlas of Wholesale Grocery Trading Areas" (Bureau of Foreign and Domestic Commerce), the overlapping of the areas is indicated where it is known to exist. In the words of the Atlas, "Trading area boundaries are not high walls over which trade cannot pass, rather, they mark spheres of greatest influence, frequently bordered by regions from which the center in question draws some trade but which are primarily related to some other center."

[2] In 1935, The National Wholesale Druggists' Association published "Distribution through Drug Channels in the 84 Wholesale Trading Areas." This study presented wholesale drug trading area maps and pertinent market data.

"Again, for goods sold to the dry goods and department store trades the picture is very different from either of the others. There are 681 places having large department stores and 632 additional places with smaller ones. About these places it is possible to group surrounding counties in retail shopping areas which will have few exact parallels with the areas for jobbing either drugs or groceries.[1]

"By constructing in the manner thus outlined a series of subdivisions of the country into various types of homogeneous areas or markets, tributary to centers, it might be possible to find a number of centers appearing on all or several of the lists which would then become a recognizable group of general trading centers. But in the meantime, groupings of trading areas about recognizable centers for the various individual trades seem to offer the best results.

"Great progress has been made in the development of figures for use in sales quotas. The quantitative figures have been increased in variety and trustworthiness; the qualitative figures giving some idea of purchasing power have also been made much more useful; and, finally, the grouping of these figures in conformity with the sales practice and experience of the individual concern has shown marked advance. Instead of measuring sales in a given place by comparison with the population of a political group, they are now being measured more nearly by the people who ought to be reached from that place with due consideration of their ability to buy."[2]

Retail Trade Areas.—Although retail trade areas may be laid out on the basis of population and purchasing power principally, these and other data cannot be applied unless the following factors (statistically measured) are taken into consideration:

1. Transportation facilities (streets, highways, electric lines, railroads, etc.).
2. Facilities for communication (newspapers, telephones, etc.).
3. Competition from other dealers.
4. Business, amusement, and other attractions.
5. Topographical and climatic conditions, and their effect.
6. The psychological effect of distance and the mobility of trade.
7. Special barriers, hazards, etc.

In connection with the above, it is obvious that the type of commodity, the size of the store, the delivery service, the advertising policies, etc., must be carefully considered, and that it is possible to influence the factors that would naturally determine the size of the territory. Ordi-

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[1] The J. Walter Thompson Company has prepared a detailed tabulation of these retail shopping areas for the entire country.

[2] From Paul T. Cherington, "Some Recent Developments in Market Analysis," an address delivered before the Taylor Society.

narily, the territory should be laid out on the basis of the best existing conditions, allowing, however, for a reasonable amount of advertising and other influences which can be brought to bear in a practical manner.

Consumer Market Data Handbook.—While a number of secondary sources of data may have to be used in determining market areas and sales territories, and while much original information may have to be gathered, there is one published source with which all students directly or indirectly interested in commodity market analysis should be familiar, because of its value as an aid in analyzing the geographical distribution of the market for a certain product. This source is the "Consumer Market Data Handbook," compiled and published by the United States Department of Commerce, Bureau of Foreign and Domestic Commerce.[1]

In selecting the items for the Handbook, an effort was made to include in each type the items that would be of greatest value to those interested in marketing to the different classes of consumers. The information was compiled from both governmental and private business sources and includes the following information for the 3,073 counties in the United States: population and dwellings; retail and wholesale sales; value of products manufactured; number of farms; employment and payrolls; retail sales by types of stores; income tax returns; automobile registration; radios; residential telephones; domestic electric meters; magazine circulation; and farm values.

The county was selected as the statistical unit primarily because it is the smallest political unit for which satisfactory statistics are available. The county unit was strictly adhered to with the exception of a few instances. The counties are listed in such a manner that it is convenient to retabulate the figures to fit the particular needs of the individual organization in laying out its market areas. The map in Exhibit 202 indicates how the country can be subdivided into sales territories.

The different kinds of county data included in the handbook will be used in many different ways, varying according to the different types of commodities, sizes of distributors, operating policies, etc. The manufacturer of electrical household equipment may wish to lay out his sales territories on such a basis that the number of electrical consumers in each territory will be about equal. Another may wish to weight the data for each territory according to its purchasing power, or according to the population which represents a potential market. Another would have the problem of variations in the success of the different salesmen

[1] First published in 1929 as the "Market Data Handbook of the United States." This was followed by "General Consumer Market Statistics" in 1932, and by the "Consumer Market Data Handbook" in 1936 and 1938. Brief descriptions have been given previously on pp. 44 and 446. An "Industrial Market Data Handbook" is in preparation.

and the territories would be unequal in importance. In all of these instances, however, the manufacturer would have a basis for knowing where his markets exist. Likewise, the tire manufacturer has a basis for knowing where the tire market is when he refers to the automobile registration figures. The figures on farm values may show the farm machinery manufacturer where his market is. The manufacturer of a safety device for mine hoists knows where the minerals are being produced, and so on. It is not, of course, essential that the distribution

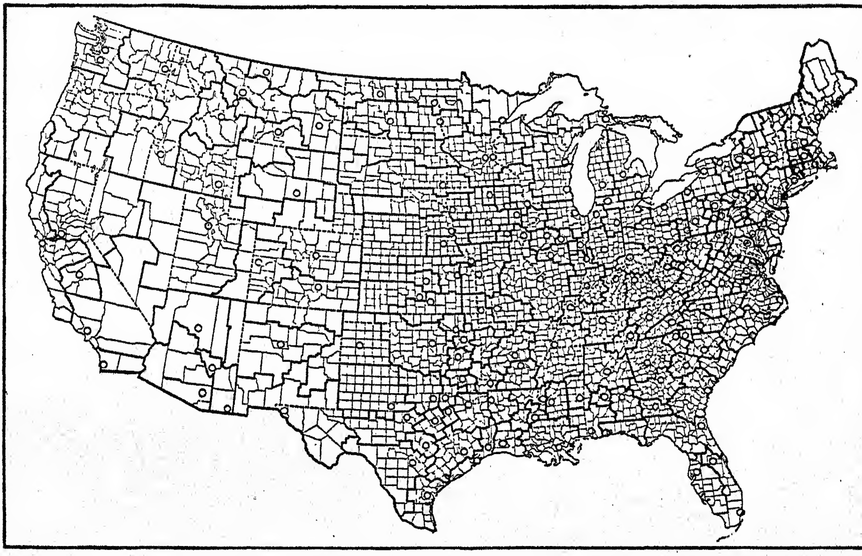

EXHIBIT 202.—"Market Data Handbook" wholesale grocery territories map (1929). This map indicates how the country can be divided into sales territories on the basis of economic units by combining the counties around central distributing cities. (The original map is 27 by 42½ inches in size and shows the names of the counties and principal cities.) A similar but more detailed map is included in the "Atlas of Wholesale Grocery Trading Areas," published by the United States Bureau of Foreign and Domestic Commerce, March 1938 (see first footnote page 496).

be national in scope. Each section of the country, a state, or even a part of a state, can be much more readily handled as a market if the distribution of that market is understood. A manufacturer of feed for dairy cattle, for instance, may be interested only in the figures on the dairy products in the State of Virginia, but the distribution of this industry tells him where his market is, and he immediately has the basis of deciding upon the best receiving points, transportation systems, etc.

Determining the Consumer's Point of View.—In view of the competition that exists in marketing most commodities at the present time, it is important to determine definitely what selling points make the greatest appeal to the consumer. The method is ordinarily simple although the

application is often difficult. All of the known selling points are listed and a large number of consumers are asked to state their preferences. Data are compiled from these by classes and the seller is able to determine the most satisfactory combinations for the different classes of demand.

In making these audits of public opinion, great care and considerable skill are necessary if one is to uncover the real truth. It is not enough to take the individual's word as to his reason for his preference for certain commodities, places of buying, services, and other factors upon which he bases his choice. This is because he bases his decisions more upon emotion and habit than he does upon reason. Consequently, considerable attention must be given to watching the buyer and recording exactly what he does in actual practice. If, for instance, a questionnaire is filled out by the average user of household goods regarding why a certain article is purchased and which will show whether advertising, dealer recommendation, or experience in use is the most powerful buying motive involved, we are likely to find that, according to the answers, experience in use ranks highest even in a case where the subtle use of advertising is almost entirely responsible for the sales of the product. As a rule, a person does not realize how much he is being influenced by suggestion in advertising, and the reasons that are given for buying a certain product may not be real reasons at all. Consequently, one of the most important points in making a statistical market analysis is the recognition of these difficulties. Judgment and experience, of course, are required in solving such problems, but many wrong conclusions will be avoided if it is realized that the difficulties exist.

Fitting the Product to the Market.—A careful statistical study of what the public wants is often extremely valuable in strengthening a concern's competitive position. Many staple commodities, such as children's underwear, household cleansers, brushes and brooms, or bacon, bread, cheese, and similar food products, or water heaters and similar equipment, are not materially affected by the style element, and their sales are determined largely by the number of relatively permanent selling points in relation to price. It is, therefore, of great importance to break such products down into their elements of desirability and then combine as many of these elements as possible into one product. Infants' underwear becomes a question of wool, cotton, silk, rayon, or mixtures; buttons or tapes; light or heavy; colored or white; softness; shrinkage; durability; price; etc. Brooms become a question of weight, durability, appearance, etc., in relation to price. Water heaters become a question of instantaneous or storage, service, size, and price. Other products can be broken down into their component parts in the same way; and by collecting information by means of questionnaires or through field enumerators, the selling concern will be able to provide the most desirable

product by classes of people and by localities. Exhibit 203 shows a simple questionnaire that was used in analyzing the market for water heaters.

It is obvious that an opinion based on a representative sample, such as those just referred to, will often cause decisions to be made in a manner which will differ widely from those which will be based upon the opinions

LETTERHEAD

January 10, 1931.

With the idea of extending the market for his product, a manufacturer of a medium-priced water heater, making both storage and instantaneous types, has asked us to secure some information for him about the possible market for his product in New England.

We feel that you will be willing to take the time necessary to answer the brief questions below, as the information furnished now may be of real benefit to you at a later date, since the sales policy will be based upon the facts obtained.

Assuring you of our appreciation of your cooperation,

(Signature)

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1. What type of hot water heater do you prefer?
   Instantaneous?__________ Storage?__________
   Why?__________
2. What priced heater do you most frequently install in the average home?
   $__________ Price range $__________ to $__________
3. (*a*) Are you thoroughly satisfied with the present method of marketing heaters through the plumber?__________
   (*b*) Would you prefer to buy direct from manufacturer?__________
   Why?__________
   (*c*) If bought direct, should heater be installed by
   Plumber?__________ or Heater Company?__________
   Why?__________
4. Remarks__________
   NAME__________ ADDRESS__________
   (It is not necessary to sign this questionnaire.)

EXHIBIT 203.—Questionnaire sent to builders in an analysis of the market for water heaters.

of one or a few. Individual opinions on the part of the management will often differ from those of the majority of consumers to an extent which is great enough to interfere seriously with the profits of the enterprise. It is often the case, especially in small enterprises, that the opinion of the manager is based to greater extent upon friendly compliments than upon cold facts regarding the opinions of unsympathetic buyers. If, at her tea, Mrs. Jones is highly complimented on her home-made candy and is told that she should start a candy factory and store, she should not be too credulous, for she may find that the public has different opinions

should she actually start the enterprise. Such cases are very common in the business world, where enthusiastic inventors of all types of things take friendly compliments as indications of the markets for their products, only to find later, through the expensive process of trial and error, what they could have found out at small cost by collecting unbiased statistical data—namely, that there is virtually no demand for the product.

Statistics in Advertising.—Statistical investigations are valuable aids in planning an advertising campaign, in carrying it out, and in checking results. Before proceeding with an advertising or selling campaign, a concern needs to know the answers to the following questions: Who are the users of its product? Where do they live? How many are there? How much may they be expected to buy? What are the substitutes for its product, and how satisfactory are these substitutes? What are the preferences, likes, and dislikes in connection with its product and its competitors? It is obvious that such information is of utmost importance. The questions, of course, cannot be answered perfectly, but statistical facts and analyses are a great aid in securing better answers than could be obtained without them.

In answering the above questions, one ordinarily needs data on the size, density, and distribution of population, and facts which indicate the characteristics of the people such as race, age, religion, and education. Any illiteracy, of course, directly limits the effectiveness of advertising. One also needs data on incomes and on the distribution of the family purchasing power among the different products. By using the sampling process, either with questionnaires or personal enumerators, much information can be gathered at a relatively low cost which will indicate the answers to the last two questions listed above, and although the answers will not be based upon complete information, they will be much more reliable than mere opinions, and much less costly than experimenting with the market.

Careful statistical studies to determine selling points are being used with substantial success to increase the efficiency of advertising. Such investigations are helpful in deciding what appeals are the strongest, what illustrations are the best, what size is the most effective, and how frequently the advertisement should appear, as well as in determining to a considerable extent what the results are to be before the money is spent.

One method of checking up on results of different advertisements in different mediums is through keeping careful records of the inquiries. The different advertisements in the different magazines are keyed in such a way that the inquirer indicates the particular advertisement which caused him to send in the inquiry. For instance, the address of the advertiser may contain "Department 263." This, of course, is not

really a part of the address but is a key which indicates that the inquirer saw the advertisement in a certain magazine, each individual advertisement in each magazine also being designated by a different "department" number. Another method, which is used to determine whether or not an advertisement is being read, is to offer, for instance, a complimentary booklet on an interesting subject and at the same time to tell the reader to send for the booklet, giving its particular number. The general subject of the booklet may be advertised in several different magazines, but in each of the different magazines the booklet will bear a different number, and consequently the advertiser will be able to check the relative effectiveness of the different mediums.

In distributing advertising material and circular letters by direct mail, it is often advisable to test out the material by sending it to a small proportion of the list first. If a certain percentage of inquiries is received, the advertiser has an indication of what is likely to happen if the material is sent to the remainder of the list. In this way, the advertiser often realizes a saving by finding that it is advisable to revise the letter or circular or to prepare a new one.

Laboratory tests of advertising are often made by psychologists for the purpose of testing effectiveness. These tests are made both before and after the advertising is run. There are many difficulties in making these tests, however, which interfere with getting accurate results. Most of them arise from the fact that the conditions under which the average person actually reads advertising cannot be duplicated in a laboratory. The reader ordinarily cannot have the same attitude toward advertisements under laboratory conditions, where tests are being made, that he has when casually reading a magazine. Even with these difficulties, however, the advertiser is able to gain much valuable information from the records obtained, which he can use to advantage in preparing and revising copy.

Another important test in advertising and selling is that of going directly to the consumer and discussing the product with him. An investigation which was made for a soap manufacturer illustrates the value of this kind of information. The manufacturer had made every effort to make this soap as good scientifically as it was possible for expert chemists to make it. He had also spent considerable sums in getting a well-designed cake and an appropriate package. He wished to combine in this product as many qualities as possible that would make it a popular soap for women. It was widely advertised and readily available in neighborhood stores. It was offered at a reasonable price, but it would not sell. He then conducted an investigation with a representative number of women who tried out the soap and discovered a number of objections among which the most important was the odor of the perfume.

This perfume had been included as a matter of course by the man who had made up the formula, and while it had not been thought objectionable by the men who worked with the soap 86 per cent of the women in the test objected to the odor and suggested that a different one should be used and 64 per cent said that the odor was much too strong for soap.

The foregoing methods of testing advertisements are also valuable in checking up on packages, window displays, counter displays, and the like, where their success depends upon their effect upon the consumer.

Advertising is costly, and it is generally thought that much money is wasted in this method of creating demand. For this reason, the leading publications which sell advertising prepare statistical data for the purpose of showing those who are buying the advertising just what kind of a market is covered, and about what results can be expected. Reference has been made (page 445) to some of the more widely applicable work of the Curtis Publishing Company, the Crowell Publishing Company, and the International Magazine Company. Much of the market analysis work, however, of these and other magazine and newspaper publishers is designed to show their advertisers the sizes and types of the markets reached through their particular advertisements.

When magazines and newspapers are attempting to determine the nature and extent of the markets served, it is ordinarily necessary to collect a considerable amount of primary data. An example of a survey of this type, made by a college newspaper in analyzing the market that it served, is furnished by an investigation conducted by the *California Daily Bruin*. The questionnaire that was used is presented in Exhibit 204. A few of the results will be discussed in order to indicate how they were used. The survey indicated an annual expenditure by the students of approximately $1,500,000 for clothing. Naturally, the local department stores and other dealers in commodities used by these students would be interested in this market. According to the survey, practically all of the students in the university read their college newspaper, and it was also read by the families of 62.5 per cent of the students. Ninety-four per cent of the students said that they read the *Daily Bruin* advertisements, while only 24.5 per cent said that they read the advertisements of the down-town newspapers. With these figures, those who were selling *Bruin* advertising space could point out the great advantage of advertising in this medium, as a method of reaching this market. In the same manner, the automobile and other figures were analyzed and presented to prospective purchasers of advertising space.

Sales Management Statistics.—The use of statistics in selling has increased rapidly during recent years, though there are still many sales managers who do not understand their use and who are skeptical of their

"CALIFORNIA DAILY BRUIN" QUESTIONNAIRE

1. Check the section you live in

Hollywood......Beverly Hills......Wilshire......Beach Towns..........
(Santa Monica and nearby towns)

Other Sections......

2. Where do you shop most? (Check three in each of *A*, *B*, and *C*.)

A	*B*	*C*
Army & Navy Dept. Store		
Barker Brothers		Baker's
Broadway Dept. Store	Harris & Frank	Florsheim
Bullock's	Desmond's	French Slipper Shoppe
Coulter's	Silverwood's	Gude's
Dyas	Foreman & Clark	Hackleman & Long
Maxime's	Hamner & Son	I. Miller
May Co.	Jacoby's	Innes
J. J. Haggarty	Mullen & Bluett	Walk-Over
Robinson's	Phelps-Terkel	Wetherby-Kayser
Sears Roebuck		Star
Walker's Fifth Street Store		Young's

Others..

D. Estimate amount spent on clothes yearly..........................

3. Do you shop in Hollywood?.....
4. Do you shop in the Los Angeles downtown district?............
5. What is your usual method of transportation?................
6. Do you drive a car?............ Own one?...What make?.......
....
7. Is the *Bruin* read by members of your family?..................
8. Do you read ads in the *Bruin?*....
....
9. Do you read any of the dailies?..
....
Do you read the ads in them?
....
10. Do you buy tailor-made clothes? ....Ready-made?..........
11. Are you accustomed to dealing with specialty shops or with department stores?............... From preference or convenience?
....
12. Do you ordinarily confine your trade to one store provided it meets your requirements?.......
13. Do you shop without regard to the reputation and service of the company?...................
14. Do you find that advertising influences you in deciding where to buy?.......................
15. Is quality or price the deciding factor in your judgments?.......
....
16. Do you ordinarily pay cash or charge your purchases?.........

....

NAME (optional) ADDRESS SEX

EXHIBIT 204.—Questionnaire used in analyzing a university market and in determining the value of a college newspaper as an advertising medium.

value. The general manager of a certain steel-forging business in a recent discussion of the question "Is the statistician a real help to the sales manager?" states: "I know a sales manager who almost lost his job a few years ago. He was and is a great volume builder. His men know how to sell merchandise and they get sizable orders. The house is never short of orders. But the problem which his management must face is that at the end of the month there is no worth-while profit to compensate for all the work done. This house tried to get this sales manager to install a statistical department. But the suggestion was not received in a kindly manner. 'Too much fool overhead expense now,' was his reply. 'Just leave us alone. Give us better goods and, if possible, get your costs down. You've got us carrying so much overhead now that you are choking us to death. Don't talk about increasing the overhead.'" However, an investigation was made which showed that the company's difficulties lay in the fact that the business was being obtained from a territory so broad that, without a proper price differential to include freight and without recognizing and adjusting the traveling expenses in the distant territory, the company would continue in trouble. The selling policy was revised and an advertising program was initiated. Within two years enough additional business was developed in the home territory, at a good profit, to make the house independent of its distant business.[1]

Many department stores, as well as other institutions, keep careful monthly, weekly, and daily records of sales. As soon as the figures for the current week are available, a form is filled out comparing them with the sales of the previous week and with those of the same week of the previous year. Ordinarily, the week of the previous year referred to is the same week according to the calendar—that is, the twenty-fifth week of this year would be compared with the twenty-fifth week of last year. However, this rule cannot be followed arbitrarily when similar weeks, as far as trade is concerned, do not come at the same time every year, and, in order to make the analysis more complete, one would compare the week preceding Easter Sunday of the current year with the week preceding Easter Sunday of other years. Monthly analyses are also made by comparing the sales for the current month with the sales for the preceding month and with the sales of the same month of the preceding year. And daily comparisons are made by contrasting corresponding Mondays, Tuesdays, etc. in previous weeks, months, and years. Tendencies and changes over a longer period of time may be analyzed, as indicated in the previous discussions of long-time trends.

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[1] Deute, A. H., General Manager, The Billings and Spencer Company, in *Printers' Ink*, March 7, 1929.

In setting sales quotas, it is extremely important that the sales manager know the condition of the business cycle in the industry as accurately as possible. If the quota is the principal basis of paying salesmen, and it is based on prosperous conditions, salesmen will not receive a fair income during a period of depression. If the quota is based on a depressed market, the salesmen will receive such large amounts in prosperous times that they will be greatly out of proportion to just incomes.

Forecasting Sales.—In making a forecast of sales, one must first find out as much as possible about the past and present conditions which affect future conditions in the market for the particular concern's products. Then the forecast is made by applying the information in making the schedule of sales expectations. But a sales forecast should not be made by a mere mechanical application of information, although definite statistical procedure may be followed in arriving at the estimates. The forecast itself is not ordinarily as definite as it appears to be, and its best use depends upon a complete understanding of its real value. It is a systematic estimate which gives the sales manager or other executive a better idea of what to prepare for than he would have without it, but as a rule so many things can happen, which cannot be foreseen, that the sales forecast should not be followed blindly.

The information that is to serve as the basis of a sales forecast is gathered in various ways. Sometimes the necessary data are furnished by dealers. Sometimes they are collected by salesmen in the field. In other instances the investigations are based on information secured directly by the home office. In most instances, it is advisable to use the reliable secondary data that are available on such questions as population, purchasing power, financial conditions, building activity, retail trade conditions, and the like, even though extensive primary investigations are also made.

Field Investigations—Primary Forecasting Data.—Personal investigations in the field, when they can be made, are often of great value in connection with making sales forecasts. These may be carried on in a number of ways. A company's representatives, when calling on the trade, may report upon the sales expectations of each dealer for the following year. From these data on sales expectations, along with the representatives' own observations, a sales estimate for each district may be made.

When gathering information of the above type, definite questionnaires or schedules should be filled out for each dealer, showing not only what the dealer himself expects his sales to be, but what the representative believes is a reasonable expectation. Exhibit 205 presents a dealer's sales estimate questionnaire for gathering information to be used by a manufacturer in making a sales forecast. This form includes dealer's

and representative's estimates, sales for the current year, and stocks at the beginning and end of the year—all by classes and types of products.

After obtaining the estimates of dealers' sales, the compilation of the company sales budget for the entire district and for the country as a whole is undertaken. In this procedure, however, the dealers' sales amounts

DEALER'S ESTIMATE OF SALES

For year__________

Firm name__________ Location__________ Date__________

Estimate by__________ Position__________ Co. Representative__________

	Dealer's sales estimate for next year	Sales this year (approximate)	Budget for this year	Representative's sales estimate for next year	Expected stock holdover (at selling price) end of this year	Stock holdover (at selling price) beginning of this year
Product I Grade *A*						
Grade *B*						
Grade *C*						
Product II Grade *A*						
Grade *B*						
Etc.						
Total						

EXHIBIT 205.—Dealer's sales-estimate questionnaire for gathering information used by a manufacturer in making a sales forecast.

must be converted into the manufacturing company's sales basis. In other words, the dealers' gross margins and the jobbers' gross margins must be eliminated. This can be done by deducting the average or standard mark-up from the sales estimate for each product, but as this may vary among different products, the percentage mark-up deduction may need to be computed for each type of article. Also, the dealers' holdover stocks must be taken into consideration in estimating the manufacturer's sales. In this connection, it is important to note that an

increase in the stock held over does not necessarily mean that the dealer will not order to fill the entire expected sales requirements. Larger sales volume usually requires larger total holdover, although it usually results in a smaller *percentage* of holdover for inventory requirements. The important feature in making adjustments of manufacturers' sales estimates because of dealers' holdover stocks, is *to observe variations from a standard percentage of holdover stock.* Thus, if product II, Grade *A* sales are estimated for the next year at $5,000 for a given dealer and he has goods on hand of that grade and product of $4,000 at the sales price, it is probable that he will buy only enough during the coming year to end the year with a normal balance. Thus, if 20 per cent of the annual sales is a normal holdover, this dealer could be expected to order the product of this grade in the amount of $2,000 at his selling prices, as this would leave a $1,000 holdover at the end of the next year.

The example above indicates how an analysis of expected sales is made on the basis of dealers' and representatives' estimates. A similar analysis of sales probabilities can sometimes be made by determining the expected sales according to salesmen's estimates.

To illustrate, we may consider the case of a furnace manufacturing company which has a retail department handling sales and installations of furnaces in new buildings and replacements in old buildings. Sales are made by salesmen who follow up every "lead" evidencing a proposed new building or the remodeling of an old building. These salesmen establish contacts with architects, builders, and contractors. In estimating the sales of the retail or installation department for the following month, quarter, or year, the estimates of the individual salesmen should be indicative of the immediate consumer demand, although their estimates may not accurately forecast sales of a period far in the future.

In using salesmen's estimates to assist in the preparation of a sales forecast, a careful analysis of the past estimates of each salesman and of his actual sales must be made. Each salesman must be made to realize that his estimate is more than a prediction; it is a quota expected from him. In making his estimate he will need to refer to his record of sales in progress and in anticipation. His sales for a similar period in a previous year will be important for comparison, but his knowledge of current conditions and current prospects should be most significant. Accuracy in making a sales forecast through the salesmen's estimates is dependent upon their realization of their responsibility for careful estimating. Obviously, to make the sales forecast definitely valuable to the manufacturer, the salesmen's estimates must be by types of products as well as in total.

Collecting Forecasting Data at the Sales Office.—Many companies may not find it financially possible, or even advisable, to send representa-

tives into the field to call on dealers. Also, until their salesmen are properly trained, the salesmen's estimates may be unreliable. Therefore, it may be necessary to limit the investigation of conditions in the different

COUNTY		STATE		TERR. NO.	
1930 POP	% URBAN	DENSITY	PRINCIPAL CITY		
1020 POP.	ALL FAMILIES		ALL DWELLINGS		
% INC OR DEC	% WHITE	% COLORED	SQ. MILES		
NUMBER OF FARMS	WITH RUNNING WATER		WITH ELECTRICITY		%
NUMBER OF FARMS WITH GAS	%	% FARMS IMPROVED		AV ACREAGE	
NUMBER OF FARMS WITH INCOME OF $					
NUMBER OF FARMS FULLY OWNED	WHITE				
FIRE WOOD CUT	CORDS	AV PER FARM	COAL MINED, TONS		
GAS CONSUMERS	RATE	CITY		% FAMILIES	
ALL DWELLINGS WITH ELECT	% FAMILIES				
COMPETITION					
VALUE ADDED BY MANUF	%	YEAR	CROPS	%	YEAR
MINES & FISHERY	%	L. S & L. S PRODUCTS	%	YEAR	
TOTAL INCOME	BUYING POWER PER FAMILY		INCOME PER FARM	INDEX	
NO. OF DEALERS SOLD	HOWE.	G M	DEPT.	FURN	P P INDEX
					S OF L INDEX
					BUYING POWER PER CAPITA

Front

YEAR	STOVES	%	CABS	OVENS	HEATERS				WICKS	C. S. & H.	
					WATER	ROOM	%	CIRCU.			
	QNT'Y	FAMS	QNT'Y	QNT'Y	QNT'Y	QNT'Y	FAMS	QNT'Y	QNT'Y	VALUE	

Back

EXHIBIT 206.—County analysis card for sales-quota determination. (*Courtesy, Perfection Stove Company.*)

territories to information collected in the companies' offices. The practice of the Perfection Stove Company is illustrative of this method of surveying sales probabilities. This practice will now be described in connection with Exhibit 206.

"The unit accepted by the company for the accumulation of data and for purposes of forecasting sales expectations is the *county*. This unit was chosen because the product of the company—oil heaters—obtains its best distribution in rural sections, where buying power is good and fuel conditions are favorable to the use of oil. A detailed analysis has been made, for each county, of all conditions which seem to affect the sale of its product, and the resulting data are entered on a 'county card' (Exhibit 206).

"On these cards, the shipments to dealers are tabulated by counties, a report being sent to each district manager every three months, showing the county totals by items for his district. The manager has a duplicate set of the cards and is advised of any additional information or changes. The card provides for the leading competition the company has to meet. The information, therefore, was obtained mainly from a 'poll' of oil-stove users, conducted by magazines and journals in which the company advertises. To supplement this condition of competition, the company's own salesmen check up from time to time by questioning the dealers they call upon. It is readily admitted by the company that these data are not complete, yet they do give an exceptionally good 'cross-section' view of county conditions and are proving in practice accurate bases from which to work. The latest government and other reports of crop, financial, and trade conditions are carefully analyzed and used to modify the expectation of business to be done, as well as to reveal the sections where special advertising or intensive work is required.

"The county card, on the reverse side, shows what business has been done for several years back, and there are now being accumulated, by means of the punched card system, monthly shipments. This knowledge of general conditions and best sales is considered essential for a reasonable estimate of prospective business.

"As soon after November 1 as the October statistics are completed, the statistical and research department makes up detailed statements covering the 10 months' business, January to October, plus an estimate from November to December. These statements are then forwarded to each district manager for a preliminary estimate of the next year's business. The estimates, after being carefully reviewed, are translated into a manufacturing budget so that the cost department may develop a manufacturing program and figure standard costs for inventory purposes. As it is a six weeks' job to make entirely new standards for a possible new line of product, the necessity of beginning this work by November 15 is clearly seen.

"After the preliminary sales estimates have been reviewed and the manufacturing program decided upon, the budget officer visits the district offices in turn and works out with each manager a revised sales

budget, adjusting the ratios of the various products and increasing or decreasing items which appear to be out of line. Upon the budget officer's return to Cleveland the figures are again carefully reviewed and then approved by the general sales manager and the directors."[1]

Market Trends.—In solving marketing problems, business men are constantly thinking of the long-time trend and its relation to the other movements. The two principal trends to be considered in this connection are the trend of the market for the product and the trend of the particular company's sales of that particular product. Though the business man constantly has long-time trends in mind and must consider them, it is difficult to determine them accurately.

It is of utmost importance that changes and faults in the basic trend be discovered promptly so that they will not be mistaken for cyclical variations. Often these may be discovered through investigations and analyses which are not statistical. This is illustrated by the wholesale dry goods and grocery situation in Los Angeles in 1926. Department-store sales were increasing steadily, while wholesale dry goods sales showed a marked tendency to decline. An investigation indicated a shift to the effect that department stores were buying more of their merchandise directly from the manufacturer and less from the wholesaler. Another investigation indicated that a similar situation existed in the grocery trade in Los Angeles—that is, while the city had been growing steadily, wholesale grocery sales had declined. The explanation was that chain grocers had a much larger proportion of the retail trade in 1926 than they had in 1921, and while chain stores were buying a considerable proportion of their groceries from jobbers in 1921, they were buying practically all of these goods directly in 1926. In addition to this, the tendency to consume a larger proportion of fresh fruits and vegetables had also influenced the basic trend of wholesale grocery sales.

Seasonal Variation.—It is generally appreciated that seasonal variation plays an important part in many types of distribution. Exhibit 157, page 327, shows the Christmas peak in shoe sales very distinctly, and Exhibit 155, page 324, shows the summer peak in gasoline consumption. A statistical determination of the average seasonal variation aids materially in making plans for the year's business. Seasonal variation often occurs in a market as a whole and in a concern's business as a whole, but, if the best results are to be obtained in making constructive analyses, it is often necessary to break a concern's business up into minor parts and to study the variation in small groups or single lines of commodities.

In many instances, seasonal peaks and valleys in a concern's business can be smoothed out to a considerable extent, with profitable results.

[1] Bruere, Henry, and Arthur Lazarus, "Applied Budgeting," pp. 123–126, McGraw-Hill Book Company, Inc., New York, 1928.

In other instances, the causes of the seasonal variation are more fundamental and it is advisable to fit the business carefully to the market conditions as they exist. In both cases, it is obvious that it is necessary to have a thorough understanding of the seasonal variation, and, in this connection, statistical methods serve a very useful purpose.

Marketing and the Business Cycle.—In no field is the proper analysis of the business cycle of any greater importance than in marketing. All that has been said in past chapters regarding the analysis of the business cycle is of great importance in market analysis. Undoubtedly, this is already appreciated by the reader, because several of the examples used in illustrating business cycles in previous discussions of this subject were distinctly marketing problems. Some of the greatest losses in handling commodities are due to misjudging the cyclical conditions of the market. The 1920 silk situation, for instance (see page 272), furnished a striking example of lack of information, wrong information, and incorrect analysis regarding the cyclical condition of the market.

Similar problems were faced by those who were engaged in other lines of wholesale and retail trade. "In the autumn of 1919, the merchandise manager of a department store in New England began to make plans for the spring trade in one of his important lines. He knew the prices of raw and finished goods had been advancing steadily, and retailers had encountered difficulties in securing prompt deliveries. He was convinced that he must place his orders well in advance of his needs. His store, like that of other retailers, had sold an unprecedented amount of goods. Trade rumors appeared to indicate that the supply of raw materials was less than normal, and it was so difficult to secure deliveries that retailers duplicated orders with different manufacturers in order to be assured of the delivery of part of their requirements.

"The merchandise manager thus decided to place in the autumn an order for the full requirements of the anticipated spring trade in 1920, for the purpose of avoiding the loss of prestige which the store would face if it were unable to supply its customers. To finance its purchases the company negotiated a bank loan at a time when money rates were rising, when wholesale and retail prices of all commodities were tending upward, and when the rapid advance—although the manager did not know this fact—had led to a considerable amount of speculation in the goods he wished to buy. The loan was obtained, the order given, and the result, of course, was a heavy loss.

"It should here be pointed out that the retailer must carry stocks on hand. He must expect to make some losses in a period of depression. Some retailers believe that it is wise to build up a reserve in a rising market, fixing prices on a replacement value; they believe that it is good business to take losses early in a falling market by the same process of

determining prices by replacement values. If information were provided to enable the retailer to change his present habits and buy 'from hand to mouth' at the top of the boom and in larger quantities in a period of depression, bank credits would be relieved and cancellations decreased. It is the natural tendency at present to overstock in boom times and to stop buying in depression. This tendency is strengthened by the difficulty of obtaining deliveries in a period of boom.

"The following instance of two manufacturers in the shoe industry illustrates the difference between what may be readily recognized as a conservative policy in one period of the business cycle and what just as clearly becomes a speculative policy in the other extreme of the cycle.

"The first manufacturer of shoes, well along in the period of expansion, received more than normal orders from his customers. He knew the industry was overexpanded but he lacked any basis for judging how to handle his business at this time. Supplies of leather were apparently very small; congested railroad conditions made it imperative to have his raw material shipped to him several weeks before it was usually necessary; prices of leather were rapidly rising. The men in the industry did not question the price at which leather could be purchased, but seemed willing to pay any price for raw material laid down at the factory. Under these conditions the manufacturer followed what must have seemed to him a conservative policy by purchasing raw material enough to cover his orders. Then came the break in leather prices; cancellations poured in, and he was unable to liquidate and experienced an enormous loss on his inventory of raw material.

"Another manufacturer, under the same market conditions and at the same stage of the business cycle, convinced himself through a study of conditions that his orders were inflated and that it would be unwise to cover with raw material. He cut his orders to 25 per cent of his requirements. Other executives in his plant and particularly his sales organization opposed this policy. They prophesied not only that he would have to cover at high prices but that he would probably not be able to secure raw material at all and consequently his relative position in the industry would be destroyed. After the break this shoe manufacturer was able to purchase raw material to fill his requirements at much reduced prices, and he was able to send his sales organization into the field with a line of shoes produced at low cost, in competition with his competitors who were attempting to liquidate high-priced inventories.

"It was not because one of these managers was conservative that he was more successful in coping with the problem of cyclical losses. The distinction that should be emphasized is that the manager who knew and acted with reference to the fundamental facts of the relationship of his

particular business to his industry and who had a sound knowledge of general business conditions from study of statistical data of trade generally was able, to a large extent, through policies planned in 1919 and 1920, to avoid the difficulties of 1921, while the manager who ran his business without reference to such factors was gambling in his most important decisions.

"A problem which business men faced in 1920 and 1921 was the widespread uncertainty caused by cancellation of orders. From one standpoint these cancellations were entirely unjustifiable. Legally the orders constituted contracts and should have been carried out. It is important, however, to consider why widespread cancellations occurred. Their extent was undoubtedly due to at least two causes, both intensified by the wide fluctuations in that particular cycle.

"First, in the period of inflation of 1919 and 1920 an unusual and perhaps unprecedented delay in securing deliveries was caused by the breakdown of transportation facilities near the peak of the boom. Many business men not only placed orders earlier than usual but also duplicated them, or ordered in larger quantities, with the expectation of having deliveries scaled down. When the slump came they had strong temptations to cancel.

"Second, price levels had been on such a high plane when orders were placed, and the fall in prices was so violent, that many business men faced the alternative of cancellation or bankruptcy. Under these conditions it was wiser for the seller to accept cancellation than to deliver material to a customer who could not pay.

"From the illustrations of business experience which we have reviewed, it is evident that knowledge of one's own business should be strengthened by knowledge of the conditions in the industry of which it is a part, and by information about current and future trends in general business conditions. The business man is placed thereby in a strong position to exercise judgment based on facts rather than on guesses, speculations, or approximations.

"First, he must have available for his use current facts about general business conditions throughout the country and knowledge of the probable future trend of general business conditions.

"Second, he must have the basic facts about his industry. Because his particular business is influenced by conditions affecting his entire industry, he must be in a position, with others in his industry, to study its peculiar industrial problems.

"Third, he must secure enough facts about his own business to give him not merely statistics but a proper basis for judgment as to his general policies.

"Fourth, he must inform himself in regard to the general credit situation and especially the attitude of his own bank toward extensions of loans."[1]

Importance of Random Movements.—In analyzing market conditions, it is of utmost importance that random movements be identified as quickly as possible so that they will not be mistaken for cyclical variations. As is the case in studying faults in the trend (previously discussed) these random movements may often be discovered through investigations and analyses which are not statistical. Random variations are particularly characteristic of the changes in market conditions because of the effect of styles, fads, and similar influences, which vary widely from time to time and are difficult to analyze.

Price Situation.—Among the more important parts of the market investigation is a thorough analysis of the price situation. This involves collecting adequate current and past data and analyzing them over a period of time. Trends, seasonal variations, cycles, and random movements, all may be important. Studies often must be made which compare the price relation of one industry to another or different parts of the same industry to each other.

If prices are high in a certain locality as compared with other areas, or if they are unusually low, then an attempt should be made to find out the reasons for this condition and whether the condition appears to be permanent or temporary. Since prices are determined largely by supply and demand, they should also be analyzed in relation to the purchasing power of the buyers. This is necessary in order to determine the quality of the product that should be pushed in any particular market.

In determining whether a price should be lowered or raised, it is often well to compare the prices of the product under consideration with those of some of the commonly published price indexes. In this way, one often has a good indication of the general demand conditions.

Importance of Allowing for Price Changes.—Any statistical comparison of sales or other data based on dollar valuation is likely to be misleading unless the effect of price changes is thoroughly understood. The statistician of the Graybar Electric Company, writing in 1929, explains this difficulty in terms of his company's experience: "The sharp general rise in prices brought about by the war, which affected those in the electrical supply field, led the Graybar Electric Company to weight its sales statistics so as to show physical volume of sales as early as 1918–

[1] From "Business Cycles and Unemployment" report and recommendations of a committee of the President's Conference on Unemployment; short report published by the United States Department of Commerce (pp. 3–9), and full report published by McGraw-Hill Book Company, Inc., New York, 1923 (pp. xiii–xviii).

1919. The upward swing of prices had become so pronounced by 1919–1920 that had not this been done in some of the studies made, estimates of sales based on dollar valuation would have been 100 per cent or so out of the way on the basis of physical volume. And the studies would have been more confusing than helpful to the sales department and almost altogether valueless to other departments.

"This matter of price fluctuation is quite generally given consideration in connection with sales statistics. But there is a tendency to give it consideration only occasionally. If sales show an increase over last month in dollar value, many sales managers are satisfied to let well enough alone for the time being. And they do not ask the statistical department for an adjustment until the end of the year, or the end of several years, when there may be a rude awakening. A salesman who doubles the volume of his sales in a period of a year or so, as some of the Graybar salesmen did in 1916 and 1917, and again in 1919 and 1920, naturally likes to think as well of his accomplishment as possible, and so does his sales manager. But it does no good to keep him on a pedestal of false dollar value only to knock it out from under him at the end, as is inevitable if any great changes in prices occur. Nor is it any more fair to him to leave him in a needlessly deep pit if prices are declining. Therefore, it is not only important that adjustments of dollar volume to physical volume should be made, but that the adjustment should be a continual monthly operation.

"To take a representative commodity by way of example, an average carload of iron conduit, which is a staple electrical supply handled by the Graybar Company, was worth $1,412 in 1914. The average cost in 1920 had shot up to $3,135, and the cost at present (1929) for the same quantity has dropped to $2,570. Since the requirement of this commodity per unit of building construction has not changed materially in the meantime, the efficiency of the sales department is more properly measured by the units of merchandise than by dollar value, and it is apparent that the adjustment should be checked at frequent periods. This is even more true in the case of the service and warehouse departments whose efficiency is wholly measured by the physical volume of merchandise handled. Their expenses are measured in current dollars, but for the same dollar they have to handle 22 per cent more conduit than they did in 1920. For, in 1920, sales of $100,000 worth of conduit represented only 32 carloads, whereas in 1929 that amount of sales represents 39 carloads; and in 1914 it represented 71. And this is but one example of price fluctuations in commodities in our field.

"Of equal importance is the manner in which the adjustment is made. The Graybar Company has worked out a series of commodity indices which it employs for this purpose, based on price changes in 19 represen-

tative groups of the 60,000 items that it carries. In each group, key commodities are selected. And the commodity index for each group is based on the average selling price of these commodities during each month. This index is applied directly to statistics relating to individual groups of commodities; then, for some purposes, a composite commodity index is employed which is arrived at by averaging the group indices and weighting them in accordance with the importance of each group in terms of sales."[1]

Transportation—Time and Cost.—Freight charges in marketing often amount to a considerable proportion of the final cost of the product. Weakness or strength in a particular market often depends upon small differences in transportation costs. When shipping goods long distances, it often happens that there are several routes to be considered. Water transportation is ordinarily less costly than rail transportation. Rail transportation for long distances is ordinarily less costly than truck transportation, while truck transportation competes strongly for freight that is hauled relatively short distances. An adequate compilation of data on the subject will ordinarily answer any question of cost quite clearly.

Freight rates, however, do not always constitute the only factor in deciding the method of transportation that is to be used. Time is always of considerable importance and sometimes it is of very great importance. In shipping goods from one coast of the United States to the other, water freight rates are low as compared with rail rates but the greater time involved often prevents the use of this method. This is especially true in shipping perishable products. It is obvious that transportation requirements vary greatly with different commodities, but the finding of the most economical method in the long run is greatly facilitated by the careful use of statistics.

Locating Stores.—Analyses of store locations vary widely with the different types of stores. Some of the principal problems involved in locating a retail drug store and in locating a department store will be discussed briefly at this point.

Most of the merchandise handled by a retail drug store is classed as convenience goods; that is, the different products are purchased at a convenient location. Consequently, the trade depends upon those who find it convenient to make their purchases there. Statistical checks, showing the number of persons which pass given points, will help to determine the most favorable locations where the trade depends on drop-in customers as it does in many downtown locations. If it is to be a neighborhood store, the number of people living in the immediate vicinity

[1] GARDINER, J. H., Statistician, Graybar Electric Company, abstracted from "Getting the True Sales Story from Statistics," *Sales Management and Advertisers' Weekly*, p. 327, February 9, 1929.

has an important bearing. In making these surveys, careful judgment must be exercised in order that the data may be classified so that they will be most significant. It is important to know how many persons pass a location during the different hours of the day. Such a classification indicates to a considerable extent whether they are laborers going to work, or business people who go to work a little later, or shoppers who do most of their buying still later in the morning. In the afternoon, the first heavy movement of traffic is ordinarily shopping traffic, which is followed by business people and workmen going home, and this in turn is followed still later by the going and coming of the theater crowds. Automobile traffic on heavily congested streets is ordinarily of but little importance as drop-in trade. In fact, the net effect of heavy through-traffic may be negative, since it discourages people from crossing the street and serves as a barrier against the market on the opposite side of the street. Other factors, such as purchasing power, are just as important in this connection as they are in connection with the problems previously discussed.

The procedure in analyzing the location for a department store is quite different from that just mentioned. The number of people passing a given location may be small and might indicate a poor location for a drug store, but this fact would not indicate whether or not the location would be a good one for a department store. In selecting a site for a department store, the problem is much more extensive and depends upon such factors as the present and potential accessibility in relation to the location of present and future purchasing power. It also depends upon its nearness to similar stores in the shopping district. Such factors as width of street, parking facilities, street car service, and the like are also of great importance.

Specific Selection of Prospects.—After making a general analysis of demand, the next step is to make a specific selection of the prospects to be followed up. This procedure will be explained by referring to the experience of the Standard Conveyor Company, which, by reducing its prospects' list 67 per cent increased its sales 15 per cent. In referring to this the president stated: "Had anyone asked us a year or so ago how well we knew our market, it is quite possible that we would have pointed to our old prospect list of 120,000 and said that was the whole of it. It was—and then some.

"To-day, with our list of 39,000, we name as prospects only those companies which are specifically interested in what we, or our competitors, make, and which can use our products in their factories.

"That is my idea of knowing your market. When you know that—when you know where your goods can be sold—the actual selling of them is a clearly defined problem.

"Every business has its individual peculiarities. If it is to be successful, it must have a definite place, and while its activities might enlarge, it cannot get outside that place. Our own business consists of the manufacture and sale of various conveyors, which are of no value to some industries, but invaluable to others. The foremost element in deciding this is labor. A 12-man shop in a back room, naturally, would have no use for an elaborate conveyor installation which would keep production moving steadily.

"Yet there was a time when we were expending sales effort on prospects which were not prospects at all, because they had not, and never would have, a use for our conveyors.

"Our sales promotion manager worked nearly a year to devise a new method of checking our future sales. Taking our old prospect list, he checked it up with Dun and Bradstreet and learned that a certain rating would average a certain number of employed workers. This seldom deviated. As our product is either useful or of little use, dependent on the number of men employed, right there we had the means of making a list which would consist only of plants that would have a definite need for our products. Not every business can use that measuring stick—but almost every one can find some similar way of picking a real prospect.

"When we had pared down our list, we checked the total number with the investigations of others and found it a reasonable one—it approximately agreed, for instance, with the industrial coverage of certain publications.

"This was narrowing our selling effort to a point where it could be felt. It was a matter of clearing the decks for action. Our action was to compile an industrial census, telling us the whole story about each one of these 39,000 companies. When the census was completed, we had in graphic detail the fact that there are about 30 major industries which we can serve. We had them divided as to industry and territory; if we wanted to know how many automobile manufacturers, for instance, were in a given territory, our industrial census gave us the exact information.

"With this information at hand, when it came to selling, instead of sowing our seeds broadcast, we were dropping them in well-defined rows—what any regulated business should do.

"Our own selling is done through territorial branch offices, and, through our industrial census, we have set quotas for each salesman. The element of the salesman entered into the setting of these quotas to some extent. His work for the year before was considered. But the main thing was—what should he be able to do?

"Some companies hold their contact with the market entirely through their commercial travelers. Salesmen are given geographic territories.

Armed with a prospect list and such inquiries as have happened into the office, they are sent out on the road. Spurred on by the burning enthusiasm of a go-getter sales manager, they are given such indefinite instructions as to work their territories thoroughly and do their best to increase sales. It is a matter of 'hurrah' selling entirely: 'Smith sold so much last month,' they are told, 'now you go out and sell this much more.' The men try it—and sometimes succeed, for enthusiasm is indeed a wonderful thing.

"Yet this is not constructive salesmanship. It is like blindly shooting at a covey of quail with a shotgun: because they fly close and your gun scatters wide, you may bring down a bird or two.

"In this kind of organization an office conference is usually held to determine production. The sales manager brings forth his records. 'Business conditions in the Midwest are good,' he reports, 'and the salesmen are cleaning up. With any luck at all, their sales should be such-and-such. In the West, the men are averaging so-and-so.' And down through the list of territories he goes, adding up such-and-such, and so-and-so, and this-and-that, until he has figured what production should be—based on what salesmen have been selling.

"What a slip-shod way of doing business! What an unkind way—unkind to every man in the organization. A production figure is set, but it is not a true production figure. The men in the factory are working on the achievements of the men on the road a year ago. Their jobs, their whole lives, depend on it. The men on the road are content to average last year's mark, or perhaps, carried away by the sales manager's gusts of enthusiasm, to do a little better.

"How much better it would be to call the salesman aside, and give him a quota of sales. 'You can sell this much,' you would tell him, 'because in your territory there are so many industries which are particularly adapted to the use of our products. Here are their names.'

"One of our salesmen came in not long ago, complaining about the quota we had set for him. 'It's too high,' he said. 'You've got me down to sell 7,000 feet of gravity conveyor this year, while last year I only sold half that much and was supposed to have had a good year.'

"I called in our sales manager and together the three of us went over this man's territory, with our industrial census before us. 'Look here,' I said, pointing to the census, 'in your territory there are 46 lumber concerns. Now last year you didn't sell a single lumber company, though every one of them is a potential user of our conveyors. Here are automobile manufacturers—how many feet of conveyor did you sell automobile companies last year?'

"In that manner we went down through the list.

"'I'm glad I complained, so that I could get set right,' he admitted. 'It's a matter of directing energies. I was sort of in the dark, and this thing brightens up the way. I believe I can sell more than my quota.' I thought he could, too—and he did."[1]

Questions and Problems

1. Why are marketing analyses important to manufacturers and traders?

2. Before attempting to build up a new market, what data on the product and on the consumers should be collected?

3. Are primary or secondary data most often used in market analyses?

4. What type of study of the market should first be made? How extensive should it be?

5. How accurately can the demand for, and supply of, a product be measured?

6. How can indexes or indicators of demand and supply be obtained for specific products? Illustrate.

7. How are population and purchasing-power data used in determining the extent and distribution of demand?

8. How can data on commodity stocks be obtained?

9. Discuss the problem of determining market areas and sales territories.

10. What factors must be considered in laying out retail trade areas?

11. Explain how the "Market Data Handbook of the United States" can be used in making market analyses.

12. How would you determine what the public wants in a particular product? Are questionnaires reliable for obtaining the consumer's reasons for purchase? Discuss the difficulties involved in collecting such data.

13. In fitting a product to the market, what kinds of investigations should be made?

14. What does a concern need to know before proceeding with an advertising campaign? How can the information be secured?

15. What types of data and analyses are particularly important to the sales management officials in: (*a*) national distributing organizations, and (*b*) department stores?

16. What is the nature of a sales forecast, and how should data be obtained for it?

17. How does stock holdover by dealers affect the manufacturer's volume of sales, and how can the latter estimate his sales from dealers' estimates?

18. How are salesmen's estimates used in the preparation of sales forecasts?

19. What data does the Perfection Stove Company use in making sales forecasts, and how does it get the information?

20. Why is it important that basic trends be determined as accurately as possible in market analyses?

21. How are seasonal variation studies used in improving one's position in the market?

22. Why is careful analysis of the business cycle so important in the field of marketing?

23. How may the business cycle affect department store purchases and sales?

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[1] DONAHOWER, H. L., President, Standard Conveyor Company, abstracted from "We Threw 67% of Our Prospects Overboard—and Our Sales Increased 15%," *System*, p. 577, November 1926.

24. Is a conservative policy of purchasing adequate when a boom period reaches a high point? How can inflation in size of orders be ascertained? Why are order contracts sometimes not enforced in a period of depression?

25. What four fields of business facts must be studied to judge the position of a business in regard to business cycles?

26. Discuss the importance of discovering random movements in market analyses.

27. What types of price analyses should be made in connection with investigations of the market?

28. Discuss the effect of price changes in terms of the experience of the Graybar Electric Company.

29. Why is it important to make studies of time and cost of transportation?

30. In locating retail stores, what data should be collected? Discuss the differences in analyzing drug store and department store locations.

31. How did the Standard Conveyor Company obtain an efficient list of prospective customers? How does it direct its salesmen?

32. Using the "Market Data Handbook," make a report on the market for household electrical equipment in the United States or in a certain area.[1]

33. Using the "Market Data Handbook," analyze the market in a certain area or in the United States as a whole for certain products in which you are interested.[1]

34. Select a certain street corner in your city, and analyze it as a location for a retail drug store.[1]

35. Select a certain street corner in your city, and analyze it as a location for a department store.[1]

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[1] See Appendix IV, pp. 671 to 712, for suggestions for organizing and outlining a report.

CHAPTER XXII

REAL ESTATE ANALYSIS

Practically every one is interested in the problem of analyzing real estate. This interest may be from the point of view of determining general values or from the point of view of determining what is best for one's own particular purposes. The problems of land are basic to most economic activities. Its value varies in accordance with the importance of its use, and while it may be of considerable value for use in agriculture or mineral production, it ordinarily has its greatest value when used as building sites.

Use of Statistics in Real Estate.—In order that one may determine as definitely as possible whether, for instance, one piece of real estate is more valuable or more desirable than another, all costs considered, it is necessary to have as many definite measurements of conditions as can reasonably be obtained. There are so many factors involved in determining the value of a piece of real estate that it is always impossible to do so with 100 per cent accuracy, but often the collection of only a few of the outstanding statistical facts will enable one to make a much better decision than could be made without them. More specific uses of statistics in real estate will be suggested from time to time throughout the chapter.

Defining the Problem and Planning the Investigation.—All that has been said regarding the definition of the problem in connection with previous discussions applies with equal force to a problem of real estate analysis. Before undertaking a survey, the specific question to be considered should be carefully outlined so that plans can be made to include the collection of as much as possible of the important information. Since there are so many factors involved, it often happens that a preliminary analysis is of great value either in showing that it is not advisable or necessary to investigate further, or in showing that additional information should be included, which was not considered in the original plan. A preliminary survey ordinarily can be made in such a way that it will not be necessary to duplicate the effort when a more detailed survey is made later.

Supply and Demand.—In analyzing real estate situations, business men often find simple statements of the relation of supply and demand

very valuable when such are possible. Examples of reports of this type are the dwelling and office building vacancy surveys made in a number of cities.

Statements of real estate vacancy situations are ordinarily most valuable when accompanied by practical interpretations of their significance. In other words, what does a vacancy of 5, 10, 15, or 20 per cent mean? In Los Angeles, for instance, studies of the relation of supply to demand in office buildings showed a vacancy of 20.3 per cent on December 31, 1926, and indicated a vacancy of 25 per cent early in 1927. The question naturally arose as to whether or not a vacancy as high as 25 per cent was actually serious in a city with a rate of growth as rapid as that of Los Angeles. The significance of such a vacancy factor was made somewhat clearer by making estimates of how long the supply would probably last if no additional buildings were added except those already under construction. These estimates indicated in 1926 that even if no more new office space was placed upon the market, except that which was then under construction, there would be practically no shortage by 1930 and there might be a substantial vacancy at that time. In this way, the situation was stated in a practical manner so that its significance was easily appreciated. (The methods of making these estimates were explained in detail in Chapter XVII, pages 368 to 372.)

When making analyses such as the foregoing, however, it should be fully realized that the relation between supply and demand may not always control the situation. At the time the above Los Angeles survey was made, it was obvious that a number of new buildings would be constructed in the face of such an unfavorable situation because of long-time lease agreements made during the 1923 to 1924 boom, which called for the erection of height limit office buildings during the following few years.

Forecasting Demand.—Representative problems of estimating future demand will be considered by using various subdivision situations as illustrations. The methods are of general interest and will indicate how other problems can be analyzed.

During the years from 1921 to 1929 there was widespread city land speculation throughout the country. In many cities the population increases had been large, but they did not justify all the gains in land prices. Population growth is a very important factor in determining real values of land, but it only partly controls the changes in prices. A study of this relation was made in connection with a group of Wisconsin cities (where conditions were relatively stable), which showed that in cities with less than 15 per cent increases in population from 1902 to 1922 the increases in land prices (after correcting for changes in the purchasing power of the dollar) were roughly from 60 to 70 per cent. This indicates that even in stable communities, the increases in land prices

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may be due to factors other than population growth and changes in the value of the dollar.

Although great profits have been made in certain instances, the subdivision of city lots ordinarily involves large risks and danger of unusual loss. And, no matter how great the risk, it is usually accompanied by hopes of great gain on the part of a large number of persons who are not able to distinguish between sound and unsound values, and who believe that they are making safe investments. These errors in judgment are usually caused by inaccurate and inadequate information regarding the normal possibilities of future demand. "There is always an element of uncertainty in such expansion due to the fallibility of human judgment and the lack of information in regard to the community's development. But this element of uncertainty can be and is being reduced by more careful analysis of the growth and movement of population, and of the trend of values, which reveals the condition of the market for home, factory, or business sites. In so far as real estate dealers and owners seek and take advantage of more complete information of this character, their forecasts of the needs of the community will become more accurate, and urban developments based on these forecasts will become investments rather than bad speculation."[1]

Forecasts of future demand which are based upon population forecasts are of considerable value in making future plans for projects which are related to real estate values. Methods of estimating future population have been explained previously. The using of these estimates in analyzing and forecasting real estate conditions will now be discussed.

The making of demand estimates on the basis of population forecasts can be illustrated by referring to a survey of the Los Angeles subdivision situation which was made in 1924.[2] Population forecasts were made indicating that Los Angeles City[3] would have a population of 1,250,000 in 1930, and 1,500,000 (minimum) or 2,000,000 (maximum) in 1940. From two independent bases, the relation of population to occupied lots was determined. (In this study, a lot was considered as occupied if it had a building on it, and it was necessary to make a correction for vacancies in buildings when estimating future demand on the basis of population.) Dividing the above estimates of future population by the number of persons per occupied lot (corrected), the estimated number of lots needed to 1930 was 54,855, and the minimum and maximum estimates of the number needed to 1940 was 107,367 and 212,390, respectively.

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[1] Ely, Richard T., and Edward W. Morehouse, "Elements of Land Economics," p. 94, The Macmillan Company, New York, 1924.

[2] Eberle & Riggleman Economic Service, Vol. II, No. 11, March 16, 1925.

[3] Excluding outlying districts.

The survey showed that in Los Angeles 197,208 lots were occupied while 156,557 were vacant. From these figures it was easy to estimate the relation of the supply already in existence to the probable future demand. In other words, the figures indicated that six years later (in 1930) 101,702 of the then vacant lots would still be vacant if no more lots were subdivided, since there was a probable demand for only 54,855 out of the total vacant of 156,557. Carrying the estimate forward to 1940, the figures indicated that if no more lots were subdivided, 49,190 lots would still be vacant on the basis of the minimum population estimate and there would be a shortage of 55,833 on the basis of the maximum. While such figures indicated a very unfavorable investment situation on the whole, it was obvious that still greater allowances would have to be made because it was certain that many more new subdivisions would be laid out. This point is important because there is a strong tendency to lay out new subdivisions and sell them through well-organized aggressive sales methods, while owners of individual lots are unable to apply such aggressive sales efforts, and, consequently, their lots remain unsold and undeveloped.

It is difficult to say just what proportion of lots should remain vacant for the best interests of a community under the present economic system. The ideal of 100 per cent occupancy is, of course, not possible. Furthermore, the proportion vacant would normally be larger in a rapidly growing community where there is greater chance of gain due to the increase in land values. It does not seem necessary from an economic point of view, however, that a community should carry an overhead burden in vacant lots of 44.25 per cent, as was the case in Los Angeles City (excluding outlying portions) in 1924. In the outlying districts, 62.12 per cent of the lots were vacant and it was probable (on the basis of the minimum and maximum demand estimates) that between 40 and 20 per cent would be vacant 16 years later, in 1940, even if no more lots were subdivided. Students of business economics readily appreciate that such a tremendous unnecessary overhead burden in the form of frozen investments in unproductive property has an unfavorable effect upon the business of the community.

Reasons for Expansion.—In making a statistical analysis of real estate, it should be realized that the subdivision of lots is economically legitimate only when they are needed as sites for additional homes, businesses, industries, or public purposes. This, of course, includes the margin of vacant lots necessary to provide for the laying out of large tracts, etc. An excessive proportion of vacant lots is an economic drain on a community and has no advantages. If capital is invested in productive industries instead of in unproductive lots, the result is to attract more persons to the territory because of the added demand which is

created for labor, capital, and management, which, in turn, creates a demand for more site land.

Ripening Costs in Land Utilization.—Land, which is taken out of agricultural use, subdivided into lots, and held until it is needed for a home, factory, business, or public site, is said to be ripening into use.[1] That is, a certain portion of land must be held vacant during the transition from a lower form of use to a higher form of use. To convert farm land into occupied city lots takes time and waiting. This is necessary, providing the subdivision itself is necessary.

The reason why many investments in lots are so poor is that the costs of waiting are underestimated. A long period may be necessary for the lots to ripen into use, while the investor too often expects to get his principal back, with interest and profit, in a short time. The average person who invests in lots does not keep careful accounts and does not make careful estimates of the costs of carrying vacant land. Hence, there is a tendency to underestimate the costs of taxes and special assessments and to neglect to make due allowance for a fair rate of compound interest.

It is not generally realized that the costs of carrying vacant land through a ripening period are so great that a sound financial position as well as foresight based on careful forecasting is necessary if the investment is to be profitable over a period of time. Enormous profits are made on land at certain times in certain localities, and these instances are given wide publicity and emphasis in the "high-pressure" selling methods of real estate salesmen. Underestimates of the costs and overestimates of the increases in land values on the part of the average person place him in an attitude of mind where he is likely to make many unprofitable investments.

Costs of Carrying Vacant Lots.—In the Los Angeles vacant lot survey previously mentioned, the cost to the community of carrying the vacant lots was estimated. Based upon the assessed valuation (corrected for differences between assessed values and real values), the value of the vacant lots in Los Angeles County was then $1,100,000,000. Since the cost of carrying unproductive city lots averages not less than 10 per cent per annum, the cost of carrying the lots vacant in Los Angeles County in 1924 was at least $100,000,000 annually. If any increase occurred in land values during the year, the above cost would, of course, be reduced by the amount of this increase. The significance of the vacant lot situation may be better realized by making a comparison with other industries, which shows that the total value of vacant lots in Los Angeles County in 1924 was approximately four times as great as the amount of

[1] See Richard T. Ely, and Edward W. Morehouse, "Elements of Land Economics," pp. 95–97, The Macmillan Company, New York, 1924.

capital invested in manufacturing industries of that county at the time of the 1919 census of manufactures.

Lots as Investments.—Although, in certain instances, fortunes have been made because of increases in land values, investment in city land, especially if it is vacant, requires careful and skillful analysis and selection if the net gain is to be greater than the compounded interest on a savings account. A careful investigation covering vacant tracts in New York City for a 42-year period showed that the holding of vacant land is not a profitable form of investment, except for short periods of time in exceptionally favorable locations. Similar conclusions have been reached in other studies of land values. The publicity given to the small proportion of extremely profitable land investments and the general lack of adequate knowledge regarding carrying costs influence the small investor or speculator, who is not able to discriminate between good and bad locations, to pay prices for vacant lots which discount increases in value for many years to come, and actually leave him a practical certainty of heavy loss when all costs are counted. While the average person believes that it is a simple problem to make good investments in city land, this is probably one of the most technical and involved types of investment.

One very important factor, which must be recognized in investing in lots, is the aggressiveness of other lot holders in the community. That is, for instance, a subdivider, who has the necessary financial resources, may lay out lots in a poor location. These lots may be occupied before other lots in a better location, because of the deliberate and well-planned advertising and selling methods which actually serve to create a preference for the less desirable property. The less desirable property then becomes the more desirable property.

The rapidity with which costs accumulate should also be realized. Carrying costs on vacant lots are seldom less than 10 per cent annually. On a 10 per cent basis, since an amount doubles in approximately 8 years at this rate when compounded annually, a lot owner who purchased a lot at $2,000 would have to receive $4,000 at the end of 8 years, or $8,000 at the end of 16 years, in order to receive back his original investment, carrying costs, and a fair rate of interest.

Speculation in Site Land.—Much of the buying of land is speculation of the most pronounced kind. In many of these instances, of course, the buyer feels that he is making a conservative investment. Many trades take place at prices which cannot be justified on the basis of accurate data. For instance, in a certain community, it may be possible to build and fill one apartment house. If this is done, however, the tendency is for all of the other owners of similar lots to feel that their lots are worth large amounts as sites for similar buildings, even though there is little probability of filling them.

The pressure of speculative building is an important factor in connection with site land prices. If one buys an apartment-house lot at a price which is justified only as an apartment-house site, he cannot hope to receive a fair return upon his investment unless he places an apartment house upon it. If he does place an apartment house upon the lot, it may be impossible to fill it. A survey of the situation in Los Angeles in 1926, for instance, showed that within a two-mile radius of Westlake Park there were enough apartment-house lots to care for over eight times the apartment-dwelling population of the city.

One of the principal reasons for the inflated prices of many residential neighborhood lots is found in the magic effect of the words "income property" or "business property," regardless of whether or not there is any reasonable possibility of using the property for purposes which will yield an income. Referring again to the above-mentioned survey of Los Angeles, it was shown (1926) that there were enough apartment-house lots within a six-mile radius of the intersection at Pico and Hoover Streets to accommodate almost three times the total population of the city, if one placed an average of only five families on each lot.

The same kind of maladjustment often occurs in neighborhood business property. In Hollywood, for instance, only 7.0 per cent of the occupied land was actually used for business sites (1926) while 16.3 per cent was zoned for business and held at business site prices. In some areas, the proportion of apartment-house dwellers will increase to the extent that all of the business property will be used, but in others the restrictions are such that the population in the area can never increase to the extent that all of the business property can be used for business purposes.

The efforts of the better class of realtors, city planning commissions, and others who are attempting to prevent the zoning of land for purposes for which it cannot be used for a great number of years, if at all, should receive the strong support of all who are interested in sound land values. If a community's development is to be the most rapid and sound, the speculators who are holding land for practically impossible purposes should be replaced by those who will actually use the property as home sites or for other economic purposes.

Trends, Seasonal Variations, Cycles, and Random Movements.—The importance of considering trends, seasonal variations, cycles, and random movements has been emphasized from time to time in connection with previous discussions. In most analyses, it is not necessary actually to compute these changes, but it is necessary in nearly all cases that the movements be appreciated at least in a general way. Often, if the data are presented graphically, these different types of variations can be observed to a degree that is accurate enough for practical purposes.

Since wrong assumptions regarding the trends and cycles have been responsible for so many mistakes in analyzing real estate, these movements will now be considered in some detail.

Long-time Trends in Real Estate.—There are many conspicuous examples of steep upward trends in real estate values. It is extremely important, in analyzing real estate values, that one discover any changes in the trend as soon as they occur, if possible, and that one does not confuse the upward swing of the business cycle with the long-time trend. It is generally known that there have been large and rapid increases in the values of lots in the large American cities during the last decade, but it is not generally known that some of the values were less at the time of the 1929 boom than they were 20 years earlier. Los Angeles, for instance, enjoyed a very rapid rate of growth from 1910 to 1930, but this should not have been taken to mean that all values increased as rapidly as those which were commonly given publicity. In an investigation of down-town business lot values in this city, made in 1926,[1] data on the assessed valuations of a representative number of corner lots were gathered and the average increases over a 20-year period were computed for each of eight different sections. The increases varied from practically nothing in one of the sections (which is the oldest part of the Los Angeles down-town business section) to an increase of nearly 700 per cent in one of the newer down-town sections, where the lots were worth practically 800 per cent of their 1907 value. A large part of the increase in this last section was undoubtedly cyclical, though many purchasers proceeded on the assumption that it was principally basic trend. The opportunities to gain through long-time increases in city land values may be good in many locations, but the investor should take care that he does not purchase a lot in a location which has reached its peak and which is not only experiencing a cyclical decline, but which has also had an unfavorable change in the long-time trend.

When considering trends in vacant land values, it is important to determine whether or not there is any net gain over a period of time—that is, whether or not the increases due to long-time trend more than offset the carrying costs. The fabulous increases in certain down-town New York land values are commonly referred to by real estate salesmen when selling property in large cities. However, there is another side to the question which is not so commonly discussed. A careful investigation has been made covering the history of vacant tracts in the city of New York for the years 1880 to 1921 together with the history of the number of tracts that were subdivided and sold as lots. This report gives the following conclusions: "(1) A large part, and in some cases,

[1] Eberle & Riggleman Economic Service, Vol. III, No. 47, November 22, 1926.

all of the increase in the value of vacant land is offset by actual payments to the city in the form of taxes and special assessments with interest thereon. (2) These charges have increased rapidly within recent years, and at the same time the rate of increase in land values has fallen. (3) The holding of vacant land is not a profitable form of investment, except for short periods of time in exceptionally favorable locations. (4) Purchasers of outlying vacant lots at public auction almost invariably pay prices which discount future increases in value many years to come, and leave to the purchaser a practical certainty of heavy loss."[1]

The increases in the prices of lots during the period from 1913 to 1920 were emphasized by the rising general price level. This must be carefully observed in analyzing trends of real estate prices, as well as trends of other prices.

The foregoing discussion does not mean that investment in city land is not profitable, but it does mean that a careful analysis should be made of long-time trends before investing. And it means that the up-swing of a business cycle must not be confused with the trend. Many very profitable investments have been made in city land and there is no reason to believe that they cannot continue to be made if the locations are carefully and skillfully analyzed.

Real Estate Cycles.—Almost every major cycle of prosperity in this country is accompanied by a high degree of real estate activity. This often takes the form of a boom in certain localities. In fact, some of our greatest booms have been those that were based on real estate speculation. The "crowd," including some of the most highly successful citizens of the area concerned, is carried away by the enthusiasm which accompanies the vast increases of wealth that are thrust upon the lucky ones. Those who actually understand the economics of the situation are in the minority.

An outstanding characteristic of both real estate and building cycles is that they are several times as great in amplitude and length as the usual general business cycle (see Exhibit 207, which shows real estate cycles since 1800 and building cycles since 1830). Real estate and building cycles tend to be from 15 to 20 years in length, while the general business cycle tends to be only a few years in length. Recognition of the difference between the tendencies of real estate and building cycles on the one hand and those of the general business cycle on the other is imperative in any analysis of real estate conditions. An outstanding aid in analyzing current cyclical conditions in real estate is *The Real Estate Analyst*, a national economic service devoted entirely to problems of real estate, which was

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[1] Arner, G. B. L., "Land Values in New York City," *Quarterly Journal of Economics*, Vol. XXXVI, pp. 545–580, August 1922.

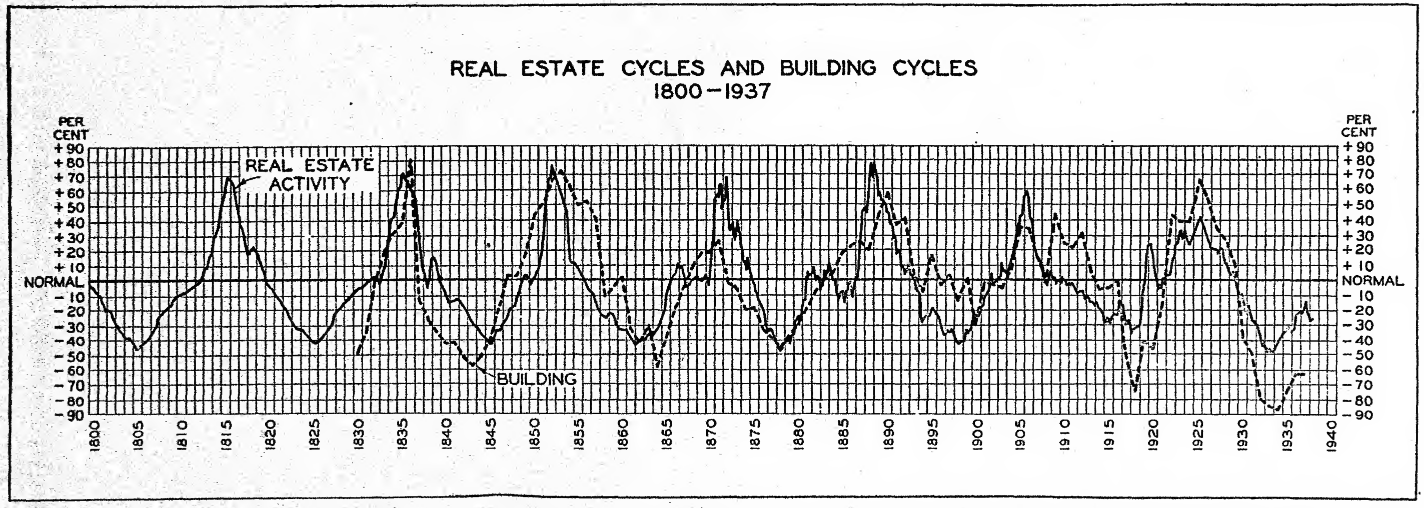

EXHIBIT 207.—Long cycles of great amplitude characterize the fluctuations in building and real estate activity as shown by the above chart. (*Real estate curve, courtesy, Roy Wenzlick and Real Estate Analysts, Inc.; building curve, see Appendix VI.*)

described briefly in Chapter XVII on Business Forecasting, page 391.

There have been many real estate booms in the United States during the past 100 years. From 1834 to 1837 the valuation of the property in Mobile, Alabama, increased from $4,000,000 to $27,000,000. After the panic of 1837 the values decreased more rapidly than they had risen. During this same period (a few years before 1837) people were coming into the State of Illinois so rapidly that it was believed that cities would spread all over the prairie and that there would be no farming land left. City lots were laid out on what are to-day the most isolated farms. Lands were sold for more than their present value and for more than fifty times their value at that date.[1] In 1919, farms were selling in Iowa at $300 to $400 per acre. In 1930, these same farms were being sold at from $100 to $150 per acre.

One of the most spectacular real estate booms of recent years was the Florida boom of 1925. In Miami, where the boom centered, business lots were sold at prices which were greater than the prices for the best lots in cities from ten to fifteen times as large. It was estimated that one subdivision alone near Miami would require approximately one billion dollars of annual income to support the residential land values, whereas the entire annual income of the United States at that time was only approximately seventy-three billion dollars. The Florida boom also reached a very high stage in many of the smaller towns and villages. For instance, in Lake Wales, an interior town of 2,747 persons (1925 state census), a business lot about two blocks from the center of its business district was sold for $800 per front foot and another lot was sold for $1,000 per front foot. In towns of this size, a business enterprise can seldom produce enough in a specific location to warrant paying more than a few thousand dollars for the entire lot. From the overoptimistic situation which accompanied the boom, Florida passed abruptly into a severe period of depression. With her undeveloped land, mineral, climate, and beach resources, Florida undoubtedly has a steep upward tendency in her long-time growth, and the depressed conditions are as unrepresentative as were the conditions of the preceding boom.

Random Movements in Real Estate.—In analyzing real estate cycles, it is not only necessary to make the usual distinction between minor cycles, which recur every few years, and major cycles, which occur only once or twice during a business generation and sometimes they do not occur this often, but it is also necessary to identify the random movements. The type of boom which a community may experience due

[1] See Ivan Wright, "Farm Mortgage Finance," p. 5, McGraw-Hill Book Company, Inc., New York, 1923.

to the building of a railroad or the opening up of new lands in some instances may never be repeated, and consequently it is important to recognize that such a boom is not cyclical. More extreme examples of random movements are the booms which accompany the development and exhaustion of natural resources, such as the oil booms in Oklahoma, Texas, and California, or the mining booms which caused such cities as Rhyolite, Nevada, to flourish and decline. (Rhyolite, once a city of several thousand persons, now has no human inhabitants.)

The determination of whether or not a boom is cyclical depends upon whether or not the natural conditions and other factors are such that the condition can be repeated. Florida, for instance, still has the same climate, the same advantageous location, and the same other economic factors which constituted the principal bases for the development resulting in the boom of 1925, and, therefore, it is practically certain that other cycles will follow, though it may be some time before another boom occurs which will be as extreme as the last one. Rhyolite, on the other hand, no longer has the mineral resources, as far as is known, and has no other bases for development. Its boom was distinctly a random one, and consequently cannot be expected to recur. In the development of most communities, however, cyclical and random movements occur together, and while it is not possible to isolate the two movements completely, they should be determined as accurately as possible.

Control of Real Estate Cycles.—In analyzing an economic situation in real estate activity, it is important to note the degree of control that is exercised by those who appreciate the real situation. In a boom, there are many who have had wealth thrust upon them, but who believe that their success is due to their ability, and these men do not hesitate to give advice and make recommendations which are thoroughly unsound, but which are readily believed by the unwary public. While those who appreciated the situation in the Florida boom published much sound economic information for the guidance of buyers, they had but little actual control over the unsound publicity and exaggerations that were so widely broadcast. In this boom, one of Florida's newly made millionaires, posing as an authority on the economic questions of the area, described the port at Miami as the "future great Pan-American port of the Atlantic," which will "grow into something far greater than New Orleans, Galveston, or Savannah," and many persons believed that this development would take place within a few years. (In 1926, the New Orleans foreign tonnage was approximately 75 times that of Miami.)

An example of a city which grew rapidly after the World War, but in a remarkably steady manner, is San Diego. There was no boom to correspond with the Los Angeles boom of 1923 or the Florida boom of 1925. The growth of San Diego was accompanied by a much more

conservative attitude than that which characterizes a speculative boom. When activity reached a high point in the cycle in 1926, certain interests made exaggerated claims regarding the fortunes that could be made in San Diego real estate, but there was a very strong sentiment against such practices, which was effective in preventing inflation of the type that was characteristic of the Los Angeles and Florida booms. It is said that much of the acreage that was annexed to the city was added for the purpose of preventing improper street work, building, etc. An idea of the difference between the 1926 prosperity in San Diego and the 1923 boom in Los Angeles is given by the per capita building statistics which were $150 per capita in San Diego and $250 per capita in Los Angeles. By 1928, both cities had declined to approximately $100 per capita.

Land and Building Vacancy Statistics.—The importance of knowing the relation between the supply of and the demand for various types of land and building space in connection with analyzing retail and other markets for commodities, investment conditions, and general business conditions, as well as the real estate situation, has been indicated previously. To determine such relations, it is ordinarily necessary to make a field survey. Often the problem of expense makes it impossible to make a complete enumeration, and the method of sampling must be used. When determining the vacancy situation in apartment houses and office buildings, it is necessary to cover practically all, or at least a large proportion, of the individual buildings, if the figures are to be representative. A substantial field sample, however, of single dwellings, neighborhood stores, and other small buildings which are present in large numbers, will indicate the vacancy situation in these structures accurately enough for many purposes. In some instances, other data can be used as an index of vacancies. For example, the percentage of idle gas meters has been used in certain Ohio cities, and, in London, England, the records of the percentages of idle water meters furnish an index of vacancies since 1871.[1]

Most of the vacancy (or occupancy) surveys that have been made in the various communities were directed by local real estate boards, but such surveys have also been made by post offices, police departments, housing associations, universities, banks, public utility companies, and other organizations. Very often, however, the best results can be obtained by a joint committee representing the different interests in the community. Some of the more important points involved in making vacancy surveys of buildings will now be considered. This description of

[1] For the London index of idle water meters, see George F. Warren and Frank A. Pearson, "World Prices and the Building Industry," pp. 127–129, John Wiley & Sons, Inc., New York, 1937.

the methods and technique of making building vacancy surveys indicates the procedure that may be followed in a cooperative project directed by a joint committee, and as such will serve as an example in making many other types of surveys which could not be undertaken by an individual concern or organization. The need for various types of primary data that may be gathered in this manner quite commonly arises when an economic survey of a community for development, marketing, or other purposes is undertaken along the lines suggested in the outline for a city commercial and industrial survey presented in Appendix IV, pages 671–712, inclusive. Under the cooperative plan the first step to be taken by the various interested parties is to organize a survey committee.[1]

Organizing an Occupancy Survey Committee.—A local occupancy survey committee should include interested representatives from such organizations as the local real estate board, the city planning commission, the post office, the city government, mortgage bankers' association, chamber of commerce, building owners' and managers' association, apartment house association, builders' exchange, telephone and other public utility companies, health councils, and university bureaus of economic research. There are many other organizations which might have a definite interest in the compilation of occupancy data. This committee might well be organized on a permanent basis with a chairman, vice-chairman, secretary, or other officers.

Planning the Survey.—In making a vacancy survey, it is important to lay out a careful plan of how the work is to be done and what is to be included with due regard for the probable use that will be made of the data. In making this plan, one should carefully consider the following questions: What will be the scope of the survey? How will it be financed? How will the field data be collected? How will the data be tabulated? And how will the data be presented and reported? It is desirable in many instances to include an analysis with the report for those who wish to use it as an aid in making their own special applications of the data.

Financing an Occupancy Survey.—In making an occupancy survey, it is important to utilize available information, as far as possible, in order to keep down expenses. In most instances a successful survey depends upon a considerable amount of contributed help. Such assistance can often be contributed by business, governmental, and civic organizations with very little sacrifice, where it would be impossible to raise the funds necessary to hire regular enumerators. With the proper organization and use of contributed help, a survey of a large city, which otherwise would cost several thousand dollars, can often be completed at a cost

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[1] This description of methods of making an occupancy survey is abstracted in part from "How to Make an Occupancy Survey," prepared by the United States Department of Commerce and published by the National Conference on Construction.

of a few hundred dollars, and a survey of a small city can often be made with practically no cash outlay.

Conditions will vary from city to city and from time to time in the same city, but a rough idea of the cost involved may be obtained from estimates of the cost of a survey of St. Louis, which covered the number of units occupied, vacant, and under construction, and the number of extra families, all by classes of buildings for 128 districts of the city. These estimates are presented in Exhibit 208.

Determining the Information to Be Gathered.—Before an occupancy survey is started, careful consideration should be given to determining

COST OF MAKING OCCUPANCY SURVEY IN ST. LOUIS

	Actual cost	Estimated cost if all work were paid for
Supervision	$ 00*	$ 1,000.00
Gathering field data	00†	12,000.00
Labor for punching cards	50.75	50.75
Sorting and tabulating	00‡	300.00
Copying and checking tables	309.00	309.00
Printing forms for field records	187.34	187.34
Printing reports (3,000)	108.25	108.25
Other costs (maps, drawings, etc)	150.00	150.00
Total	805.34	14,105.34

* Contributed by Albert Wenzlick Real Estate Company, Research Department.

† Work done by Post Office. (Because of the familiarity of letter carriers with conditions in their districts, they are sometimes able to fill out the occupancy survey cards with but relatively little effort and at but little or no extra cost to the Post Office.)

‡ Use of sorting and tabulating machines contributed by International Business Machines Corporation.

EXHIBIT 208.—The above table illustrates how, by use of contributed help, surveys can be made which might be difficult if not impossible to finance on a direct cash basis.

just what information is desired. Frequently omission of desirable information is discovered after the survey has been completed. Care should be taken to guard against any such omissions. In determining the types of information that the survey should show, it is well to set up dummy tables with appropriate headings and stub-items, and then, working backward from these tables, make up the schedules and other forms in such a way that the desired information may be procured.

Unit of Measurement.—In making practical occupancy surveys of residential and neighborhood buildings, it is necessary to use a unit of measurement which is easy to apply in the field and which is readily understood in analyzing the data. Ideally, it might be desirable to use such units as square feet of usable area, but practically, especially in the case of residential buildings, it is doubtful if the interpretations and

applications would be any better than when a simple count is made on the basis of accommodation units. That is, for most purposes in making occupancy surveys of residential and neighborhood business buildings (but not large office and industrial buildings), it is practical and usually desirable, in order to save expense, to collect the data on a basis of the number of units. In residential buildings the unit is the space designed for and ordinarily occupied by one family. In neighborhood stores and neighborhood office buildings, a unit is the space ordinarily designed for and occupied by one concern.[1]

Area to Be Included.—One of the first questions to be decided is that of the territory to be covered. It is important to give due consideration to the outlying districts. These districts are often excluded to save expense and later when back information on the outlying areas is needed for comparative purposes, it is not available. The official boundaries of cities are not ordinarily satisfactory as boundaries of the areas which should be covered by occupancy surveys.

Areas by Which Data Are to Be Recorded.—In analyzing local situations it is very important to have the data recorded by small subdivisions of a city. Different districts in a city do not grow at the same rate, and conditions vary to a great extent from one part of the city to another. One part of a city may be declining while another is growing, and still another is standing still.

The number of small areas into which a city should be divided will vary with the size of the city. It is probable that the average area should contain not more than 5,000 or 10,000 people, allowing for increases for the next decade or so.[2] Population and area of these unit districts will vary considerably in any one city, but an attempt should be made to lay out each district so that the conditions within it will be as uniform as possible. In determining these unit areas, it is not feasible to use postal districts or city wards because their boundaries are often changed. Probably the best unit area basis for taking vacancy surveys is the "census tract." The boundaries of census tracts are permanent, which makes the area a valuable one, not only for the collection of census and occupancy data, but also for the collection of other types of data by the various organizations in the city.

Date of the Survey.—In determining when to make an occupancy survey, there are many points which should be considered. In the first

[1] It is desirable in the more complete surveys of store, office building, or industrial space, especially if the buildings are large, to record space on the basis of square feet of rentable area rather than units.

[2] The average population per census tract in the nineteen cities which are on a census tract basis is from approximately 2,000 to approximately 16,000. Ten of these cities, however, have tracts with an average population between 3,000 and 7,000.

place, consideration should be given to whether the survey is to be taken annually or at other intervals. The value of a survey is greatest if it is taken at regular frequent intervals so that comparisons can be made with past dates and so that trends and changes can be studied.

Every month in the year has been used as a survey date, but the most popular dates are in the autumn and in the early spring. A survey taken in January or February gives late data for the use of real estate men and others who are analyzing the spring market for building space. However, the financing of large-scale building projects and, to a certain extent, other building projects, is ordinarily arranged before the end of the calendar year, and if the vacancy information is to be used in building up a case with a financial institution, or if it is to be used by the financial institutions in analyzing conditions, it must be available before the first of January.

Classification by Types of Structures.—It is important to report vacancy conditions by types of structures. The classification of major types of residential and neighborhood-business buildings might be as follows: Single houses ("freestanding"), horizontal double houses[1] (sometimes called "semi-detached"), row houses,[1] vertical two-family houses (one family over the other), flats, apartment houses, living quarters over stores, neighborhood stores, and neighborhood offices.[2] Local conditions will undoubtedly make necessary some variation in these classifications. Sometimes one or more classifications can be dropped, and sometimes it may be necessary to add one or two classes in order to meet local requirements. In many instances it will be advisable to subdivide the preceding major classes.

Standardized Terms for Types of Dwellings.—In cooperation with the National Association of Real Estate Boards, the Mortgage Bankers' Association, the National Association of Building Owners and Managers, the American Statistical Association, and other organizations interested in improving occupancy surveys, the United States Department of Commerce has suggested the dwelling designations shown in Exhibit 209. In arriving at these standards, the secretaries of real estate boards and other interested individuals throughout the country cooperated in dividing all residential buildings into the major types illustrated. In many cities it will be advisable to have subclasses under these major types. For instance, in Los Angeles it is desirable to classify single houses on the rear of the lot separately from single houses on the front of the lot for the reason that there is a major distinction between the desirability of these two subdivisions of the single dwelling class. In St. Louis and other

[1] In some cities units are owned separately, in others this is not customary.

[2] In most of the large cities the local office building owners' and managers' associations have made extensive surveys of office space occupancy on a rentable area basis.

MAJOR CLASSES OF DWELLINGS

THE TERM "UNIT," AS USED BELOW, DESIGNATES ONE FAMILY ACCOMMODATION

(1)


Typical characteristics: single building for occupancy by one family - ordinarily 1 or 2 stories.

STANDARD TERM: Single house

LOCAL TERM:____________________

Note: Sometimes it is advisable to make separate classifications for houses on the front of the lot and houses on the rear of the lot. As a matter of convenience, single houses which are used as rooming houses, or which contain light housekeeping accommodations where major structural alterations have not been made are included in the classification, "single houses."

(5)

Typical characteristics: heat, not furnished - no janitor service - frequently individual outside entrances - ordinarily 3 to 6 unit capacity and 2 stories, but sometimes greater number of units and as high as 3 and even 4 stories. (The heavy dashed lines show the divisions between units.)

STANDARD TERM: Flat

LOCAL TERM:____________________

Note: See note at bottom of section 6.

(2)

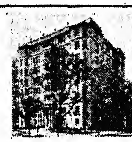

Typical characteristics: two family units standing side by side - ordinarily 1 or 2 stories. (The heavy dashed line shows the division between units.)

STANDARD TERM: Horizontal double house

LOCAL TERM:____________________

(6)

Typical characteristics: heat furnished - has janitor service - joint main entrance - usually has elevator service (but not always) - ordinarily more than 10 family units.

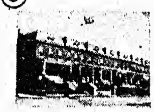


STANDARD TERM: Apartment house

LOCAL TERM:____________________

Note: It is suggested that the division between class 5 and class 6 in northern cities may be made on the single basis of whether or not heat is furnished. In southern cities the distinction may be made on the basis of whether or not janitor service is furnished.

(3)



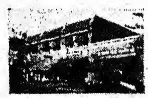
Typical characteristics: several units built together in a row - individual outside entrances.

STANDARD TERM: Row house

LOCAL TERM:____________________

(7)

Residential quarters over stores - usually one story over store. (Do not include apartment houses which have stores on the ground floor.)



STANDARD TERM: Living quarters over stores

LOCAL TERM:____________________

(4)


Typical characteristics: two family units - one over the other. (The heavy dashed line shows the division between units.)

STANDARD TERM: Vertical two-family house

LOCAL TERM:____________________

(8)

Other important types in your city that cannot be included in the classes illustrated above.

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EXHIBIT 209.—Questionnaire used by the United States Department of Commerce in obtaining information on local terms applied to different types of residential structures. The "standard terms" are those which were recommended for types of dwellings for standardized vacancy surveys. This system provides for the addition of any special types which may be of local importance.

cities it is advisable to distinguish between two-family flats and all other flats. Likewise, in many cities it is desirable to keep regular apartment houses and apartment hotels separate.

It is obvious that there are many line cases or "in between" types, which are not readily classified according to the types listed in Exhibit 209. Unless such a type is an important one it is suggested that it be included in the most nearly related class. If this classification is not feasible, it will be necessary to carry an additional class. In Los Angeles, for example, the bungalow court is common. Although it might be possible to include the bungalow court in one of the major classes shown in Exhibit 209, its inclusion probably would not be desirable, because, from the point of view of land occupied, size of building, class of tenant, etc., it is a distinct type. In this city, therefore, it is undoubtedly advisable to add a class called "bungalow courts." In New York it may be desirable to add another class to take care of tenement houses; but in many cities it would be possible to include tenements with flats or apartment houses by merging them or by making them a subclass.

In most cities there are several kinds of dwellings which might be considered as separate classes, but which, for practical purposes, are difficult to classify satisfactorily because of changes in use. For instance, there is the rooming house, the fraternity house, the house with light housekeeping rooms, and the like which might be recorded separately for certain purposes. Because of the difficulties involved, however, it is usually customary to include each with the most nearly related type of dwelling. Unless there are strong reasons to the contrary, therefore, the rooming house or the house with light housekeeping rooms, if it is a single house structurally, would be included with the single family houses, and, if it is a double or row house structurally, it would be included with double or row houses.

In working out the classifications presented in Exhibit 209, an attempt was made to standardize the classifications that could be used for the entire country and, at the same time, to avoid interference with local customs and usages. The classifications are based upon the distinctions that are most commonly made between the different types of buildings. They are not based upon structural differences alone. The distinction between a flat and an apartment house is often made according to qualities other than type of structure, although size of building is usually closely related to these other qualities. For instance, in the class of buildings called "flats," ordinarily heat is not furnished and there is no janitor service; frequently they have individual outside entrances, and they are usually from three to six units in capacity. Apartment houses, however, as a rule have heat furnished; they have janitor service; they have one or more joint entrances; they commonly have elevator

service; and they are usually larger buildings than flats. Ordinarily it will not make much difference in which class doubtful cases are placed because they are closely related to both classes. There should be no difficulty, however, in differentiating between most apartment houses and most flats.

Occupied and Vacant Units.—It is important to have the total number of occupied as well as vacant units in order that percentage relations may be determined.

The question sometimes arises as to just what the words "occupied" and "vacant" mean. In most cases there will be no difficulty, but in some instances questions will arise. If a house is temporarily closed, while the occupant is on a vacation, it should not be counted as vacant unless the space is actually on the market. If, however, certain unoccupied space is under lease, and is considered as "not vacant" by the owner, and if at the same time the lessee is trying to sublet the property, it should be considered by the enumerator as vacant. One should watch for such conditions when obtaining vacancy information from apartment house managers and office building managers, since in some instances they consider their buildings fully occupied if all of the space is leased even though tenants may have moved out and the space may actually be on the market.

Buildings under Construction.—It is obvious that data on buildings under construction are important in analyzing the vacancy situation. These buildings should not be counted as vacant because they are not ready for occupancy. Such data indicate the immediately potential supply and, if no change occurs in demand, the supply of vacant space will be augmented to the extent of these buildings when completed.

Again we have the question of classification. When should a building be classified as "under construction"? A portion of an apartment house may be furnished and actually occupied before the remainder is ready for occupancy. In such a case the units which are vacant and not ready for occupancy should be recorded as "under construction." Another question that often causes difficulty is: Just when does a building start to be under construction? If the excavation work has just begun, is the building under construction? Inasmuch as excavation work is sometimes completed long before the erection of the building is commenced, it is probable that, in making an occupancy survey, the best rule is to consider that a building is not under construction until the use of building materials has actually been started.

Doubling-up.—During periods of depression, when incomes are curtailed, large numbers of families give up maintaining separate housekeeping quarters and move in with friends or relatives in order to reduce expenses. Thus, vacancies sometimes increase substantially with no

new building and with no decrease in population. It is obvious that the number of extra families is an important figure which should be obtained if possible. The cooperation of the post offices is a very valuable aid in securing this information for the different types of dwelling units. The postmen probably know better than any group in a city when outside families are temporarily living in a unit with another family.

For general occupancy survey purposes, it is probably advisable to keep all records on doubling-up in such a way that the individual family cannot be identified even on the field card. The field card described in Chapter II, page 26, provides for recording totals for each class by blocks. In case any question is raised as to whether or not the gathering of information on doubling-up necessitates inquiring too much into private family affairs, the explanation can be given that the data are recorded only as totals for blocks, and that no record of individual families is made even on the original field cards.

Other Classifications.—There are other classifications under which occupancy data can be grouped if facilities for gathering and handling the data are available. Among those which have been found to be of value are: "size of units in number of rooms by types of buildings"; "rental or value of units by types of buildings"; "age of building"; and "equipment," especially in apartment houses.

Necessity for Definition and Simplification of Terms.—The importance of excluding unnecessary classifications and information, especially in the initial surveys, cannot be overemphasized. It is very difficult to make clean-cut and accurate definitions of many of the terms which must necessarily be used in making occupancy surveys, and the addition of any terms that would not be widely used in a given city should be discouraged.

In the preceding discussion an attempt has been made to limit the number of items and classifications to those which are most important. The preceding classifications are suggested only after a considerable amount of study, carried on in cooperation with individuals and organizations in various parts of the country that have made occupancy surveys. Often those in charge of making an occupancy survey are requested to include highly specialized classifications which would have only a very limited use. In such instances, it is usually desirable to have a special investigation made, which should be paid for by the individual or concern requesting the specialized information.

The preceding description of classifications has been given in considerable detail, in order to illustrate the types of problems that must be considered in statistical surveys and how practical approaches can be made to their solution. These examples are representative of the usual difficulties that confront those whose responsibility it is to plan a statistical survey,

and are indicative of the care that must be taken in deciding upon such details before starting the work of gathering the data.

Gathering the Data.—The problem of gathering occupancy data centers mainly on the item of personnel needed for field work. To facilitate the field work the classifications by which the data are to be gathered should be as simple as possible, and their number should be limited to the minimum consistent with the use to be made of the data. It is also important to provide the proper schedule form for the use of the enumerators in the field.

As indicated previously, many different methods have been used to secure the personnel necessary for making vacancy surveys. Among the most important are:

Paid enumerators.
Help contributed by real estate boards.
University students (course work).
Police.
Help contributed by public utilities.
Post office letter carriers.

The above, it is understood, applies only to the personnel to be used in the field for gathering the data and may not include the clerical staff which is necessary after the field data have been gathered.

Paid enumerators are seldom used as the sole method of gathering occupancy data, because of the expense involved. It is ordinarily easier to secure contributions in the form of personnel than it is to obtain the money necessary to pay enumerators. When a large number of enumerators are wanted for one or two days' work, it is sometimes difficult to secure those who are sufficiently reliable for their work to be accepted. Of considerable assistance in this regard, are the employment offices in colleges and universities which often have lists of students who are familiar enough with this type of work to make good enumerators and who are glad to have work for a day or so. Another source, which has been found to be helpful, is the waiting list of street-car motormen and conductors maintained by the street-car companies. The usual type of employment agency ordinarily is not in position to furnish help of the type required for such investigations.

In many instances the problem of securing personnel has been solved by the real estate men directly, either by contributing their own time or the time of their employees. The desired information is gathered for certain specified sections of the city which are assigned to them. Some of the larger universities which have practical courses in statistics are located in cities where it is desired to make occupancy surveys, and a practical laboratory problem can often be worked out in which the students take part in gathering the field data. The city police depart-

ment has in some instances made occupancy surveys in which the original information was gathered directly by the police force. In connection with their work of analyzing local conditions, the public utility companies have often helped in various ways to gather information such as occupancy data.

Probably no type of personnel can do a better job of gathering occupancy survey field data than the Post Office Department's letter carriers. These men are more familiar with the conditions in their districts and can record the data more accurately and with less difficulty throughout a city than any of the other classes of enumerators mentioned in the preceding list. This is particularly true in connection with gathering data on extra families to determine the extent of "doubling-up." During recent years the Post Office Department has cooperated under certain conditions in making occupancy surveys. In most of the cities which have made occupancy surveys, the local groups have taken advantage of this assistance.

The Field Schedule Form.—A convenient schedule form for recording the desired information in detail should be provided by the local group having charge of the survey. An example of such a form with complete instructions has already been described on pages 26 and 27 in connection with Exhibits 3*A* and 3*B*. This form includes spaces for recording the number of units "occupied," "vacant," and "under construction," and the number of "extra families," by classes of buildings, and is designed to be filled out for each side of every block. It is important to confine the use of a card to one side of a block for convenience in assigning the cards to subdistricts or census tracts, and to simplify enumeration and checking. This is especially important when the information is collected by letter carriers, for the reason that it is a convenient method of breaking up the letter carrier's route. It would be very difficult for a carrier, for instance, to collect the information by entire blocks, because such a system would not fit in well with the layout of his route.

Compilation and Tabulation.—The compilation and tabulation work in this particular type of vacancy survey has already been described in detail in Chapter IV, pages 51–57, as examples of hand and machine tabulation methods. Exhibit 7, on page 52, shows a master working table form which provides space for the data to be compiled and for the corresponding percentages showing the proportions occupied, vacant, and under construction, and the proportion occupied by extra families. From this table, or from the machine tabulations of equivalent data, the desired special-purpose tables can be compiled, such as the one in Exhibit 210.

Analyzing Real Estate Vacancy Data.—As indicated previously, the residential vacancy survey has been described in some detail to provide an example which will serve as a general guide in making other real estate

TOTAL NUMBER OF LIVING UNITS AND NUMBER VACANT, WITH PERCENTAGES VACANT BY TYPES, FOR 33 DISTRICTS OF THE CITY AND COUNTY OF DENVER, SEPTEMBER 1937

District	Total number of units	Total number vacant	Percentages vacant					
			Total	Single residences	Double residences	2-family residences	Terraces	Apartments
A	2,790	20	0.7	0.8	0.0	0.0	0.0	2.6
B	6,540	63	0.9	0.8	0.9	3.5	0.0	4.1
C	2,777	44	1.6	1.1	0.9	2.4	2.5	4.0
D	1,887	8	0.4	0.6	0.4	0.0	0.0	0.0
E	2,626	19	0.7	0.8	0.0	0.0	...	...
F	3,035	35	1.2	0.6	0.8	0.8	3.2	5.3
G & H	1,708	10	0.6	0.5	2.0	0.0	1.7	0.0
I	970	7	0.7	0.9	0.0	0.0	0.0	0.0
J	1,242	6	0.5	0.5	5.9	0.0	0.0	0.0
K & L	2,009	40	2.0	0.8	3.3	0.0	2.4	2.5
M	6,490	46	0.7	0.5	0.4	0.2	0.5	2.5
N	1,411	18	1.3	0.7	2.5	7.2	0.8	1.2
O	4,692	72	1.5	1.5	1.0	2.2	1.2	2.6
P	4,988	68	1.4	0.2	0.6	0.0	2.8	1.5
Q	5,583	72	1.3	2.0	0.5	0.0	0.0	1.3
R	3,305	39	1.2	1.3	0.5	0.8	0.9	1.8
S	3,517	21	0.6	0.6	0.4	2.4	0.8	0.8
T	3,811	44	1.2	1.2	0.8	0.0	0.0	0.0
U	3,255	40	1.2	1.3	1.1	1.2	0.5	1.4
V	3,666	67	1.8	1.8	3.6	1.2	2.6	1.3
W	2,608	29	1.1	1.2	0.9	0.0	0.0	0.0
X	1,154	3	0.3	0.1	0.0	0.0	1.5	0.0
Y	3,441	49	1.4	1.5	2.1	0.0	1.0	0.0
Z	3,113	60	1.9	1.7	1.4	0.0	0.0	3.1
A-1	1,047	8	0.8	0.8	1.4	0.0	0.0	...
B-1	620	5	0.8	0.7	...	0.0	...	...
C-1	707	7	1.0	1.0	0.0	0.0	...	1.4
D-1	694	12	1.7	1.6	0.0	0.0	...	20.0
E-1	3,067	62	2.0	2.0	2.8	0.0	0.0	1.7
F-1	1,598	25	1.6	1.7	0.0	0.0	...	0.0
G-1	723	9	1.2	1.3	0.0	0.0	0.0	0.0
All districts	85,074	1,008	1.2	1.1	1.0	1.0	1.2	1.8

EXHIBIT 210.—An example of a summary table that can be compiled from the master working table suggested in Exhibit 7, page 52. Classification of vacancy statistics by small sections of a city or metropolitan area is of great importance in their practical appiication. (*Courtesy, School of Commerce, University of Denver, and Denver Real Estate Exchange.*)

surveys. The analysis of real estate vacancy data requires their careful interpretation in relation to many other indicators of local business situations. The position in the business cycle, the tendency of the long-time growth, questions of seasonal variations and random movements—all are important. There is, of course, a very direct relation to new construction, but there also appears to be a tendency for vacancies to increase when construction falls off, for the reason that there is an inevitable crowding and reduction in the use of space when incomes decline.

The tendency to "double-up" should be very carefully considered in any analysis of vacancy figures, even though adequate statistical data may be lacking. The importance of this is indicated by the 1931 St. Louis survey, in which data on extra families were collected.[1] In this city, in the spring of 1931, there were more than two and one-half times as many extra families living in single houses as there were vacant single houses available. It was likely, however, that when these families again occupied separate quarters, they would move into the multiple family dwellings as well as into single family houses. It was obvious that the number of vacant units of all types (19,206) would be substantially reduced if the 11,597 extra families again occupied separate quarters. In other words, if the number of extra families were subtracted from the number of vacancies, the vacancy percentage would be reduced from almost 9 to a little over 3.

It is difficult to say what percentage of vacancies is best for all concerned. Conservative practice requires, however, that the usual types of income buildings should be constructed in such a way that a 90 per cent occupancy will net a profit. From the point of view of the individual concern that is planning to build an office building, a high vacancy does not mean that a new building should not be built at all, but it does mean that if the building is built, it should be so superior in relation to rental rates that there will be no difficulty in attracting tenants who are occupying other space. If a superior building and service cannot be offered at competitive prices with a resulting net profit to the owners, then undoubtedly the project should wait until these conditions can be met.

In dwelling structures, there is considerable competition between the different classes as well as within each class. That is, an apartment house competes to a certain extent with single family houses as well as with other apartments. Again, supply and demand seem to have freer play in the single family house situation and ordinarily a tendency toward increased vacancies would be accompanied by considerable cutting of

[1] Greater St. Louis Occupancy Survey, Spring, 1931, made by the St. Louis Post Office under the direction of Delbert S. Wenzlick.

rents. These and other such variations do not present any unusual difficulties, however, if the principles previously discussed are clearly understood.

Rents and Prices.—Data on rents and prices are very significant when analyzed in relation to vacancies. Changes in rents and prices are the real indication of the effects of changes in the vacancy situation. Rising rents ordinarily indicate an accumulating demand for new space. Rent indexes for various cities throughout the country are compiled by the United States Bureau of Labor Statistics, the National Industrial Conference Board, and the Real Estate Analyst.

Real Estate Appraisal or Valuation.—The appraisal or valuation of real property is becoming a specialty which requires skillful use of statistics. The typical appraisal involves such statistical problems as measuring the rate of growth of cities and the corresponding rate of absorption of land into various kinds of uses, the prediction of future growth and distribution of population, the determination of building life and building mortality, and the testing of the validity of capitalization rates. Appraisers also use the various statistical enumeration and sampling devices in their surveys to secure data on vacancies, rentals, expenses, building activity, subdivision activity, and other factors, which they use in making valuations.

Real properties (sites, sites and buildings, leasehold estates, leased fees, or other units capable of separate ownership and independent operation or enjoyment) have become so complex, and their values so difficult to determine, that the valuation of the appraiser who determines values on the basis of opinion alone is no longer considered reliable. Good judgment and experience, of course, are more important than ever, but they must be based upon the use of scientific statistical methods.

The technique of making appraisals cannot be expressed in complete detail in the limited space which can be given to the subject in this volume. Consequently, the discussion will be limited to some of the more important points that are of general interest, and the reader is referred to special works on this subject when a more complete description is required.[1]

General Principles Involved in Valuation.—In approaching the study of principles of appraisal, two broad outstanding subjects should be carefully distinguished. First, there is the subject of *method*, which has

[1] See Frederick M. Babcock, "The Valuation of Real Estate," McGraw-Hill Book Company, Inc., New York, 1932; John A. Zangerle, "Principles of Real Estate Appraising," Prentice-Hall, Inc., New York, 1927; and other books on appraisals listed in Appendix IX beginning on p. 761. See also Frederick M. Babcock, "Common Errors in Appraisal Method—An Analysis," *National Real Estate Journal*, November 24, 1930, and other articles on appraisals published in this magazine.

to do with the devices of logic and reasoning used in appraising. Second, there is the subject of *data,* which involves the proper selection, collection, and handling of information. The use of the right method will not result in correct appraisal if the data are inaccurate or inadequate. Likewise, the use of accurate and appropriate data will not result in correct appraisal if the wrong method is used.

It is also important to appreciate the distinction between the *theoretical method* of valuation and the *practical methods* of valuation. The theoretical method is dictated by economic theory. Whatever creates value phenomena in real estate becomes a part of the theoretical valuation method. The practical methods should be in accord with the theoretical method in so far as possible. Many classes of properties cannot be appraised, except roughly, and by methods which very poorly approximate the correct theoretical method.

Finally, in any study of the principles of appraisal, it is necessary at the outset to distinguish sharply between *cost* and *value.* The importance of making this distinction is not generally realized, and there are still many mortgage men and real estate dealers who consider a dollar of cost to represent a dollar of value, even though in their own experiences there may have been many instances where clearly the cost of the property had a very remote, if any, relation to its value.

The Theoretical Valuation Method.—The fact that real properties have utility gives them market values. The values are exchange values produced by bidding in the market. In such bidding, prospective purchasers weigh anticipated benefits of ownership against prices; thus, we say that present capital value is the result of anticipated future benefit or the expected future income stream. Anticipated returns can be in monetary or amenity form. The present worth of the future returns expected constitutes present value. Hence, the theoretical method of valuation comprises the prediction of future returns and the determination of their present value. Three elements are involved: (1) an expected or anticipated return in the form of an income or flow of amenities; (2) an interest rate reflecting risk or the degree of certainty of the prediction; and (3) the resulting value. This method parallels the operation of the real estate market.

Practical Valuation Methods.—In practice there are three general approaches in making valuations:

1. The income method.
2. The comparison method.
3. The cost method.

The first method is the most accurate and correct. The last two sometimes have to be used in practice when it is impossible to use the income method. These three methods will now be discussed briefly.

Under *the income method*, property is valued by discovering the present worth of a predicted earning expectancy. That is, future earnings are predicted and their present capital value is discovered by discounting. The income method is practicable for use where the property is of such a character that it produces (or can be expected to produce) predictable earnings in money form. It parallels, so far as possible, the correct theoretical method and is, therefore, the most accurate of the various methods. The result of the calculation is the total value of the property. If it is desired to distribute the total between land and building (or other fractional parts of the property), the separation is made by dividing the total net earnings between the parts. In the case of a building representing the highest and best use of the land, an amount is deducted from the total predicted income sufficient to return the investment in the building during its useful commercial life and to pay, in addition, a fair rate of interest return upon that portion of the building investment which remains invested from year to year. The remaining net income is capitalized to discover the land value—that is, the investment which will yield the remaining income at a fair rate of interest is determined. In cases where the building does not represent the highest and best use of the land, the building value is made residual. In all cases, the process comprises a distribution of the total value. It is never sound practice to combine fractional appraisals to determine total value.

Under *the comparison method*, the property being appraised is contrasted and compared with similar properties which have been sold. This method of appraisal does not correct for booms and depressions and should be resorted to only when the first method cannot be applied.

Revenues.		
First floor, rental value	$ 28,500	
Upper floors, rental value	122,300	$150,800
Vacancy and contingency allowance (5 per cent for first floor, 15 per cent for upper floors)		19,770
Effective gross revenue estimate		$131,030
Expenses.[1]		
Heat, light, and power	$ 12,480	
Janitor's department	10,290	
Elevator department	3,110	
Maintenance and repair	7,125	
General expense	8,905	
Taxes	26,180	68,090
Estimated normal annual net earnings		$62,940

[1] Depreciation should not be included as an operating expense in estimating the net income to be capitalized.

It, therefore, has but a limited application. Its most common use is in determining the values of individual family dwellings. The future returns are in the form of amenities and the present importance of the future amenities can be determined only by assuming that market prices are fair estimates of their value. This method is considered one step down from the pure theoretical method. However, it will be noted that the income method requires the use of the comparison device (in determining the capitalization rates).

Under *the cost method*, it is assumed that the value of a property is equivalent or directly related to the cost of reproducing it. This method departs radically from the sound valuation theory. As a practical matter, it is impossible to estimate accrued depreciation and obsolescence directly. However, this method is the one commonly used in the valuation of many kinds of factories and industrial properties. With all of its inaccuracies, no better method is available in such cases because of the utter impossibility of ascertaining the portion of the income of the manufacturing business that should be allocated to its real estate. This method is generally used in the appraisal of "service" properties, such as schools, courthouses, and churches, where it is assumed that the maximum value of the properties to their owners cannot exceed the cost of replacing them. In such instances, there is no practical way of determining the deduction necessary to arrive at a figure representing "value."

An Illustration of Valuation Procedure.—An instance of the procedure in determining the value of a property, based upon methods previously discussed, will now be described briefly.

An office building constructed in 1913 has a gross building area per floor of 10,000 square feet and its height is 80 feet, making a total cubical content of 800,000 cubic feet. At the present time, this type of building costs 60 cents per cubic foot, and a fair estimate of the replacement cost of construction new is $480,000. Carrying charges during construction, the cost of financing, etc., would amount to about $125,000, making a total estimated required investment, exclusive of land, of $605,000. This amount does not represent the present value of the building. It is an estimate of the total cost, exclusive of land, which would be required to replace the building in new condition at prevailing prices.

By means of a study of the prospective rental market, leases in force in the building, the competitive position the building occupies, probable operating expenses, etc., the appraiser predicts average yearly revenues and expenses as indicated at the bottom of page 551. It is estimated that the building, if new, would have a total commercial life of 45 years. It is also estimated that the building, because of its present condition and position in the rental market, will have a remaining commercial life of about 20 years. The valuation then proceeds as follows:

Estimated normal annual net earnings	$62,940	
Annual level sum for 45 years required to return $605,000 and pay $8\frac{1}{2}$ per cent on the declining investment residues.	52,768	
Residual return on land	$10,172	
Value of the land at 6 per cent		$169,533
Value of building: present value of $52,768 per year for 20 years at $8\frac{1}{2}$ per cent		499,362
Total valuation		$668,895

In the above example, for the sake of simplicity, the earnings have been estimated on the basis of the average rate for the remaining life of the building. It is better practice to predict an earning expectancy, which declines from the full earning power predicted for the following year to a level of earnings just sufficient to pay the return necessary to support the land in the last year of the life of the building. Another point that should be noted in connection with the preceding illustration is that the actual age of the building was not an element in determining the present depreciated value. The fact that the building was built in 1913 had no bearing upon its present value. The value was determined strictly on the basis of future productivity.

Appraisal Data Requirements.—As previously indicated, appraisal accuracy not only depends upon selecting the right valuation methods, but also upon having accurate, adequate, and appropriate data. The collection and tabulation of data is probably the greatest task of the appraiser. The data necessary in valuation may be classified as follows:

1. *Data Pertaining to City Growth and Districts.*—This includes past and present statistics on number and characteristics of population; purchasing power; trend of development; building activity; city planning, zoning, and other restrictions; public utility services; transportation facilities, including streets and highways; special developments, conditions, etc.; and such influencing factors as business development, shopping and amusement facilities, educational facilities, general improvement program, public confidence, climatic conditions, drainage, general public standing and financial rating of owners in district, and indications of the progressiveness of the community.

2. *Data Pertaining to Architecture.*—This involves much information and knowledge which only an architect possesses and, consequently, the services of an architect are often necessary in doing appraisal work. It is important to have information which will show what types of architecture are sound in relation to style and design, so that fads and other factors, which are only temporarily popular, may be discovered. Productivity and functional design are closely related.

3. *Data Pertaining to Obsolescence.*—This includes statistical and other data which will show to what extent certain factors tend to cause buildings

to decline in value, so that the rates of obsolescence may be computed with some degree of accuracy.

4. *Data Pertaining to Revenue Productivity.*—This includes data pertaining to rental rates, etc., of similar or competing buildings.

5. *Data Pertaining to Expenses.*—This includes data on current operating expenses, taxes, etc.

6. *Data Pertaining to Costs.*—This includes not only building costs, but all costs involved in the development of land, carrying charges, financing costs, etc.

7. *Data Pertaining to the General Market for Investment.*—This includes information on real estate, income, foreclosures, and the like, as well as data on mortgage rates, security markets, and other information of assistance in determining the relative desirability of real estate properties as investments.

8. *Data Pertaining to the Local Real Estate Market.*—This includes statistics and other information on the vacancy situation; probable resale possibilities; transfer prices; numbers of speculators, developers, and investors; and asking and offer prices.

9. *Data Pertaining to the Property.*—This includes a general description of the location, legal description, size and shape of land, description of building, description of leases, details regarding encumbrances, statement of special assessments, and present use of premises.

Among the more important sources of information which would be used in compiling the data listed above are the following:

1. Past and current operating statements and lease rolls of the property being appraised, and of other buildings of similar character.

2. General sources, such as owners and tenants of near-by properties, local real estate brokers, and others interested in local developments.

3. Local government departments, such as city building department, city engineer's office, city or county zoning or planning commission, county assessor's office, city street department, city and county boards of education, and other departments which may have information.

4. Private sources, such as banks, title companies, insurance companies, public utility companies, architects, builders, and construction companies.

5. Trade and other associations, such as real estate boards, building managers' associations, building material associations, engineers' and architects' associations, and chambers of commerce.

6. Primary or original surveys made by the appraiser himself. This is undoubtedly the most important source of all. It is always necessary for an appraiser to examine property personally.

Although the better appraisers are rapidly becoming more skillful in applying the available rental, vacancy, overproduction, subdivision,

and other statistics in their efforts to increase the accuracy of their valuations, it should be appreciated that one of the greatest difficulties in making an appraisal is to secure sufficient, satisfactory, and adequate data. The problem of securing the proper information, for instance, on the long-time trend and economic cycle in a certain city or in a section of a city is particularly difficult. The bases of the values of real properties are to be found in their future productivities. The appraiser, therefore, not only finds it necessary to use statistical procedure in his appraisal methods, but he must also make practical applications in the form of business forecasts, production analyses, marketing analyses, and investment analyses as discussed in other chapters of this book.

Questions and Problems

1. Why is it important to make statistical analyses of real estate conditions?

2. What use can be made of data on building and lot vacancies?

3. How can the future demand for lots and buildings be forecast?

4. In growing communities, what is the significance of vacant lots?

5. Why is an excessive vacancy in lots an unsound condition?

6. When is it economically justifiable to lay out new lots?

7. When is land said to be "ripening into use"?

8. How do carrying costs during the period of ripening into use compare with purchase prices?

9. What is the difference between real estate investment and real estate speculation?

10. What is the influence of speculative building on land prices?

11. Is the holding of vacant land for long periods of time usually profitable? Why?

12. Describe the real estate cycle. Explain the mistakes that are commonly made in confusing cyclical movements of real estate prices with their long-time trends.

13. Discuss land booms. Are they influenced by cycles in general business?

14 Do random movements or temporary conditions ever affect real estate prices? Illustrate.

15. How do inaccurate statements and unsound judgments by representative promoters and speculators affect the severity of a real estate boom? Discuss the possibilities of controlling the real estate cycle.

16. How would you proceed in collecting data on vacancies in dwellings, and how would you classify dwellings for this purpose?

17. Why is it important to have residential vacancy data classified by the different kinds of dwellings and by small areas?

18. How would you proceed in collecting information on office-building vacancies?

19. Discuss the problem of tabulating vacancy data.

20. What are some of the principal points to be observed in analyzing vacancy data?

21. Why are statistics important in making appraisals of real property?

22. In connection with appraisals, distinguish between method and data; theoretical method and practical methods; and cost and value.

23. Explain the theoretical method of valuation.

24. Describe three practical methods of valuation.

25. Illustrate in detail an instance of determining the value of a business property.

26. In making real estate appraisals, information should be compiled upon what subjects, and what are some of the specific facts that should be obtained?

27. What sources can be used in obtaining data for use in making valuations?

28. Select a certain lot in your city, and analyze the trends of factors that affect its value.

29. Make a survey of vacant residential buildings in your city, and analyze the data gathered. Use the sampling process if necessary.

30. Make a vacancy survey of the store buildings in your city, and analyze the data gathered. Use the sampling process if necessary.

31. Prepare a detailed plan with forms for making a complete vacancy survey of residential and business buildings in your city.

32. Analyze the vacant lot situation in your city.

33. Select a property in your city, and make an appraisal of it.

CHAPTER XXIII

INVESTMENT ANALYSIS

It is the purpose of this chapter to discuss the practical use of statistics in analyzing investments. Many severe losses have been sustained because investors did not realize the risks that they were taking, and were not able to use relatively simple statistical precautions, such as those suggested in the following pages. The discussion will be confined principally to procedures to be used in determining whether or not an investment is desirable by helping to answer such questions as: Is the concern's financial position safe? Is there a good market for the company's products? Has the management shown itself to be capable in times of stress? Do the earnings compare favorably with earnings of similar companies? Methods of getting information on these and similar questions from the point of view of the investor will be considered.

Opportunities in Investment.—At all times, whether in prosperity or depression, there are some industries which are in a strong position in relation to both volume of business and profits. At the same time, other industries are simply marking time or are slowly declining as a result of changes in economic conditions, poor management, or changes in the demand for their products or services. The making of successful investments depends upon the ability to select the most promising companies in soundly established but growing industries, and the ability to determine a favorable time and price at which to buy.

The demands created by changes in the standard of living and by the constant growth of certain industries furnish opportunities to make profitable investments which will have large capital appreciations as well as satisfactory dividend or interest incomes. One of the purposes of statistical analysis in investment is to help one to select those securities which, by reason of peculiarly favorable circumstances, can be expected to increase in value and yet give adequate safety at all times. In this connection it should be emphasized that consistent investment success is attained *not* by constant day-to-day trading and shifting of securities in the market for quick profits, but by buying securities of the best managed enterprises in soundly expanding industries and by holding them for long-pull appreciation in value. It is also to be recognized that, during periods of general business depression and low earnings, statistical analyses are essential in order to select the securities of companies

which will maintain earnings adequate to maintain dividend or interest payments, and which will enter succeeding periods of recovery and prosperity in a position to take advantage of the more favorable business conditions.

Place of Statistics in Investment Analysis.—As a general rule, the average business man will have but little information upon good prospective investments unless he makes a definite effort to get the information himself from reliable sources. However, he may have considerable favorable (but unreliable) information on weak or unsound securities. The reason for this is that much selling effort is necessary in the case of new or weak securities and the better investments are not pushed so hard as the poor investments. In fact, some of the strongest sales efforts are made in connection with very weak or even fraudulent securities. And these efforts are successful chiefly because of the ignorance and lack of analytical methods on the part of prospective investors. A good rule in investing is never to buy anything that any one tries to sell. In other words, one should buy established securities as the result of independent or unbiased investigation or advice.

"In no other field of business has more been said and written—and less actually understood—than in that of investments and stock speculation. Superficial 'analyses' of conditions, fanciful theories for guessing short swings, tips, rumors of mergers, pools, 'insiders' activities, and a hundred other things are to be found on every hand. Everywhere advice is dispensed freely and in great quantities. Generally, the more cheaply it is secured the more expensive it proves in the end."[1] Definite statistical facts, if properly analyzed and understood, will greatly reduce losses of hard-earned savings due to investing in weak or worthless securities.

Those Who Will Not Study Should Be Content with Low Yields.—It does not seem reasonable that people will work hard to save up a few hundred dollars and then make an "investment" in securities about which they actually know nothing except what salesmen tell them. Such investors expect high yields or fortunes, when in reality they often receive no return and lose their original investments. Only those who make careful analyses of their investments have the right to expect higher than the usual incomes. Others, as a rule, should confine their investments to conservative securities recommended by reliable investment counselors, or to such securities as those in which savings banks may legally invest their deposits.[2] The latter ordinarily include the interest-bearing obligations of the United States Government, certain state and municipal bonds,

[1] From Foreword to "Consistent Investment Success," copyright 1928, by The Brookmire Economic Service, Inc., New York.

[2] It is not to be assumed, however, that all investments which are legal for savings banks are desirable for the conservative investor.

conservative real estate mortgages, and certain bonds of well-established railroad companies with favorable records of stable earnings. Of course, absolute security is unattainable. Three of the Great Powers which took part in the World War are no longer in existence, although bonds of such governments were considered very safe before the war.

Distinction between Investment and Speculation.—There is no well-defined distinction which will enable us to place all investment in one class and all speculation in another. There is no question but that a person wishes to make an investment when he walks into a reliable broker's office with cash or a certified check to make an outright purchase of stocks or bonds, with the expectation of receiving a regular reasonable return in the form of interest or dividends, and with the expectation of receiving back at a later date the amount paid. If, on the other hand, he wishes to open a margin account and buy widely fluctuating stocks which he would expect to sell in a few days or weeks at a profit, there is no question but that he wishes to speculate.

The degree of risk and the amount of gain expected furnish indications of investment and speculation. If the risk is great, usually one is speculating, and if the risk is low, usually one is making an investment. It often happens, however, that what one thinks is an investment, is speculation of the most pronounced kind. Statistical checks, skillfully used, will usually correct this difficulty.

Investors often select their stocks in such a way that they can reasonably expect them to appreciate over a period of time (say at least a few years). Speculators ordinarily expect their stocks to appreciate in a few days or weeks. The first class (investors) should select their stocks (or have them selected) on the basis of skilled analyses of the conditions of the companies and their present and probable earning powers. The second class (speculators) select their stocks chiefly on the basis of hunches and tips regarding the day-to-day erratic movements of the stock market.

In this chapter, we are concerned only with the problems of investment. And while investments can be made in other things, our discussion will be confined chiefly to investments in stocks and bonds, although mortgage loans and real estate will be briefly considered. Much of the discussion of stock and bond analysis applies also to mortgage-loan securities.

The Relation of the Investor to the Investment.—Before analyzing particular securities, the investor should make an analysis of his own particular requirements. Some people can afford to take greater risks than others. Young persons, as a rule, can afford to take greater risks than older persons. Single persons ordinarily can take greater risks than those who have families to support. Investments for widows and

orphans commonly require the maximum safety. Those who have independent incomes can take greater risks than those who do not. Some people are temperamentally unfitted to assume risks, and the uncertainty may even affect their health, while others enjoy taking a chance and are good losers in case the venture is unsuccessful. Some investors are properly trained and are willing to make careful analyses, while others are not.

As the amount of the risk assumed depends to a considerable extent upon the type of security purchased, we shall now consider fundamental differences in the securities available for investment.

Investment Analysis.—An investment analysis should be made from at least three points of view. First, the nature and rights specified in the security purchased must be determined, particularly in relation to other securities which the concern has issued. Second, conditions in the security markets as a whole must be studied in order to know general market trends. And, third, the companies issuing the securities must be investigated thoroughly to determine the real value of the rights stated in their securities, and to determine whether the market prices of the individual securities make them really attractive as an investment. These three points of view will be considered in the order given.

Stocks and Bonds.—It is assumed that the reader has a preliminary knowledge of securities and that it will not be necessary to explain such terms as "stocks" and "bonds." It is believed advisable, however, to review some of their principal features from an investment point of view.

In a corporation having bonds, preferred stock, and common stock, the bondholders ordinarily have the first claim on assets and a claim on gross revenue prior to stockholders' claims. The preferred stockholders have the first claim after the bondholders have been satisfied (assuming that there are no unpaid accounts or other prior obligations), and the common stockholders have the last or residual claim.

Bonds are usually mortgage loans bearing a fixed rate of interest, although there are a number of bond issues known as "debentures" which are not secured by specific pledges of property. The bondholder ordinarily does not participate in the earnings even if the company becomes exceptionally prosperous.[1] However, the prosperity of the company affects the safety of the bond principal and interest, and frequently specific bonds are considered to be sound investments because of the continued prosperity of the company rather than on account of the liquidating value of the property pledged.

Common stocks represent ownership of the enterprise, which includes the right to earnings. With the increased chances to gain, there is also

[1] There are, however, occasional exceptions to this rule as pointed out on p. 562.

an increased risk as to principal and income. But because the profits above a stated rate on the preferred stock ordinarily go to the common stockholders, large fortunes have been realized by those who have owned common stock shares in successful businesses.

From an investment point of view, preferred stocks lie in between bonds and common stocks. Preferred stocks represent shares in the ownership, but ordinarily they bear fixed rates of dividends. Since the security is less than that of bonds, the dividend rates are higher than bond interest rates. When considering preferred stocks, the investor should carefully investigate the conditions under which they are issued, as, for instance, whether they are cumulative or non-cumulative; whether the dividend rate is limited to a certain percentage or whether the stock participates in the dividends; whether the stock is redeemable by the company at a certain premium; and whether it is convertible into common stock. All other rights and privileges of the stockholder (appearing on the face of the certificate) should be carefully studied by the investor. The investor should know these conditions before becoming a stockholder, although he may never wish to exercise any of his rights, especially if he has been successful in selecting a prosperous company. Above all, the investor should endeavor to select only the companies which are and may reasonably be expected to remain successful. This involves the statistical analyses which will be discussed later in the chapter.

It should be pointed out that there is a wide variation in the attractiveness of specific investments in each class of the securities just discussed. Some bonds are safe while others are not. A specific concern's bonds are ordinarily safer than its common stock, but the common stocks of some concerns are much safer than the bonds of others. A stock, which is scientifically selected, can easily be safer both as to principal and income than a bond or preferred stock which is bought with no investigation. For example, the common stock of the American Telephone and Telegraph Company, which has an unbroken dividend record of 38 years, is a more sound investment from the standpoint of security and return on the investment than either preferred stock or bonds in comparatively new companies which may or may not prove to be successful. The financial records and the prospects for future success are the determining factors.

Variations in Types of Bond Investments.—Even among bonds there are many variations in the attractiveness of the investment, owing to differences in the types of bonds issued and to differences in the types of organizations issuing the bonds.

An investigation to determine the particular characteristics of, and rights offered in, a given bond, and the comparison of these characteristics and rights with those of other available bonds is the first step in a bond

investment analysis. Bonds may be classified on the basis of (1) purpose for which issued; (2) security of principal; and (3) payment of interest and principal. Bonds are referred to as bridge bonds, equipment bonds, improvement bonds, etc., referring to the purpose to be accomplished or the use of the money. Classified according to security of principal,, bonds may be (*a*) prior-lien bonds, which take precedence in their claim over mortgage bonds; (*b*) mortgage bonds protected by mortgages on the property, which may be first, second, third mortgages, etc.; (*c*) debenture bonds, having no security except the company's promise to pay; (*d*) collateral trust bonds, which are secured by stocks, bonds, or mortgages of other companies owned by the company issuing the collateral trust bond; or (*e*) government bonds, issued by city, county, state, or national governments. Bonds classified according to the payment of interest and principal may be: (1) income bonds upon which interest is paid at the discretion of the board of directors; (2) profit-sharing or participating bonds, which share in profits in addition to receiving a fixed rate of interest; (3) tax-free covenant bonds, upon which the company agrees to pay the Federal income tax to the extent of 2 per cent on the interest upon authorization of the recipient of the interest;[1] (4) tax-exempt bonds—the interest on municipal and state bonds is free from Federal income taxation; (5) convertible bonds, that is, bonds which carry the right under specified conditions to be converted into some other security, such as preferred stock or common stock; and (6) perpetual bonds, which have no date of maturity when the principal must be repaid to the holder.

United States and Other Government Bonds.—The bonds of the United States Government have been considered to be the safest investment available, and probably will continue to be so regarded unless the total amount issued, including short-term notes and certificates of indebtedness, should become so large that investors doubt the ability of that government to meet interest requirements and repay principal. The total of these obligations outstanding on June 30, 1938, was approximately $37,165,000,000, and they are secured by the wealth of the nation, which has been estimated at from 250 to 360 billion dollars during the past decade,[2] and by the annual income of the nation, which was estimated at approximately $67,500,000,000 for the year 1937.[3] Interest paid on the United States Government obligations issued before 1917 and on all Treasury Notes, Treasury Bills, Treasury Certificates of Indebtedness, and obligations issued under the Federal Farm Loan Act

[1] These bonds have not been issued since December 31, 1933.

[2] See Moody's "Manual of Investments—Government Securities," pp. 24–25, 1938.

[3] National Industrial Conference Board.

is free from income tax, and the interest on Treasury Bonds and Savings Bonds is partially tax free. The tax position of a security is important in determining its rank as a desirable investment. A 4 per cent bond, subject to a 50 per cent tax, yields a net return of 2.5 per cent, which is less than the yield of a 3 per cent government tax-free bond.

The bonds of Great Britain, Dominion of Canada, the Scandinavian countries, and Switzerland also have had a good reputation. In many instances, however, the bonds of foreign governments are very unsafe investments. Before purchasing such securities, careful analyses should be made of past records of political stability, government policies, burdens of taxation, etc., as well as the history of the particular bond under consideration. The importance of securing especially sound and impartial advice when purchasing foreign investments can not be too strongly emphasized. No income-tax advantages are obtained by residents of the United States who purchase foreign bonds.

State Bonds.—The desirability of state bonds as an investment depends upon: (*a*) the good faith of the people of the state in paying their debts as shown by their past records (many state debts have been repudiated in the history of the United States but there has been no repudiation in recent years); (*b*) the wealth of the state compared to the size of the debt and the total population; (*c*) the laws regulating the state's debts; and (*d*) the tax position. State bonds are exempt from Federal taxation and usually from state and municipal taxation in the state where issued. State bonds, in general, rank next to Federal government bonds as safe and conservative investments, but all state bonds are not considered equally good.

Municipal Bonds.—Most municipal bonds[1] rank high as conservative investments, but greater care must be exercised in investing in municipal bonds than in either United States Government or state bonds. Towns sometimes develop rapidly as the result of a boom, and decline later almost as rapidly. Cities have more frequently repudiated their debts because of loss of ability to pay than have states. Furthermore, municipal bonds are sometimes declared illegal.

The size of the city's debt relative to the value of the taxable property is, of course, a basic factor in determining the desirability of its bonds. Two general tests of the desirability of municipal bonds are: (*a*) whether they are legal investments for savings banks, and (*b*) whether they are offered for sale by well-established and reliable investment bankers.

[1] In this discussion, the classification of municipal bonds is limited to direct, full obligations of towns and cities of—say—10,000 people and over. Bonds issued by smaller localities or portions of municipalities are generally considered as municipal bonds, but there are many types of these bonds to which the present discussion is not applicable.

Nevertheless, an investment in municipal bonds should not be made as a result of these general tests alone, but an investigation should always be made of the financial condition of the city or district issuing the bonds, and the property securing them.

Corporation Bonds.—Corporation bonds are usually secured by a mortgage under the terms of which specific property is pledged as security for the payment of both principal and interest. The mortgage, however, may not necessarily be a first lien on the property. Bonds, moreover, are rarely known as third- or fourth-mortgage bonds, since such a title would indicate too plainly the inferior lien. The investor, consequently, must be on his guard. Second-mortgage bonds, for instance, are often termed "first refunding"; third-mortgage bonds are designated "general mortgage," "consolidated," etc.

It is one thing to know definitely the exact property which is pledged as security for a bond, but it is quite another thing to determine its real value. From an investment viewpoint, corporation earning power is the true measure of value. The ratio of fixed charges to the net income available for the payment of these charges reflects the security of the bonds. In determining the number of times interest charges should be earned to reflect a satisfactory condition, one must consider the type of business activity in which the corporation is engaged. In railroad companies, 1.50 to 1.75 has been considered satisfactory for the largest companies; public utilities require 1.75 to 2.00;[1] and industrials should earn at least twice the interest charges even in the most unfavorable years. Proper accounting principles must be used in determining net income available for interest charges; otherwise a false impression will result.

With the exception of the bonds of some of our leading corporations, industrial bonds have not proved very popular. In the first place, the conservative investor finds no appeal in industrial securities because of the speculative element. And the more venturesome individual expects a larger return than industrial bonds assure. This is reasonable because of the sacrifice in security which he makes. In the second place, there are relatively few issues of industrial bonds because of the instability of earnings of the average company.

Investment Characteristics of Preferred and Common Stocks.—Other things being equal, preferred stock is a safer form of investment than common. The preference is usually as to both dividends and assets, and from an investment standpoint, this advantage tends to offset the limited income return. Common stocks often have a fair investment rating, in many cases superior to the bonds of other corporations which are not so prosperous. The test of such a security is not so much in

[1] A lower rate may be applied in cases of proven stability of earnings.

the amount of earnings in prosperous years, as it is in the amount in times of depression. The stock of any company that is able year in and year out, through dull times as well as busy, to show substantial earnings and regular dividends may be considered as offering a good opportunity for a permanent investment.

To the extent that potential appreciation in value is desirable in investment, stocks surpass bonds. Certain types of investment stocks have inherently a better opportunity for appreciation than others. Preferred stocks which enjoy a participation in income privilege are in this class. Common stocks, which have gained an investment rating, offer the best opportunities from the standpoint of potential appreciation, because the income which the common stockholders may receive is limited only by the earnings of the company. But common stocks purchased during prosperous times usually contain a greater element of possible depreciation than appreciation. The increased earnings of a company during a period of prosperity cause its stock to sell at higher prices, and subsequent declines in business conditions, assuming that prosperous times can never be constant, result in lower prices.

Little likelihood exists that bonds (except those with conversion privileges) will ever sell at a large premium as compared with common stocks, because of the fundamental difference that bond income is, as a rule, limited, while common-stock income is ordinarily unlimited. When the return on stocks is limited, as on preferred stocks, the price will tend to keep within the same limitations as bond prices.

Form of Capitalization.—In selecting an investment in stocks or bonds, the capitalization of the company is one of the controlling factors. In a company which has issued bonds, preferred stock, and common stock, part of the revenues must be used to pay bond interest and part must be used to pay preferred-stock dividends before anything is available for common-stock dividends (or for the surplus). If dividends of, say, 6 per cent are paid on the common stock of such a company during a period of prosperity, it can easily happen that there will be nothing at all for common-stock dividends during a period of depression. Other things being equal, therefore, there is considerably more risk in investing in the common stock of concerns heavily capitalized with fixed obligations than there is in buying the common stock of a concern whose capitalization consists entirely of common stock.

If the capitalization of a concern consists entirely of common stock and business becomes depressed, there will be no bonds and preferred stock to absorb any profits that may be earned, and they can be distributed as reduced common-stock dividends. (Of course, if bonds and preferred stocks represent only a small proportion of the capitalization, this objection may not be important.)

While bonds and preferred stock have greater security of both principal and income than the common stock of the same concern, they bear only a fixed rate of income and often are not so desirable as the common stock of equally prosperous companies whose entire capitalization is represented by common stock. Common stock shares in the earnings and common stockholders get the full benefit of a concern's prosperity whether the earnings are paid out in dividends or left in the surplus account.

Perhaps the other side of the question should be mentioned at this point as there are those who claim that the common stock of a concern with a high proportion of its capitalization in the form of bonds and preferred stocks is the best buy, because only a limited proportion of the profits made on the capital furnished by the bond and preferred stockholders go to the holders of these securities. The rest goes to the holders of the common stock. Such would be the case when the concern is prosperous, but it is equally true that the common-stock dividends may be entirely wiped out in times of depression because of the prior claims of the bonds and preferred stock. The chance of increased earnings carries with it a greatly increased risk. The principle is the same as in buying on a margin. Safety is sacrificed for the chance to secure a greater gain. If a careful analysis shows the common stocks of the two types of concerns to be equally desirable in other respects, the conservative business man, who wishes to reduce his risk and at the same time share the profits, will buy the common stock of the company which has the smaller proportion of bonds and preferred stock.

The purpose of the foregoing brief discussion of the different types of stocks and bonds has been merely to point out general characteristics and principles. Analyses to determine the relative position and rights of stocks and bonds of a given company are sometimes very complicated; a single corporation may have several classes of bonds, several classes of preferred stock, and several classes of common stock, and some may be convertible to other classes and some may be redeemable. Obviously, these conditions should be carefully investigated in any investment analysis.

Investments in Real Estate Mortgages and in Real Estate.—Popular forms of investment in the United States include real estate mortgage loans and the actual real estate. The preceding chapter on real estate analysis describes methods of determining the values of real estate and probable trends in these values, as well as other factors which are important to the investor. Since these mortgage loans have real estate for their security, similar analyses should be made in determining their safety and desirability. One of the chief objections to both mortgages and real estate as forms of investment is the lack of a ready market.

They cannot be disposed of on a moment's notice, as a rule, so readily as can well-selected bonds or stocks.

Security of a mortgage loan depends upon two factors: the priority of the lien, and the margin of equity. Upon receiving a mortgage, the lender, or mortgagee, has a lien upon the property. As the owner of such lien he may acquire the property in case of failure to pay the loan. There may be, however, other claims which take precedence over his. The second factor determining the safety of the principal is the difference between the amount of the loan and the value of the property. In large cities, the limit of safety for real estate mortgages is generally regarded as $66\frac{2}{3}$ per cent and in smaller towns 25 to 40 per cent. A farm mortgage on improved farms rarely should exceed 50 per cent.

Six of the important elements of risk that must be considered in connection with mortgage investments are: (1) inaccurate appraisal, (2) influence of depressions and panics, (3) neighborhood changes, (4) deterioration of buildings, (5) losses in foreclosure, and (6) danger of forced sales. Each of these should be carefully analyzed by the lender.

Active Listed Securities.—The careful investor will usually do well to confine a major portion of his investments to readily salable securities that are listed on responsible exchanges. The reason for advising that the securities be both listed and readily salable is that the business investor often wishes to have a ready market as one of the requirements in his investment. He may neither wish nor expect to sell the securities at a moment's notice, but he wishes to have an investment liquid enough that he may be able to sell it readily if this becomes desirable. Active listed securities are also better collateral for loans, as the bank can sell them readily in case the loan is not paid. If the investor is certain that he will not wish to dispose of the securities, this feature is not so important, and he may find a satisfactory investment in such securities as bonds of small cities which are unlisted and for which there is no active market.

The fact that a stock or bond is listed does not indicate that it is active. In 1930, less than one-quarter of the stocks listed on the New York Stock Exchange were active. To be readily salable, the stock need not be an active one, as there are a number of high dividend paying stocks that are inactive, either because the price is so high, or the issue is so small. Ordinarily there would be very little difficulty in disposing of such stocks on the exchange. However, they may not be desirable investments.

In connection with market activity, it should be pointed out that there is no definite relation between activity and investment value. A reasonable degree of activity is a desirable quality, but many excellent securities are rather inactive, while many very active securities are of little investment value.

Sources of Data on Stocks and Bonds.—All of the larger newspapers of the country publish data upon stocks and bonds, such as daily price quotations, volume of stock or bond sales, dividend rates, dates dividends are declared, and short summaries of the current profit and loss statements. Much more complete information is carried in such specialized dailies as *The Wall Street Journal* or the *Boston News Bureau*. One of the best sources is the *Commercial and Financial Chronicle*. *Poor's Manual* and *Moody's Manual* give detailed information on many companies. Specialized service is furnished by the Standard Statistics Company, The Brookmire Economic Service, Babson's Statistical Organization, and others. The investor can also get some information from his bank,

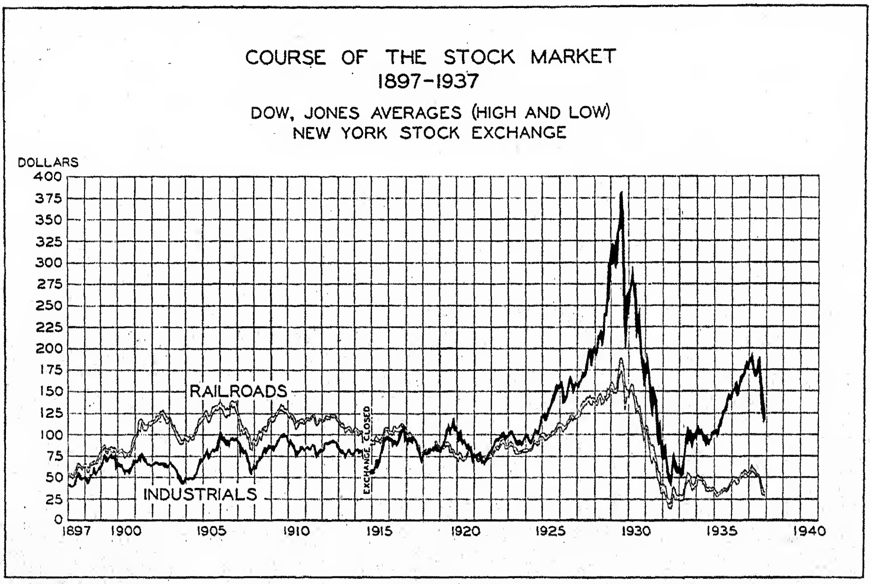

EXHIBIT 211.—Cyclical swings of average stock prices. (The industrial list, as shown above, was changed from 12 stocks to 20 as of December 1914 with the resulting effect of lowering the position of the index approximately 25 per cent. On October 1, 1928, the number of industrial stocks was changed from 20 to 30 without changing the average price.)

and if he is not already familiar with the various investment banking and brokerage houses, he should seek his banker's advice when selecting such agencies. This last point, the selection of an investment banker or broker, is very important, for there are many houses in most of the large cities whose practices and standards are not at all satisfactory from the point of view of making safe and sound investments.

Investing in Relation to Long-time or Cyclical Swings.—It should be emphasized again that the profits made through appreciation, as considered in this discussion, are based upon changes which take place

over a considerable period of time, such as several months or a few years.[1] In such analyses we are not concerned with attempts to make profits due to day-to-day or week-to-week fluctuations. The fundamental underlying movements in business, like the tides, can be analyzed and their movements can be forecast. But the short-time day-to-day fluctuations, like the waves, are subject to various surface conditions, which often have no relation to the deeper-seated forces. Probably most of those who deal in stocks are trying to make profits from day-to-day or week-to-week fluctuations, but this type of dealing, on the whole, should be classed as speculation and not as investment. In this discussion, we are interested in the fluctuations which can be accounted for on an economic basis, such as increases in earning power.

The opportunity to take advantage of the cyclical swings in the stock market is indicated graphically in Exhibit 211, which shows the Dow-

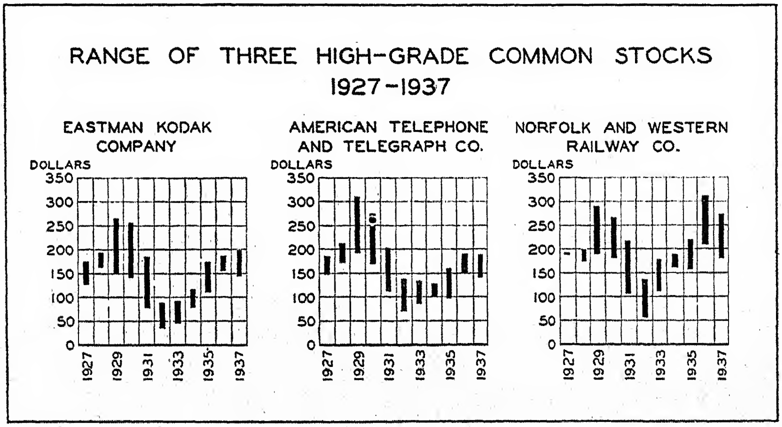

EXHIBIT 212.—How changes in the prices of three stocks (an industrial, a public utility, and a railroad) have offered opportunities for gain.

Jones averages for both industrials and railroads from 1897 to 1937. Every few years (or sometimes months) one can buy when prices are relatively low and sell when prices have increased from 50 to 100 per cent, providing he makes the statistical analyses that will enable him to buy and sell the right stocks at the right times.

Exhibit 211 shows the average prices of groups of representative stocks. Ordinarily, however, one is interested in the swings of individual stocks when he wishes to buy.[2] The opportunities to profit from the

[1] The term "long-time swings" should not be confused with "long-time trend." Long-time swings is a term commonly used in investment circles to distinguish the changes that take place over a few months or years from the day-to-day and week-to-week fluctuations. The terms "long-time pull" and "long pull" are also commonly used in this connection.

[2] If one wishes to invest in the average of a number of high-grade securities he can do so by investing in one of the better investment trusts.

cyclical up-swings in prices of three individual stocks (an industrial, a public utility, and a railroad) can be appreciated by referring to Exhibit 212. A study of the high and low figures in the table of Exhibit 214, on pages 582 and 583, will also indicate the investment opportunities furnished by 36 oil stocks.

It is important, as a rule, to select stocks of concerns whose basic trends are distinctly upward rather than downward. Unless this is done the declining trend will offset part, if not all, of the cyclical gains.

From previous discussions of the effect of business cycles, seasonal variations, trends, and unusual movements upon business data, the reason for considering these factors in analyzing a company and the price of its securities should be apparent. Ordinarily, however, the more refined methods discussed earlier in this book will not have to be applied. If one understands the computation and the use of these methods, he can consider the various forces or movements at considerable length without actually making the detailed computations.

Security Price Indexes.—In making a general study of security prices, an index of the security market is particularly useful. Also, in studying the price of an individual stock (or bond) it is necessary to make comparisons with the general price level of all stocks (or bonds) and with the price level of a group of similar stocks (or bonds), in order to determine whether the individual price has been affected by market conditions in like manner and degree. In recent years, certain indexes of security prices have been computed and are published regularly to show monthly, weekly, and even daily composite changes. Several of these indexes will now be discussed.

The Dow-Jones Index of Stock Prices.—The oldest security price index is that of Dow, Jones & Company, present publishers of *The Wall Street Journal*. This index was started in 1884 as a simple average of the daily closing prices of 11 common stocks. In 1897, this index was changed into two indexes: one, the simple average of the daily closing prices of 20 railroad stocks; and the other, a similar average of 12 industrial stocks. In 1916, the number of industrial stocks was increased to 20[1] and in 1928 to 30. In 1930, a new average of 20 utility stocks was added. The railroad and utility averages are still unweighted averages—that is, they are obtained by adding together the closing prices of the day of the 20 selected stocks in each average and dividing each total by 20. The industrial average is now an index, which, though not weighted, permits the occasional shifting of stocks in the average to accommodate changed conditions, stock dividends, etc., without destroying the continuity of the index. These averages are published in many financial papers and magazines. Similar indexes of bond prices are also published.

[1] For the period from December 1914 to October 1916, quotations are available for both the 12- and 20-stock series. (See Exhibit 211, page 569.)

The New York Times Index.—The indexes published by *The New York Times* are prepared as simple averages of the daily high, low, and closing prices, respectively, of 25 railroad stocks, of 25 industrial stocks, and of the 50 stocks combined. There are nine series—three each for railroad, industrial, and the combined stocks, showing, respectively, the course of the high, the low, and the closing prices. Like the Dow-Jones Index, there is no system of weighting the prices included; each index is stated in terms of dollars to show the average high, the average low, or the average closing price of each group. The New York Times Index was started in 1911.

Indexes of the Standard Statistics Company.—In 1923, the Standard Statistics Company introduced a weighted index based on the market prices of 228 stocks and weighted by the number of shares of each stock outstanding at the date of each price quotation. In 1929, the index was revised to include 406 issues, and subsequent additions have been made so that in June 1938 there were 420 issues included in the index. In addition to the index of price movement of all stocks, this company publishes group indexes to indicate separately the movement in industrial, in railroad, and in public utility stock prices. These groups are further subdivided into 62 groups to give the movements according to companies producing automobiles, food products, coal, copper, oil, leather, etc. The indexes are based on weekly quotations (usually Wednesday closing prices) and the purpose is to show broad market tendencies rather than day-to-day fluctuations. They are expressed as percentage relatives with the year 1926 as a base period.

In addition to the weekly indexes of stock prices, the company publishes a daily index, compiled in the same manner as the weekly, but including only 50 industrial stocks, 20 railroads, and 20 public utilities. The industrial stocks included are representative of all major industries, and they have been chosen with a view to making the daily index closely similar to the weekly in direction and time of movements.

Several supplementary indexes are prepared by the Standard Statistics Company. These include: (1) weekly indexes of the price and yield of 20 high-grade industrial preferred stocks; (2) weekly index of the prices of 19 New York City bank stocks; (3) weekly index of the prices of 18 fire insurance company stocks; (4) weekly index of 18 investment trusts' shares; and (5) weekly indexes of the price and yield of high-grade bonds, based upon 67 issues subdivided into groups of 15 industrials, 15 utilities, 15 railroads, 15 municipal bonds, and 7 United States Treasury Bonds. Also a daily index of 60 bond prices is prepared, with supplemental indexes for 20 industrial, 20 railroad, and 20 utility bonds.

Investigation of the Concern Whose Securities Are under Consideration.—The most important consideration in choosing securities, whether

they are stocks, bonds, or mortgages, is not the conditions and rights stated in the security, nor is it the price in relation to other securities having like rights and conditions. Although these are important in choosing an investment, an investigation of the company or enterprise whose securities are under consideration is of even greater significance.

In the remainder of this chapter, the principal factors ordinarily to be considered in investigating business concerns from the investment point of view, will be discussed. These factors may be briefly listed as follows:

1. Nature of the Industry.
   - *a.* Kind.
   - *b.* Stability and history.
2. Location Relative to
   - *a.* Markets.
   - *b.* Sources of materials.
   - *c.* Labor.
   - *d.* Capital.
   - *e.* Transportation facilities.
   - *f.* Related industries.
3. Management.
   - *a.* Character.
   - *b.* Success and standing.
4. Markets.
   - *a.* In the industry.
   - *b.* For the concern.
5. Capital Structure and Ownership.
   - *a.* Control.
   - *b.* Distribution of securities.
6. Growth of the Concern.
   - *a.* Reason of, in past.
   - *b.* Future prospect.
7. Financial Statements.
   - *a.* Analysis of.
   - *b.* Ratio analysis.
   - *c.* Accounting policy.
8. Earnings Records and Reality of Earnings.
9. Stock and Bond Prices; Financial Policies.
   - *a.* History.
   - *b.* Yield.
   - *c.* Dividend and other financial policies.
10. Safety of Principal and Income.
    - *a.* Appraisal of risk.
    - *b.* Adequacy of return in relation to risk.
11. Business Cycles, Seasonal Variations, Trends, Unusual Movements.
    - *a.* Of the concern.
    - *b.* In the industry.
    - *c.* In general.

Nature of the Industry.—Usually the first consideration in choosing the securities of a concern for an investment is the nature of the industry

in which it is engaged. Industries having government franchises, under which competition is limited, and permanent operation for a fair return has been aided by state or national regulation, are not comparable with other industries. Therefore, the problems in choosing the security of a railroad, a telephone, or a gas and electric corporation are not the same as those encountered in investing in an oil company, a lumber operation, or a department store. Even in picking the securities of these last mentioned types of companies, the important points to investigate vary because of differences in the industries. Oil companies may produce in the oil fields; they may also refine; and they may distribute to the retail trade through their own stations. Lumber-producing companies usually do not retail their products to the consumers. Department stores deal only with consumers and, usually, only within a limited area. Because of such differences in the nature of the business of these companies, it is not possible to compare their investment attractiveness solely on the basis of their financial statements at a given moment of time. Particularly, the conditions must be examined to determine the stability of the industry and its prospects for the future.

Companies that manufacture luxuries ordinarily are more hazardous, from an investment standpoint, than those that produce necessities. Temporarily, the profits earned by producers of amusement equipment, such as sporting goods, may be large; but continuing profits are not so probable in such industries as in sugar refining or cigarette manufacturing.

Concerns manufacturing consumers' goods usually suffer less in depression periods than companies manufacturing capital goods which are used in plant expansions and in the creation of new industrial enterprises. Often such capital expenditures, as for new buildings, equipment, and even the maintenance of existing equipment, can be deferred for several years, whereas goods for direct consumption usually have a more steady demand.

Often the best opportunities for investment in an industry exist when conditions appear to be at their worst. In industries that have proved to be indispensable, the securities of the strongest companies are sometimes available at particularly low prices, because the industry has suffered a severe or extended period of depression. During such a depression many investors do not recognize that readjustments are taking place—competitors are being eliminated and overexpansion is being corrected. These factors often make the securities of the surviving companies particularly desirable at the moment when short-sighted holders of these securities are most discouraged. At such times, if the careful investor determines beyond all reasonable doubt that the industry is a permanent one and that the prospects for better conditions are not too far in the future, he is assuming a low risk in purchasing securities

of the strongest companies. Obviously, such analyses to determine the permanence and the future prospects of the industry must be made carefully, and they require the collection of much statistical data on conditions in the industry and the use of the products.

Location of Enterprise.—When selecting an investment, it is important to consider the location of the enterprise in relation to its markets, its sources of raw materials, capital, labor, and such other considerations as the general reputation of the community. In many instances, an enterprise cannot have the best location from the point of view of all of these factors, but it should be well located in relation to at least some of them. Often it is desirable to gather data which will show whether the industry should be located near one factor or another if these factors are scattered. Industries vary widely in this respect. A lumber mill ordinarily will be located near the timber supply so that it will be necessary to transport only the commercially valuable portions of the logs. Bread manufacturing, on the other hand, must take place near the market because of the high rate of deterioration of the finished product.

Another important consideration is location in relation to transportation facilities. This involves a study of waterways, railroads, highways, airmail routes, and any other method of transportation or communication of importance to the concern.

Some communities have developed in such a way that they present advantages for certain kinds of industries as compared with communities which do not have such development. An automobile manufacturer, for instance, finds a trained labor supply, sources of capital in sympathy with the automobile industry, and the necessary related industries in and around Detroit. New England is noted for textiles, Ohio for iron and steel, Minneapolis for flour milling, and Los Angeles for motion pictures. Anything as unusual as a cotton textile mill in North Dakota or an automobile factory in Arizona should be investigated with more than the usual care before considering it as an investment.

Necessity for Checking up on the Management.—In the preceding outline of factors to be considered, one of the principal items is management. It may seem that this factor would be the only one that need be considered, and that the checks upon the success of any management need not be statistical. This is not the case, however, for it is in connection with checking the management that statistics serve one of their most useful purposes; it not uncommonly happens that when the management is apparently successful, conditions are actually very unsound. A representative illustration of this is the 1920 difficulty of the Goodyear Tire and Rubber Company,[1] described in Chapter XIII. The phenomenal

[1] After the 1920 reorganization, the Goodyear Company prospered, and in the depression of the 1930's it was one of the strongest of the rubber companies.

growth of this concern and its large surplus caused many investors to believe that the company was in the hands of an exceptionally capable management. A check-up of the management in 1919, however, would have indicated that the company had recklessly expanded its fixed investment at high costs, and that it had a large merchandise inventory which had been accumulated at high prices during the prosperity period of the business cycle. The prudent investor, with these facts at hand, would know that the company would be in no condition to pass through a period of depression in industry. It not infrequently happens that a management which is successful in building up an industry is deceived by its own optimism into thinking that an unsound financial structure is safe and strong.

Markets for the Concern's Products.—One of the most important factors to be analyzed in making investments is the market for a concern's products (or services). Under our present economic organization, a company has less control over this factor than over many of the others, and consequently the outlook for the market for a concern's product must be favorable at the outset. Ordinarily a company can change its methods of manufacturing, or even its financial organization, for these are internal questions which can be controlled within the concern. But the market depends upon external conditions and, ordinarily, successful development depends upon meeting the conditions of the market rather than upon trying to change the market to fit the product. To be sure, such forces as advertising or salesmanship can be used to create demand, but competition is so keen that even where they are used to their fullest extent, the product must be one which will definitely satisfy the demand of the trade if the concern is to benefit from a lasting market.

It is in analyzing the conditions of the market for a concern's product that the investor has one of the greatest opportunities. A few years ago the securities of certain agricultural machinery manufacturers were low priced because the poor implement and machinery market resulted in small or no profits, with corresponding dividends. Farmers had not been buying machinery nor implements since the agricultural depression of 1920. An analysis of the market showed that the farm purchasing power was increasing and that the need for new farm equipment was greater than it had been for several years. A substantial decline in the raising of horses also indicated that there would be an increase in the demand for substitutes in the form of tractors. The indicated improvement in the market for agricultural equipment took place, and the securities of the established manufacturers correspondingly increased in value.

Suggestions for making surveys of the market for a concern's products were given in the previous chapter on Marketing Analysis. Though the investor's point of view is somewhat different from the point of view of

the sales manager, the principles are the same, and need not be discussed again at this point. As in the previous chapter, the market should be analyzed for the industry as a whole as well as for the individual concern.

Capital Structure and Ownership.—Inquiry regarding the ownership and capital structure is important for several reasons. One should know something about the policies of the group of shareholders that controls the organization, and how widely the ownership is distributed. It often happens that two large groups of stockholders with two widely different policies of operation and management are constantly fighting for control. As a rule, such a condition is undesirable, for it makes consistent progress difficult. Again, it is very important to know what proportion of the ownership is held by the management, since the condition is ordinarily more sound when the management (if an able one) has a considerable share in the ownership than when outside interests are the principal owners. An instance of this is the case of a certain large building-materials manufacturer. In this concern, which was doing a very good business, a few large stockholders, who were not particularly interested in the firm's future, sold their shares to a competitor who thereby gained control. Upon this change in control, an assessment was levied upon the stock, and, at the same time, the manufacturing plant of the company was closed. Furthermore, it was understood that in the future the company would produce only raw materials, to be sold to the company which had acquired control at a price so low that it would be impossible ever to pay dividends on the stock of the first company. Consequently, the stock of the minority stockholders became practically worthless, and most of it was forfeited because the assessments were not paid. Although the minority stockholders instituted legal proceedings, they were not successful. In this particular case, an analysis of the ownership before investment would have indicated that this danger existed.

In connection with capital structure and ownership policies, the form of capitalization—that is, the proportions of bonds, preferred stock, common stocks, etc.—should be considered, as discussed previously. Earnings records and stock prices will be discussed later.

Growth of an Enterprise.—The security of an investment depends considerably upon the stability of the enterprise. For instance, one would not wish to make an investment in a concern whose business may be legislated out of existence. As a rule, the important risks which are typical of the industry are not difficult to discover, and if the investor is conservative, he will avoid those where the risks are great, although there may be unusual chances for gain if the concern is fortunate.

The success of many concerns depends upon near-by natural resources, and they cannot prosper after these resources have been exhausted. The

supplies of some natural resources, such as standing timber, are known, while others, such as natural gas, are unknown. There are many instances of factories being built which depended upon natural gas as a fuel, where most of the investment was lost because the supply lasted only a few years. In other cases, the concern's success has depended upon the ownership of a certain patent, and the prosperity of the concern was limited to the life of the patent.

Some concerns have grown because of superior judgment and keen analysis of their problems by their management. Others have grown because of extremely fortunate conditions for which they are not in the least responsible. Other things being equal, the concerns in the first group are more likely to continue to make progress than those in the second.

Ordinarily, the conditions most desirable from an investment point of view will be found in connection with large concerns rather than with small ones, although this is not always the case. Most of the securities that are well known and have a ready market are those of the larger concerns. The conservative investor will take care to select investments only in the enterprises which are of a reasonably permanent nature and which can profit by the general expansion or long-time growth of business as a whole.

Analyses of Financial Statements.—Financial statements published by corporations are of two kinds. One type, the balance sheet with supporting exhibits, reports the financial condition at a given date. The other type, the profit and loss statement and supporting information, states the results of operations over a period of time. By studying the facts presented in these statements, considering the two types of statements individually and also in relation to each other, the results of the management, the growth of the company, the changes in the financial structure, and other important information can be obtained.

Unfortunately, all companies do not publish complete balance sheets and profit and loss statements. Also, the financial magazines and newspapers do not always reprint the complete statements issued by the leading companies, but condense these reports to a point where inadequate information is given. When possible, investors should obtain complete financial statements over a period of several years and analyze them carefully. It is strange that many purchasers of securities in the past have investigated their purchases much less carefully than they would investigate an automobile or a suit of clothes before purchasing. Yet, in buying securities, they should rely even less upon sales talk than in buying automobiles and clothing. Unfortunately, but few security salesmen have been trained to make real analyses of their merchandise.

In the paragraphs which follow, several procedures in analyzing statements will be presented. The first methods considered will deal with the balance sheet.

The 100 Per Cent or Common-basis Statement.—Comparison of the financial condition of one corporation with another is difficult unless their balance sheets are reduced to a common basis. This can easily be done

	Company *A*		Company *B*	
	Amount	Per cent	Amount	Per cent
Assets				
Cash	$4,000	4.0	$12,000	2.4
Receivables	15,000	15.0	50,000	10.0
Merchandise	10,000	10.0	40,000	8.0
Total current	$29,000	29.0	$102,000	20.4
Fixed assets	69,000	69.0	383,000	76.6
Deferred charges	2,000	2.0	15,000	3.0
Total assets	$100,000	100.0	$500,000	100.0
Liabilities and net worth				
Notes payable	$6,000	6.0	$10,000	2.0
Accounts payable	8,000	8.0	20,000	4.0
Other current	3,000	3.0	2,000	.4
Total current	$17,000	17.0	$32,000	6.4
Bonds, etc	20,000	20.0	150,000	30.0
Total liabilities	$37,000	37.0	$182,000	36.4
Net worth	63,000	63.0	318,000	63.6
Total liabilities and net worth	$100,000	100.0	$500,000	100.0

EXHIBIT 213.—A simple illustration of the 100 per cent or common-basis statement.

by converting the dollar values of each to percentages of the total of each side of their respective balance sheets. The procedure is illustrated in Exhibit 213. It will be seen that, when reduced to percentages of the whole, each item in the statement can readily be compared with similar items on other statements.

It is particularly desirable to compare the 100 per cent or common-basis statement of a given concern with the usual or average statement of like nature prepared for similar concerns. Data have been assembled by various organizations, such as the Robert Morris Associates (Lansdowne, Pennsylvania) and various bureaus of business research of universities,

which show the composite assets and liabilities of many companies. Some of the statements published by these research organizations present data for groups of unclassified industrial concerns, whereas other tables are given for groups of companies classed as drugs, lumber, boots and shoes, wholesale dry goods, etc. Obviously, the latter type of table is very useful in comparing the condition of a given concern with other companies in the same industry.

This form of analysis is also useful in studying one corporation over a period of years. By reducing each item on the balance sheet at each balance sheet date to a percentage base, the significance of the changes in items from year to year can be seen. For example, if merchandise inventories have increased in amount over a period of years, by determining the percentage which the amount at each year bears to the total assets, it can be seen whether or not the inventory has increased out of proportion to the increases in other assets. And usually it is not the amount of increase in an asset or a liability that is important; it is the change in relationship to the total that is significant.

Use of Ratios in Analyzing Statements.—While the determination of a concern's strength or weakness cannot be removed from human judgment and be based solely upon mathematical calculation, nevertheless, ratios are of considerable value to investors in interpreting the accounts. The eight most important ratios in analyzing financial statements are:

1. Current.
2. Worth—debt.
3. Worth—fixed.
4. Merchandise-receivables.
5. Sales—merchandise.
6. Sales—receivables.
7. Sales—fixed.
8. Sales—worth.

These eight ratios may be classified in two groups: the first four pertain to the status of a business (shown by the balance sheet) at a certain time, while the second four are significant because they measure the changes effected between statement dates.

The current ratio is the ratio of current assets to current liabilities. This ratio has been widely used by bankers when making loans. Before the World War, a statement which showed current assets and current liabilities in the relation of 2:1 was ordinarily considered to be a safe risk. It is now generally realized, however, that such a relation may be safer than necessary in a period of depression and distinctly unsafe in a period of prosperity. This ratio has its chief significance when compared to the same ratio of other concerns in the same industry operating under similar conditions.

Even under normal price conditions the current ratio alone is not conclusive as to the financial strength of any business. It must be considered along with other ratios rather than as a definite criterion in itself. A concern with a current ratio somewhat below the prescribed 2:1

may be in better financial condition than one having a much better current ratio. If the latter has a heavy bond issue outstanding and the former has no long-term obligations, obviously the difference in their current ratios is not conclusive of their relative strength or weakness. Also, in railroads and in public utilities particularly, the current ratio is of relatively minor significance where the general credit standing of the company is sound, because in such instances the current debt can be eliminated by the issue of bonds or preferred stock.

The second ratio listed above, that of net worth to total debt, is considered by some to be equally as important as the current ratio. This ratio shows the proportion of owned to borrowed capital. It is impossible to state a definite ratio as the ideal, however, as a ratio may be conservative for one industry and yet be unreasonable for another. In the case of a railroad, for instance, a ratio of 1:1 might not be alarming if there were uniformity of income to meet fixed charges on indebtedness, but in most manufacturing concerns the ratio of 1:1 would not ordinarily be very safe.

The ratio of net worth to fixed assets is one which shows the liquidating protection afforded to creditors. If this ratio is high—that is, if the net worth is large in comparison with fixed assets, the creditors are given greater protection than where the ratio is low. Comparison of this ratio at various dates for the same company shows whether or not the policy of the company is to give the creditors better protection or to increase the extent to which creditors are financing the plant assets.

The ratio of merchandise to receivables (trade notes and accounts receivable) is really a check on the current ratio, discussed above. It shows the relative composition of the main items of current assets.

The four other ratios mentioned above—namely, sales in comparison (1) with receivables, (2) with merchandise, (3) with fixed assets, and (4) with net worth—are methods of measuring how productive the business has been in relation to individual assets and in relation to the total net assets or net worth, which is the excess of assets over liabilities.

It is not within the scope of this chapter to discuss these or other ratios in detail, as such a discussion would require an entire book. Those mentioned constitute only a few of the most useful. And in order to use these ratios satisfactorily, considerable practice in computing and studying them in different companies is necessary. Particularly, it is difficult to determine the importance to be given to the different ratios. For credit purposes (that is, for those planning to become creditors), it has been suggested that the following weights be applied to the ratios in determining an index of credit strength:[1]

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[1] Wall, Alexander, and Raymond W. Duning, "Ratio Analysis of Financial Statements," p. 158, Harper & Brothers, New York, 1928.

Ratio	*Relative value or weight (per cent)*
Current	25
Worth—debt	25
Worth—fixed	15
Sales—receivables	10
Sales—merchandise	10
Sales—fixed	10
Sales—worth	5
Total value of all ratios	100

When data on profits are available, two additional ratios—profits on sales and profits on net worth—have been found highly important. When these ratios are included in the index of credit strength, obviously, an adjustment of the above weights is necessary.

Earnings Records.—One writer has said, "All elements of investment value may be reduced, ultimately, to the earning capacity—the economic reason to be—of the enterprise."[1] In analyzing an investment, one should determine whether or not the company's statement of earnings is significant and acceptable. In this connection, it is very important to study the earnings statement together with the balance sheet. Such a study should include a comparison of current reports with those of earlier years, in order to determine whether or not the concern is making progress. The results will be indicative of the stability of earning power and character of management. Such a comparison indicates the normal condition and assists in indicating whether the current report covers an abnormal, normal, or subnormal year of earning power. It is difficult to say which, on the whole, is the more important, the earnings statement or the balance sheet. It is really necessary to use both and to analyze the features of each in the light of the other.

In considering an investment in the common stock of a company, it is essential that both the capitalization (as revealed by its balance sheet) and the market price per share of the common stock be considered in relation to the record of earnings. Exhibit 214 presents an example of a tabulation of data which are very helpful in analyzing common-stock investment opportunities. In this example, data are given for 36 oil stocks on dividends, price ranges, net earnings per common share, gross operating income, assets, capitalization, book value per common share, working capital, and major operating fields. Statements like that shown in Exhibit 214 furnish valuable information, not only in considering one company, but also in comparing the common-stock investment possibilities of different companies. When such comparisons are made within

[1] Dewing, Arthur Stone, "The Financial Policy of Corporations," The Ronald Press Company, New York, 1926.

RECORD OF 36 OIL

Dividends on common stock				Price range common stock				Corporations		Common-stock data					
										Net earnings per common share					
Total 1937 payments	Yield based on 1937 payments	Latest paid or payable		1929–1938 through June 21			Ratio; price to 1937 earnings			First quarter		Fiscal year	Years		
										1938	1937		1937	1936	1935
$	√	$	Date	High	Low	Last	X		Par Val.	$	$		$	$	$
2.00	3.1	Q0.50	4-30-38	125½	11¼	65¼	21.5	Amerada Corp.	No	0.55	0.66	Dec.	3.04[71]	2.52	2.28[71]
1.00	4.5	Q0.25	6-15-38	77⅞	8⅜	22	6.3	Atlantic Refining	25	0.71[22]	0.69[22]	Dec.	3.51	2.53	1 49
1.00	6.9	Q0.25	5- 2-38	49⅜[21]	3	14½	17.1	Barnsdall Oil	5	0.41	0.26	Dec.	0.85[71]	1.01	0.35
0.90	9.9	Q0.20	8-15-38	45	4	9⅛	6.2	Consolidated Oil Corp.	No	......	......	Dec.	1.48	1 18	0.74
1.50	5.1	0.25	6-30-38	49	3⅝	29½	9.9	Continental Oil Co. (Del.)	5	0.39	0.45	Dec.	2.97	1.78[77]	1.57[77]
1.00	4.8	S0.50[30]	6-10-38	39	1⅝	20⅞	13.0	*Creole Petroleum	5	......	......	Dec.	1.61	1 23	0 79
1.00	2.6	0.25	7- 1-38	209[21]	23[21]	39	11.1	*Gulf Oil Corp.	25	......	......	Dec.	3.51	2.90[71]	1.16[1]
0	0	10%Stk.[176]	10-17-30	116⅞[21]	1½	7¼	7.7	Houston Oil v.t.c.	25	0.32	0.31	Dec.	0.94	d0.03	0.27
2.00	3.1	0.37½	7- 1-38	128[21]	33	64	12.3	*Humble Oil & Refining	No	......	......	Dec.	5.22	3.80	2.67
1.25	7.4	S0.62½	[30]6- 1-38	41	6¼	16⅞	17.2	*Imperial Oil, Ltd. (cpn.)	No	......	......	Dec.	0.98[71]	0 95[71]	0.94[71]
2.50	10.3	S1.25[30]	6- 1-38	39¾	7⅜	24⅜	. .	*Internat'l Petroleum (cpn.)	No	......	......	June	......	1.81[19]	1.65[19]
1.50	6.9	Q0.25	7-25-38	38⅛	⅜	21⅛	10.1	*Lion Oil Refining	No	[6]1.04[176]	[6]0.73[175]	Dec.	2.17	1.43[71]	d0.22
1.50	8.8	0.25	6- 1-38	39⅛	3¾	17	5.9	Mid-Continent Petroleum	10	0.25	0.66	Dec.	2.86	2.57	1 31
1.00	9.5	0.50	12-15-37	32	4¾	10½	8.0	Ohio Oil Co.	No	0.07	0.40	Dec.	1.31	0.70	0.34
0.75	6.6	0.75	12-15-37	29¾	2⅛	11⅜	8.2	Pacific Western Oil	10	0.20	0 27	Dec.	1.39	1.58[71]	0.66
2.75	7.7	Q0.50	6- 1-38	64	2	35⅜	6.6	Phillips Petroleum	No	0.52	1.26	Dec.	5.42	4.02	3.23
1.65	8.6	Q0.35	6-30-38	37	6	19½	6.7	Plymouth Oil	5	0.94[175]	0.83[175]	Dec.	2.85	1 66	0.70
0.25	2.4	0.25	12- 1-37	30¾	2½	10⅝	4.9	Pure Oil Co.	No	......	......	Dec.	2.15	1 63	2.00
1.00	4.7	Q0.25	6-15-38	69⅜	5½	21¼	10.8	Seaboard Oil of Delaware	No	0.37	0 48	Dec.	1.97	2 00	1.34
1.00	7.0	0.35	7-15-38	34¾	2½	14¼	9.9	Shell Union Oil	No	0.19	0.24	Dec.	1 44	1 35[77]	0.37
1.50	5.8	0.50	7-25-38	60⅜	2	26	4.3	Skelly Oil Co.	15	0.56	1.43	Dec.	6.07	4 42	2.17
0.80	5.5	0.25	3-15-38	48⅛[21]	5½	14½	8.0	Socony-Vacuum Oil	15	......	......	Dec.	1.82	1.38	0.72[11]
3.65	10.7	Q0.50[30]	6-30-38	69⅞	9½	34	7.3	*South Penn Oil	25	......	......	Dec.	4.64	3 59[71]	1.93
2.00	7.2	Q0.35[30]	6-15-38	81⅞	15⅛	27⅞	8.8	Standard Oil of California	No	0.55	0.61	Dec.	3.17	1 79	1.43
2.30	8.0	Q0.25	6-15-38	63	13½	28⅞	7.9	Standard Oil (Indiana)	25	......	......	Dec.	3.66	3.09	1 98
1.50	9.0	Q0.25	6-15-38	46⅞	8⅜	16⅝	10.3	*Standard Oil (Kentucky)	10	......	......	Dec.	1.61	1.46	0.84
2.50	5.1	S1.00[30]	6-15-38	84⅞	10⅞	49⅞	8.8	Standard Oil (New Jersey)	25	......	......	Dec.	5.64	3.73	2.43
1.50	7.9	Q0.25	6-15-38	129	11½	19	5.2	*Standard Oil of Ohio	25	......	......	Dec.	3.67[11]	4.77[32]	2 77
1 00[31]	. . -[112]	Q0.25	6-15-38	91	24⅜	49½	12.8	Sun Oil Co.	No	......	......	Dec.	3.86	3.01[1]	2.81[1]
0	0	Q0.50	12-20-20	24	½	2⅜	8.2	Superior Oil Corp.	1	0.08	0.03	Dec.	0.32	0 14	0 25
2.25	5.5	Q0.50	7- 1-38	71⅞	9½	40⅝	8.1	Texas Corporation	25	0.56[23]	1 00[23]	Dec.	5.02	4 10	1.85[4]
0.10	2.9	0.10	12-23-37	9⅝	2	3½	5.1	Texas Gulf Producing	No	0.20[11]	0 25[11]	Dec.	0.68[11]	0 93	0 90[71]
0.40	4.3	Q0.10	6- 1-38	23⅞	1⅜	9¼	8.3	Texas Pacific Coal & Oil	10	0.27[6]	0 25[6]	Dec.	1 11	0 72	0.03
1.20	8.9	Q0.25	6- 1-38	23½	2	13½	6.5	Tide Water Assoc. Oil	10	0.42	0.38	Dec.	2 08	1 03[62]	0.73
1.40	7.3	Q0.30	5-10-38	57	8	19½	7 5	Union Oil of California	25	0.49[23]	0.47[23]	Dec.	2 58	1.40	1 15
0	0	0 25[177]	5-10-28	29⅞	1	2⅜	8.9	Wilcox Oil & Gas	5	......	......	Dec.	0.24	d0 06	d0.71

—Explanatory notes—

Fiscal year: When fiscal year ends January through June, earnings are listed under preceding year.

Earnings: Wherever possible, earnings are calculated after deduction of undistributed profits tax, and they are specifically noted when such tax materially affects earnings per common share.

d Deficit.
Q Quarterly
S Semi-annual.
* Listed on N Y Curb.
[1] Adjusted.
[4] Before Adjust. on Foreign Exch.
[6] Before Federal Taxes.
[11] After Extraordinary Charges.
[12] After Inventory Adjustment.
[19] Both Classes of Stock.
[20] All Classes and Par Values.
[21] Old Stock.
[22] Preliminary Report.
[23] Indicated.
[29] Not Reported.
[30] Including Extras.
[31] Also Extra Stock Dividend.
[32] After Undist. Profits Tax.
[37] Dec. 31, 1937.
[38] Before Extraordinary Adjustments
[42] Predecessor Companies.

EXHIBIT 214.—An example of a tabulation prepared to facilitare the study of certain fac-
Stock Digest Service, 89

STOCKS, JUNE 1938

Common-stock data: Net earnings per common share, 1934	1933	1929	Gross operating income 1937	1936	Total assets as of Dec. 31, 1937	Capitalization: Funded and long-term debt	Preferred shares outstanding	Common shares outstanding	Book value per common share Dec. 31, 1937	Working capital: Cash and equivalent	Current assets	Current liabilities	Major operating fields: Producing	Refining	Transporting	Marketing	Natural gas	Chemicals
$	$	$	Million $		Mil. $	Million $	Thousands		$	Million $								
2.20	0 50	3 11	13.9	10.6	20.3	0	0	789	20.62	3 01[76]	5.13	1 42[37]	√	—	—	—	√	—
2.07[71]	2.46[71]	6.20	131 2	113 1	186.2	0.5	148	2,664	55.25	5.99	46 06	23 23[37]	√	√	√	√	—	—
d0.47[71]	d0 86	3.25[21]	13.8	11 1	25 1	3 9	0	2,250	7 84	1 21[76]	4.91	3.45[37]	√	—	√	—	√	√
d0.02	d0 05	2.82	241.8	215 4	352.3	48.6	54	13,915	18 72	30.37	105.09	26 96[37]	√	√	√	√	√	—
1 02	0 18[77]	1 90	89.2	75 8	104.7[104]	0	0	4,683	20 39[104]	7.00	38.82	8 70[104]	√	√	√	√	—	—
0 76	0 59[71]	0 41[48]	30 8	24.2	76.7	0	0	6,974	10 11	0 78[76]	23.22	6.15[37]	√	—	√	—	—	—
[1]0.31[71]	d1.25[1]	4 92[1]	278.7	235 8	560 4	72.9	0	9,076	38.67	31 28	158 36	40.00[37]	√	√	√	√	—	√
d0.29	d0.93[71]	1.09[1]	8.1	6.3	51.8	7.2[72]	358[52]	1,099	28.43	1.41[76]	3.25	1.06[37]	√	√	√	√	√	—
2.45	2.33	3.65[1]	213.3	155.7	329.4	6.0	0	8,988	30.91	6.91	52.99	44 74[37]	√	√	√	√	—	—
0.96[71]	0.53[12]	0.99	......[25]	......[25]	178 8	0	0	26,965	5.63	31.34	72.85	8.61[37]	√	√	√	√	—	—
1.49[19]	....[25]	.....[25]	34.5[178]	29.6[178]	156.1[138]	0	200[144]	14,324	[198]8.62[138]	46.16	62.34	6.22[138]	√	√	√	√	√	—
0.05[71]	d0.90	4.39	10.6	9.4	13.7	3.8	0	435	18.46[126]	0.25	3 62	2 45[126]	√	√	√	√	—	—
0.45	d0.95	4.14	42.0	40.4	65.4	0	0	1,858	32.73	7 26	22.33	3.93[37]	√	√	√	√	—	—
0.32	d0 54	2.57[1]	64.2	53 0	138.9	0	548	6,563	11 72	7 86	34.49	5 72[37]	√	√	√	√	√	—
0 63	d0.01	1.82	3.9	3.8	22 3	5.3	0	1,000	16.02	0.40[76]	0 92	0.60[37]	√	—	—	—	—	—
1 39	0.36[71]	5 20	118 7	105 1	212.5	25 1	0	4,449	38.10	12.21[76]	45.85	15.10[37]	√	√	√	√	√	√
0 64	0.05	2 13	7.4	5.2	15.9	1 7	0	1,020	10.72	1.57	2.43	1.70[37]	√	√	—	—	√	—
d0 97	d0.53	1.52	119.1	106.1	178.4	2 7	721[20]	3,982	21.57	10.46[76]	40.72	14.43[37]	√	√	√	√	√	—
1.11	1 03	d0.02	7 7	6.4	12.5	0	0	1,244	9.37	......[25]	3.24	0.87[104]	√	—	√	—	√	—
d0.24	d0.49[71]	1.26	260.3	231 7	380.2	61.0	341	13,071	19.03	15.97	85.95	31.15[37]	√	√	√	√	√	√
0 31	d1.26[71]	5.28	41.5	35.7	56.5	10.7	66	1,004	32.75	3.45[76]	12.24	4.42[37]	√	√	√	√	√	—
0 77	0.73	2 23[42]	552.8	498.5	905.2	128 1	0	31,151	21.60	98.59	299.84	76.20[37]	√	√	√	√	√	√
1 63[12]	0.43[12]	4.28	37 1	30.9	40.3	0	0	1,000	36.93	6.73	14.25	2.25[37]	√	√	√	√	—	—
1.41	0.58	3.63	192.1	153.3	596 3	0	0	13,005	43.94	20.51	79.88	18.35[37]	√	√	√	√	√	√
1.25[38]	1.14	4.66	365.5	331.2	735.1	2.6	0	15,267	43.46	53.29	208.17	51.30[37]	√	√	√	√	—	—
1.00	0.94	2.58	53.1	45.5	40.1	0	0	2,605	13.20	8.67	19.30	5.68[37]	√	—	—	√	√	—
1 70[90]	0.97	4.76	1,308.9	1,162.1	2,060.8	155.9	0[48]	26,225	49.29	205.93	672.07	237.14[37]	√	√	√	√	√	—
d3.34[11]	d1.89	6.63	57.9	51.4	63.8	1.2	120	754	55.74	7.50	18.76	8.13[37]	—	√	√	√	—	—
2.61[1]	2.75[1]	3.43[1]	133.3	105.4	128.4	9.4	100	2,316	39 79	6.51	37.48	13.67[37]	√	√	√	√	—	—
0.28	d0.36	0.97	1.5	1.1	7.5	1.8	0	1,389	2.96	0.27[76]	0.68	0.60[37]	√	—	—	—	√	—
0.64	d0.05[38]	4.90	376.2	337 5	614.8	65.7	0	10,875	44.66	44.43	180.51	53.45[37]	√	√	√	√	√	√
0.91	0.58	.....	2.1	2.0	9.9[104]	0 4	0	890	10.25[104]	0.15[76]	0.39	0.19[104]	√	—	—	—	—	—
d0.28	d0 63	0.84	4.0	3.3	10.6	0	0	888	11.55	0.92[76]	1.67	0.31[37]	√	—	√	√	—	—
0.35	0.61[11]	1.73	145.2	124.6	203.9	39.2	500	6,370	14.36	13.38	58.46	16.44[37]	√	√	√	√	√	—
0.66	0.45[71]	3.42[1]	85.3	67.6	165.5	18.0	0	4,666	20.65	16.15[76]	50.15	[23]8.25[104]	√	√	√	√	√	—
d1.09	d1 25	[6]1.21[11]	3.4	3.4	5.1	0.7	1	486	7.65	0.09[76]	0.70	0.41[37]	√	√	√	√	—	—

—Explanatory notes—

Funded debt: Includes subsidiary obligations, and guaranteed bonds.
Cash and equivalent: Includes call loans, time deposits, and marketable securities at balance sheet value unless market value is lower;—Excludes special deposits and company's own securities.

[48] Parent Company Only.
[52] $25 Par Value.
[62] Before Extraordinary Charges.
[71] Incl. Extraordinary Income.
[72] Exclud. Securities Called.
[76] Cash Only.
[77] Exclud. Extraordinary Income.
[96] After Extraordinary Income.
[104] Mar. 31, 1938.
[112] Not Calculated.
[126] Apr. 30, 1938.
[138] June 30, 1937
[144] $2.50 Par Value.
[175] Four Months Ended April 30.
[176] Preferred Dividend Accumulations Approximate $3,086,922.
[177] Preferred Dividend Accumulations Approximate $67,277.
[178] For 1937 and 1936 Fiscal Years Ending June 30.

tors pertinent to the investment values of a group of common stocks. (*Courtesy, Monthly Broad Street, New York.*)

industries (as in this exhibit), as well as between industries, the investor can more readily determine the relative advantages of the common stock that he is considering.

Not many published profit and loss statements contain more than a report of net earnings, income from outside sources, total income, and total deductions therefrom. Such a report calls for an even closer study of the balance sheet in order to discover what reserves have been allowed, what deductions have been made for depreciation, etc., before the figure for net income was determined. Complete statements of income and expenses are highly desirable and often can be obtained if the company has a wide distribution of its securities. Only by obtaining the details of income and expenses can it be determined whether the company is increasing its net income through a greater output, or by the reduction of costs, or by both. A figure of value in this connection is the operating ratio—that is, the proportion of net income which is consumed by expenses. A comparison of this ratio, for the concern under consideration, with those of other companies similarly situated often discloses the relative efficiency of the management and the relative situation of the concern in question.

The history of the earnings should be studied over a period that is long enough to carry them through at least one period of depression. For, as a rule, the true test of a concern's management and stability depends upon the manner in which profits are earned during times of stress. As indicated previously, earnings often fluctuate very widely as a concern passes through the different phases of the business cycle.

Reality of Earnings.—While the income statement is often considered to be the most important indication of the value of the securities of a company, it should not be accepted without verification by comparison with balance-sheet changes. The earnings of a company must be verified in two ways to determine whether or not they are real. First, they must be verified from the standpoint of accuracy and conservatism in accounting, and, second, they must be verified from the standpoint of ultimate availability to the security holders. Verification from the first point of view has been more common in the past than from the second, for many analysts have failed to consider what is done with the earnings of companies—that is, they have failed to perceive whether earnings are profitably or unprofitably invested in additional fixed or working assets when not used in the reduction of liabilities or distributed as dividends. One of the best signs of a sound concern, from an investment point of view, is a steady growth in business financed wholly out of surplus earnings.

In checking the accuracy of earnings statements, it is first necessary to see that the reported earnings, less dividends declared during the

period, represent the change in earned surplus from the beginning to the end of the period, as indicated in the balance sheets of these two dates. Losses suffered are sometimes charged directly against earned surplus, so that the earnings statement does not reveal the exact change in earned surplus during the period. Yet such losses may materially affect the accumulated profits available for dividends.

Furthermore, changes in methods of computing certain expenses can often be detected by analyzing balance-sheet changes, and the conservatism of the company in comparison with other companies can be seen in this manner. For instance, depreciation and depletion expenses as recorded in the profit and loss statement must be compared with the properties operated. These items are particularly important in railroad and other public utility companies, and in all companies having a large physical plant. Unless an adequate rate of depreciation (and of depletion, where wasting assets are operated) is used, the profits of the period will be overstated, and, conversely, if this rate is overstated the profits will be too low. Also, maintenance expenses should bear a reasonable relation to the amount of plant property operated, and to the extent to which it is operated in a given year (which is indicated by the gross revenues in the case of railroads and public utilities).

Where reliable statements of depreciation and other rates used by various companies are published, comparison with these rates should be made in studying the statements of any company. For example, depreciation rates on locomotives, passenger cars, freight cars, and work equipment used by the Atchison, Topeka and Santa Fe Railway recently have been at from 2.9 to 4.8 per cent, depending upon the construction and type of equipment, whereas the Great Northern Railway uses from 2.7 to 2.9 per cent and the Union Pacific has adopted rates varying from 3.2 to 4.7 per cent.[1] In analyzing the earnings of a railroad, reference to the rates used by companies operating under conditions similar to those of the road under consideration will indicate the relative conservatism of the company in recording depreciation expense.

Maintenance expenses, as reported in income statements, often vary from year to year in an unreasonable manner. In order to determine the adequacy of the reported maintenance expense, this expense, as shown in the profit and loss statements over several years, should be ascertained and expressed as a percentage of depreciable plant (or on a mileage basis, or rate per locomotive, etc., in the case of a railroad). For instance, if $3,000 per mile is spent for maintenance of way by a railroad in one year, and $500 per mile is all that is recorded the next year, obviously the earnings of the two periods are not comparable. One authority states:

[1] Mundy's "Earning Power of Railroads," pp. 28–29, published by Jas. H. Oliphant & Company, New York, 1936.

" . . . when under fairly normal conditions a railroad's maintenance of way expenses average below say $1,500 per mile of road these expenses should receive careful scrutiny by the intending investor."[1]

While these examples of analysis to determine the accuracy and conservatism of corporations in stating their earnings are very important, it is to be noted that the above remarks are only a suggestion of this type of analysis. Detailed comparisons of many items of expense among different companies, and for the company under consideration over a period of years, are necessary in a complete analysis to determine the soundness of income statements.

Even when it has been determined that the earnings of a given period are stated conservatively, these earnings may never be realized by the stockholders. In other words, they may never become real to the investors in common stock, because they may never be passed on to them, but may be reinvested in a plant which fails to produce proportionate profits.

The statements of the Studebaker Corporation from December 31, 1922, to December 31, 1928, afford an illustration of profits reinvested in a plant which has not as yet increased the earnings of the company. During the six-year period, plant assets, as shown on the company's balance sheets, increased from approximately 64.8 million to 86.3 million dollars, or 33.2 per cent. This increase was accomplished by the investment of profits. The surplus and the proprietorship reserves increased in approximately the same amount as the plant—namely, from 14.7 million on December 31, 1922, to 38.6 million on December 31, 1928.

Yet this increase in plant did not produce greater profits or more business. Earnings in 1928 (a particularly good year in the automobile industry) were $13,947,181 as compared with $13,773,869 in 1924. The intervening years produced reported earnings of approximately 16.6 million in 1925, 13 million in 1926, and 12 million in 1927. From the standpoint of sales in dollars, in no year since 1923 has a volume been obtained that equaled the amount of 166 million obtained that year; in 1928 the sales volume was 157.7 million. Also, in units of cars produced, there was a decline from 145,167 cars in 1923 to 136,205 in 1928. In short, there is no evidence that the earnings in excess of the portion distributed to the stockholders were put into productive use. Although the book value of the plant increased, its apparent productive value did not increase proportionately.[2]

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[1] Mundy's "Earning Power of Railroads," p. 23.

[2] At the end of 1929, after acquisition of the Pierce-Arrow properties, the plant accounts of Studebaker Corporation were reduced to 79.8 million. Apparently, the plant was reappraised and the values of plant items were reduced in the merger of the two companies. Earnings for 1929 were reported as 11.9 million, but the company

In comparing the Studebaker Corporation with other companies, it is interesting to note that the Chrysler Corporation earned almost 22 million dollars in the first nine months of 1928 with a smaller plant account than Studebaker, and the Packard Motor Car Company earned nearly 22 million dollars in the year 1928 with a plant account of less than 31 million dollars. These instances give further evidence of the unproductive policy which the Studebaker Corporation pursued during the years 1923–1928 in investing earnings in the plant. While Studebaker was putting surplus into a plant to manufacture many of the parts usually assembled by automobile manufacturers, other companies were assembling these parts at lower costs than those of Studebaker. Thus, it is evident that the earnings not distributed by the company during the six-year period cannot be considered as real earnings, and if future earnings are used in the same way, they will not be real.

As it is a common practice of companies to reinvest earnings in plant, it is always important to analyze the results of such investments. If earnings are increased by this procedure, the practice is sound, but if the earnings so invested are unproductive, they are lost to the stockholders.

Stock and Bond Prices in Relation to Earnings.—The history of stock and bond prices over a considerable period of years and a comparison of the prices of individual securities at a given time are necessary considerations in making investments. In such studies, it is not the price alone that is important; it is the relation of the price to earnings that is significant.

In making investments in bonds, other than government issues, the number of times the bond interest charges have been earned by the company over the past years and the expectations for the current and future years are of first importance. Similarly, in considering preferred stocks, an investor should ascertain the number of times the dividend requirements of each stock have been earned by the respective company, and the regularity or irregularity of such earnings in relation to the dividend requirements over a period of years. For example, if a bond or a preferred stock has a record of earnings which shows that at least twice the interest or dividend requirements have been earned each year for the past several years, it is to be expected that such a bond or preferred stock will sell on a higher price basis than others which show earnings that sometimes fall slightly below requirements.

Common stocks also should be studied from the standpoint of earnings per share in determining the price at which a certain stock should be bought and in choosing the particular stocks that appear to be the most favorable for purchase. Exhibit 215 presents the high and the low prices

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failed to weather the ensuing depression and, after suffering a loss of 8.7 million in 1932, it was placed in receivership on March 18, 1933, and subsequently was reorganized.

per share of 44 common stocks for the year 1937, together with the reported earnings of that year on a per share basis. These data are

RATIOS OF STOCK PRICES TO EARNINGS—1937
44 Representative Common Stocks

Companies	1937 earnings per common share	1937 prices per share		Ratios of prices to earnings	
		High	Low	1937 High	1937 Low
Agricultural implements:					
J. I. Case Co.	$13.44	$191¾	$80	14.27	5.95
Caterpillar Tractor	5.25	100	40	19.05	7.62
Deere & Co.	4.91	27	19½	5.50	3.97
International Harvester	6.24	120	53½	19.23	8.57
Automobile manufacturers:					
Chrysler Corporation	11.96	135¼	46⅛	11.31	3.86
General Motors	4.52	70½	28⅝	15.60	6.33
Building materials:					
Crane Co.	3.96	56½	22½	14.27	5.68
Johns-Manville	5.81	155	65½	26.68	11.27
Owens-Illinois Glass	3.15	103¾	51¼	32.94	16.27
Sherwin-Williams	8.65	154¾	72¼	17.89	8.35
Chemicals:					
Air Reduction	2.86	80¼	44½	28.06	15.56
Monsanto Chemical	4.43	107½	71	24.27	16.03
Union Carbide & Carbon	4.80	111	61¼	23.13	12.76
Electrical equipment:					
General Electric	2.23	64⅞	34	29.09	15.25
Westinghouse Electric	8.28	167⅝	87⅛	20.24	10.52
Foods:					
Beechnut Packing Co.	6.28	114¾	90¾	18.27	14.45
Borden Co.	1.43	28	16	19.58	11.19
Coca Cola Co.	5.80	170½	93¾	29.40	16.16
Corn Products	2.52	71¼	50½	28.27	20.04
General Foods	1.77	44¼	28⅛	25.00	15.89
United Fruit	4.26	86¾	52	20.36	12.21
Machinery and tools:					
Allis Chalmers	4.79	83½	34	17.43	7.10
Food Machinery Corp	4.21	58	27	13.78	6.41
Mesta Machine	4.88	72¼	33¾	14.81	6.92
Merchandise:					
Best & Co.	3.99	62¾	29	15.73	7.27
J. C. Penny Co.	6.61	103¾	57½	15.70	8.70
Sears Roebuck	6.00	98⅝	49¾	16.44	8.29
F. W. Woolworth Co.	3.44	65⅜	34	19.00	9.88
Mining:					
American Smelting	6.65	105¾	41	15.90	6.17
Anaconda Copper	3.69	69½	24½	18.83	6.64
International Nickel	3.31	73⅜	37	22.17	11.18
National Lead	.94	44	18	46.81	19.15
Oils:					
Atlantic Refining	3.51	37	18	10.54	5.13
Continental Oil (Del.)	2.98	49	24	16.44	8.05
Standard Oil of New Jersey	6.06	76	42	12.54	6.93
Texas Corporation	5.09	65⅛	34¾	12.79	6.83
Public utilities:					
Consolidated Edison	2.17	49⅞	21⅛	22.98	9.74
Pacific Gas & Electric	2.71	38	22	14.02	8.12
Southern California Edison	2.20	32½	17¾	14.77	8.07
Steels:					
American Rolling Mill	2.80	45¼	15½	16.16	5.54
Bethlehem Steel	7.74	105½	41	13.63	5.30
National Steel	9.02	99¼	55	11.00	6.10
U. S. Steel	8.59	126½	48½	14.73	5.65
Youngstown Sheet & Tube	7.03	101⅞	34⅛	14.49	4.85

EXHIBIT 215.—The use of ratios, such as those presented above, facilitates analyses of the general investment situation as well as the price ranges, in relation to earnings, of individual securities. (*Data from The Fitch Stock Record, July* 1, 1938.)

particularly useful when converted to ratios of the respective high and low prices to earnings, as has been done in the columns at the right in Exhibit

215. These price-earnings ratios are also called "prices as multiples of earnings," and they show, for example, that J. I. Case Co. common stock sold in 1937 as high as 14.27 times the 1937 earnings and as low as 5.95 times those earnings.

In Exhibit 215, the stocks have been grouped according to industry classification, and it is interesting to note that the common stocks of companies in certain industries sell on a much higher basis than those in other industries. Particularly, it will be observed that stocks of chemical, electrical equipment, and food industries usually sell on a relatively high times-earnings basis. Within each group, considerable variation in the relative prices is also evident, as can be seen by comparing the stock of Chrysler Corporation with that of General Motors.

The type of analysis shown in Exhibit 215 can be expanded to show a comparison of the price ranges of individual stocks, in relation to the earnings, over a period of years. For example, a study of the price-earnings ratios of United Fruit Company stock from 1929 to 1936 shows:

Year	High	Low
1929	23.4	14.6
1930	24.8	11.0
1931	29.2	7.5
1932	16.5	5.2
1933	21.4	7.3
1934	18.6	14.2
1935	26.1	17.0
1936	17.8	13.6

To use this information in connection with a proposed purchase of the stock, the earnings of the current year, and prospects for the following year, should be considered. If, for example, the stock can be purchased at 60, is it high or low in relation to expected earnings? If the current-year earnings are estimated at \$3.50 to \$4.00 per share, at what price might it be expected or be possible that United Fruit will sell?

Although accurate estimates of future earnings often cannot be obtained, it is essential that the latest and best information be considered before buying. Estimates are made for many stocks by the Standard Statistics Company, Inc., and are published in the *Investors Guide Stock Reports*. Periodic reports of earnings for many companies are published in the *Standard Earnings Bulletin*.[1]

[1] United Fruit Company earnings (preliminary) for the first quarter of 1938 are shown at 87 cents per share, as compared with \$1.17 in the first quarter of 1937, in the July 1938 *Standard Earnings Bulletin*. However, the July 18, 1938, issue of the

A further check on expected earnings of the current year can be obtained by investigating the earnings records of a company in relation to the Federal Reserve Board Index of Industrial Production. If, for example, a company earned $2.00 per share in a year when this production index averaged 80, it may be reasonable to expect a similar current-year result if the current estimate of industrial production for the year is approximately 80.

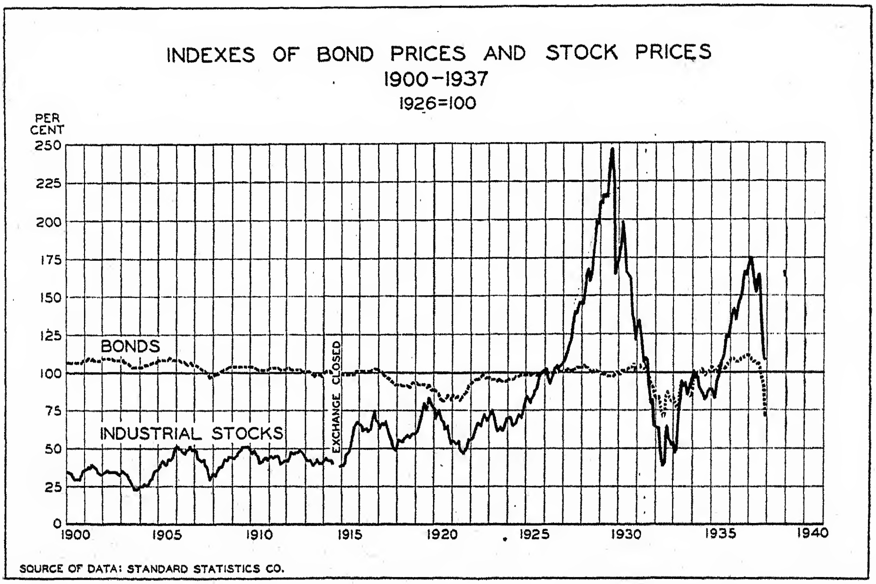

EXHIBIT 216.—An understanding of the differences in the cylical movements of prices of stocks and bonds, as shown in the above chart, is fundamental to investment analysis. (The Standard Bond Price Index has been converted to percentage relatives.)

In considering stock and bond prices, it is important to note that stock prices fluctuate much more widely than bond prices, and that their movements do not always correspond in time and in direction (see Exhibit 216).[1] The best time to buy stocks may not be the best time to buy bonds and *vice versa*. For instance, the bond market steadily declined during 1928 and most of 1929, so that at the peak of stock prices in 1929, bond prices were at the lowest level in five years. Ordinarily, however, the effects of easy and tight money situations are reflected in both stock

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Investors Guide Stock Reports estimates that the 1938 earnings for the year will not dip badly below 1937 earnings.

[1] The stock price index in Exhibit 216 is the Standard Statistics Company's *Monthly Industrial Stock Price Index* and the bond price index is this company's *Bond Price Index*. The stock index is regularly stated in percentage relatives (1926 = 100) and the bond index is stated in dollars. For the purposes of Exhibit 216 the bond index has also been converted to percentages of a 1926 base.

and bond prices, but with much more pronounced results in stock prices for reasons previously explained (note the time correspondence of the cyclical dips of 1903, 1907, 1913, 1921, 1923, 1932, and 1937 in Exhibit 216).

The preceding discussion applies to the stock and bond price situation in general. When specific issues are considered, it is ordinarily possible for the investor to shift funds from stocks when stock prices are high to certain good bonds which may be purchased on relatively favorable bases. Then, when stock prices decline, the funds can be shifted back again to carefully selected stocks. It is very important in making investments to realize that at all times many stocks and bonds are out of line with the average prices. In relation to the average, some are overvalued and some are undervalued, and it is necessary to analyze carefully the individual issues and select those which can be purchased at the most favorable prices.

Computation of Bond Yields.—Investors in bonds do not always realize what they are buying for the price that they pay for a bond. For example, when a person pays par for a bond he is buying not only the right to receive back the principal of the bond at a future date, but he is also buying the right to interest payments during the period to maturity. The value of the bond is, therefore, the present worth of the principal plus the present worth of every interest coupon. For example, if one buys a 5-year 6 per cent bond at par, interest payable semiannually, what one really buys is 10 coupons of $30 each and one coupon of $1,000. Obviously, money due six months to five years hence is not worth par to-day. One would not pay $300 for these 10 coupons and $1,000 for the principal, or a total of $1,300 for this bond now. What one really does in buying this bond is to discount these 10 little promissory notes and the one big promissory note and figure them at to-day's value, which is as follows:

	29.13	for first coupon, due in	6 months
	28.29	for second coupon, due in	12 months
	27.45	for third coupon, due in	18 months
	26.65	for fourth coupon, due in	24 months
	25.88	for fifth coupon, due in	30 months
	25.12	for sixth coupon, due in	36 months
	24.39	for seventh coupon, due in	42 months
	23.68	for eighth coupon, due in	48 months
	22.99	for ninth coupon, due in	54 months
	22.33	for tenth coupon, due in	60 months
and	744.09	for the principal due	5 years hence.
	$1,000.00	for the 10 coupons and principal.	

In brief, when one buys on March 1, 1938, a 6 per cent bond due March 1, 1943, one really pays

A further check on expected earnings of the current year can be obtained by investigating the earnings records of a company in relation to the Federal Reserve Board Index of Industrial Production. If, for example, a company earned $2.00 per share in a year when this production index averaged 80, it may be reasonable to expect a similar current-year result if the current estimate of industrial production for the year is approximately 80.

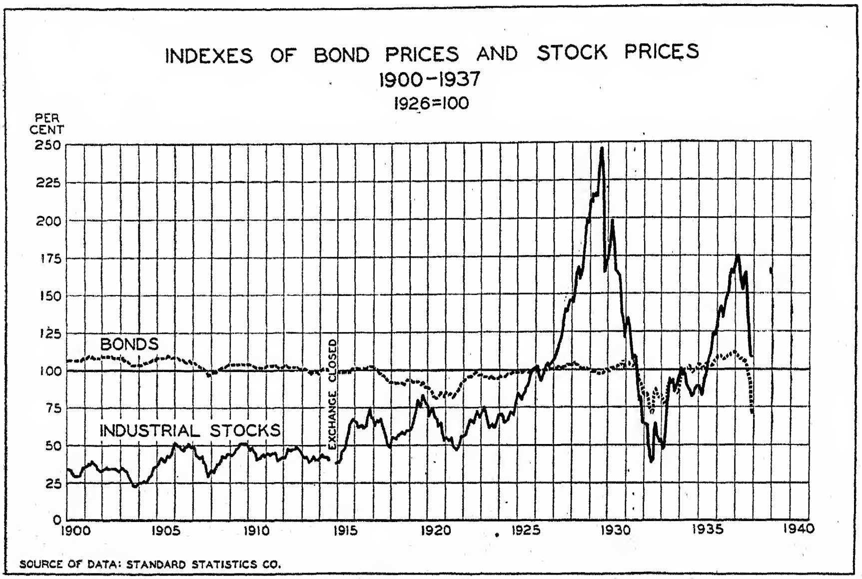

EXHIBIT 216.—An understanding of the differences in the cylical movements of prices of stocks and bonds, as shown in the above chart, is fundamental to investment analysis. (The Standard Bond Price Index has been converted to percentage relatives.)

In considering stock and bond prices, it is important to note that stock prices fluctuate much more widely than bond prices, and that their movements do not always correspond in time and in direction (see Exhibit 216).[1] The best time to buy stocks may not be the best time to buy bonds and *vice versa*. For instance, the bond market steadily declined during 1928 and most of 1929, so that at the peak of stock prices in 1929, bond prices were at the lowest level in five years. Ordinarily, however, the effects of easy and tight money situations are reflected in both stock

Investors Guide Stock Reports estimates that the 1938 earnings for the year will not dip badly below 1937 earnings.

[1] The stock price index in Exhibit 216 is the Standard Statistics Company's *Monthly Industrial Stock Price Index* and the bond price index is this company's *Bond Price Index*. The stock index is regularly stated in percentage relatives (1926 = 100) and the bond index is stated in dollars. For the purposes of Exhibit 216 the bond index has also been converted to percentages of a 1926 base.

and bond prices, but with much more pronounced results in stock prices for reasons previously explained (note the time correspondence of the cyclical dips of 1903, 1907, 1913, 1921, 1923, 1932, and 1937 in Exhibit 216).

The preceding discussion applies to the stock and bond price situation in general. When specific issues are considered, it is ordinarily possible for the investor to shift funds from stocks when stock prices are high to certain good bonds which may be purchased on relatively favorable bases. Then, when stock prices decline, the funds can be shifted back again to carefully selected stocks. It is very important in making investments to realize that at all times many stocks and bonds are out of line with the average prices. In relation to the average, some are overvalued and some are undervalued, and it is necessary to analyze carefully the individual issues and select those which can be purchased at the most favorable prices.

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	25.12	for sixth coupon, due in	36 months
	24.39	for seventh coupon, due in	42 months
	23.68	for eighth coupon, due in	48 months
	22.99	for ninth coupon, due in	54 months
	22.33	for tenth coupon, due in	60 months
and	744.09	for the principal due	5 years hence.
	$1,000.00	for the 10 coupons and principal.	

In brief, when one buys on March 1, 1938, a 6 per cent bond due March 1, 1943, one really pays

	\$744.09	for the principal
and	255.91	for the coupons or interest.
	\$1,000.00	for the bond.

It may seem strange that in many bonds the present worth of the interest exceeds the present worth of the principal. This is true of most bonds with a maturity of over 12 years, and of all bonds with a maturity of over 20 years. For example, a 20-year 6 per cent bond purchased at par of \$1,000 at date of issue is worth \$693.44 as to interest coupons alone, and \$306.56 as to principal. That is, in paying \$1,000 one is paying over two-thirds of this amount for the right to receive interest semi-annually for 20 years and only one-third for the right to receive back the principal. This illustrates the importance of considering the earning power of a concern when buying bonds as well as when buying stock. An even more striking example in the case of bonds is afforded in the West Shore First 4's, due in 2361. The present worth of the entire principal of \$1,000,000 of these bonds is now approximately \$.06. That is, \$1,000,000 due in 423 years is worth to-day, on a 4 per cent basis, compounding semiannually, only six cents.

Safety of Principal and Income.—Safety of principal and income has been emphasized throughout the preceding discussion, but it is believed that it is well to point out again some of the important factors in this connection. It has been estimated by the Federal Trade Commission that \$500,000,000 per year are paid for worthless securities. With all of the protection that is provided by the government and private institutions, these enormous sums are lost annually because the investors have not made sufficient checks regarding the safety of their investments. Every investor will, of course, decide for himself how much information he should get and from what sources he will get it. But, on the whole, the type of information which comes to investors in the form of advertising or in connection with selling effort is likely to be of the type that should be investigated and compared with other securities which are not being pushed, before an investment is made. On the whole, the best stocks and other types of investments are not conspicuous. If any one is making an unusual effort to sell a certain security, it is usually because of the large commission which he himself will make, rather than because of the desire to furnish the investor with the best security that he could select.

Diversification.—Most investors recognize the necessity of diversifying investments in order to reduce their risk. This applies with equal force to large and small investors. This principle, of course, should not be carried too far, as it is possible for investors so to diversify their investments that they cannot give sufficient attention to each one. But if an investment is spread over some 10 or 20 carefully selected securities,

it is usually safer than if the entire investment is placed in one security, even though the care given to its selection is equal to the aggregate used in the diversified selection.

Questions and Problems

1. In making investments, why is it important to have definite statistical facts regarding the companies whose securities are under consideration? Is investment success to be attained by constant shifting of stocks or bonds, or by holding them for relatively long periods?

2. How would you distinguish between investment and speculation?

3. Should all people undertake the same risks? How would you determine the risk suitable to the individual?

4. What are three points of view from which analyses and investigations may be made in investment studies?

5. Describe briefly the different classes of corporation securities. Are all equally desirable?

6. Explain the classification of bonds on the three bases of (1) purpose to be accomplished, (2) security of principal, and (3) payment of interest and principal; and name common bond titles under each classification.

7. What characteristics determine the attractiveness of various government, state, and municipal bonds?

8. How can the value and priority of the security for corporation bonds be determined? Why is the ratio of net income to interest expense important in choosing a corporation bond investment?

9. What are the relative advantages and disadvantages of preferred and common stocks? Are they better than bonds as investments?

10. In buying a security of a certain company, what would you need to know concerning the capitalization of that company? In buying a certain common stock, is it more desirable that there be bonds outstanding, or that there be no claims prior to the given stock purchased? Discuss.

11. What are the disadvantages of real estate mortgages as forms of investment, and what are the risks in making this type of investment?

12. What is meant by a "listed" stock? Are all listed securities actively traded in? Is it better to buy extremely active securities, inactive listed securities, or moderately active issues? Discuss.

13. What are some of the best sources of data on stocks and bonds?

14. Explain "investing in relation to long-time or cyclical swings."

15. Describe briefly the Dow-Jones Index of stock prices, the New York Times Index, and the indexes of the Standard Statistics Company.

16. Name 11 principal factors to be studied in investigating the company whose issues are under consideration.

17. Why is it necessary, in investment analysis, to consider the nature of the industry to which a concern belongs?

18. Why is it important to check up on the management?

19. Of what interest are the markets for a concern's products in connection with analyzing its securities?

20. Of what significance is the location of an enterprise?

21. How should capital structure and ownership be considered when studying a firm's stocks or bonds?

22. To what extent does the security of an investment depend upon the growth and stability of an enterprise?

23. Discuss the analysis of financial statements.

24. Explain the 100 per cent or common-basis statement.

25. What are eight important ratios that are used in analyzing financial statements? Explain each.

26. In analyzing earnings statements, what items of expense are particularly important?

27. Why is it important to note the results of reinvestment of earnings of a company? Does the amount of earnings always increase when earnings are put back into the business?

28. Discuss "reality of earnings" from two points of view.

29. Discuss the importance of analyzing stock and bond prices and dividend policies. What ratios are valuable in such studies?

30. Are bond and stock prices affected in the same manner by changes in the business cycle? Explain.

31. In buying a bond due in 100 years, is the ultimate value of the security back of the bond as important as the earning ability of the assets?

32. How does a premium paid affect the bond yield?

33. Does the public often buy poor or worthless securities? Discuss.

34. Is diversification of investments advisable? Why?

35. On the basis of a careful analysis, make a recommendation for investing $10,000 of your own funds.

36. From the point of view of making long-swing investments, analyze the common stock of the following companies: American Can Company, Bethlehem Steel Company, General Electric Company, General Food Corporation, General Motors Corporation, International Harvester Company, National Biscuit Company, Standard Oil Company (New Jersey), United States Steel Corporation, American Telephone and Telegraph Company, and Atchison, Topeka and Santa Fe Railway Company.

CHAPTER XXIV

STATISTICS IN BANKING

Banks feel the effects of the conditions in many kinds of business and constantly need information as to the trend in a wide range of activities. They are intimately interested in the success of their depositors and borrowers, and are often able to help them substantially by furnishing them with important information through their statistical departments. Many banks now have statistical departments within their organizations for the purpose of gathering and analyzing information, not only on their own internal operations, but on general business conditions and on any line of activity in which they might be directly or indirectly interested.

It will be the purpose of this discussion to point out some of the more important ways in which statistics are used in banking. The point of view in this chapter will not be that of the bank alone. It is the aim not only to point out how the banker uses statistics in operating his bank, but also to indicate how the client uses statistics in connection with his banking problems. Obviously, the limited scope of this discussion will not permit going into great detail. This introduction to the subject, however, should indicate or suggest many possible uses that are not discussed.

Scope of a Bank's Statistical Work.—As indicated above, the statistical work of a bank covers a very wide range of information, of value not only to the bank itself but to the bank's clients. This work includes the collection of data and their analysis in determining the condition of the cycles in general business and in different lines of business. It includes the collection of information on, and the analysis of, population and purchasing power, production methods, labor conditions, commodity markets, real estate situations, and investments. Furthermore, it covers the analyses of individual concerns' markets and internal conditions, as shown by financial statements and other data. And, finally, it includes the analyses of the bank's own internal condition in relation to the preceding factors.

The work of a bank's statistical department requires that it constantly watch statistical publications of the Federal, state, county, city, and foreign governments; of commercial, trade, and other associations; and of private organizations. This material must be indexed and filed in such a manner that it can be referred to on a moment's notice if demanded

later. The proper use of such sources requires a good knowledge of their relative validity. Data of current interest must be analyzed and presented as soon as possible to the proper officials. Some statistical departments make a daily digest of the contents of a number of daily papers and other current sources of information, and these digests are distributed throughout the bank for the information of the officers. Any information which the statistician knows is of immediate interest to an officer in the bank is dispatched to him at once and with comments if necessary.

Bank's Interest in Customers' Success.—Since a bank's success depends to a large extent upon the success of its clients, it naturally follows that it is anxious and willing to help its clients in every way possible. One way in which a bank is in a good position to furnish valuable assistance is in giving advice in connection with problems that can be solved on the basis of superior statistical information. While most of the information gathered is of direct interest to the bank, much of it is for the use of the bank's clients only, and often the bank will prepare a special report for an individual client. By gathering the information of common interest on a wholesale basis, the bank is able to serve many clients. Probably but few individual business concerns could afford to gather the information for themselves, but by distributing the cost over a large group, the cost per client is low. The compensation that the banks receive for their services in furnishing information is represented by increased business from its clients. There is no direct charge, and, as far as the client is concerned, this service ordinarily costs him nothing. Many banks are equipped to furnish a rather complete statistical service, and the client who does not take advantage of it is missing a valuable opportunity.

It should also be pointed out, however, that banks frequently find themselves doing an amount of service for a customer that is hardly justified. The question of the extent to which a bank is justified in analyzing a customer's business is, of course, a very difficult one to decide. It undoubtedly would be a fairer procedure if a bank would require a customer to pay for a specialized analysis, particularly if a large or doubtful line of credit is involved. Nevertheless, competition between banks tends to force them to make such analyses at their own expense.

The Bank's Obligation.—Banks are ordinarily looked up to in a community as the best advisers regarding financial and other business conditions—not only of the community, but of the individual concerns in the community. Since the banks ordinarily expect to enjoy the respect of their clients and others in the community because of their superior judgment regarding economic matters, it becomes a duty and obligation of these institutions to collect accurate information, make sound analyses,

and apply the results of their findings in such a way that the business of the community will tend to be stabilized along sound lines. If the banker appreciates the true situation of the business cycle and regulates the granting of credit upon a scientific basis, he can do much to reduce unemployment and the other bad effects of periods of extreme depression.

In order, however, to make the skillful analyses required above, the banker must use scientific methods in finding the facts and in analyzing them. Some banks still use such inflexible methods of analysis as the 2:1 ratio of current assets to current liabilities, regardless of the type of business, the kinds of goods, or the position in the business cycle. If the business is an old established retail store selling staple food commodities, its risk is quite different from that of the real estate concern promoting an outlying subdivision. Luxuries and style goods fluctuate to a much greater extent than staples. Finally, a 2:1 ratio in a period of depression might be much safer than necessary, whereas a 2:1 ratio in a period of prosperity might be extremely dangerous, as explained in the preceding chapter.

It is, of course, relatively simple for a bank to be so conservative that it never takes any chances, but in such an instance, the bank is not fulfilling its obligation to the borrower by furnishing him funds in the proper relation to his business. A commercial bank loan system should be fair to the borrower as well as to the lender—that is, the bank should be in position to give the assets of the borrower the highest possible rating commensurate with the safety of the lending institution. This is the only fair procedure. From the point of view of sound economic development, it is just as objectionable for the borrower to be granted less than that to which he is entitled, as it is for the bank to lend excessive amounts.

The banker requires accurate data and analyses in order to render service that is fair to all concerned. He must have accurate information on the integrity of the borrower; the ability of the borrower to repay; the type of business; the kinds of goods and the markets for them; the amount and condition of assets and liabilities; and management policies. And, finally, he must have an appreciation of trends, and cyclical, seasonal, and random movements in the individual concern, in the industry, and in business in general.

Obviously, the use of statistics is imperative in analyzing the preceding factors. In such analyses, of course, statistics can never take the place of good judgment, but they can be of great aid in helping bankers to improve the accuracy of their decisions.

Practically all of the surveys and analyses discussed in the preceding chapters are of direct as well as indirect value to banks. Their application in carrying on the business of a bank will now be discussed briefly.

Business Forecasting.—No group of business men has a greater interest in what business conditions are likely to be than the bankers. It is particularly important that they do not lend too much when business is dangerously inflated. It is a general rule that in periods of depression many banks fail because they have not correctly analyzed the general business situation, or the conditions in the overexpanded individual concerns to which they have made loans that cannot be repaid.

To no class of business men is it more important that business be stabilized and that booms do not develop than it is to the bankers. When difficulties develop after a period of overexpansion, loans cannot be paid to the bank and, at the same time, the depositors demand their money. Such a situation often places a bank in a position from which it cannot recover. This, incidentally, indicates again why banks find it good business to furnish information on business conditions. Not only is it to the bank's interest to keep from overexpanding its own business in a period of inflation, but it is also of direct interest to the bank to do what it can to keep its clients from overexpanding.

Population and Purchasing-power Statistics.—Population and purchasing-power data are of great importance to banks in analyzing fundamental conditions, and they are also of distinct value for special purposes, such as determining locations for branches. If a community has a rapid rate of growth, the situation is favorable to the expansion of the business that depends upon local conditions, and this, in turn, is favorable to the expansion of a bank's business. If a community ceases to grow, much of the financing which would otherwise be necessary will not be required. To be of greatest value to a bank, population data should include non-permanent residents as well as permanent by various classifications, and the figures should be up to date, especially if a community is changing rapidly in size. Appreciating the value of such data, banks in some instances have taken a very active interest in improving the accuracy of estimates of both population and purchasing power.

Production and Labor Statistics.—Though banks often keep and analyze data upon their own production and labor problems, the greatest use made of production and labor statistics is in connection with analyzing the situations of their clients. These data (discussed in detail in Chapter XX) are important in connection with determining the position of an industry, the position of a client in the industry, and in connection with helping the client solve his own problems. The bank, of course, ordinarily does not make the production studies for the client, but its understanding of production data enables it to analyze the client's position with greater accuracy, and it can often assist the client by making helpful suggestions.

Marketing Analyses.—In making loans, it is of utmost importance, if the bank is to be of greatest service, that it know something about

general marketing conditions and about the special conditions of its client's market. If the market for a client's product is well established and sound, the client is obviously a much better risk than one whose market is erratic and doubtful.

Many examples could be cited showing the importance of market analyses to banks. Careful analyses of the market for farm products in 1919 would have saved many middle-western banks, as well as the farmers themselves, from failure in the depression of 1921. In one instance, an analysis of a certain market for cement saved a bank from making an unsafe loan and this, in turn, undoubtedly saved the manufacturer from having the losses that would have resulted had he expanded into that territory. Another analysis made of the market for a new type of insulating material showed that it would be impractical for the manufacturer in question to undertake the venture, and it is believed that this knowledge not only kept the bank from making an unwise loan, but kept the manufacturer from entering into a doubtful undertaking. The preceding discussion applies principally to domestic markets. Some of the larger banks are able to be of still greater assistance in connection with international markets, and can give the prospective exporter or importer much valuable information on foreign trade.

Real Estate Analysis.—Practically all banks and mortgage houses are deeply interested in real estate conditions. If they are not lending directly upon the security of real estate, or if they do not own real estate themselves, they are at least interested in the security of their clients' real estate holdings, and in the effect that real estate conditions may have upon general conditions in the locality. It has often happened that the increase in the assets of an industrial or merchandising concern has been largely in the write-up of the realty values, and it is of utmost importance to the bank making loans to such concerns to know whether or not these increases are real, and fundamentally sound in relation to the long-time trend. This was especially true during the 1920–1930 decade because of the widespread real estate speculation. In some communities, there have been sound and steady increases in realty values, and in the income which has been realized from the sound use of real estate. In other communities, inflationary booms have left them carrying tremendous overhead burdens in the form of investments in unproductive property, which has an unfavorable effect upon, not only banks, but all lines of business. Probably no class of business men is giving greater attention to analyzing the ups and downs in real estate activity than the bankers.

Investment Analyses.—All that has been said previously (Chapter XXIII) regarding the analysis of investments is of vital interest to bankers. Not only must the bank know how to invest some of its own funds, but it is often called upon to give advice to investors as part of its

business. It must often make loans with securities as collateral, and it must also extend loans to concerns which have portions of their assets in various types of investments. This point will be considered at greater length in the discussion of credit analysis.

The Line of Credit.—Business concerns should ordinarily arrange in advance for adequate bank loans. The more conservative managers usually arrange for credit in excess of the probable maximum amount that they expect to require. If a concern's general credit standing is satisfactory, it may be granted a single loan or it may be granted "a line of credit." Banks use many types of methods in determining what a concern's line of credit should be. Definite statistical information upon a concern's condition, however, is a very valuable aid in credit analysis.

In granting a line of credit, the bank ordinarily imposes certain conditions upon the concern. Important among these are the reports that the concern is required to make regarding its financial condition and any other situation upon which the bank may wish information for use in determining whether or not the line of credit should be granted or changed. Furthermore, the condition of the bank itself may change so that it is necessary to revise the concern's line of credit. Such a situation is often forced upon a bank by changes in business conditions. In such an instance, adequate information on business conditions is not only necessary to the bank in relation to its own operations, but it is also valuable for the purpose of showing a client the reason for curtailing his line of credit.

Credit Analysis.—The actual work of making credit analyses of individual concerns is usually a duty of the bank's credit department. The bank's statistical department furnishes much valuable data which are useful in this connection, but detailed statistics on the individual concern are usually collected by the credit department directly.

Keeping the officers of a bank informed as to the standing of its borrowing customers is a very important duty of the bank's credit department. Credit files must be revised frequently, and the changes in customers' standings must be noted accordingly in order that any questions can be answered promptly. While, in this discussion, the emphasis is being placed upon statistical data, it should be appreciated that the credit department uses all kinds of information from many and various sources. Some banks have even gone so far as to have their credit departments carry on investigations concerning prospects before they solicit their accounts. When the information that a credit department is obtaining regarding a client shows that his condition is more favorable than it was formerly, the bank can increase the customer's credit limits if requested, or any doubt that may have existed regarding the present limit may be removed. If, on the other hand, it is shown

that the customer's situation is less favorable, it may be necessary to decrease the credit limit, discontinue the account, or have a confidential discussion with the client which will clear up the situation. In all such instances, it is the work of the credit department to provide the necessary data for the guidance of the bank's officials in making their decisions.

The larger the bank, the more difficult it is and the more complicated it is to gather the necessary credit information and to keep it in proper order. In small banks, complicated records are usually not kept, and the cashier may rely upon his personal knowledge of the customers and his general knowledge of the business situation to furnish him the necessary credit information. In large banks, however, personal contacts and general information are not sufficient, and an elaborate system of collecting and filing statistical and other credit information becomes a necessity, if the bank is to operate on a sound basis, and at the same time furnish the maximum of service that is consistent with the situation of the specific concern and the general condition of business.

Information Required from Borrowers by Banks.—The nature of the principal statistical and other items required by a bank when a loan is desired is indicated in the following list:[1]

1. Kind of business—type of business organization—and names of owners and managers.
2. An up-to-date balance sheet, and balance sheets for the preceding few years, if possible.
3. Income statements for the same period.
4. The average inventories carried, valued at cost and at market.
5. The amounts of notes and accounts receivable by dates due.
6. A schedule of investments showing dividends or interest payments and the present market values.
7. A statement of any notes receivable, accounts receivable, or merchandise which may be placed as security for indebtedness.
8. Explanation of unusual balance-sheet items.
9. A statement of the seasonal variation of the liabilities of the borrower.
10. A description of the client's methods of borrowing, other lines of credit, etc.
11. A statement of all types of insurance carried.
12. A complete description of the methods of handling depreciation and obsolescence, with the rates and the manner of computation.
13. A description of the policy followed in disposing of the earnings of the business.
14. An analysis of surplus indicating what disposition is made of it.
15. A description of the ownership of real estate and leases.
16. A description of construction plans and how projects are to be financed.
17. Description of other business interests of the owners and managers of the borrowing company.
18. An analysis of the relations with, and conditions of, subsidiary companies.
19. An analysis of the concern's unfilled orders.

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[1] Abstracted in part from "Financial Handbook," The Ronald Press Company, New York, 1925, pp. 1198–1200.

20. Analysis of the borrower's market in general.
21. A market analysis of the borrower's products and selling policies.
22. An analysis of the relation between labor and management.
23. An analysis of trends, seasonal variations, and cycles in production and sales.
24. An analysis of the probable effects of business conditions on the concern's business.

If the bank has the preceding information, it has a basis for judging whether or not the borrower will be able to repay a loan at maturity. And it knows whether the loan must be secured or whether it may be unsecured. It also has a good index of managerial ability, which is a very important consideration in granting credit. The character of the individual managers, the political situation, climatic conditions, and the geographical location must also be considered along with the preceding information, as they have an important bearing upon the interpretation of the foregoing list of items in determining whether or not a loan should be granted.

Analysis of Borrower's Statements.—A bank requires financial statements from a borrower for use as an aid in determining the risk involved. For this reason it is also important that the borrower know what constitutes a good financial statement in order that he may present a good case when he requests a loan. Of course, the most careful and complete analysis of a financial statement cannot cover the whole risk, because factors other than those shown in such a statement may entirely offset a poor or good financial showing. In a pamphlet issued by a national organization of bank credit men, it is estimated that on the average the financial strength displayed by the borrower's statement affects about 40 per cent of the credit decision; the economic risk, which is the effect upon a business of general economic conditions, affects about 20 per cent of the credit decision; and the moral risk, which covers both integrity and all phases of ability, probably affects about 40 per cent of the credit decision. These three elements, with the relative importance indicated, cover the whole analysis of the risk and make up the total of 100 per cent of the credit decision.[1]

Eight ratios which are helpful to banks in analyzing financial statements have been discussed in the preceding chapter. The ratio of quick assets to current liabilities is commonly used by banks in analyzing borrowers' statements. When properly interpreted in relation to the kind of business, type of commodity, and condition in the business cycle, this ratio indicates how well the borrower can meet his current obligations

[1] From a pamphlet entitled "Financial Statements" issued by the Robert Morris Associates (the national organization of bank credit men). See also Alexander Wall and Raymond W. Duning, "Ratio Analysis of Financial Statements," especially Chaps. I and XII, Harper & Brothers, New York, 1928.

even if an emergency arises. The point of safety of this ratio varies widely from one business to another and from one period in the business cycle to another. For instance, a concern handling principally women's style goods in a period of prosperity should have quick assets amounting to several times the current liabilities, while a concern handling staple food commodities in a period of depression may have a ratio less than 2:1 and be safe. Proper interpretation of this ratio, however, requires several other supplementary ratios.

The worth-to-debt ratio, secured by dividing the net worth by the total debt, shows the balance between the source and the ownership of the funds being used in the business. As the debt increases in relation to the funds invested by the stockholders, the company becomes more dependent for working capital upon its creditors and more susceptible to the conditions of depression. The lower the ratio, the more the owners must rely on their moral risk, and anything that will raise a question regarding the moral risk may result in disaster to the firm. If, over a period of time, this ratio is declining, it indicates that the concern is becoming weaker. If it is rising, it indicates increasing strength.

The worth-to-fixed-assets ratio, which is secured by dividing the net worth by the fixed assets, indicates the dollars of net worth or stockholders' investment for every dollar of fixed assets. The margin above 100 per cent indicates the proportion over and above the fixed capital investment that the stockholders have provided as active working capital possible of investment in liquid assets. If this ratio shows a tendency to increase over a period and is well above 100 per cent, the bank knows that the client's situation is favorable as far as this ratio is concerned. If the ratio has been declining and is not well above 100 per cent, the bank knows that it must investigate the risk more closely in order to determine whether or not it is advisable to make the loan.

The ratio of merchandise to receivables (merchandise inventory divided by the total of accounts and bills receivable) indicates the dollars of merchandise that there are for every dollar of receivables. Among other things, this ratio indicates to the bank whether or not there is a greater or less amount proportionally of profits figured as part of the current assets.

The ratio of sales to merchandise (net sales divided by the total merchandise inventory) gives the dollars of sales for every dollar reported as inventory. From this ratio, the banker knows something about the freshness and salability of the merchandise, and it is particularly important as a means of indicating an overinventoried condition or an accumulation of stale merchandise.

The sales-to-receivables ratio (net annual sales divided by total accounts and bills receivable) indicates the dollars of sales per year for

every dollar classed as receivables. The higher the amount of the sales for every dollar still uncollected, the greater will be the liquidity of the receivables. As this ratio rises, the length of the collection period decreases and the banker knows that the period of risk through bad debts is shortened and that the danger from possible credit troubles is decreased. The higher the ratio, within reasonable limits, the more desirable are the receivables.

The sales-to-fixed-assets ratio (net sales divided by net total fixed assets) indicates the dollars of net sales for every dollar invested in plant or other non-liquid assets. The banker uses this ratio in analyzing the comparative gross earning capacity of the plant investment.

The sales-to-worth ratio (net sales divided by the net worth) shows the dollars of sales for every dollar of stockholders' invested capital. This ratio serves the banker as a valuable check upon the management. If the money placed in the business by the stockholders is turned over too slowly, it may indicate a distinct lack of progress. On the other hand, if the ratio is too high, it may indicate that the company is going ahead too fast for its capital investment, and that it might be in difficulties, should business conditions become unfavorable.

To be of greatest significance, the above ratios should be computed and compared over a period of time. The trends and tendencies shown are often more important than the actual position indicated at a particular date. Trends, seasonal variations, cycles, and random movements in the individual concern should be carefully considered and compared with indexes of general business conditions in the area concerned.

The above discussion indicates the importance of the borrower's financial statement to the banker. Obviously, it is important that the client thoroughly understand the interrelations of the items in his statement, not only in order that he can make every effort to have his business in such a condition that his statement will be favorable when he applies for a loan, but also in order that he can conduct his business along sounder lines than he could without making such analyses.

Analyzing a Bank Statement.—Bank-statement analysis is of widespread interest and significance. Not only must a bank watch its own condition and the condition of other banks with which it does business, but the client must also watch the condition of his bank from the point of view of his own safety.

Aside from the specific personal interest in particular institutions indicated above, it is also quite necessary to analyze bank statements in connection with determining general money and credit conditions in a community. The importance of studying the money and credit situation in analyzing general business conditions has been emphasized previously.

Owing to the requirements of state and Federal banking laws, bank statements are more readily available than those of most other institutions. Several times a year the Comptroller of the Currency calls for a statement from all national banks. At the same time, the state bank examiners ordinarily call for a statement also. The calls come without warning, and the date for which the statement is to be made is ordinarily

THE NATIONAL CITY BANK OF NEW YORK
INCLUDING DOMESTIC AND FOREIGN BRANCHES
CONDENSED STATEMENT OF CONDITION AS OF MARCH 31, 1938

ASSETS		
Cash and Due from Banks and Bankers		$ 634,261,100.79
United States Government Obligations (Direct or Fully Guaranteed)		370,621,979.70
State and Municipal Bonds		106,417,185.73
Other Bonds and Securities		116,710,578.15
Loans, Discounts and Bankers' Acceptances		569,370,705.22
Customers' Liability Account of Acceptances		13,032,603.85
Stock in Federal Reserve Bank		3,675,000.00
Ownership of International Banking Corporation (Including Paris Office)		8,000,000.00
Bank Premises		49,685,947.85
Other Assets		13,651,097.71
Total		$1,885,426,199.00
LIABILITIES		
Deposits		$1,694,472,206.09
Liability as Acceptor, Endorser or Maker on Acceptances and Bills	$46,141,933.27	
Less: Own Acceptances in Portfolio	14,952,636.89	31,189,296.38
Items in Transit with Branches		10,315,190.11
Reserves for:		
Unearned Discount and Other Unearned Income		3,608,182.89
Interest, Taxes, Other Accrued Expenses, etc		8,117,390.27
Dividend		1,550,000.00
Capital	$77,500,000.00	
Surplus	45,000,000.00	
Undivided Profits	13,673,933.26	136,173,933.26
Total		$1,885,426,199.00

EXHIBIT 217.—A condensed statement of a bank's condition.

a few days earlier so that there is no opportunity to alter the books. That is, a call might come in on March 5 for a statement as of March 1. These call statements are required by law to be published. A condensed statement is usually published for the information of the general public. And for the purposes discussed in this chapter, a condensed statement is usually the most convenient and is otherwise satisfactory. A representative condensed statement is illustrated by Exhibit 217. Data for determining the average condition over past periods by states, counties,

and cities may be obtained from the annual reports of the Comptroller of the Currency and from the reports of the state banking departments.

Whether a commercial bank statement is analyzed from the point of view of the bank or the point of view of the client, one of the more important considerations is the relative liquidity of the assets. In other words, one should know the condition of the bank's assets in relation to how well it can meet its immediate obligations, which under normal conditions should be mostly deposits payable on demand. To determine this, requires the answering of a number of questions, and foremost among these is whether or not the bank is maintaining a sufficient reserve.

Reserve.—The term "reserve," as it is used in banking, refers to cash or its equivalent. The "legal reserve" of a national bank can consist only of deposits in the Federal Reserve bank. The "actual reserve" includes, in addition to the Federal Reserve bank deposit, the cash in the vault, and deposits in banks in reserve or central reserve cities. "Demand deposits" include all deposit accounts subject to check, demand certificates of deposit, outstanding certified checks, and outstanding cashiers' checks. The total of the actual reserve should be compared with the total of demand-deposit obligations. While the actual reserve is a much smaller figure than total deposits, it should be large enough to care for any demands that may be made. The proportion depends somewhat upon how liquid the other assets are, and upon economic conditions in general. An indication of the normal position to be maintained may be gained by considering the average of a large number of banks. Data compiled from the annual report of the Comptroller of the Currency show that on June 30, 1928, the actual reserve for 7,691 national banks was approximately one-third as large as the total demand obligations. The same report shows that the combined actual reserve of 15,078 state (commercial) banks was also approximately one-third as large as the combined demand obligations. Similar compilations for these two classes of banks, made as of June 30, 1921, also show practically the same relation. During the last few years, however, the situation has been very unusual, and reserves have been exceptionally large in relation to deposits. Obviously, the cash reserve that a bank should maintain in times of depression should be as large, if not larger, than in times of prosperity. But if this proportion runs much below one-third in a commercial bank, one should take immediate steps to determine whether or not the condition is serious. Such an investigation should indicate the steps that should be taken.[1]

[1] In the above computation, "total actual reserve" includes "due from banks," "lawful reserve with Federal Reserve bank," "checks and other cash items," "exchanges for clearing house," and "cash on hand." "Total demand obligations" includes "due to all banks," "certified checks and cashiers' checks," "dividend

A national bank's lending power depends upon the ratio of its reserve to its net demand deposits. According to the statutory requirements, the Federal Reserve bank deposit for a country bank should be 7 per cent of the net demand deposits; for a reserve city bank, 10 per cent; and for a central reserve city bank, 13 per cent. The bank must also carry a legal reserve of 3 per cent against its time deposits, which are the deposit accounts and certificates payable in 30 days or more. The Board of Governors of the Federal Reserve System, however, has the authority to increase these requirements up to twice the percentages given. (In August 1938, the requirements for net demand deposits were 12 per cent for country banks, 17½ per cent for reserve city banks, and 22¾ per cent for central reserve banks. For time deposits the requirements were 5 per cent.)[1]

Deposits.—In the preceding discussion of a bank's actual reserve, it is necessary to consider the deposit accounts, because these deposits represent the claims that the assets must be in condition to meet. The bank's primary responsibility is to its depositors, and it must be prepared to pay them in full the funds that it holds for them. The establishment of the Federal Deposit Insurance Corporation does not relieve banks of this responsibility. "Deposit insurance is only an assurance to the public that the banks will discharge their responsibility to depositors, and deposit insurance could not succeed if the banks neglected this responsibility." (Deposit insurance covers only deposits of $5,000 or less.)[2] It is not correct to say that the lending power of a bank is represented by the sum of its time and demand deposits, plus its outstanding notes and net-worth items (capital stock, surplus, and undivided profits). The deposits are offset partly by the loans that the bank has already made and therefore represent partially used lending power. The strength or soundness of a bank cannot be gaged by the volume of deposits or by the total resources. Deposits represent the bank's liabilities, and if they increase as a result of loans, the bank's position will grow weaker. To a bank, deposits are the same as accounts payable are to a commercial concern, and the bank's strength should not be judged by its volume of deposits any more than the soundness of a commercial concern can be judged by its accounts payable. Of course, the bank's position becomes

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checks outstanding," "individual deposits subject to check," "demand certificates of deposit," "state, county or other municipal demand deposits," and "other demand deposits." When "net demand deposits" are used, balances due from banks other than the Federal Reserve bank and foreign banks, "our checks," and checks on other banks, including clearing house exchanges, are deducted from the gross figure.

[1] See the *Federal Reserve Bulletin*, p. 715, August 1938.

[2] See Goldenweiser, E. A., "The Function of Deposit Banking," *Journal of the American Statistical Association*, p. 303, June 1938.

stronger if the deposits increase as a result of deposits of actual cash, but this is because of the actual cash and not because of the increasing deposit liability.[1]

Loans and Discounts.—Under normal conditions, most of the assets of a commercial bank should consist of "loans and discounts." These holdings should consist principally of self-liquidating commercial paper, running less than six months and distributed as to maturity so that some of the notes are constantly falling due. In addition to determining whether the bank has a sound proportion of the different types of loans, one should also find out as much as possible about the quality of these loans. Obviously, this cannot be determined from a bank statement, but must be based on other information.

As far as our analyses are concerned, the term "loans and discounts" may be taken to have the same meaning that the term "loans" has in common business usage. The use of the two words arises from the practice of charging *interest* on loans, which is added at the end of the period, while in the case of *discounts*, the charge for the use of the money is deducted at the beginning of the period.

Investments.—Under normal conditions, the bond holdings of a bank should be relatively small. When money is very easy, or when deposits are coming in faster than they can be loaned out, a bank may have to increase its bond investments in order that it may have some income from the money it holds. Such bonds, however, should be readily salable and of certain security. As a rule, banks should not invest in corporate stocks, as they represent shares of ownership rather than first claims on the assets, and their values fluctuate too widely. In recent years, conditions have made necessary a number of changes in policies and regulations. In the spring of 1938, the Regulations of the Comptroller of the Currency governing bank investment policy were revised to provide that "in addition to such securities as have been authorized heretofore, member banks of the Federal Reserve System may purchase investment securities, of such sound value or so secured as reasonably to assure payment, issued by established commercial or industrial businesses or enterprises that can demonstrate the ability to service such securities, without requiring that the securities be offered for public distribution or that they must be readily marketable, provided, however, that they mature not later than 10 years after the date of issuance and that 75 per cent of the principal be amortized by maturity through substantial, periodic payments, none of which would be required during the first year."[2]

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[1] See "Financial Handbook," pp. 1227 and 1228, The Ronald Press Company, New York, 1927.

[2] *Federal Reserve Bulletin*, p. 563, July 1938.

Fixed Assets.—The remaining assets of a bank are represented by such items as the bank building, furniture, and fixtures. These are fixed and not liquid. They should constitute a very small proportion of the total. Some bankers hold that they should not amount to more than 20 per cent of the amount of the capital, surplus, and undivided profits. If the bank owns other real estate, each item should be examined carefully, as it probably has been acquired as the result of non-payments of loans. Ordinarily, such holdings should be disposed of at the earliest opportunity. A national bank can own, within certain limits, the building that it occupies, but any other real estate acquired because of defaulted loans must be disposed of within five years. Sometimes the buildings owned by state banks represent too large a proportion of the assets. In some states, the buildings cannot be carried at a value greater than one-fourth of the amount represented by the capital stock.

Rediscounts and Bills Payable.—The demands made upon banks by their customers are ordinarily subject to high cyclical and seasonal variations. If the amounts of rediscounts and bills payable due the Federal Reserve banks in cyclical or seasonal emergencies are high, it is not a cause for question, but if these liabilities remain heavy the year around and for several years, it is a sign that the bank is overexpanded.

Analysis over a Period of Time.—From the preceding discussion, it is obvious that it is very important to note the changes that take place in various items over a period of time. An analysis of a single statement does not present a complete picture of the bank's position. It should be made over a number of years in order to determine the trend in a bank's activities and changes in the bank's policy. These changes over the period should be compared with those of other banks in the community. And, finally, after the various ratios and data have been placed upon a systematic basis over a period of time for the various banks, they should be carefully compared with the available indexes of business conditions in the areas concerned.

Bank Letters.—Many of the leading banks publish monthly letters or bulletins which contain valuable statistical data pertaining to local or national business. Two of the outstanding letters of this type are the monthly letter of the National City Bank of New York and the Cleveland Trust Company *Business Bulletin.* These particular letters are written principally from a national point of view. Others, such as the *Monthly Summary,* issued by the Security-First National Bank of Los Angeles, report upon local conditions. In mentioning bank letters, however, the point should be made that while many banks issue careful reports containing valuable statistics, there are also many which issue monthly bulletins that are of but little, if any, practical value in analyzing a business situation.

Statistical Work of the Federal Reserve Board and Banks.—In discussing banking statistics, special mention should be made of the statistical work of the Federal Reserve System. This includes, not only the work of the Division of Research and Statistics of the Federal Reserve Board, but also that of the statistical departments of the 12 Federal Reserve banks.

While the Federal Reserve System statistics relate in considerable part to the general credit situation and operation of the money market, they also cover a wide range of other factors, of interest, not only to banks, but to business men in general. Among the more important series published regularly in the *Federal Reserve Bulletin* are: reserve bank credit, gold stocks, money in circulation, and member bank reserve balances; analysis of changes in monetary gold stocks; discount rates and money rates; member bank credit; bank balances; bankers' acceptances and commercial paper outstanding; brokers' loans; commodity prices, security prices and security issues; production, employment, and trade; industrial production; factory employment and pay rolls; building; trade and distribution; bank suspension; gold holdings of central banks and government; gold exports and imports; condition of central banks in foreign countries; discount rates of foreign central banks; foreign money rates; foreign exchange rates; treasury and capital market statistics; and price movements in principal countries.

While some of the above series are quoted from other sources, many of them are collected directly through the Federal Reserve System and are compiled or converted into index numbers by the Division of Research and Statistics of the Federal Reserve Board. Further details on these series may be found by referring to the current issues of the *Federal Reserve Bulletin.*

In addition to cooperating with the Federal Reserve Board statistical department, the 12 Federal Reserve banks often carry on special studies of their own. A few examples will be mentioned in order to indicate the type of work done. The Federal Reserve Bank of New York has published frequent analyses of the principal factors affecting the movement of rates in the call-loan market, including in particular an account of the effect of year-end withdrawals by out-of-town lenders. The Federal Reserve Bank of Minneapolis has published figures showing how the total credit extended by country banks is divided, with particular reference to the part represented by loans to customers. This bank has also made studies of the trend of deposits of country banks, with separate figures for different sections in the Minneapolis district (mixed farming, wheat raising, etc.) and according to the size of the community in which the bank is located. The measurement of the volume of production in the individual Federal Reserve districts, primarily in terms of physical

volume, is illustrated by material that has been published in the monthly reviews of the Federal Reserve Banks of Cleveland, Minneapolis, and San Francisco. All of the Federal Reserve Banks have given careful attention to reporting figures on retail trade. The Federal Reserve Bank of New York has published a comparison between the increase in velocity of New York City deposits and the increase in loans made to brokers by lenders other than banks. Compilation and publication of figures relating to the earnings of corporations have been carried on by the Federal Reserve Banks of New York, Philadelphia, and Cleveland. The Federal Reserve Bank of Minneapolis has given considerable attention to the estimating of farm income. Other examples of the many valuable kinds of statistical studies made by the Federal Reserve System will be found by referring to the *Federal Reserve Bulletin* or to the monthly letters of the Federal Reserve banks.

In addition to the published statistical information mentioned above, which is readily available to all banks as well as other business men, the Federal Reserve System collects and compiles much information which is primarily for the use of Federal Reserve officials and member banks.

Questions and Problems

1. Why are banks interested in business statistics?

2. What is the scope of a bank's statistical work?

3. Of what significance are statistics in connection with a bank's interest in the success of its customers?

4. Discuss the bank's obligation to a community as an adviser on financial and business conditions.

5. Discuss the banker's interest in business forecasting.

6. Of what importance to banks is information on population and purchasing power?

7. Why is a bank interested in production and labor statistics?

8. How does a bank use marketing analyses?

9. Why are banks concerned with real estate conditions?

10. Is investment analysis important to banks? Why?

11. How is a line of credit determined?

12. Discuss the work of credit analysis.

13. What information is required from borrowers by banks?

14. How is a borrower's statement analyzed?

15. The ratios of quick assets to current liabilities, worth to debt, worth to fixed assets, merchandise to receivables, sales to merchandise, sales to receivables, sales to fixed assets, and sales to worth are of what significance to a banker in analyzing a borrower's statement?

16. How is a bank statement analyzed? Of what importance is such an analysis?

17. What are "call statements"? Of what value are they to the business man?

18. From the point of view of statistical analysis, what does a bank's "reserve" indicate?

19. What is shown by loan and discount data, and by deposit data?

20. Explain the "ratio of loans to deposits." How is this ratio interpreted?

21. What analyses should be made of a bank's bond holdings?

22. What relation should a bank's fixed assets bear to its total assets? Explain.

23. Discuss the relation of a bank's deposits to the sum of its capital stock, surplus, and undivided profits.

24. What may be indicated by the item "overdrafts" on a bank statement?

25. What is the significance of a bank's rediscounts and bills payable?

26. Why is it important to analyze bank statements over a period of time?

27. Discuss the importance of bank letters or bulletins as sources of statistics.

28. Discuss the statistical work of the Federal Reserve Board and banks.

29. In cooperation with a certain bank in your city, prepare a detailed plan of its present and projected statistical work.

CHAPTER XXV

EXECUTIVE CONTROL AND MANAGEMENT STATISTICS

In preceding chapters, analyses of business problems in many fields of activity have been considered. Methods and illustrations of such analyses have been presented in connection with forecasting, budgeting, marketing, production, real estate, investment, and banking. In several of the discussions, the problems have been problems of control, and the methods described have been methods of effecting adequate control over production, marketing, and other operations. In these discussions, the analyses have been localized to the particular department of the business under consideration. In the present chapter, it is intended to review briefly the use of statistical analyses and methods in effecting control over the functions of each department or part of the business, and also to discuss the use of statistical methods in controlling the organization as a whole.

Statistical Control.—"Statistical control is a method of arriving at executive decisions through analysis of the facts which relate to a particular business. To be genuine it must begin with the desire of the executive to act in accordance with the results of such analysis. It is the function of the statistician to assemble the facts, all the facts, both external and internal, which are pertinent, and nothing but the facts; to analyze these facts in such manner as to indicate obvious courses of action; and where obvious courses of action cannot be indicated, to determine as accurately as possible the probable range of results under different plans of procedure. But here the work of the statistician ends. Only the executive can translate facts into executive performance.

"The practical statistician sees in statistical control a method of improving business administration that has very definite possibilities and equally definite limitations. On the other hand, the business executive, who is unskilled in statistics, is very apt either to condemn statistical control *in toto*, or to expect far too much from it. These differences in attitude arise very largely from the failure to make a clear distinction between the unorganized, relatively aimless statistics of the average business, and those organized and purposeful statistics which are required for genuine statistical control.

"In any well-managed business undertaking, and particularly in one of large scale, there is, and must be, consciously or unconsciously, a very

great dependence upon statistics. Books of account, pay rolls, stock records, sales records, production schedules, cost records, purchasing schedules, and innumerable other entries and summaries of current and prospective operations are nothing more or less than the statistics of the business. In some special enterprises, notably those concerned with the various forms of insurance, and to a considerable extent in banking, the statistical element predominates. Furthermore, there is rarely even a minor executive in any business who does not, knowingly or otherwise, base his decisions very largely upon the statistics of his operations."[1]

Before considering methods and tools used in executive control, it is first necessary to view the internal organization structure through which executive control from the offices of the chief executives can be effected. After a view of the organization structure, the relation of the control functions to be exercised in each department to those of the major executives can more easily be seen.

Internal Organization.—Business organizations vary not only in size, but also in types of internal organization. Briefly, there are four main types of internal structure: the line organization, the functional organization, the department organization, and the line-and-staff organization.

In small concerns, the line organization is predominant. This type is characterized by a line of authority and responsibility which extends from the top to the lowest class of duties. Thus, the chief executive usually exerts a directing influence throughout the entire organization, and in a small organization this is often accomplished by direct personal contact with the minor executives and even with the minor employees. In order to be successful in managing such an organization, the chief executive and his subordinates ordinarily must have a wide range of technical knowledge and ability.

The functional type of organization is characterized by a division of the management so that each executive supervises and manages one major function. In other words, in this type of organization, the management is characterized by a division of responsibility according to types of work or functions performed by the employees. Workers in any one part of the business may have several "foremen" or minor executives over them, each of whom exercises supervision and control over these workers according to the particular part of their duties they are performing.[2] Above the foremen or minor executives are specialized major

[1] Rorty, M. C., "The Statistical Control of Business Activities," *Harvard Business Review*, Vol. 1, No. 2, pp. 154–166, January 1923.

[2] Thus, in a machine shop organized on a functional plan, Frederick W. Taylor had four types of bosses: gang bosses, speed bosses, inspectors, and repair bosses. The gang boss had charge of all work up to the time that the material was set in the machine; he was responsible for *starting* the planned production (getting the materials

executives that supervise and manage the operations related to their particular specialty. The duties of the chief executive or president of such an organization are more in connection with the choice of the functional executives and the coordination of their activities than in supervision over the performance of their technical duties.

The third type of organization is found in such institutions as department stores. In these stores, it is usual to deviate from the ordinary

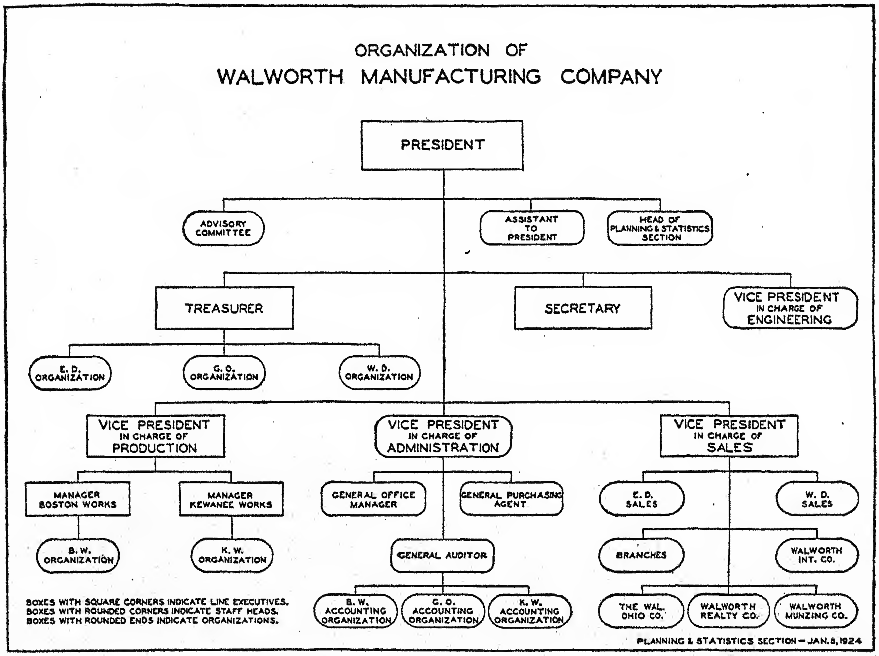

EXHIBIT 218.—Chart of internal organization. (*Redrawn from Bulletin of the Taylor Society, Vol. IX, No.* 3.)

type of line organization in which authority extends from the top to the bottom through a line of intermediate executives. Instead, the merchandising organization is divided into many separate units or *departments*, each in charge of a department manager or "buyer." The result is similar to a group of many stores, except that the department managers

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and setting them in the machines) up to the point where operation of the machine began. The speed boss was responsible for the machine process and his duties were to obtain the best quality of machine work in the quickest possible time. The inspector, however, was given the authority to pass upon the quality of the work and to see that standards of quality were maintained. The repair boss was required to see that the workers maintained their machines and auxiliary equipment in good repair. Thus, each machine worker was under four bosses.

are responsible to a central authority or general manager and also are supervised and controlled to a certain extent by several functional departments which exist outside of the "operating" departments. Thus, there is usually a separate publicity or advertising department through which the advertising policies, methods, and plans are made for the entire store. Also, there are merchandise managers who plan, supervise, and check the work of the department managers, and often a personnel department is in charge of the hiring of all employees and the problems of employee welfare. Because of the existence of these and other functional departments, the department type of operation may be said to include a hybrid or unusual line organization with which certain features of functional operation have been combined.

Large manufacturing companies usually exhibit more nearly the characteristics of what is known as the "line-and-staff organization." In this type there are line executives who are responsible for the performance of work operations. They are assisted in an advisory capacity by technicians, or specialists known as "staff" men. This is illustrated in Exhibit 218, which presents the organization of the Walworth Manufacturing Company. The chart shows that the line executives under the president are in charge of the operating parts of the company, which include the production, sales and financial departments, and the secretary's office. The staff heads in this company have charge of planning and also of recording and checking up on the results of operations. However, the staff heads in this and in all line-and-staff organizations are not in an executive position in relation to the workers in the line organization. They act in an advisory capacity to or through the various executives.

Relation of the Chief Executive to the Internal Organization.—In a corporation, whether the type of internal organization be line, functional, departmental, or line-and-staff, there must always be one executive who is responsible to the board of directors for the operation of the concern. Through this executive, the policies of the board are put into effect, and, through him, the board is informed of the results of operation. Therefore, every president or chief executive must have methods of carrying out the policies determined upon by the board of directors, and methods of checking the results obtained.

In order to control effectively the operations of the business, it is essential, first, that adequate plans be made. Second, it is necessary that the progress of the planned operations be carefully recorded in the departments responsible for the performance. Third, the actual performance must be measured and recorded in the general accounting records of the organization. And fourth, the records of actual results must be analyzed, compared, and interpreted in relation to the planned operations. Obviously, the plans determined upon will be most effective

if they are prepared in detail and if the form in which they are prepared is standardized. In order to compare actual performances with the planned performances, the form in which the plans are made and the form in which the report upon actual performance is made must be closely related. Therefore, it is usually a function of the comptroller's department to prepare the reports of planned operations and also those showing the actual operations. The progress of the work in each operating department, however, must be carefully watched by the executive in charge of such departments. Therefore, the progress charts of production, of sales, of planned construction, etc., must be maintained in the departments which perform these respective operations.

Statistics and Accounts.—"From a business standpoint, it is impossible to adopt a narrow definition of statistics. A practical separation must, of course, be made between statistics and accounts, and, in fact, it is very necessary that no attempt be made to burden the regular accounting forces with the changing details of statistical analyses. The two classes of work are radically different. In general, the accounts will serve their full purpose if they accurately reveal the financial status of the business and the financial results of current operations, and in addition locate general responsibility for any marked variations from normal performance. But beyond this point it is not profitable, even where it is possible, to carry the rigidity of the balance sheet and the double entry. Cost analyses, operating reports, projections of future financial and operating results, and many other statements and summaries, all require flexibility of handling and must be treated from a statistical rather than an accounting standpoint. Furthermore, the accounts themselves are the materials from which many statistical studies are derived, so that with the special field of the accounts once clearly defined, it is not too much to say that all pertinent numerical facts, and many that are not strictly numerical, whether within or without the business, whether relating to the past, the present, or the future, fall within the scope of business statistics.

"It is this very pervasiveness of the statistical element in modern business that is most confusing to the executive who is seeking to avail himself of improved statistical technique in the handling of his affairs. Yet it is not impossible to establish a working distinction between business statistics in bulk, and those special statistics and statistical studies which form the elements of statistical control. This distinction is, however, largely based on the attitude of the executive.

"It is possible to have reams of statistical reports and hundreds of charts and diagrams without securing even the beginnings of statistical control. If the statistics grow out of the absolute needs of the business and are looked upon as necessary evils rather than as positive and

constructive factors, statistical control does not exist. Furthermore, even the most competent of statisticians will have little success in establishing a basis for statistical control if he is not in working contact with a responsible executive who will make his studies effective."[1]

The relation of the chief executive to the comptroller's department, through which the plan of operations and the record of actual operations are made, will now be considered. Then the relation of the chief executive to the operating departments which carefully record the progress of the work will be discussed.

Comptrollers' Reports for the Chief Executive.—While the plan of operations may ordinarily best be prepared in the comptroller's department, it should be recalled (see Chapter XVIII, Budgeting) that the preparation of these budget plans requires the cooperation of all of the major and minor executives in the organization, and a budget will be effective only if those who are to carry out the budget plans have taken part in its preparation. The comptroller's department collects and summarizes the tentative plans suggested by the other executives. In order to coordinate these plans, many organizations have found it advisable to install a budget committee consisting of the major departmental executives, and often minor executives are included. The comptroller is usually the chairman of such a committee. When the budget has been passed by the committee it is submitted to the president as a tentative budget. Final adoption of the budget, however, should ordinarily rest with the board of directors.

Although the chief executive usually does not meet with the budget committee, he will influence the preparation of the tentative budget according to the need that may exist at any time. Thus, the tentative budget will ordinarily be in accord with the policies of the chief executive and the board of directors, so that delays in its adoption will seldom occur after it leaves the committee.

The president or chief executive is also dependent upon the comptroller's department for a complete and accurate statement of the actual results of operations. Accounting reports must be prepared to serve both the departments whose operations they reflect and the major executives who supervise the organization as a whole. Accurate and adequate accounting for the actual results obtained in each department is obviously a prerequisite of accurate and adequate reports. Thus, the duties of the accounting staff include: (1) *the making of complete records* of the transactions, (2) *the preparation of operating reports* which summarize the transactions of each department and the entire organiza-

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[1] Rorty, M. C., "The Statistical Control of Business Activities," *Harvard Business Review*, Vol. I, No. 2, pp. 154–166, January 1923.

tion, and (3) *the preparation of financial statements* which present the financial condition at the end of each operating period. In this discussion, it is not practical to describe the details of an adequate accounting system for recording the transactions. However, a few of the reports, which should be prepared from the accounting records for the use of the executives, will now be mentioned.

The profit and loss statement of a company is the operating report which summarizes the actual business results. For the executive's use of this statement, supplementary reports should be prepared showing details of the marketing, or sales operations, the production and purchases operations, the labor or personnel department operations, and expenses of management according to the divisions of management, such as the credit department, the treasurer's department, and the general executive offices. The form and size of each of these statements in the large organization must be adequate to indicate the responsibility for the results obtained, and also they must agree with the form and size of the statements prepared for the planned operations to facilitate significant comparison.

The statement of assets, liabilities, and net worth (that is, the balance sheet) should be prepared monthly by the comptroller's department for the higher executives and the board of directors of the company. In addition to this statement, which is a general summary of the financial condition, there should be supplementary statements setting forth details of important assets, liabilities, and surplus. Analyses of accounts receivable, of inventories, and of accounts payable are important in controlling the finances of the corporation. Without such analyses of the present financial condition, the plans of future operations cannot be prepared intelligently.

Operating Departments' Reports for the Higher Executives.—In each operating department, special records must be maintained in addition to records of operations prepared for the comptroller. Some of these records have been discussed in previous chapters, particularly in Chapter XX (Production and Labor Statistics) and in Chapter XXI (Marketing Analysis). It should be observed here that they are not only essential to the management within the department, but they also serve the higher executives as measurements of current operating progress. Therefore, the progress reports and the detailed sales reports prepared within respective departments must be currently available to the higher executives such as the general manager, the vice presidents, and the president. In addition to these progress reports, the chief executives should be furnished with reports on research and experimental activities. In the sales department, marketing studies are often carried on by sales executives both as an aid in the preparation of the budget and in deter-

mining the effectiveness of the products. These reports should be immediately available to the major executives.

Comparison of Planned Operations with Actual Performance.—In order to assist the major executives and directors in measuring the actual results in relation to the budget plans, the comptroller's department[1] should prepare statements which show comparisons and explain the causes of variation in actual performance from planned performance. The operating reports previously mentioned should, therefore, be prepared in an additional form which presents both the figures of planned income, expenses, etc., and the actual figures.

Mere comparisons, however, do not assist materially in accomplishing an improvement in the results, unless the variations of the actual data from the planned data are intelligently accounted for. Analyses of variations in sales, in production, in expenses, and in net profits must be made as bases for intelligent interpretation. A variation in sales, for example, may appear from a comparison of the budgeted sales items and the actual results. But an increase in the physical *volume* of sales over the planned volume may actually have taken place, although the *dollars* of sales have decreased, owing to an unexpected decrease in selling prices. Therefore, the variation in total sales dollars from planned sales dollars should be explained by a statement which will measure in exact terms the extent to which the increase in physical volume was offset by price declines. Such a statement will present a computation of what the total volume would have been if no price decline had taken place, and the total effect, in dollars, of the price decline.

In comparing actual production costs with planned costs, analyses to determine the causes of variations are particularly necessary. Were total costs greater because of a larger volume or because of higher costs? Were material costs higher than anticipated, or did labor or overhead costs increase? Was the increase in material cost due to a general advance in material prices or to the use of more expensive materials? Did wage rates increase, or were the laborers less productive? Are the overhead expense rates carefully and accurately determined, or are they rough approximations? Such questions as these must be satisfactorily answered in interpreting variations in production costs.

Also, the actual expenses of selling, of administration, and of financing must be carefully compared with the items in the budget plans. In accounting for variations between the budget and the actual expenses, it is important to note that the *ratio of expense*, based upon sales, is an

[1] Where a separate statistics and planning organization exists, this section or department usually prepares the analytical statements referred to in this discussion. When the statistics and planning section is not a part of the comptroller's department, it works in close cooperation with this department.

important basis for comparisons rather than dollars of expense. In other words, if sales have fallen below or have exceeded budgeted sales, many items of selling expense and of administration should be adjusted accordingly in order to obtain the budgeted rate of net profit on sales.

Comparison of Actual Results with Standards of Performance.—As experience in production, marketing, and management is obtained, corporations can develop standards by which actual results can be measured to determine efficiency. In the production department, *standard costs* are useful as an accounting method and as a basis of measuring attainment. Similarly, standards of advertising expenses, sales salaries, and the various administrative expenses in relation to sales should be determined. These standards are not necessarily the same as the budget figures adopted for any particular period. Therefore, separate comparisons of the actual data with the standards, determined on the basis of past experience, may be necessary in addition to the comparisons with budgeted data, because "standard ratios" represent a more permanent or longer-period relationship.

In the management of the organization as a whole, many other ratios that measure the effectiveness of the management may be computed and compared with standard ratios. These management ratios are both of financial and of operating nature. Several have been mentioned previously, such as the eight financial ratios for analyzing financial statements discussed in Chapter XXIII (Investment Analysis), and in Chapter XXIV (Statistics in Banking), all of which are also important to the management. A list of useful management ratios is as follows:[1]

1. Measures of Earnings.
   a. The return earned on net worth.
   b. The ratio of surplus net profits to sales.
   c. Earnings on common stockholders' investments.
   d. Operating profits earned on total capital used.
   e. Ratio of operating profits to sales.
   f. Margin of gross profits earned on sales.
2. Measures of Costs and Expenses.
   a. Ratios of costs and expenses to sales.
   b. The cost of borrowed capital.
   c. The cost of total capital used.
3. Turnovers.
   a. Turnover of total capital used.
   b. Turnover of inventories.
   c. Turnover of accounts receivable.
   d. Turnover on fixed property investment.

[1] Bliss, J. H., "Management through Accounts," p. 68, The Ronald Press Company, New York, 1924. A detailed discussion of these ratios will be found in this book, and a discussion of similar ratios is contained in "Management's Handbook" pp. 236–258.

4. Financial Relationships.
   a. Net working capital and ratio.
   b. Manner in which capital is invested.
   c. Sources from which capital is secured.
   d. Proportion of earnings left in the business.

The Executive in Relation to External Statistics.—It is the duty of the management to see that facts and events occurring outside of the individual organization are properly recognized and considered in making executive decisions. The problems of management are not entirely a matter of internal relationships. The major executives must so organize their internal management duties that their attention can be given to the interpretation of the effect and importance of such subjects as general business conditions, conditions in their industry, and the indications of future conditions. From previous discussions, it will be appreciated that the procedure indicated as advisable—as a result of business forecasts, studies of the labor situation, analyses of the market, analyses of real estate conditions, or investigations of financial conditions—often involves serious questions of executive policy, and that, in such instances, the decision as to what course to follow can be made only by the management. It follows, therefore, that while the point of view in the preceding discussions of methods of making analyses (on the basis of external statistics) of general business, population and purchasing power, production, labor, markets, real estate, investments, and banking was principally that of the investigator, nevertheless, the application of such statistics is largely an executive problem; and the management is interested in any or all of these types of surveys when they will help to solve a related problem. From the point of view of the executive looking down upon his organization, all of the different classes of statistics previously discussed in this book are management statistics.

As an example of specifically relating an individual organization to external statistics, let us suppose that we are analyzing competition with a view to increasing our competitive strength. One method of finding where one can increase one's competitive strength is that of comparing the costs of doing business. Such reports as those of the Harvard Bureau of Business Research furnish much information which can be used in making these studies. These reports furnish data on the average conditions in retail grocery, shoe, hardware, drug, jewelry, tire, general merchandise, and department stores, as well as for wholesale automotive equipment, grocery, drug, and dry-goods stores. If, for instance, we are operating a retail grocery store, and our salaries and wages constitute 15 per cent of our net sales, we find by referring to these reports that we are paying considerably more than the average (10.9 per cent), which indicates that this is a point which should be investigated with a view to

reducing operating expenses. Suppose also that our losses from bad debts are 5 per cent. This also indicates an opportunity for substantial improvement as the average loss is only .4 per cent. Our stock turn, however, of 12.5 is better than the average (10), which indicates that our greatest efforts to make improvements need not be applied here.

If we are operating a small department store, similar data to that presented above enable us to analyze our situation with a view to meeting the competition of the large department stores. We find that the gross margin (difference between cost of goods and sales) is 32.2 per cent in the large stores, while it is only 28.6 per cent in the smaller stores, but in spite of the lower operating expenses in small stores, the large stores had a net profit of 2.3 per cent while the small stores had a profit of only .5 per cent. This indicates that the large department stores pay less for their merchandise on account of their greater buying power. We find, however, that the large department store has a lower interest charge and a higher turnover, which indicates that we might improve our situations in this respect by studying the methods of the large department stores.

Similar comparisons can be carried out in the different kinds of wholesale trades. If, for instance, we are operating a wholesale drug concern and rent is 3.0 per cent of our net sales, we would be paying a considerably higher proportion than the average, which is only .5 per cent, and it may be advisable for us to investigate this matter quite thoroughly. It may be that rents have increased in our location until it is no longer advisable for us to pay them, providing we can find a more economical location which is satisfactory for the wholesale drug business.

Manufacturers, realtors, bankers, and other business men can make similar comparisons in instances where data are available.

The Statistical Organization.—The amount and kind of statistical work that can be carried on varies directly with the size of the concern. When a large concern employs experts to work out certain problems statistically, it is relatively easy to distribute the expense over the large organization. If a small firm wishes to make the same statistical investigations, the cost may be out of all proportion to the size of the concern. Problems which would be complicated in a large concern, however, are often simple in a small concern, and, consequently, where an involved investigation might have to be made to solve the problem in a large concern, a relatively simple investigation might solve the problem as adequately in a small one.

"Contrary to general belief, the element of cost of statistical work is a relatively minor one in any organization of substantial size. A single, well-qualified statistician with one or two clerks, when working under the right executive, can soon demonstrate in a very practical way the profit to be derived from a further expansion of statistical activities.

Furthermore, there are few large-scale organizations in which a critical study of existing expenditures for unorganized bulk statistics and a proper reorganization of existing statistical work will not effect economies sufficient to pay for a centralized statistical office.

"It is, nevertheless, to the interest of both the statistician and the executive to establish statistical control gradually and with a careful testing of each step. On this basis each addition to the statistical organization and each element of statistical work will constitute a proven source of profit, rather than an item of expense. On the other hand, if an attempt is made to create a large statistical organization over night, without well-established relations with the executives, there is great danger of repeating the experience recently brought to the writer's attention, where a very competent statistical force gave repeated warnings of dangers ahead and was then suddenly disbanded, as a measure of economy, when the disregard of its warning led to disaster.

"With a policy of gradual development once adopted and with immediate out-of-pocket expense thereby minimized, the next problem is that of choosing the chief statistician and establishing his relations to the balance of the organization. As previously indicated, the statistician must report directly to an executive who is in thorough sympathy with the idea of statistical control and is capable of valuing the statistician's work and translating it into terms of definite proposals for action. It is also desirable for this executive to be in position to put into effect, in the routine statistical work of the various units of the organization, the suggestions for improvements that will naturally originate with the central statistical office.

"In selecting the chief statistician, there are two opposite possibilities. In either case the ultimate aim must be to combine, in this staff specialist, a broad, practical knowledge of the business with skill in statistical technique. This aim may be realized, in one way, by choosing a man already familiar with the business, who has an analytical mind and is capable of acquiring a knowledge of statistical methods, sufficient at least, to permit him to direct the work of statistical specialists of junior rank. The alternative procedure is to select a man of the proper general caliber, already skilled in statistics, and to give him an opportunity to familiarize himself with the details of the business, while at the same time he gradually develops the machinery of statistical control. A compromise plan is to make a temporary assignment of a man skilled in the business, with the future chief statistician working 'pro tem' as his assistant. Any one of these plans will give satisfactory results if it is kept clearly in mind that the qualities required in the chief statistician, in order of importance, are good judgment, analytical ability, knowledge of the business, and statistical skill.

"After the selection of the chief statistician and the assignment of the central statistical office to a definite position in the business, further developments in statistical organization will depend very largely upon the gradual education of departmental and other executives in the effective use of statistical studies.

Centralization of Control.—"The principle to be followed is that of centralization of statistical control in matters of methods and general policy, combined with decentralization in matters lying within the field of individual departments or organization units. Ordinarily it will be found that there are existing employees in each important unit of the organization who are doing work of a statistical or semistatistical nature and are relied upon by the executives of such units for much of the information upon which their day-by-day decisions are based. Such employees need not be designated as statisticians, but they should, nevertheless, be brought at a very early date into direct contact and cooperative relations with the chief statistician.

"In line with this principle of decentralization of statistical work, it is especially important to keep in mind the close relation which, as previously indicated, must exist between statistics and executive responsibility. The chief statistician must initially make many studies which, in a sense, will be critical of existing performance. His success will not be measured, however, by the immediate value of such criticisms, but by his ability to establish within the various units of the organization such methods of statistical control as may tend to obviate the need for criticism. The chief statistician must feel, therefore, that he has gained rather than lost, whenever a departmental or unit executive expresses a desire to have any portion of the detailed statistical work transferred to his immediate supervision.

"With a general plan adopted for the development of the statistical organization, many of the details of the actual statistical work and of the assignment of such work within the organization will settle themselves on the basis of day-to-day experience."[1]

Routing Data through an Organization for Use in Preparing Reports for the Executives.—In order to show how practically all of the various types of internal and external statistics previously described in this book may be gathered and used regularly in the management of a specific enterprise, a statistical routing diagram of a large manufacturing concern is presented in Exhibit 219.

The numbers in the small circles in the diagram refer to the various reports that are compiled. The topics of these reports are given in the list which accompanies the diagram.

[1] Rorty, M. C., "The Statistical Control of Business Activities," *Harvard Business Review*, Vol. I, No. 2, pp. 154–166, January 1923.

In following the data through the organization according to the diagram, it will be noted that the sales department, on the basis of its analyses of the market, sends forecasts of sales (in quantities) to the production department. Knowing something about the probable volumes of the different products that will be required, the production

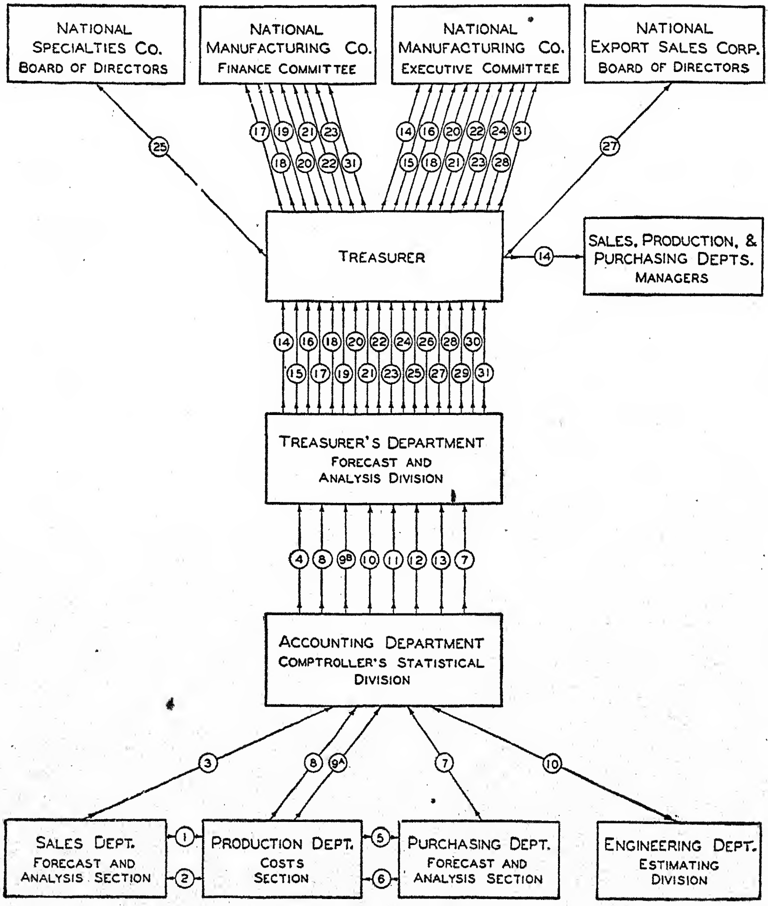

EXHIBIT 219A.—Routing of data through an organization for the purpose of preparing forecasts and financial reports to be submitted by the treasurer to the executives designated. For key to numbers, see Exhibit 219B. (*Names of companies changed.*)

department figures the mill cost of manufacture and forwards it to the sales department. Then the sales department is able to prepare report number 3 (forecast of selling price, freight and delivery, and selling expenses), which is forwarded to the comptroller's statistical division of the accounting department where the forecasts of the various departments are assembled and any necessary computations are made.

The production department sends report number 5 (a forecast of the consumption of materials and supplies in manufacture in quantities) to the purchasing department, and the purchasing department sends report 6 (forecast of material prices) to the production department, which uses it in computing costs. The purchasing department's forecast of pur-

KEY TO CHART

1. Forecast of Sales, quantities.
2. Forecast of Mill Cost of Manufacture.
3. Forecast of Selling Price, Freight and Delivery, and Selling Expense.
4. Forecasts assembled and computed. Sales in quantities and value; Mill Cost of Sales; Freight and Delivery, Selling Expense, Administrative and Net Receipts.
5. Forecast of Consumption of Materials and Supplies in Manufacture, quantities.
6. Forecast of Material Prices.
7. Forecast of Purchases, value.
8. Forecast of Production Capacity, units.

9*a*. Forecast of Consumption of Materials and Supplies in Manufacture as percentages of Mill Cost of Manufacture.

9*b*. Forecast of Consumption of Materials and Supplies in Manufacture, value.

10. Forecast of Construction, value.
11. Forecast of Investment (Working Capital in detail).
12. Forecast of Taxes, Bonus, Administrative, etc.
13. Actual figures (see detail).
14. Executive Committee Series, Books I and II.
15. Analysis of Executive Committee Series.
16. Variance of Forecasts.
17. Executive Committee Series, Summary—Pamphlets.
18. Cash Position by months, National Manufacturing Company.
19. Cash Position by months, National Manufacturing Company, Amendment.
20. Cash Position Condensed, National Manufacturing Company.
21. Capital Position, National Manufacturing Company.
22. Dividend Recommendation (after action by Finance Committee)
23. Consolidated Investment and Earnings Statement.
24. Bonus Calculation.
25. Forecast of Cash Position by months, National Specialties Company.
26. Forecast of Cash Position by months, National Specialties Company, Amendment.
27. Forecast of Cash Position by months, National Export Sales Corporation.
28. Forecast of Earnings, National Export Sales Corporation.
29. Treasurer's Book of Diagrams.
30. Forecast of Cash Position by months, National Building Corporation.
31. Special Reports—Intermittent.

Responsibility for items 1 to 10, inclusive, rests with General Statistical Committee; for items 11 to 13, inclusive, with Comptroller's Statistical Division; while items 14 to 31, inclusive, represent Reports prepared by the Forecast and Analysis Division of the Treasurer's Department, which are based on the Forecasted and Actual data covered by items 1 to 13, inclusive, together with additional information obtained within the Treasurer's Department.

EXHIBIT 219*B*.—Key to numbers on diagram shown in Exhibit 219*A* on opposite page.

chases in value (report 7), the production department's forecast of production capacity and consumption (reports 8 and 9*a*), and the engineering department's forecast of construction (report 10), all are sent to the comptroller's statistical division to be combined with other forecasts.

Then, after the proper combinations and computations are made, reports 4, 8, 9*b*, 10, 11, 12, 13, and 7 (see topical list in Exhibit 219*B*) are forwarded to the forecast and analysis division of the treasurer's depart-

ment. In that division, the internal reports are analyzed along with external data such as that furnished by the economic services and other statistical reporting agencies, and forecasts are prepared for reports 14 to 31, inclusive (see list in Exhibit 219*B*).

Reports 14 to 31 are then forwarded to the treasurer's office. After the treasurer's final approval, certain of the reports are submitted to the finance committee (see Exhibit 219*A*), others are submitted to the executive committee, one is forwarded to the directors and assistant directors of the sales production and purchasing departments, and others are submitted to the boards of directors of subsidiary companies.

In this manner, the company regularly compiles a series of statistical reports to be used as the bases of formulating many of its executive policies.

In carrying on a concern's statistical investigations and analyses, it is obvious that the statistical department (or others who do the statistical work) cannot work alone, but must cooperate closely with the other departments in the organization. This is true not only in gathering and analyzing internal data, but also in gathering and analyzing external data.

The Relation of the Statistical Department to the Management.—The responsibility of the statistical department (or division) as an aid to the management is obvious. Its position in the organization plans of different concerns, however, varies widely.

In Exhibit 219(*A* and *B*), responsibility for items 1 to 10, inclusive, rests with the central statistical committee; for items 11 to 13, inclusive, with the comptroller's statistical division; while items 14 to 31, inclusive, represent reports prepared by the forecast and analysis division of the treasurer's department. In other concerns, the statistical department may be under the direct charge of a vice president, an assistant to the president, the comptroller, or other responsible official, but wherever the responsibility is placed, the statistical staff must not only collect, tabulate, and chart the data, but also it should apply methods of analyzing and interpreting the data through the use of such devices as averages, measures of dispersion, computation of composite index numbers, determination and elimination of trend and seasonal influences, and the measurement of correlation. And, finally, the results must be presented in such a manner that the significant points are easily seen and understood by those for whom the investigations and analyses are made.[1]

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[1] After the data have been gathered and analyzed, careful attention must be given to the presentation of the material. Often good data and good analyses are not appreciated because they are not presented simply and clearly. When long, detailed tables are presented with a report, it is usually well to prepare summary tables which can be included in the text of the analysis, by which the executive can see the signifi-

Proper analysis of the facts collected in the field of general business or in a particular industry requires much study and experience. The development of a staff trained both in economic principles and in statistical technique is, therefore, an important part of business management.

The relation of the statistical department to the management also depends to a considerable extent upon how well the executives understand the use of statistics and statistical analyses. While statistical reports are successfully used by many executives who have not had technical training, it is true, nevertheless, that the better the executives of the business understand the statistical methods and recognize the economic principles which are employed by the statisticians in their analyses, the better they will be able to apply the statistical information gathered. The factors to be recognized and the methods of interpreting them are so numerous that blind acceptance of the conclusions of any one analyst is not to be advocated. The executives who are responsible for directing an organization must understand how the results are obtained, at least in a general way, if they are to get the greatest value from statistical data and analyses.

As mentioned from time to time in the various chapters of this book, the increasing competition among business institutions makes more accurate decisions necessary if a concern is to prosper. Consequently, in most business concerns that have real statistical departments, the statistician's problems are those which involve questions of management. This close relation of the statistical department to the management presents both a challenge and an opportunity for practically minded business statisticians to work directly with senior and junior executives in improving the bases upon which the management makes its important decisions.

What the business man is interested in is the results, but they must be presented in such a way that he can understand the procedure well enough to check them in a general way with his own judgment and experience. If a trend has been fitted by the method of least squares, for instance, it may not be necessary that the executive understand how to calculate the line, but he must ordinarily have a general appreciation of the nature of long-time trend. When the executive cannot use the statistician's work, it may not be the fault of statistics but the fault of the statistician. Too often there is a lack of cooperation because the

cant points at a glance. Charts and maps are valuable aids in presenting discussions. Ordinarily the reports should be neatly typed on a good paper and bound in a suitable cover. Practically speaking, a good appearance is impressive and is often taken as an index of the care and skill that has been used in the other work connected with making the report. Further information on preparing statistical reports for business executives is given in the appendix (see p. 713).

statistician does not know enough about matters other than statistics, and he is considered an outsider by those familiar with the practical side of the business.

The difference between the statistician's profession and such professions as chemistry, engineering, or law must be thoroughly appreciated by both the statistician and the business man. The statistician's work ordinarily involves questions of management, with which the executive feels that he himself is more familiar than anyone else, while, on the other hand, he may not feel familiar enough with such fields as chemistry, engineering, or law to make authoritative decisions. The executive may not understand chemistry, but he will accept the statement of a skilled industrial chemist that a certain process will produce a desired result, for the chemist knows that a certain reaction will take place and he can prove his process experimentally. The business statistician, however, has no such direct appeal to the confidence of the executive; his results are usually much less definite and depend upon the reactions of human beings who often are affected differently by the same stimuli. What the business statistician does is to interpret the meanings of figures relating to matters with which the executive has had long and intimate contact. Consequently, the executive is not satisfied unless he can understand in a general way how the statistician reaches his findings.[1]

Although the situation has improved materially during the last few years, a better understanding of practical usableness by the students who are to become statisticians, as well as by the students who are to become executives, will do much to eliminate the poor cooperation or lack of cooperation which still exists in many instances. The statistician must use his methods in such a way and present his results in such a way that the general processes and solutions will be intelligible to those who are to base their decisions upon them. When it is imperative that highly refined methods be used, the statistician must see that his technique does not obscure his results. In the last analysis, the value of statistics depends upon the use that can be made of them.

Cooperation Necessary.—"As has previously been indicated, any successful program must be based upon a policy of gradual development, with the closest of cooperation between statistical and administrative forces. Each subordinate executive must feel that he is directing, and to a large degree is planning, his own statistics. The most fatal of all steps is to attempt to impose statistical control as a supermanagement over a skeptical and unwilling organization.

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[1] See Leonard P. Ayres, "The Dilemma of the New Statistics," Presidential Address, Eighty-eighth Annual Meeting, American Statistical Association, December 29, 1926; published in the *Journal of the American Statistical Association*, March 1927.

"Beyond this initial requirement, there is the need for a continuing practical touch in every angle of the statistical work. The actual reports and analyses will vary in type from business to business, and from month to month in any given business, but the basis for all internal statistics must be accurate and properly filed records of current and prospective activities. It is much more important to know just where and how fundamental information may be obtained, when needed, than it is to multiply routine reports and compilations. Ordinarily it will be found that the primary records need only the most inexpensive kind of revision to make them thoroughly adaptable to all further requirements.

"With this solid background of accurate records once established, it is possible to simplify greatly the reports which form the basis for routine statistical control. Each analysis, to justify itself as a routine report, must have a definite relation to executive action, and its indications must in every case be positive, either that conditions are satisfactory or that they are unsatisfactory, and, if the latter, *where* and *why*. Furthermore, the frequency of each report must be adjusted to executive requirements.

Multiplying Executive Power.—"The final test of the value of statistical control will lie in its success in multiplying the power and increasing the effectiveness of the responsible executives. It is a basic principle in business economics that the efficient size of an enterprise is determined by the point of balance between the increasing efficiency of processes and methods, and the decreasing efficiency of the human element, with growth in magnitude. When the volume of business is once great enough to permit full efficiency in processes and methods, further growth is ordinarily limited by the ability of a small group of active managers to spread their personal influence and enthusiasm throughout the organization.

"It is here, in particular, that properly planned statistics have their greatest usefulness. If the executive will ask himself specifically and in detail what items and conditions he must watch and when he must act—if he will take each recurring problem and determine what facts are necessary for his decisions—it will not be difficult for a trained statistician to set up a series of periodic or special reports of such sort as to make the majority of these decisions a matter of relatively simple routine.

"This picturing of a whole business so that the essentials stand out sharply from the mass of detail, is the essence of statistical control. Charts, tabular statements, and narrative comment may be combined according to the taste of the executive. Many experiments must be made and abandoned before satisfactory results are finally obtained. There will always be new problems to be met and new conditions to be analyzed. But, in the end, the executive who perseveres will find a new range of vision in his business, a new length of arm and certainty of

hand in his management of affairs, and a new power and responsiveness to brake or throttle in his whole organization."[1]

Questions and Problems

1. What is meant by statistical control in business?
2. What are the four main types of internal organization of business, and why must these be distinguished in considering executive control and management statistics?
3. What is the relation of the chief executive to the internal organization?
4. Why is it necessary to distinguish between statistics and accounts in statistical control?
5. In order to control effectively the operations of the business, what essentials are necessary and why?
6. Of what significance are comptroller's reports for the chief executive? What should these reports include?
7. Describe operating department reports for the chief executive.
8. How can plans of operation be compared with actual performance?
9. How can actual results be compared with standards of performance?
10. What types of external statistics are of value to the management of a concern? Explain in detail.
11. Why may all of the different classes of statistics discussed in this book be considered as "management statistics"?
12. Discuss analysis of competitive strength through comparing costs of doing business.
13. Discuss the development of a statistical organization.
14. Are statistical studies practical for small concerns? How? Why?
15. Discuss centralization of statistical control.
16. Explain how data may be routed through an organization for the purpose of preparing executive reports.
17. Explain the relation of the statistical department to the management.
18. Is the statistician's advice ordinarily accepted by the business executive in the same manner as the advice of an attorney at law, a mechanical engineer, or an industrial chemist? Explain.
19. Why is cooperation necessary throughout an organization if there is to be effective statistical control?
20. How does statistical control "multiply executive power"?
21. List and describe a set of reports which would be of practical value to the executives of an organization with which you are familiar.
22. Prepare a diagram for routing through the organization the data necessary for the reports listed in answer to Problem 21.
23. Prepare a detailed plan for organizing a statistical department in a certain concern in your city which will cooperate with you in furnishing information.

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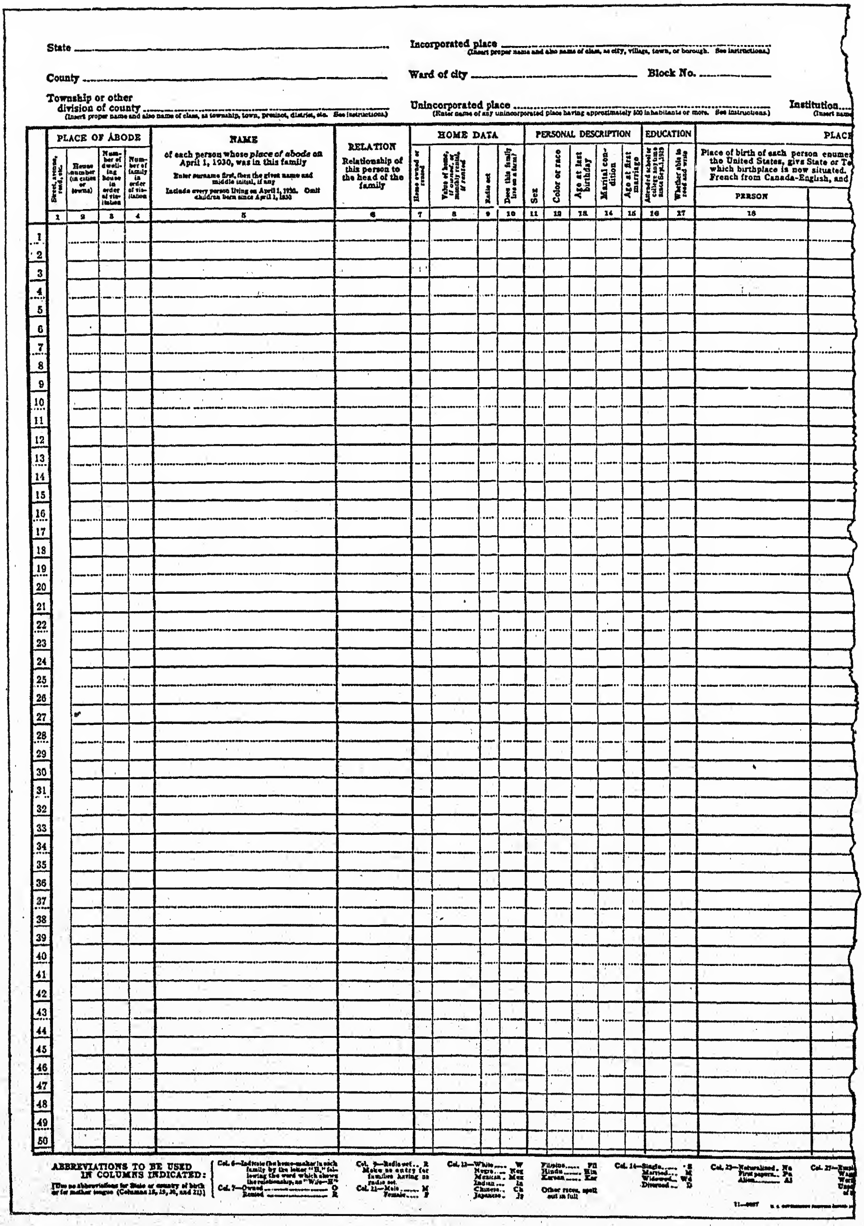
[1] Rorty, M. C., "The Statistical Control of Business Activities," *Harvard Business Review*, Vol. I, No. 2, pp. 154–166, January 1923.

APPENDICES

APPENDIX I

SCHEDULE AND QUESTIONNAIRE FORMS

The schedules and questionnaires in this section are presented for the purpose of supplementing Chapter II on Collection of Data. When it is desired to prepare schedule or questionnaire forms, it is ordinarily helpful if one can refer to representative but varying examples that have been used in actual practice. It is from this point of view that the six exhibits, which are presented on the following pages, have been selected. (These exhibits are numbered from 220 to 225 inclusive, and are self-explanatory.)



State ....

County ....

Township or other division of county ....
(Insert proper name and also name of class, as township, town, precinct, district, etc. See instructions.)

Incorporated place ....
(Insert proper name and also name of class, as city, village, town, or borough. See instructions.)

Ward of city .... Block No. ....

Unincorporated place ....
(Enter name of any unincorporated place having approximately 100 inhabitants or more. See instructions.)

Institution....

PLACE OF ABODE				NAME	RELATION	HOME DATA				PERSONAL DESCRIPTION					EDUCATION		PLACE
Street, avenue, road, etc.	House number (in cities or towns)	Number of dwelling house in order of visitation	Number of family in order of visitation	of each person whose *place of abode* on April 1, 1930, was in this family. Enter surname first, then the given name and middle initial, if any. Include every person living on April 1, 1930. Omit children born since April 1, 1930	Relationship of this person to the head of the family	Home owned or rented	Value of home, if owned, or monthly rental, if rented	Radio set	Does this family live on a farm?	Sex	Color or race	Age at last birthday	Marital condition	Age at first marriage	Attended school or college any time since Sept. 1, 1929	Whether able to read and write	Place of birth of each person enume[rated] ... the United States, give State or Te[rritory] ... which birthplace is now situated. ... French from Canada-English, and ... PERSON
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

ABBREVIATIONS TO BE USED IN COLUMNS INDICATED:

Exhibit 220.—An enumerator's schedule for the 1930 population census

Form 15-6

DEPARTMENT OF COMMERCE—BUREAU OF THE CENSUS

FIFTEENTH CENSUS OF THE UNITED STATES: 1930

POPULATION SCHEDULE

Enumeration District No. ____________

Supervisor's District No. ____________

Sheet No. ____A.

of institution, if any, and indicate the lines on which the entries are made. See instructions.)

Enumerated by me on ____________, 1930, ____________, *Enumerator.*

OF BIRTH		MOTHER TONGUE (OR NATIVE LANGUAGE) OF FOREIGN BORN				CITIZENSHIP, ETC.			OCCUPATION AND INDUSTRY				EMPLOYMENT		VETERANS			
…ated and of his or her parents. If born in …rritory. If of foreign birth, give country in (See Instructions.) Distinguish Canada-Irish Free State from Northern Ireland		Language spoken in home before coming to the United States	CODE (For office use only. Do not write in these columns)			Year of immigration to the United States	Naturalization	Whether able to speak English	OCCUPATION Trade, profession, or particular kind of work, as *spinner, salesman, riveter, teacher, etc.*	INDUSTRY Industry or business, as *cotton mill, dry-goods store, shipyard, public school, etc.*	CODE (For office use only. Do not write in this column)	Class of worker	Whether actually at work yesterday (or the last regular working day)		Whether a veteran of U. S. military or naval forces		Number of farm schedule	
FATHER	MOTHER		State or M T	Country	Native								Yes or No	If not, line number on Unemployment Schedule	Yes or No	What war or expedition?		
19	20	21	A	B	C	22	23	24	25	26	D	27	28	29	30	31	32	
																		1
																		2
																		3
																		4
																		5
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																		49
																		50

…oyer .... E
…or salary worker ... W
…ing on own account. O
…d worker, member …e family .... NP

Col. 31—World War .... WW
Spanish-American War .. Sp
Civil War Civ
Philippine Insurrection .. Phil
Boxer Rebellion .... Box
Mexican Expedition .... Mex

ENTRIES ARE REQUIRED IN THE SEVERAL COLUMNS AS FOLLOWS:

Cols. 6, 11, 12, 13, 14, 16, 18, 20, 20, and 25—For all persons.
Cols. 7, 8, 9, and 10—For heads of families only. (Col. 8 requires no entry for a farm family.)
Col. 15—For married persons only.
Col. 27—For all persons 10 years of age and over.
Cols. 21, 22, and 23—For all foreign-born persons.
Col. 24—For all persons 10 years of age and over.
Cols. 26, 27, and 28—For all persons for whom an occupation is reported in Col. 25.
Col. 30—For all males 21 years of age and over.

(reduced ⅔). (On the reverse side is space for an additional 50 names.)

MARKET SURVEY

AREA NO.______

Section No.	STORE ROOMS Total Units 1930	No. Vacant '29	No. Vacant '30	OFFICE BUILDINGS Total Rentable Area (sq. ft.) 1930	Vacant Rentable Area (sq. ft.) 1930

TO THE OWNER OR MANAGER:

The bearer, Mr ______________________ is assisting the Real Estate Board in conducting a city-wide census of occupancy in various types of buildings. Please co-operate in this survey by giving the requested information. This information will be treated as strictly confidential and will be used only in tabulating the figures for the city-wide results.

REAL ESTATE BOARD OF KANSAS CITY, MO.

By: ______________________
EXECUTIVE SECRETARY

REAL ESTATE BOARD R.E.B.K.C. KANSAS CITY

EXHIBIT 221.—An enumerator's schedule and identification card, Kansas City occupancy (or vacancy) survey. (*Courtesy, Real Estate Board of Kansas City, Missouri.*)

DEPARTMENT OF COMMERCE

BUREAU OF STANDARDS

WASHINGTON

ADDRESS REPLY TO
BUREAU OF STANDARDS

September 21, 1929

IN YOUR REPLY
REFER TO FILE NO.

Name
Position
Address

Dear Sir:

The Department of Commerce and the National Bureau of Economic Research have undertaken a joint study of the extent to which Federal, State and local public construction (including quasi-public institutions such as endowed universities and colleges) can be adjusted as a practical matter to help relieve unemployment in periods of depression and to contribute to business stability in general. In this connection we would like to have the following information from your office:

1. For the purpose of taking advantage of depression periods in the business cycle, do you attempt to accelerate or retard
   (a) new construction?__________
   (b) repairs and maintenance?__________

   Remarks:______________________________
   ____

2. For the purpose of taking advantage of slack seasons in various lines of business and employment, do you attempt to accelerate or retard
   (a) new construction?__________
   (b) repairs and maintenance?__________

   Remarks:______________________________
   ____

3. New construction, fiscal year ending ______, 1929 $__________
   " " " " " ______, 1928 $__________
   " " " " " ______, 1927 $__________
   " " " " " ______, 1926 $__________
   " " " " " ______, 1925 $__________

4. Repairs & maintenance, fiscal yr. ending ______, 1929 $__________
   " " " " " ______, 1928 $__________
   " " " " " ______, 1927 $__________
   " " " " " ______, 1926 $__________
   " " " " " ______, 1925 $__________

As the time allotted to us for making this study is short, your cooperation is requested in returning this questionnaire in the accompanying franked envelope which requires no postage, so that it may reach us not later than October 5.

Yours very truly,
(signature)

Form PW8

Division of Building and Housing.

EXHIBIT 222.—A combination letter and questionnaire. When both letter and questionnaire can be short, the combination form is ordinarily the most desirable.

JOSEPH RICHARDS COMPANY
INCORPORATED
247 PARK AVENUE • NEW YORK

OFFICE OF THE
DIRECTOR OF RESEARCH

April 23, 1925.

To the
Lady of the House.

Dear Madam:

Won't you do us a favor?

We are called upon from time to time to answer many questions about household products and we have always found that the best way to get such information is to put the matter up to the woman of the house.

So, if you will be good enough to answer the few short questions on the inside page we shall be a thousand times obliged to you. Since your answers will be used for statistical purposes only you need not sign your name. If you wish to do so, however, you may rest assured that it will be held strictly confidential.

Let us thank you in advance for any help you may give us.

Cordially,

Stanley I Clark

FM

Director of Research.

EXHIBIT 223*A*.—Letter to accompany questionnaire in Exhibit 223*B*. This letter frankly asks a favor, which is a better method than to make an attempt to offer something of doubtful interest in return for the information, or to give an unconvincing reason why the recipient should fill out the questionnaire. (*Courtesy, Richardson, Alley & Richards Company.*)

QUESTIONNAIRE

Even though your experience with window shades may be very limited, we would greatly appreciate your answering as many of the questions as possible and returning them to us.

What make or makes of window shades are you now using?

....

....

When you last purchased window shades what make did you ask for?

I asked for..

I did not ask for any make..

If you did not ask for any make, why did you not do so?

....

....

Which do you consider more important, the cloth or the roller (check one).

Cloth............................ Roller............................

Would you expect equal service and satisfaction from two different-priced window shades made by the same manufacturer?

Yes............................ No............................

With what makes of window shades or rollers are you familiar?

Shades	Rollers
....	....
....	....
....	....

Please check (✓) that particular make of window shade and roller which you like best.

What are your three favorite magazines?

First..

Second..

Third..

Do you live in the city or the suburbs—apartment or private house?

City............................ Apartment............................

Suburbs............................ Private house............................

If you own an automobile, what make is it?

Our car is a..

We do not own a car..

EXHIBIT 223*B*.—Questionnaire to accompany letter shown in Exhibit 223*A*.

Dear Sir:

With the object of securing more accurate information regarding the present relation of supply of money to volume of house building, we are sending this inquiry to a representative group of financial institutions and builders. Your courtesy in furnishing the information requested on the attached card will be greatly appreciated.

If you wish a copy of the report prepared on this investigation, kindly make a check mark in the space provided on the attached card.

John Doe & Co.

May 15, 1931

FIRST CLASS
PERMIT No. 10
(Sec. 384½ PL & R)
BOSTON, MASS.

ONE
CENT
POSTAGE

BUSINESS REPLY CARD
NO POSTAGE STAMP NECESSARY IF MAILED IN THE UNITED STATES

2c.-POSTAGE WILL BE PAID BY—

JOHN DOE & CO.
1234 MARKET ST.,
BOSTON,
MASS.

esident
ings Bank
falo, N. Y.

In my opinion, money for small home construction is as indicated below:

For first mortgages:	For second mortgages:
( ) very plentiful	( ) very plentiful
( ) plentiful	( ) plentiful
( ) scarce	( ) scarce
( ) very scarce	( ) very scarce

Remarks:____________________________

( ) Send me a report on your findings.

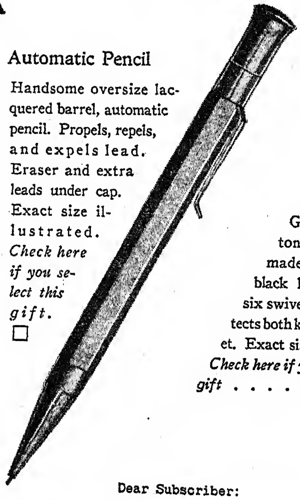
( ) I do not care to see the report.

Name:______________

Address:______________

EXHIBIT 224.—Two sides of a double post card—postage on return portion to be paid by addressee. This is a handy form, from the point of view of both sender and receiver, for certain types of short questionnaires. Business reply cards may also be enclosed with letters. The return postage on these cards is 2 cents by ordinary mail and 7 cents by air mail. (On business reply envelopes, the postage is 4 cents by ordinary mail and 7 cents by air mail.)

A

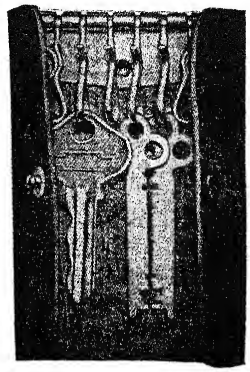


Automatic Pencil

Handsome oversize lacquered barrel, automatic pencil. Propels, repels, and expels lead. Eraser and extra leads under cap. Exact size illustrated. *Check here if you select this gift.* ☐

Key Case

Genuine Buxton Keytainer, made of selected black leather, with six swivel hooks. Protects both keys and pocket. Exact size illustrated. *Check here if you select this gift* . . . . ☐



December 1929

Dear Subscriber:

We know from past experience that Successful Farming subscribers want to co-operate with us in making the magazine increasingly better each year.

In laying final plans for 1930, we want for our guidance some general information about the Successful Farming family, so we are asking some of our readers to give us their replies to the questions in this folder.

A large number of the questions are just as they will appear in the 1930 Census schedule, which you will be required by the government to fill out within a few months. We are, therefore, particularly anxious to have your replies so we can know how our subscribers compare with farmers in general.

NOW THEN—We need this information just as soon as you can get it to us, and if you will fill out and return this folder so it will reach us before the end of December we will send you, as evidence of our appreciation, either one of the gifts illustrated above. The enclosed envelope does not need to be stamped.

Thank you.

JTM.RK

SUCCESSFUL FARMING.

Please mail the gift I have selected to me as follows:

Name____________________Town____________________R.F.D.__________State__________

(Be sure to write name and address clearly so your reward will reach you)

BMA-175-A

Exhibit 225.—A definite reward is offered to get recipients to fill out this questionnaire. Unusually long questionnaires, especially when they inquire into personal affairs to the extent that this example does, ordinarily require that something of appreciable value be offered if adequate returns are to be received. (*Courtesy, Successful Farming.*)

How many acres are there in your farm:..................... Acres How many acres are in crop land?..................... Acres In pasture? (All land used for pasture which could be plowed and used for crops without clearing, draining, or irrigating)............... Acres In all woodland?............... Acres In all other land?............... Acres How many acres in your farm do you own?..................... Acres How many acres do you rent from others?..................... Acres Do you rent from an owner to whom you are related?..................... Yes or No Do you operate your farm for others as a hired manager?..................... Yes or No What is the total value of your farm, that is, the amount for which it would sell? $............... What is the value of *all* buildings on your farm? (Included in previous question) $............................ What is the value of your home? (Included in previous question) $............................ What is the value of the implements and machinery on your farm, including automobiles, trucks, and tractors? $............................

Please give us the following information about the equipment on your farm:

	Number Owned	Year Purchased	Makes Owned	Check If You Expect to Buy Within 2 Years	Makes Expect to Buy
Automobile					
Truck					
Tractor					
Stationary Engine					
Combine					
Manure Spreader					
Electric Farm Lighting Plant					
Other Farm Lighting Plant					
Electric Refrigeration					
Milking Machine					
Cream Separator					
Radio					

What are the sizes of your trucks?..................... Tons Please check each of the following which you have in your home: Running water to the kitchen ☐ Running water to bathroom ☐ Highline electricity ☐ Telephone ☐ Will you please list the electric household equipment you have?..

....

How many head of animals do you own? Horses 2 years old and over?..................... Number Mules 2 years old and over?..................... Number Cows and heifers 2 years old and over kept mainly for milk production?............... Number Dairy cattle under 2 years of age?............... Number Beef cattle?............... Number Hogs?............... Number Sheep?............... Number Chickens?............... Number How many cows and heifers, both beef and dairy types, did you milk during all, or any part of 1929?..................... Number

(Please answer also questions on other pages)

EXHIBIT 225.—(*Continued.*)

Will you please give us the following information about the crops raised on your farm in 1929:

	Acres Harvested in 1929	Quantity Harvested in 1929 *Bushels*	Quantity Sold or to be Sold *Bushels*
Corn			
All wheat			
Oats			
Barley			
Rye			
Clover		Tons	Tons
Alfalfa		Tons	Tons
All other hay		Tons	Tons
Irish or white potatoes			
All other vegetables (Give *value* harvested and sold for these items)		$	$
All other crops (Give *value* harvested and sold for these items)		$	$

Do you use lime?........ (Yes or No) How many tons in 1929?........ (Tons) Do you use commercial fertilizers?........ (Yes or No) How many tons in 1929?........ (Tons) What was the total value of all fruits sold in 1929? $........ What was the value of all forest products (lumber, posts, etc.) sold in 1929? $........

Please give us the following information about your buildings, and building plans for the next two years:

	How Many of Each Building Have You	Check the Buildings You Are Planning to Paint Within 2 Years	Check the Buildings You Are Planning to Re-roof Within 2 Years	Check the Buildings You Are Planning to Erect Within 2 Years	Materials Will Use: (Lumber, Brick, Hollow Tile, Cement and Stone)
House					
Barn					
Poultry House					
Hog House					
Silo					
Garage					
Machine Shed					
Milk House					
Granary					
Corn Crib					
Other Buildings					

Check your preference for wall finish in your new home: Wall paper ☐ Tinted walls ☐ Painted walls ☐ What type of water system do you have?........ (Type) What make?........ (Make)

Are you planning to buy a new water system within two years?........ (Yes or No) What type?........ (Type)

What make?........ (Make)

(Please answer also questions on other pages)

EXHIBIT 225.—(*Continued.*)

What was the total production of milk on your farm in 1929?............ (Gallons) How much did you sell as whole milk?............ (Gallons)
Value? $............ How much cream did you sell as butterfat in 1929?............ (Pounds of Butterfat) Value? $............ How much
cream did you sell *not* as butterfat in 1929?............ (Gallons) Value? $............ How much butter did you churn in
1929?............ (Pounds) How much did you sell?............ (Pounds) Value? $............ Please give the number of *registered*
purebred animals owned: Dairy cattle?............ (Number) Beef cattle?............ (Number) Horses?............ (Number) Hogs?............ (Number)
Sheep?............ (Number) What modern equipment do you have in your barn? Steel stanchions ☐ Ventilators ☐ Water bowls ☐

Please give us the following information about your sales and slaughter of livestock:

	Live Animals Sold		Animals Slaughtered on Your Farm for Home Use or for Sale	Value of Meat Sold
	Number	*Value*	*Number*	
Calves under 1 year of age......................	....	$............	....	$............
Beef cattle (excluding calves)...................	....	$............	....	$............
Dairy cattle (excluding calves)..................	....	$............	....	$............
Hogs and pigs................................	....	$............	....	$............
Sheep and lambs.............................	....	$............	....	$............
Horses and colts..............................	....	$............		
Mules and mule colts..........................	....	$............		

Do you haul your livestock to market in your own trucks?............ (Yes or No) If so how long is your average haul?............ (Miles)
Do you hire your livestock trucked to market?............ (Yes or No) If so, how long is the average haul?............ (Miles) How many
chicken eggs were produced on your farm in 1929?............ (Dozens) How many did you sell?............ (Dozens) Value? $............
How many chickens did you raise in 1929?............ (Number) How many of these did you buy as baby chicks?............ (Number)
Do you use a commercial chick starter?............ (Yes or No) What brand?............ (Brand) How many chickens did you
sell alive or dressed in 1929?............ (Number) Value? $............ What was the total value of honey sold in 1929? $............
What was the total value of wool sold in 1929? $............ Please check the kind of road which adjoins your farm:
All-weather ☐ Improved dirt ☐ Unimproved dirt ☐ If on dirt, how far are you from an all-weather road?............ (Miles)
Are you a member of a co-operative marketing or purchasing organization?............ (Yes or No) Are you a member of a cow
testing or herd improvement association?............ (Yes or No) What editorial subjects in *Successful Farming* are most interesting
and helpful to you and your family?..

....

....

We would like to have you use this space to tell us of any improvements you think could be made in *Successful Farming*
to make it more helpful to you..

....

....

....

(Please answer also questions on other pages)

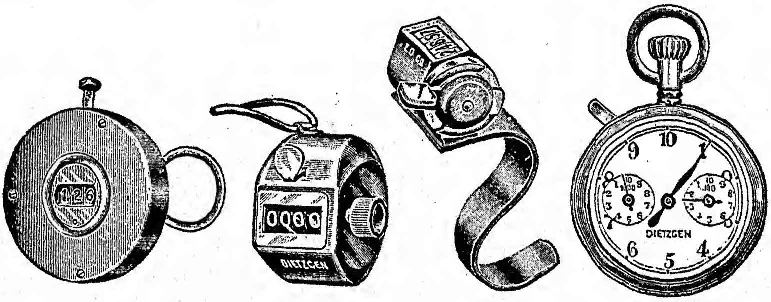
EXHIBIT 225.—(*Concluded.*)

APPENDIX II

MECHANICAL AIDS IN STATISTICAL WORK

The usual problem in business statistics requires the handling of so many figures that long-hand methods are too laborious and too expensive to be practical in carrying out such processes as counting, adding, computing, sorting, and tabulating. It is, therefore, essential that one appreciate the use of certain mechanical devices which perform these operations with far greater rapidity than they can be performed by human hands.

It is the purpose of this appendix to describe briefly several machines (some of which have been mentioned previously) which assist in collecting, sorting, tabulating, and computing. While the machines described are representative examples of those that are in common usage, many other valuable mechanical devices are also available. The reader will find many good suggestions on this subject by referring to the advertisements in business magazines.



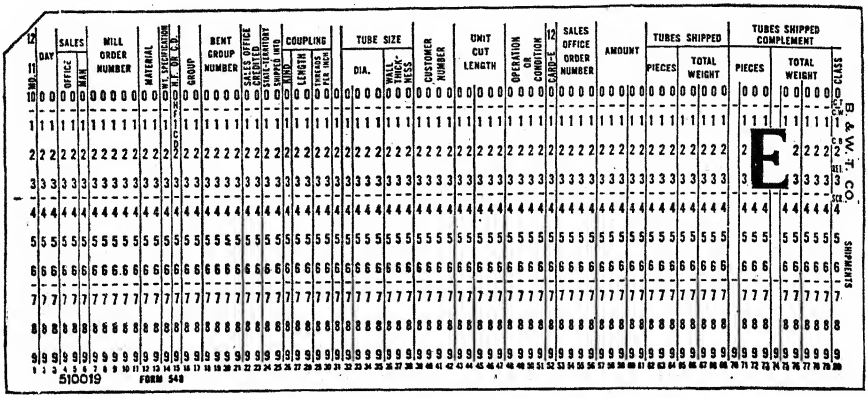
A *B* *C* *D*

EXHIBIT 226.—Counting or tallying machines. *C* is designed to fit the fingers of the left hand. *A*, *B*, and *D* may be held in either hand. The watch pattern machine (*D*) is operated by pressing the stem knob. (*Courtesy, Eugene Dietzgen Company.*)

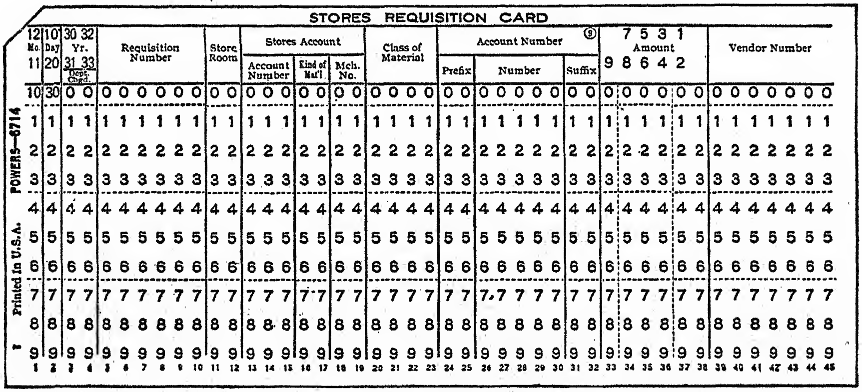
Counting Devices—the Tallying Machine.—In making traffic counts of automobiles, pedestrians, etc., in connection with such problems as the locating of stores or display rooms, a counting device known as the "tallying machine" or "counting clock" (Exhibit 226) is very useful. With this device, each vehicle or pedestrian can be enumerated by the observer by pressing the knob on the tallying machine without diverting his attention from the objects to be counted. As traffic counts require quick observation and enumeration, this simple device materially assists in obtaining complete and accurate observations.

In tabulation work, the tallying machine is often used in counting individual items which are recorded on schedules or questionnaires. In many instances, this is a more rapid method than using the tally sheet, which was described in connection with Exhibit 6 on page 50.

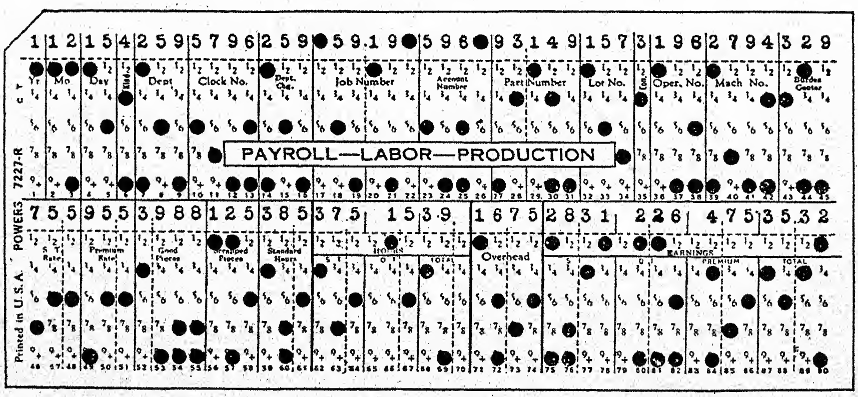
Sorting and Tabulating Machines.—When large masses of data have been collected and recorded, it is often necessary to sort and tabulate the items according to



A.—An I.B.M. 80-column "shipments" tabulating card.



B.—A Powers 45-column "stores requisition" tabulating card.



C.—A Powers 90-column "pay roll and production" tabulating card.

EXHIBIT 227.—Code cards used with I.B.M. and Powers tabulating equipment. (Full size of cards is $3\frac{1}{4}$ by $7\frac{3}{8}$ inches.)

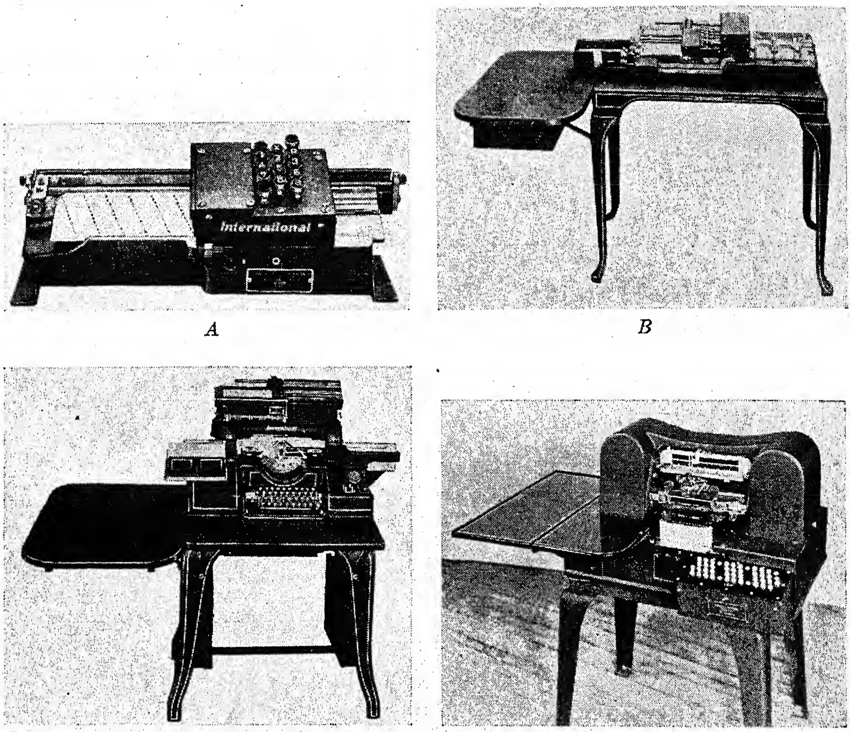
many and various classifications. As mentioned in Chapter IV (pages 53 and 54), sorting and tabulating machines are manufactured by The Electric Bookkeeping and Accounting Machine Division of the International Business Machines Corporation, and the Powers Accounting Machine Division of Remington Rand, Inc. Throughout the remainder of this discussion, the tabulating systems and equipment of these concerns will be referred to as "I.B.M." and "Powers," respectively. I.B.M. machines are not sold outright but are leased on a monthly contract basis. Powers machines are both leased and sold outright.

The principle upon which sorting and tabulating machines operate is similar to that of the player-piano. Everyone is familiar with the parchment rolls that operate player-pianos by means of holes punched on the rolls according to the notes of the composition. For mechanical sorting and tabulation, statistical data and other information which can be reduced to a numerical code are recorded on especially prepared tabulating cards by means of punched holes. The position of a hole in a card represents a figure. An electrical contact is made through the hole in the case of I.B.M. equipment, and a mechanical plunger passes through the hole in the case of Powers equipment. On the basis of the position of the contact or plunger, the cards are sorted and tabulated mechanically. Some of the tabulating machines list as well as add the data and count the cards, whereas other machines only count the number of items (or cards) and add the data according to different combinations selected. In the latter machines, the results are read from the machines, whereas, in the former, permanent listings of the items and the results are obtained.

The 80-column card, such as the one illustrated in Exhibit 227*A*, is now commonly used in the I.B.M. system, but 45- and 34-column cards are also available. In the Powers system, both 45- and 90-column cards are used, see Exhibit 227*B* and *C*. In referring to the 90-column card (Exhibit 227*C*), it will be seen that there are only six positions available for each card column. Therefore, in order to provide for the standard 10 digits (0 to 9, inclusive), numerical designations are expressed both by single-hole punchings and by combined-hole punchings. That is, the 0 and odd digits (1, 3, 5, 7, and 9) are single-hole punched, and the even digits (2, 4, 6, and 8) are combined-hole punched. Referring to the $^1_2$ position at the top of a column, if it alone is punched, the single hole will sort and tabulate on 1, but if in addition the cross (+) at the bottom of the column is punched the card will sort and tabulate on 2. This system will be made clear by a study of Exhibit 227*C*, which shows how the numbers typewritten at the top of the columns can be punched on the card.

Various kinds of machines are furnished for punching and tabulating cards. Some of the more important types are illustrated in Exhibit 228. In using the I.B.M. electric key punch, the operator presses the key for each column and as each column is punched the card is automatically moved into position for the next column punching. Pressing the keys on the Powers punch sets the column punches in position, and then, when all punches are in the proper position, the holes are made in one operation. The electric duplicating key punch is used to duplicate all data common to more than one card, such as date, branch, and salesmen. In this machine, the cards are fed from the magazine to the punching mechanism and the holes are automatically punched, duplicating data from selected columns of a card to be copied, prior to punching the remaining data by hand operation. The alphabetic punches (Parts *C* and *D*, Exhibit 228) are arranged to punch names and words as well as numbers, by means of a code. When the cards are run through the alphabetic printing tabulator, the names of the customers, or employees, etc., are printed on the tabulation sheets. The Powers punch, illustrated in Exhibit 228, Part *D*, automatically punches the one or two holes required for indicating odd and even digits on a 90-column card.

Sorting machines are illustrated in Exhibit 229. The cards are automatically placed on the conveyor which carries them past the individual pockets until they arrive at the particular pocket into which they are to be dropped according to the holes punched in the column for which the sorting gage has been set. The I.B.M. sorter (Part *A*, Exhibit 229) is available in two models. One model operates at a speed of 400 cards per minute and the other sorts 225 cards per minute. Two Powers sorting machines are available—one which sorts 45-column cards only, and one which



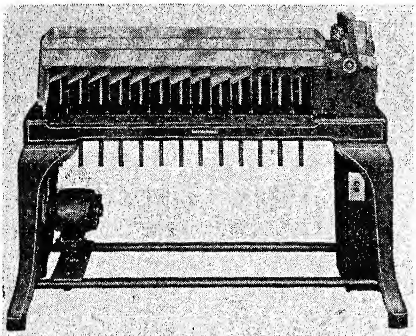
A B C D

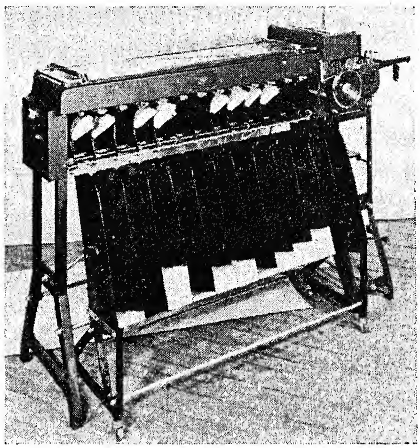
Exhibit 228.—Machines for punching tabulating cards. *A*. Electric key punch. *B*. Motor-drive duplicating key punch. *C*. Alphabetic printing punch. *D*. A 45- and 90-column alphabetic punch. (*A, B, and C, courtesy, International Business Machines Corporation; D, courtesy, Remington Rand, Inc.*)

sorts both 45- and 90-column cards, see Exhibit 229, Part *B*. The Powers machines sort 420 and 250 cards per minute.

The I.B.M. tabulator, designated as the "Electric Accounting Machine" (Part *A*, Exhibit 230), is a multiple adding, subtracting, classifying, and printing machine, which, actuated by punched tabulating cards, produces printed tabulations. It will list details from individual cards or it will print designations and totals, net totals, and accumulated net totals. On the model shown, group totals from as many as five counters can be taken simultaneously, or seven banks of information may be listed at one time, accompanied by five totals. The machine capacities for listing and accumulating vary with the individual models. The machine can be instantly transformed into a "lister" which prints in itemized detail the information recorded on

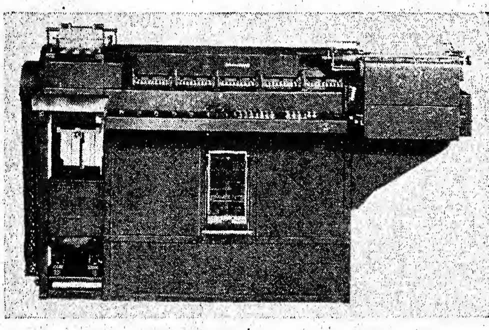
the cards, and at the same time accumulates the desired quantities, which totals are then printed in their respective positions. The machine is equipped with an auto-

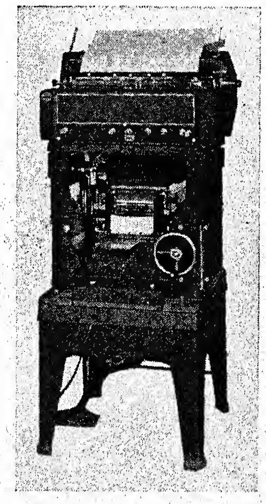




A B

EXHIBIT 229.—Horizontal sorting machines. *A*. I.B.M. sorting machine. *B*. Powers sorting machine. (*A*, *courtesy*, *International Business Machines Corporation; B*, *courtesy*, *Remington Rand*, *Inc.*)





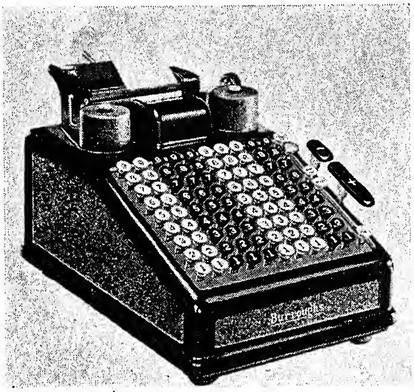
A B

EXHIBIT 230.—Tabulating machines. *A*. An I.B.M. tabulator ("Electric Accounting Machine"). *B*. A Powers 45- and 90-column tabulator. (*A*, *courtesy*, *International Business Machines Corporation; B*, *courtesy*, *Remington Rand*, *Inc.*)

matic plugboard, similar in principle to a telephone switchboard, by means of which any desired arrangement of data can be obtained from the punched cards. The machine is available in three, four, five, six, or seven bank models, each of which may be equipped with from two to five counters. These models operate at speeds of from

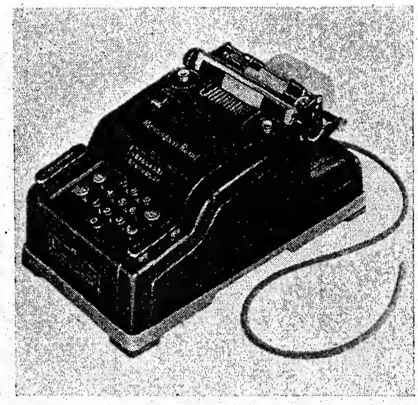
75 cards per minute, for both listing and tabulating, to a listing speed of 120 cards per minute and a tabulating speed of 150 cards per minute.

The Powers 45- and 90-column tabulator (Part *B*, Exhibit 230) has been designed to tabulate both the Powers 90-column card and the 45-column card. Information punched on cards is compiled and printed by this machine in any form desired.



A

B



C

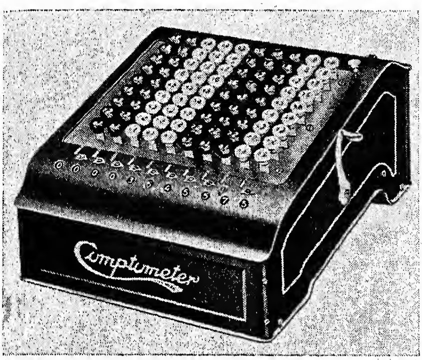
EXHIBIT 231.—Adding machines. *A*. Burroughs adding machine. *B*. Sundstrand adding machine. *C*. Remington Rand adding machine. (*A, courtesy, Burroughs Adding Machine Company; B, courtesy, Underwood Elliott Fisher Company; C, courtesy, Remington Rand, Inc.*)

This type of tabulator will perform automatically the operation of adding and direct subtracting and will produce sub or grand totals, or both. The printing capacity of the machine is in no way affected when grand totals, in addition to sub totals, are obtained. Grand totals are printed directly under the sub totals. Instead of a wiring set-up, the Powers tabulator is equipped with a "multiple translator" which is a mechanical device that makes it possible to effect a series of set-ups, whether simple or intricate, in a few seconds. One translator will of itself serve a plurality of applica-

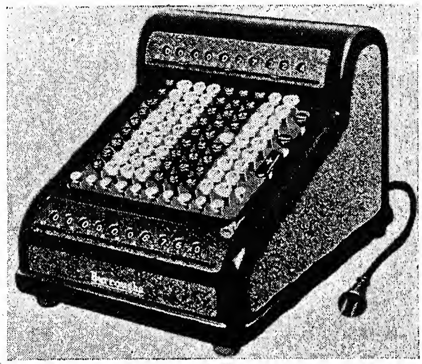
tions—enough in most instances to satisfy the requirements of the average installation. The speed at which this tabulator operates.is 100 cards per minute.

It should be appreciated that although the I.B.M. and Powers machines are similar in many respects, there are many cases where one will have an advantage over the other. For this reason many organizations use both kinds of equipment. The question as to which one should be used, however, is one which can be answered only by an experienced operator.

Adding and Calculating Machines.—Adding-listing machines are so commonly used that a detailed description need not be given here. There are two main types of such machines—those with a keyboard similar to the Burroughs and those with a keyboard similar to the Underwood Sundstrand or Remington Rand (Exhibit 231). Both types are available in the hand-operated and in the electric motor-driven models.



A



B

EXHIBIT 232.—"Key-driven" calculating machines. *A*. Comptometer. *B*. Burroughs calculator. (The models illustrated are motor driven.) (*A, courtesy, Felt and Tarrant Manufacturing Company; B, courtesy, Burroughs Adding Machine Company.*)

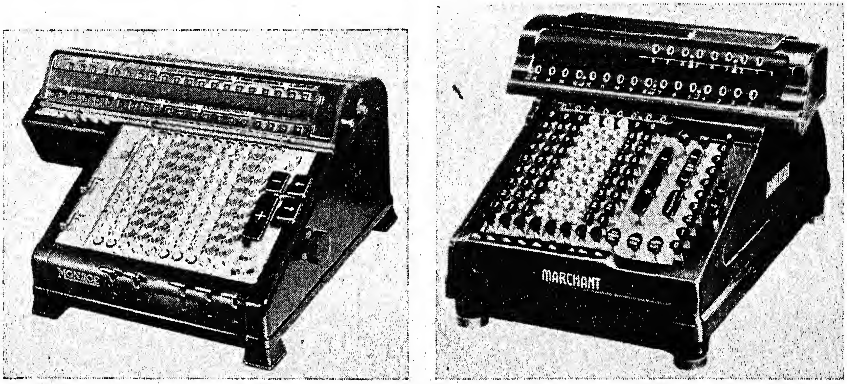
The numbers are placed in the ordinary adding machine by pressing keys and either turning a hand crank or touching an electric key which causes the motor to place the numbers of the depressed keys in the machine. Machines such as those shown in Exhibit 231 automatically list the data and the totals on tapes. These tape records are a great convenience in checking.

While adding machines are used principally for addition, they can also be used for subtraction, multiplication, and division. After setting the key for "subtraction" and placing in the machine the amount from which subtraction is to be performed, the amount to be subtracted is then placed on the keyboard and the crank turned, leaving the difference. (It is to be noted that all models are not equipped with the subtraction key.) Multiplication is performed by pressing the "repeat" key and successively adding the amount to be multiplied the desired number of times, and repeating the process in the next decimal place for each digit of the multiplier. In dividing, the divisor is successively subtracted from the amount into which it is being divided, the quotient being the number of times this subtraction is made before zero is reached, again utilizing the shift of decimals.

Calculating machines perform the same functions as the adding machines just described, though they do not list the data. However, multiplication and division can be performed with much greater ease and speed on the calculating machines, and consequently, because of this advantage, they are ordinarily preferred for such operations.

The Comptometer and the Burroughs calculators (Exhibit 232) are key-driven machines which add, subtract, multiply, and divide in much the same manner as the adding machine, except that the pressing of the keys causes the rotation of the counting mechanism and places the numbers in the counters of the machine without turning a crank or touching an electric key. In the electric machines depressing the keys actuates a motor which completes the operation. The hand or motor crank is used to "clear" the machine before starting a new computation. As with the adding machine, multiplication and division are performed as variations of addition and subtraction, respectively.

In calculators such as the Monroe and the Marchant (Exhibit 233), the hand or electrically operated crank drives the calculating mechanism. These machines are used chiefly for multiplication and division. The following description, which indicates the methods of operating these machines, is applied to the Monroe specifically,



A B

EXHIBIT 233.—"Crank-driven" calculators. *A*. Monroe calculator. *B*. Marchant calculator. (The models illustrated are electrically cranked.) (*A*, *courtesy*, *Monroe Calculating Machine Company; B*, *courtesy*, *Marchant Calculating Machine Company*.)

but the Marchant is operated similarly. The Monroe has three dials: one (upper right) to indicate the number of times the crank has been turned, one (lower) to indicate the amounts accumulated (in multiplication) or remaining (in division) after the crank has been turned, and one (upper left) for special purposes. In multiplication, the amount to be multiplied is placed on the keyboard by depressing the proper keys, and the "repeat" key is depressed. If it is desired to multiply by 653, the crank is turned three times and the entire carriage in which the counters or dials are shown is shifted one position to the right by means of a turn key on the front of the machine. The crank is then turned five times and the carriage is again shifted to the right, whereupon the crank is turned six times. The result is shown in the lower dial, and the number by which the amount on the keyboard has been multiplied (653) appears in the upper right dial, this being the number of times the crank was turned in each position of the carriage. In dividing, the dividend is first placed in the lower counter of the machine by depressing the keys and turning the crank once in the usual cranking direction. The upper counter is then cleared by turning the crank opposite it, and the keyboard is cleared by pressing the key which is provided for this purpose. The divisor is then placed on the keyboard, and with the repeat key depressed, the machine crank is turned in *reverse* direction to that used in multiplying. This subtracts the divisor amount from the dividend in the lower counter, and

when the divisor has been subtracted from the amount directly above it as many times as it can be without leaving a negative result, the carriage is shifted to the *left* and the cranking is continued. The quotient (which is the number of times the divisor on the keyboard is contained in the dividend in the lower counter) appears in the upper right counter. Care must be taken to have the divisor on the keyboard directly below the dividend, and to point off the decimal place properly in the quotient.

The Slide Rule.—The slide rule is not only an indispensable aid to the statistician and engineer, but it is also a very valuable help to the manufacturer, merchant, banker, and all others whose occupation or business involves calculations. Although the slide rule has the general reputation, among those who are not mathematically trained, of being a mysterious and complicated instrument, its operation actually is relatively simple, and it can be used readily by students in the higher grades of the elementary schools.

It will be the purpose of this discussion to describe merely the simpler operations of multiplying, dividing, squaring, and finding square root. For explanation of more complicated operations, the reader is referred to the special manuals on slide-rule operation.

If one who wishes to learn to operate the slide rule already knows how to use semilogarithmic coordinate paper, he can use this knowledge as a basis for operating the slide rule. He already knows that if he has two identical logarithmic scales and slides one along the other as in Part *A*, Exhibit 234, wherever he stops, all the numbers on the top scale will bear the same proportion to those opposite them on the bottom scale. This is a leading principle of the slide rule. Therefore, referring to the *C* and *D* scales in Part *B*, Exhibit 234, slide the slide as we will, we find that:

$$\left\{\begin{matrix}\text{Any number}\\ \text{on } C\end{matrix}\right\} : \left\{\begin{matrix}\text{The number}\\ \text{under it on } D\end{matrix}\right\} :: \left\{\begin{matrix}\text{Any other}\\ \text{number on } C\end{matrix}\right\} : \left\{\begin{matrix}\text{The number}\\ \text{under it on } D\end{matrix}\right\}.$$

Suppose then that we wish to solve the following problem:

$$5:6::8:x$$

To do this (using the *C* and *D* scales) set the slide rule as follows:

$$\begin{array}{c|c|c} C & \text{Set first term (5)} & \text{Under third term (8)} \\ \hline D & \text{over second term (6)} & \text{find fourth term (9.6)} \end{array}.$$

The above setting is illustrated in Part *B*, Exhibit 234. The slide is set so that 5 (*C* scale) is over 6 (*D* scale), and under 8 we read 9.6.

Multiplication and division resolve themselves into proportion. For instance, the proportion 1:12::5:60 is but a simple problem in multiplication, 12 and 5 being the two factors and 60 their product.

Therefore the rule for multiplication is:

$$\begin{array}{c|c|c} C & \text{Set 1} & \text{Under the other factor (5)} \\ \hline D & \text{over 1 factor (12)} & \text{read their product (60)} \end{array}.$$

In Exhibit 234, Part *B*, the slide is set for performing the above multiplication. (Note carefully that the small 2 on the *D* scale under the large 1 on the *C* scale represents 12, as it is not read alone, but with the large 1 at the extreme left.)

If we wish to multiply 12 times 9 and set 1 over 12 as illustrated in Part *B*, Exhibit 234, we find that to read under 9 we need another cycle of the logarithmic scale. To secure this effect, we simply move the slide one full cycle to the left, setting the right-

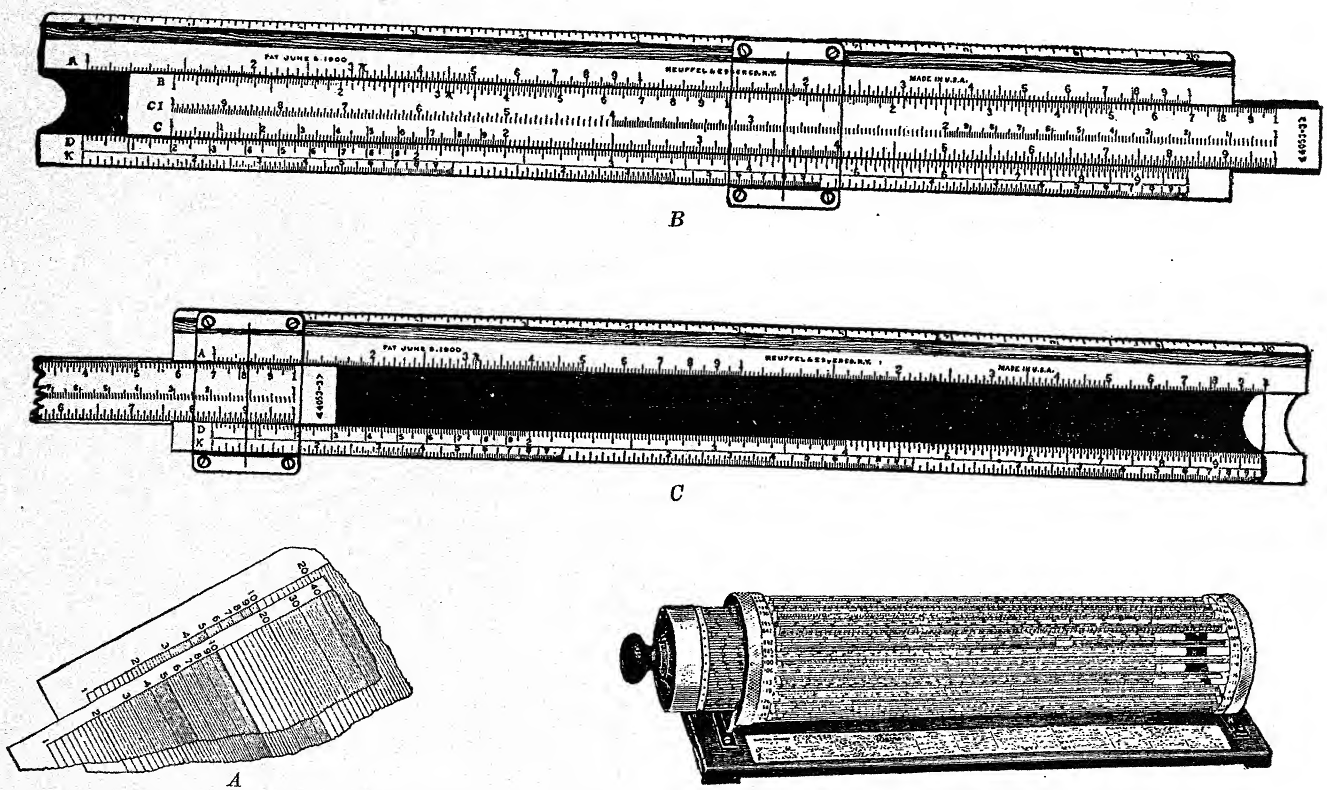


EXHIBIT 234.—Slide-rule operation. *A*. Relation of logarithmic scales. *B*. A slide rule (slide extended to the right). *C*. Slide extended to the left. *D*. Thatcher cylindrical slide rule. (*B*, *C*, *and D*, *courtesy*, *Keuffel & Esser*.)

end 1 over the 12 as illustrated in Part C, Exhibit 234. Then under 9 we read the desired product (108).

In division, the proportion discussed above (1:12::5:60) is reversed and becomes 5:60::1:12.

The rule for division is, therefore:

C	Set divisor (5)	Under 1
D	over dividend (60)	read the quotient (12)

Referring to Exhibit 234, Part B, set 5 over 60 as shown and under 1 read the quotient (12).

Some slide rules have a CI or inverted C scale. After one becomes reasonably familiar with the instrument, the use of the CI scale greatly facilitates the process of division. The rule for division with the CI scale is: Set 1 over the dividend, and under the divisor read the quotient.

A convenient setting for computing percentage relatives is:

C	Set base figure (80)	Under other figure (160)
D	over 100	read percentage relative (200)

The preceding setting is also convenient when dividing each number in a long series of numbers by one divisor. Using other terms, this setting is as follows:

C	Set divisor (5)	Under the dividend (60)
D	over 1	read the quotient (12)

In the two preceding settings the slide remains in one position for the different operations except when it is necessary to shift the slide a full length.

To square a number, use the scales designated A and D in Part B, Exhibit 234. Suppose that we wish to square 3. Set the hair-line of the indicator at 3 on the D scale. Then directly above, on the A scale, read 9, the desired square.

To find square roots, simply do the work in reverse order. To find the square root of 9, for instance, set the indicator hair-line at 9 on the A scale, being careful to use the 9 on the left-hand half of the rule, because the other 9 is really 90. Directly below on the D scale read 3, the desired root. (Note that the 9 on the left-hand half of the rule is 9, 900, 90,000 or .09, .0009, etc., while that on the right-hand half is 90, 9,000, 900,000, or .9, .009, .00009, etc.)

In using a slide rule, the operator may assume any number of ciphers preceding or following the numbers on the C and D scales. That is, 5 may be .0005, .005, .05, .5, 5, 50, 500, 5,000, etc. There are definite rules for pointing off decimal places in slide-rule computations, but the usual method is to point off by inspection.[1] One ordinarily knows that an answer is between 10 and 100, between 100 and 1,000, etc., and the slide rule enables him to locate it between these limits.

The accuracy to which the slide rule can be read is proportional to the unit length of the scales used. The 10-inch rule gives results correct to within 1 part in about 1,000, or one-tenth of 1 per cent. The 20-inch rule gives results correct to within one part in about 2,000. The Thatcher cylindrical slide rule (Part D, Exhibit 234) gives an accuracy to about 1 part in 10,000.

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[1] For descriptions of methods of pointing off in slide-rule computations, see manuals furnished by instrument companies.

APPENDIX III

STATISTICAL DRAWING AND LETTERING

A knowledge of statistical drafting is valuable from many points of view. It may serve as an immediate means of securing a position in a statistical department because it is a definite attainment which can immediately be capitalized upon by the employer, and one can show what kind of work he can do when he applies for the position. Later, when the statistical draftsman has worked his way up in the department, his knowledge of drafting will be of value in supervising the work of others. However, whether or not one ever becomes a professional chart maker, actual practice in drawing and lettering statistical charts gives valuable training in accurately and correctly reading data presented in the graphic language, and enables one to judge the work of others with greater proficiency.

This discussion on the technique of chart drawing and lettering is designed to cover briefly: (1) drawing equipment and its use; and (2) lettering.[1]

DRAWING EQUIPMENT AND ITS USE

When selecting drawing equipment, one should select the best that can be afforded, for success in charting depends to a considerable extent upon the quality of the instruments and materials used. The following list includes the instruments and materials necessary for ordinary work in graphic presentation of business statistics:

List of Instruments and Materials

Set of drawing instruments in case, including compasses with needle point, pen, pencil, and lengthening bar; dividers; bow instruments; and ruling pens.

Drawing board.

T-square.

45- and 30–60-degree triangles.

French or irregular curves and ship curves.

12-inch scale (statistician's scale preferred).

Protractor.

Lettering pens (assorted sizes), penholders.

Drawing pencils.

Art gum, erasers, and erasing shield.

Thumb tacks.

Paper.

Other equipment which may be included is mentioned in the description. Instrument dealers' and manufacturers' catalogs will furnish additional suggestions.[2]

Charting Paper.—Selection of paper depends upon the use to which it is to be put and the personal taste of the individual, but, in general, it is best to avoid cheap grades. A good paper should be strong, of uniform thickness and surface, should stretch evenly,

[1] This discussion of "Statistical Drawing and Lettering" is abstracted from John R. Riggleman, "Graphic Methods for Presenting Business Statistics," McGraw-Hill Book Company, Inc., New York, 1936. Credit should also be given to Thomas E. French's "Engineering Drawing," McGraw-Hill Book Company, Inc., New York, for many of the ideas presented in this discussion.

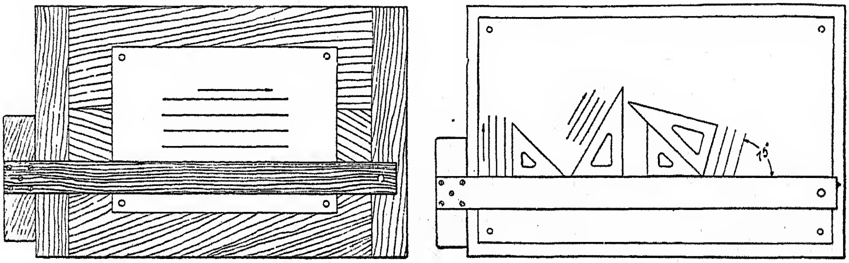
[2] See especially the catalogs of Theo. Alteneder & Sons, Philadelphia; Keuffel & Esser Company, New York; and Eugene Dietzgen Company, Chicago.

lie smooth, and allow considerable erasing without spoiling the surface. For high-class and lasting work, Whatman's paper and Reynold's bristol board are considered to be among the best. For most business graphic work, many less expensive papers are quite satisfactory if they take ink and color well and stand a reasonable amount of erasing.[1] Bond and ledger papers are also used to a considerable extent in chart making. Detailed descriptions of drawing paper with sizes and prices are given in catalogs of dealers in engineers' and draftsmen's supplies.

For making working drawings and sketches, various kinds of ready-ruled coordinate paper are available as described and illustrated on pages 78 and 79. Similar forms with months, years, etc., printed in, are also available.

Pencils.—Lead pencils are graded according to their hardness which is expressed by numbers with the letter H.[2] Personal preferences vary, but for general use in statistical charting, 3H, 4H, and 5H pencils sharpened to long conical points, have been found to meet most of the requirements in drawing. A softer pencil (2H or H) should also be provided for lettering. The chisel or wedge point is preferred by some for hard pencils as it does not wear away so fast as the conical point.

The Drawing Board.—Drawing boards are usually made of well-seasoned, clear, white pine, cleated at the ends to prevent warping (Part *A*, Exhibit 235). The ends should be perfectly straight, so that the T-square can be used accurately.



A *B*

Exhibit 235.—*A*. Use of drawing board and T-square. *B*. Use of triangles.

Paper is fastened to the drawing board by means of thumb tacks. It is usually cut larger than the drawing to be made, and should be placed on the board squarely, with the aid of the T-square, and the tacks pushed in up to the head in order that the T-square will not be obstructed.

The T-square.—T-squares are usually made of hardwood, the better grades having the edges of the blades lined with ebony or celluloid. They are used principally for drawing parallel lines horizontally and for keeping the triangle in position for drawing other lines. Horizontal lines should always be drawn from left to right, and the T-square should always be used from the left edge of the drawing board (Part *A*, Exhibit 235) as the edges are seldom parallel or at right angles to each other.[3]

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[1] Keuffel & Esser "Normal" is one of the best of the inexpensive drawing papers.

[2] Pencils commonly used are harder as the number increases, and are graded from H to 9H. Very soft pencils are graded by their softness which is designated by the letter B and a number which increases as the lead becomes softer. Thus (according to Keuffel & Esser) from soft to hard, they are 6B, 5B, 4B, 3B, BB, B, F, HB, H, HH, 3H, 4H, 5H, 6H, 7H, 8H, 9H. Different companies, however, vary in the use of this system.

[3] This would be reversed for a left-handed person, of course, who would draw from right to left and use the T-square on the right edge of the board.

Triangles.—Triangles are made of various substances, but transparent celluloid is the most popular. Vertical and sloping lines are drawn by setting the triangle against the T-square and drawing from bottom to top, as illustrated in Part *B*, Exhibit 235. Various sizes are made, but a 6- or 7-inch triangle will be found convenient for most chart work, especially when 8½- by 11-inch paper is used. As celluloid triangles warp easily, care should be taken that they lie flat or are hung up when not in use.

French or Irregular Curves.—French or irregular curves and ship curves are laid out in combinations of spirals, ellipses, and other such curves as will closely approximate those commonly met with in practice. A smoothed curve is drawn on a statistical chart by first locating all the points in the series and then so placing the irregular curve that it coincides with several of them. It is shifted from one position to another as the line is drawn. It is usually best to sketch through the points a free-hand pencil curve that is satisfactory to the eye, and then apply the irregular curve, selecting a part that will most nearly fit a portion of the line. It is very important that the draftsman indicate in advance on his chart the points where the curvature changes. He should then be careful to reverse his curve at such points. Also, it is important to select the right type of curve in accordance with the line to be drawn, and to shift this curve so as to get a long fit rather than a short one.

Scales.—Statisticians' scales are made especially for laying out statistical charts. The one shown in Exhibit 236 has four different scales: (1) one in ordinary English

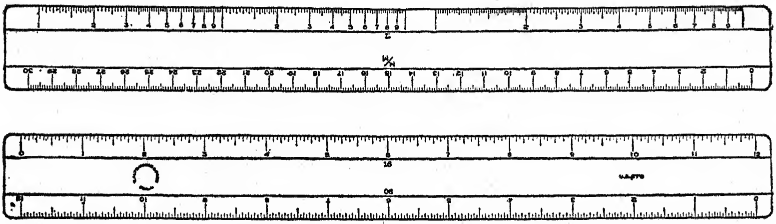


EXHIBIT 236.—A statistician's scale. (*Courtesy, Theodore Alteneder & Sons.*)

rule divisions—sixteenths of an inch; (2) one in decimal parts of an inch; (3) one in centimeters; and (4) one logarithmic. This furnishes a scale for making logarithmic graduations, and a choice of three units for making various measurements, laying off spaces, or dividing a given distance into a certain number of equal parts. These scales are made usually of plain boxwood or boxwood with white edges.[1]

There are two kinds of engineers' scales in common use—known as the "engineer's scale" of decimal parts, and the "architect's scale" of proportional feet and inches. They are made in triangular form and also in sets of flat shapes. In chart making these scales are widely used, especially by those who have had engineering training.

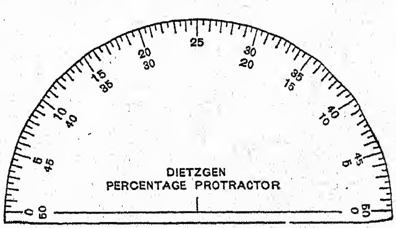


EXHIBIT 237.—A percentage protractor. (*Courtesy, Eugene Dietzgen Company.*)

Protractors.—A protractor (Exhibit 237) is a necessity in laying out "pie charts." The common protractor is a semicircle of 180 degrees, but a more convenient form for pie charting is the percentage protractor made on a basis of 100 parts to the circle.

[1] Scale number 2735 of Theo. Alteneder & Sons, Philadelphia, Pennsylvania. A triangular scale with logarithmic graduations is made by Keuffel & Esser, New York. Special scales for statisticians can be made to order by scale manufacturers.

Compasses and Dividers.—Compasses and dividers are usually included in sets of various combinations contained in a special case. The combinations vary according to the line of work and means of the user, but should contain at least a pair of compasses with pen, pencil, lengthening bar, a pair of dividers, and a ruling pen.

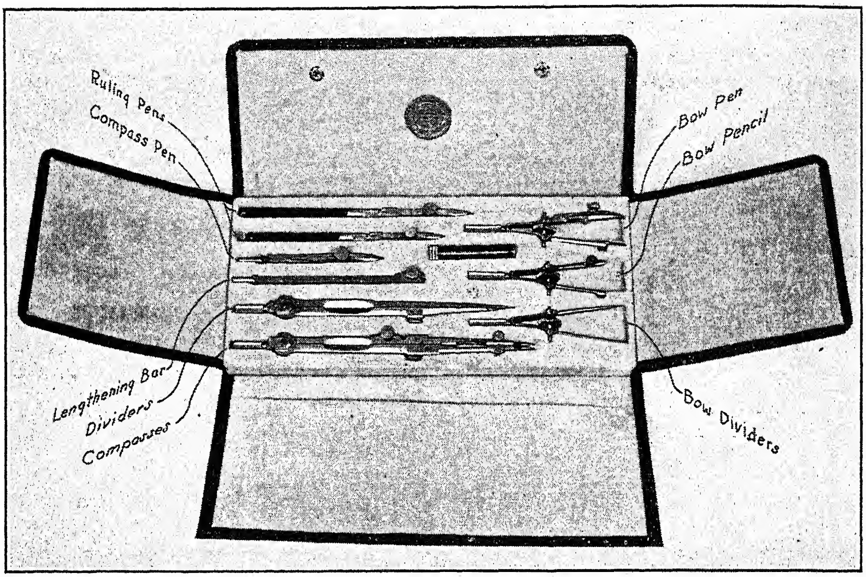


EXHIBIT 238.—Set of drawing instruments in case. (*Courtesy, Theodore Alteneder & Sons.*)

One leg of the compasses is fitted with a needle point, and the other may be fitted with either a pencil or a pen by means of a shank and socket (Exhibit 238). With most instruments the needle is separate and has a square shoulder on one or both ends from which a minute point projects. Before using compasses, this needle point should be adjusted by attaching the pen and setting the needle a trifle longer than the pen (Exhibit 239). The needle should be kept in this position and the lead, sharpened to a fine wedge or long bevel edge, should be adjusted to it. This will allow the interchanging of pen and pencil without adjusting the needle point.

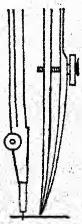


EXHIBIT 239.—Adjustment of needle point.

When drawing a circle, the compasses, held loosely between the thumb and forefinger, should be inclined slightly in the direction of revolution. Only a light pressure is necessary. For drawing circles larger than the spread of the compasses, the lengthening bar (Exhibit 238) must be inserted between the leg socket and the pen or pencil.

Dividers (Exhibit 238) are used for laying off distances, either from measurements or from other parts of the drawing, and for dividing lines into equal parts. When laying off equal distances the dividers should be turned in opposite directions each time, in order that the instrument can be operated readily with one hand.

A form of dividers known as the "hair-spring" is equipped with a screw adjustment in one leg for making fine settings. Combination instruments are made which may be used both as compasses and dividers simply by interchanging the pen or pencil point for a needle point.

The bow instruments are used as adjuncts to the compasses and dividers and have their special field of usefulness in making circles and spaces which are so small that it is difficult to make them with the large instruments. The set of three spring bow instruments includes bow dividers or spacers, bow pencil, and bow pen (Exhibit 238). Since it holds its adjustment, the bow pen is almost indispensable in duplicating a large number of small circles.

Ruling Pens.—Ruling pens are made in a number of forms, but all consist of a handle bearing two spring nibs which have a thumbscrew adjustment for regulating the width of line (Exhibit 238). Some forms have a device which allows the blades to be opened for cleaning, independently of the thumbscrew. For large work a very valuable pen is the form known as the "detail pen."

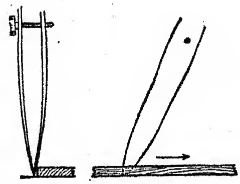


EXHIBIT 240.—Position for using ruling pen.

The ruling pen is filled by placing the ink between the nibs with a common steel pen or with the quill filler or dropper that is provided with most drawing inks. The ink should fill the pen to a height of from $\frac{3}{16}$ to $\frac{1}{4}$ inch depending upon the size of the pen; if too much is used its weight will cause it to run out upon the drawing. The width of the line is varied by changing the distance between the pen points with the thumbscrew.

The ruling pen is never used free hand, but always with a guiding edge. It should be held with the blades parallel to the edge, the handle slightly inclined in the direction of motion, and in a plane which passes through the line perpendicularly to the paper (Exhibit 240).

Ink will dry rapidly in the point of the pen when it is adjusted for fine lines and may refuse to flow after the pen has remained idle for a few seconds. If gently pinching the nibs or touching the point on the back of the finger does not start the flow, the pen should be cleaned and filled with fresh ink. Pens should be wiped clean when one is through using them, because dirty pens will not draw clean-cut lines and ink left in them will corrode the steel blades.

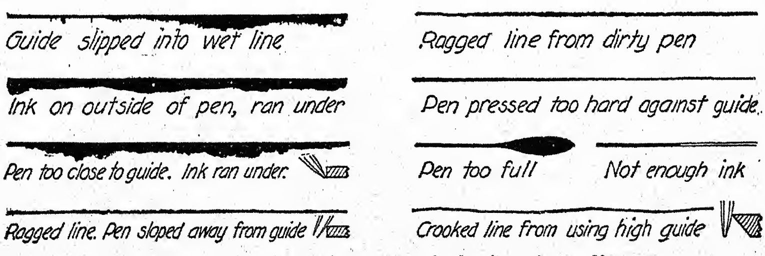


EXHIBIT 241.—Beginners' common faults in using ruling pens.

Inked lines may be imperfect through the fault of the pen, the ink, the paper, or the draftsman. Exhibit 241 shows several common faults for which the draftsman is to blame.

Ink.—Waterproof India ink is usually used for chart making, as it leaves a permanent black line. It is composed of finely divided carbon held in suspension, with shellac added to make it waterproof. Non-waterproof ink flows more freely, but is not popular because it smudges very easily. A blotter should never be used on drawing ink, as it removes part of the layer of carbon and leaves a weak line.

When they are not in use, ink bottles should be kept tightly corked to prevent the ink from thickening and drying up. Drawing ink must not be allowed to freeze, as once frozen it is useless.

Waterproof drawing inks are also made in a variety of colors. As colored inks are thinner, they flow more readily than black when used in ruling pens.

Erasers.—Although but little erasing should be necessary, a good ink eraser should be provided for erasing errors in lines or letters. In using erasers, care must be taken not to damage the paper or the surrounding work. Erasing shields, usually made of thin metal, are used to protect the work around the erasure, which is made through one of the slits.

Drawings are cleaned by rubbing with a piece of art gum, or sponge rubber. However, finished drawings should not be scrubbed all over with the art gum or rubber, as it takes the life out of the inked lines.

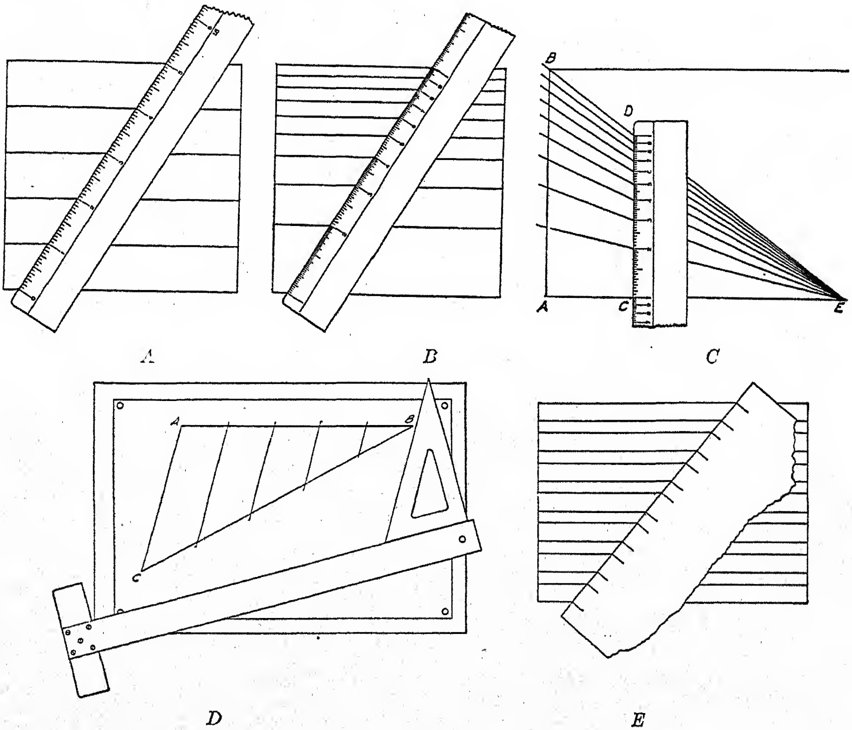


EXHIBIT 242.— Methods of dividing a given line or space.

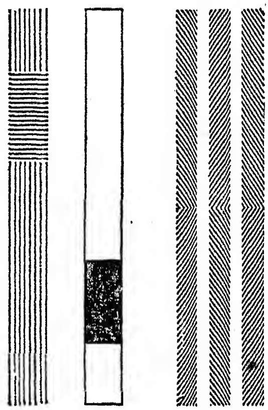
Division of Space or Lines with Scales and Dividers.—After the proportions and size of a particular chart have been laid out, a simple method of dividing space into any number of equal parts is shown in Part *A*, Exhibit 242. In this illustration, it is desired to divide the space into five equal parts, and the scale is simply rotated until the spaces can be marked off according to convenient scale intervals as shown. The same principle applies to the use of the logarithmic scale (see Part *B*, Exhibit 242). If, in such a case, the logarithmic scale is too short, a line twice the length of the scale can be laid out between the limiting lines, and each logarithmic division doubled as it is laid out along this line.

Another method which is often used in laying out logarithmic scales (as well as arithmetic scales) is that shown in Part *C*, Exhibit 242. It is desired to divide the distance *A-B* logarithmically from 1 to 10 with the scale *C-D*. Place the scale *C-D* a reasonable distance from the line *A-B* and perfectly parallel with it. Locate as

many points along C-D as are wanted on A-B. Draw the lines B-D and A-C and produce them until they intersect at E. Then lines drawn from E through the points laid off between C and D will locate the logarithmic graduations on the line A-B. This method, of course, can be used for logarithmic spacing of a distance either greater or less than the length of the scale used.

Part D, Exhibit 242, shows another simple method of dividing a line into any number of equal parts. To divide line A-B into, for instance, five equal parts, draw any line B-C, and on it step off with the dividers five convenient lengths. From C, the last point, draw a line to A. Then draw lines parallel to A-C through the points on B-C, intersecting the line A-B which will be divided thereby into five equal parts. Triangle and T-square may be used as illustrated for drawing parallel lines.

A convenient method for dividing space irregularly as, for instance, in laying out a bar chart, where the spaces between the bars are not the same as the widths of the bars, is shown in Part E, Exhibit 242. With the dividers or scale simply lay out the spaces in convenient units of the right proportions on a strip of paper somewhat longer than the space to be divided, and use the strip as illustrated (Part E, Exhibit 242).



A *B* *C*

EXHIBIT 243.—Optical illusions.

Optical Illusions.—In cross-hatching or shading areas it must be recognized that one must contend with certain optical illusions. Occasionally these illusions spoil the effect of the entire chart. Some of the most common are illustrated in Exhibit 243. Note that in A the part of the bar shaded with horizontal lines appears to be narrower than the parts shaded with vertical lines. In B, the part of the bar shaded in black appears to be narrower than the white parts. In both A and B, the sides of the bars are perfectly straight and parallel. In C, the first and second bars appear to be closer together at the ends than in the middle, and the second and third appear to be farthest apart at the ends while, as a matter of fact, the three bars are perfectly parallel.

OLD ROMAN

Roman lower case

MODERN ROMAN

Gothic

COMMERCIAL GOTHIC

EXHIBIT 244.—A few common styles of letters used by printers.

LETTERING

As far as appearance is concerned, there is no part of a chart more important than the lettering. A good drawing may be ruined by poor or careless lettering, not only in appearance but in its effect, as any evidence of carelessness or lack of skill tends to destroy confidence in the work.

Almost any consistent style of plain, neat lettering is acceptable in the ordinary course of statistical chart work. Ornate lettering has its place among artists and craftsmen, but it is neither practical nor appropriate for business charts. The most common styles used in statistical charting are the Commercial Gothic and the Modern Roman or its modifications (see Exhibit 244).

Lettering Instruments.—Many different kinds of steel writing pens are used for lettering. Exhibit 245 shows several pens which are widely used for making letters under $\frac{1}{4}$ inch high. The size of pen to be used depends upon the width of line desired for the lettering. The pen should be selected which will make the desired width without spreading the point. Steel pens may be inked by dipping them into the

bottle, but many consider it better to fill them with the quill or dropper. If too much ink is used, it has a tendency to run where strokes are joined in the letters. The ordinary cork-grip penholder is commonly used, though various other kinds are furnished to suit personal preferences.

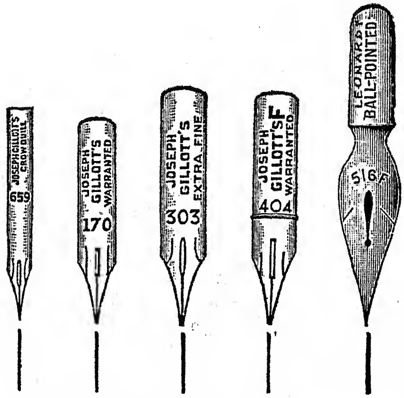


EXHIBIT 245.—Common lettering pens.

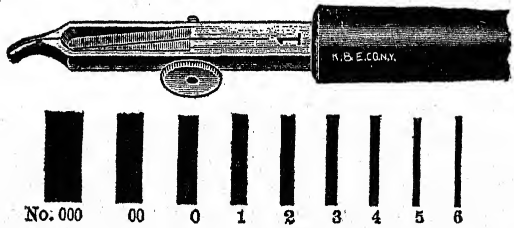


EXHIBIT 246.—Payzant lettering pens. (*Courtesy, Keuffel & Esser Company.*)

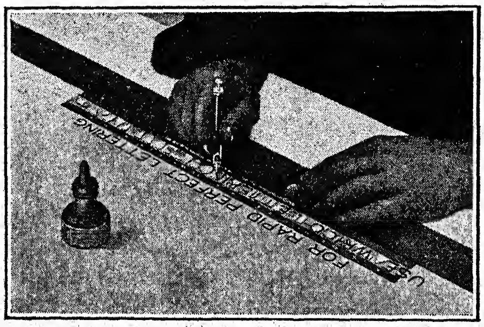


EXHIBIT 247.—Wrico lettering guide and pen. (*Courtesy, Wood-Regan Instrument Company and The Eugene Dietzgen Company.*)

A popular form of special pen for making letters from $\frac{1}{8}$ to $2\frac{1}{2}$ inches high is the Payzant pen (Exhibit 246). Uniform lines can be made easily with this pen as it is constructed to produce the same width and density of line, no matter in what directions the pen is moved over the paper.

Letters larger than $2\frac{1}{2}$ inches high, such as those used in titles of wall charts, are best made by outlining with drawing instruments and filling in, but fair letters may be made more quickly by using a brush.

After one has studied lettering and knows what sizes to use, and if one understands spacing, the Wrico lettering guides,[1] illustrated in Exhibit 247, are very satisfactory aids in making more accurate forms. A good form of pen to use with all except the smaller of these guides is the Payzant illustrated in Exhibit 246. The special pen, shown in Exhibit 247, however, is preferred by many, and it is especially necessary for the smaller lettering.

General Proportions.—There is no standard proportional relation of width to height in letters. When space for lettering is short, **COMPRESSED LETTERS,** which are very narrow in width as compared to height, are used. When it is desired to spread a short title over a long space **EXTENDED LETTERS**, which are wider than normal, are used. The most legible and stable appearance of capitals is secured by making the width about seven-eighths of the height (except I, M, and W). Letters also vary in the thickness of stroke in proportion to height. Those with heavy strokes are known as "**bold-face**" and those with narrow strokes as "light-face" letters.

In making letters, one must contend with certain optical illusions. Round letters, such as C, O, or Q, will appear to be smaller than square letters, such as E, F, or Z, if they are made exactly the same in height. This is true also of letters such as A and V which would appear to be too short at the pointed ends. In order to give the appearance of equal heights, the round or pointed ends of letters should be extended a little over the guide lines which limit the letters with square ends. If letters, such as E, H, or S, are to look well proportioned, their top and bottom halves should appear to be equal. This effect can be secured only by making the letters slightly narrower at the top than at the bottom, and by drawing the center lines slightly above the middle. This illusion will be quite evident if any printed page is turned upside down and the letter S is observed. The point must be emphasized, however, that the corrections for these illusions are very small and that overdoing them is worse than overlooking them.

Free-hand Single-stroke Lettering.—The term "single stroke" means that the width of a single stroke is the width of line used in making the letter, and "free-hand" means that no guiding instruments are employed and the pen is used as in writing. This is the most commonly used lettering, and a knowledge of its construction is very necessary for any one who makes charts. The ability to letter well can be acquired by any one who has normal control of his fingers, who will practice faithfully, and observe carefully the shapes of the letters. There is no direct relation between one's lettering and his handwriting. Many good letterers write very poorly.

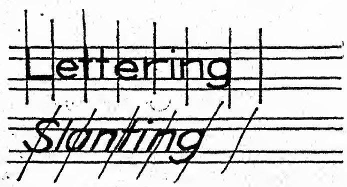
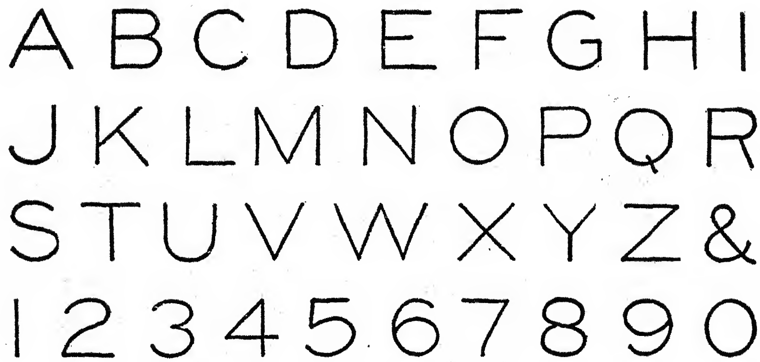


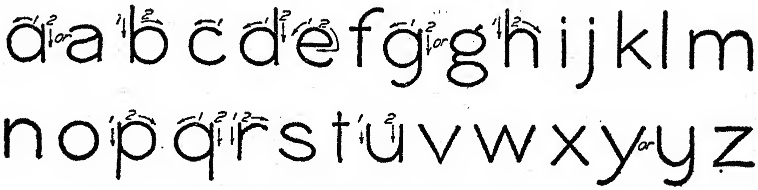
EXHIBIT 248.—Guide lines for lettering.

In lettering, the pen should be held lightly, as in writing, and the strokes drawn with a uniform motion, with a light, even pressure on the paper. The stroke should be complete in itself and should not be sketched in or retraced. The nibs of the pen should not spread. If a heavier line is desired, a coarser pen should be used. Horizontal guide lines should always be penciled in as an aid in making the tops and bottoms of the letters even and straight (Exhibit 248). Other lines may be ruled in at random to aid in making strokes vertically or at a certain slant (Exhibit 248). The letters may be made directly in ink, although penciling is necessary in composition

[1] Manufactured by The Wood-Regan Instrument Company, Inc., South Orange, New Jersey, and distributed by the Eugene Dietzgen Company, New York.

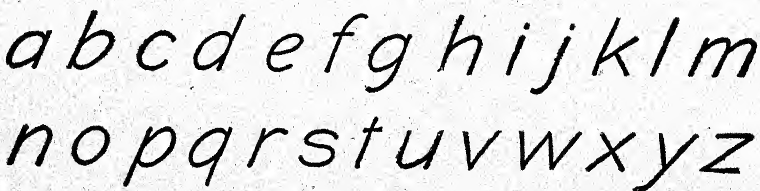


A.—Vertical capitals.



B.—Vertical lower case.

C.—Inclined capitals.



D.—Inclined lower case.

EXHIBIT 249.—Free-hand single-stroke letters. (*From "Engineering Drawing" by Thomas E. French.*)

or laying out symmetrical titles. Vertical strokes should be made downward and horizontal strokes from left to right.

Vertical Single-stroke Capitals.—The forms of these letters must be very carefully observed and should be made exactly as shown in Part *A*, Exhibit 249. They are all made the same width except M, W, I, and the numeral 1. Cross lines are drawn slightly above the center, except for A and G and the figures 4 and 9, which are a little below. The sides of the letter M should be made parallel and not sloping as in an inverted W. A very important point to observe in making the letters B, D, P, and R is that the tops and bottoms are made mainly of horizontal strokes. Figures are made the same height as capitals and sometimes slightly narrower. The backbones of the 6 and the 9 have the same curve as the cipher.

Vertical Lower Case.—The alphabet of vertical lower-case letters is made as shown in Part *B*, Exhibit 249. These letters are all of the same width, except the seven letters, f, i, j, l, m, t, and w. The bodies are made two-thirds of the height of capitals, with the ascending stems reaching the cap guide line and the descending stems reaching the same distance below (see upper part of Exhibit 248). The circle and parts of it combined with straight lines form the basis of the letters a, b, c, d, e, g, h, n, o, p, q, r, u, and y.

Inclined Capitals.—The inclined or slanting alphabet shown in Part *C*, Exhibit 249, is easier to make than the vertical, as a slight difference in inclination is not apparent, while a slight tendency from the true vertical is very noticeable. The inclination is generally made from 60 to 70 degrees to the horizontal. Special triangles are made for drawing these slope lines. Particular attention must be given to the inclined A and V in that a slope line drawn through the points of these letters should bisect the angles made by the two sides of the letters. The same kind of attention should be given to the angles in the letter W. The discussion in the preceding paragraph on vertical capitals applies equally well to the inclined capitals.

Inclined Lower Case.—The inclined lower-case letters (Part *D*, Exhibit 249) are founded upon the straight line and the ellipse instead of the circle. In other respects, they are proportioned the same as in the vertical system.

Spacing.—Definite rules for spacing letters cannot be laid down because of the varying proportions of the letters. An appearance of uniform spacing is a matter of artistic judgment and cannot be gained mechanically. If letters are spaced at equal distances along the guide lines, there will appear to be more space between letters such as AT or OC than between letters with parallel sides such as HN. To give the appearance of equal spacing, the minimum space should be used in such combinations as AT, AV, LY, or TJ, where part of one letter may even extend over part of another, while the maximum spacing should be used in straight side combinations such as HE, NB, or IR. A common mistake of beginners is making letters too narrow for their height and spacing them too far apart.

Spacing between words varies with the effect desired, but should always enable one to read the words easily. In ordinary work, the space should appear to be about 1 to $1\frac{1}{4}$ times the width of the average letter used.

It is very important to allow ample space between lines if the work is to be read easily. This space should be from $\frac{1}{2}$ to $1\frac{1}{2}$ times the height of the capitals. The crowded appearance of chart lettering is often due to the fact that not enough space has been left between lines.

In laying out a symmetrical title for a chart it usually is best to balance it on a vertical center line. The letters and spaces in the title are counted and then sketched in pencil, beginning each line with its middle letter at the center line.

Pasting on Printed Titles.—Very good results can be obtained, when making charts that are to be reproduced by a photographic process, by pasting on a title that has been set up and printed by the ordinary printing process. Several titles

can be set up and printed together at a small expense. Then these can be cut apart and mounted in the proper positions on the charts.

Typewriter Lettering.—The typewriter may sometimes be used successfully in making certain kinds of small charts. There are not, however, enough differences in sizes of type on an ordinary typewriter to give the lettering the right proportions for the various titles, captions, keys, and references usually placed on graphic work. This situation may be improved somewhat by using special machines such as the Hammond Multiplex or by using two or more machines with different sizes of type, if they are available. Letters may be typed in various colors without changing the ribbon by using colored carbon papers.

APPENDIX IV

A BASIC OUTLINE FOR INDUSTRIAL AND COMMERCIAL SURVEYS

The outline for an industrial and commercial survey of a city, presented on the following pages, indicates how statistical surveys can be and should be outlined before the actual work of collecting data is started. Not only should the survey itself be carefully thought through and outlined, but the sources to be used should also be listed in relation to the outline, as shown on pages 692 to 712, inclusive.

The outline presented on the pages which follow is not only an example which can be used indirectly as a reference in outlining a survey of practically any business statistics problem, but by making a few changes, it can be directly adapted and used in many instances in making such types of investigations as marketing analyses, factory-location surveys, analyses of the security back of investments, real estate appraisals, and studies of local business conditions. While the outline has been designed primarily to serve the broad purposes of a community or cooperative project, it will serve equally well as a basis for developing a special outline for the use of a single private firm. In preparing the outline, the intention was to make possible a choice between an intensive study of a subject of outstanding importance in the community and a more general study touching upon many subjects of potential interest for widely different purposes. It was also the intention to make the outline far more comprehensive than any survey that is likely to be made of a single city at any one time, and the details given are intended to serve as leads in stimulating thought toward the most profitable investigation that is suggested by the local situation, rather than to serve as an all-inclusive pattern. Consideration was also given to the many possible compromises between a mere "desk study" of published material and a comprehensive field survey.[1]

The detailed suggestions, which follow, will be presented in two sections: Section I—Factors to Be Covered; and Section II—Sources of Data. The numbering and lettering of the items in the outline of sources correspond directly to that of the outline of factors to be covered.

SECTION I.—FACTORS TO BE COVERED

The object here is to suggest an extensive list of important factors which might logically be included in an industrial or commercial survey of a city. Different purposes will require different arrangements and combinations of factors and differences in degree of detail.

[1] This outline (slightly condensed) is taken from "Suggestions for Use in Making a City Survey" (1938), by Ada Lillian Bush, Chief, Consumer Market Section, Marketing Research Division, Bureau of Foreign and Domestic Commerce, United States Department of Commerce. The complete work includes text material on planning, making, and using industrial and commercial surveys which will give the investigator many valuable suggestions on technique of survey procedure.

I. Historical Sketch.
   A. Origin.
      1. Date.
      2. Circumstances of first settlement.
         a. Nativity and other characteristics of settlers.
         b. Difficulties and accomplishments.
   B. Early development and factors influencing growth.
      1. Topographic and climatic influences.
      2. Trade routes and transportation.
         a. Rivers and harbors.
         b. Canals.
         c. Overland routes.
         d. Railroads.
      3. Native resources.
      4. Power, initial sources.
         a. Water.
         b. Fuel.
      5. First market outlets.
      6. Predominating human influences.
      7. Other important features.
   C. Type of city developed (industrial, commercial, educational, etc.) and analysis of reasons for that type of development.

II. Location.
   A. Geographic position.
   B. Physical features pertinent to present-day industry and trade.
      1. Topographic influences (mountains, rivers, lakes, soil, etc.).
      2. Climatic characteristics.
         a. Temperature, mean average per month, etc.
         b. Rainfall, amount and seasonal tendencies.
         c. Other (relative humidity, sunshine, wind, fogs, storms, etc.).
      3. Scenic attractions (influencing tourist trade, etc.).
   C. Location in relation to—
      1. Important resources.
         a. Raw materials (see IV).
            i. Agricultural (crop productions, dairy products, etc.).
            ii. Mineral (kind, records of production, etc.).
            iii. Forest (available supply, commercial uses, etc.).
            iv. Other.
         b. Fuel resources (production requirements, coal, oil, gas, etc.).
         c. Water supply (in quantity and quality required in processing by many types of industries—source, reliability, etc.).
      2. Labor (see VII).
      3. Markets (consider facts under "Transportation Facilities," XVI)
         a. Industrial (local, regional, national—relate to VI).
         b. Wholesale (relate to XII).
         c. Retail (relate to XIII).
         d. Export (principal commodity exports, etc.).
   D. Other (such other factors as have special significance).

III. Population.
   A. Number of persons.
      1. Total in metropolitan area (as close an estimate as can be made for the corporate city and such extensions into the surrounding area as are covered by the survey. See XIII *B*).

a. Male.
b. Female.
c. Number between eighteen and sixty-four years; and sixty-five years and over (classified as to sex).

2. Total in corporate area.

B. Percentage of total for the United States, the county, or other area (such comparisons, if any, as may apply to the purpose of survey).

C. Number of families.
1. Total in metropolitan area.
2. Total in corporate area.

D. Growth (steady—rapid) and status (established—transient).
1. Total increase or decrease in population 1920–1930.
2. Actual or estimated change in total population since 1930.
3. Other trends (shifting of population to suburban areas or such other internal and external movements as may be of significance; fundamental long-time tendencies).

E. Composition.
1. Race.
2. Native white.
3. Predominating nativity of foreign-born.

F. Literacy.

G. Other (such additional characteristics as may be important to the local situation).

IV. **Materials Required by Industry** (availability and cost factors, all important facts peculiar to the individual situation with such comparisons with other districts as apply).

A. Raw materials (see II).

B. Products of other industries (kinds readily obtainable: semi-manufactured goods, equipment, supplies, etc.).

V. **Power.**

A. Sources.
1. Purchased power from public utilities.
2. Produced by individual enterprises for own use.
a. Water.
b. Steam.
c. Internal combustion.
d. Electricity.

B. Rates and regulations.
1. At time of survey.
2. Changes within recent years.
3. Changes provided for, or under consideration.
4. Comparisons with other cities.

C. Changes in local facilities for serving industry and commerce (all important changes on which information is available).

D. Present favorable and unfavorable features of local facilities, from the point of view of the—
1. Community.
2. Supplier.
3. Individual industries.

E. Adequacy of supply to meet industrial growth.

F. Other facts of importance with respect to the local situation (provisions insuring reliability of supply, etc.).

VI. Industrial Development.

A. General production statistics and related factors.

1. Number of establishments.
2. Number of wage earners.
3. Amount of wages.
4. Cost of materials, fuel and purchased electric energy.
5. Value of products.
6. Value added by manufacture.
7. Horse power.
8. Trends in—
    a. Total value of products.
    b. Total number of wage earners.
    c. Total amount of wages.
9. Percentage of total production in the United States, in the region or state (or such comparisons with other cities as may be significant).
10. Principal markets supplied (relate to XVI and II); percentage of materials bought from local sources and percentage of manufactures sold to local agencies (if the local situation seems to warrant a direct inquiry to elicit factual data and this information is essential to consideration of the individual problem).
11. Waste products (utilization, disposal facilities).

B. Individual industries.

1. Production statistics of each important industry now in operation (present volume of production in terms of number of units, value of products, number of wage earners, etc., and trends within recent years; the national trend with respect to each industry—apparently over-developed or under-developed).
2. Number and kind of important new industries locating in the city within the period covered by the last five biennial censuses of manufacture, or other period of interest (reasons for locating).
3. Number and kind closed down or leaving the city to operate elsewhere within the period considered under *B*-2 (factual information concerning reasons, insofar as is possible).
    a. Seasonally.
    b. Temporarily (exclusive of normal seasonal closings).
    c. Permanently (obtain factual information concerning reasons for closing, insofar as possible).
4. Names of important industries that have continued in operation over a long period of years (permanence ascribed to what factors).

C. Other (such additional facts characterizing the local situation as may be essential to a comprehensive analysis of the present industrial status, the industrial advantages or disadvantages as compared with comparable cities, recent trends, and future possibilities).

VII. Labor Situation.

(NOTE: 1938 legislation on regulation of hours and wages may affect the importance of items *B* and *C* as subjects of local study. Details of application are not yet available.)

A. Adequacy of labor supply.

1. Skilled, semi-skilled and unskilled.
2. Male, female.

3. Resident, transient, predominating nationality.
4. Situation in surrounding territory (neighboring reservoir of labor from which to supplement local supply, if necessary; special conditions giving rise to this supply; type of labor available, such as unskilled farm hands, semi-skilled or skilled labor resulting from neighboring shift in industry, etc.).
5. Related factors (such as labor turnover, efficiency, training facilities).

B. Wage scale, as compared with scale in—
1. Each year of recent years.
2. Similar cities, or the region in general.

C. Laws and local regulations.
1. Wages and hours.
2. Labor conditions.
3. Changes within recent years.

D. Labor unions.
1. Organization and membership.
2. Attitude toward each other.

E. Relations between capital and labor (record as to strikes, etc.—results).

F. Special local efforts to maintain satisfactory conditions (working, living).

G. Other facts (such as the uninterrupted absence of labor troubles or the harmonious settlement of a well-known local controversy; recent improvements in methods of payment, etc.).

VIII. Employment and Unemployment (closest possible estimates where specific data can not be made available).

A. Number gainfully employed (exclusive of employees paid from relief funds).
1. Total number in metropolitan area, by—
   a. Race (white, Negro, etc.).
   b. Nativity (native white, foreign-born white).
   c. Sex.
   d. Age (under sixteen years, between sixteen and twenty-one).
2. Total number in corporate area (by each group as classified under *A*-1).
3. Gainful workers per family (average).
4. Comparisons with other cities, surrounding counties, or with the United States as a whole (the most helpful comparisons depend upon the size, type, and location of the city and purpose of survey).

B. Principal sources of employment (industrial plants, commercial establishments, etc.), in the order of their importance in number employed.

C. Recent trends toward increased or decreased employment, with respect to each principal source of employment (as concluded from such data for the current year and preceding years as are available).

D. Unemployed employables (unemployed persons able to work).
1. Number of emergency workers (workers paid from relief funds—local, state, or Federal), with such breakdown as is possible by race, nativity, sex, age, and duration of unemployment
2. Number totally unemployed (exclusive of workers paid from relief funds), classified as suggested under *D*-1, in—

a. Metropolitan area.
b. Corporate area.
3. Number partially unemployed in—
a. Metropolitan area.
b. Corporate area.
4. Percentage formerly employed in local—
a. Industry.
b. Wholesale trade.
c. Retail trade.
(Continued through all local sources of employment).
5. Percentage never employed on a full-time basis in any local occupation because of—
a. Short time in city.
b. Maturity reached during the depression.
c. Income derived from other than personal occupation.
d. Physical disabilities now remedied.
e. Other reasons.
6. Number by each type of ability (such as skilled and unskilled factory workers, carpenters, etc.).

E. Other (such other facts as may be peculiar to the local unemployment situation).

IX. **Consumer Purchasing Power** (estimated income in dollars, of individuals and families, considered in relation to cost of living).

A. Wages, salaries, commissions, etc. and business income from occupation (with a breakdown if locally feasible by the following sources of income).
1. Agriculture (such income as residents in the city may derive directly from the surrounding farm area).
2. Mining (such income as may be derived from coal mining and other mining activities by residents of the city).
3. Electric light and power, and gas.
4. Manufacturing.
5. Construction.
6. Transportation.
7. Communication.
8. Trade.
9. Finance.
10. Government, including work-relief wages and excluding work-relief wages.
11. Service.
12. Miscellaneous (all other sources of wages, salaries, commissions, etc., and business income).

B. Interest from savings accounts in—
1. Banks.
2. Postal savings.
3. Other depositories.

C. Net income from investments.
1. Owned real estate.
2. Stocks and bonds.
3. Miscellaneous (royalties, shares in cooperative establishments, etc.).

D. Other (all other known sources of local income such as direct relief, etc.).
E. Estimated per capita or per family income.
F. Distribution of family income by size of income (relative number of families having incomes below $1000, $1000–$2000, etc.).
G. Trends in income in recent years (general increase or decrease; increase or decrease in relation to changes in local cost of living; with respect to income from various industrial categories; for various types of income such as wages and salaries, investments, interest on savings, etc.; and in distribution of income by size of income).
H. Comparisons with other communities or with the United States as a whole (general conclusions to the extent of such supporting data as may be obtainable).
I. Peaks and levels (seasons or months during which local income is normally highest; normally lowest).
J. Pay-roll methods (general analysis concerning the relative proportions of full-time wage earners and salaried employees that are paid by the week, by the month, etc., and number of times in the current year when "pay days" in factories and in other important employment-giving establishments can be expected to fall on a given Monday, Tuesday, or other day of the week, and on the first week, second week, or other week of the month).
K. Other (other outstanding characteristics).

X. Living Costs, Standards, and Conditions and Related Indexes of Consumer Use of Buying Power.

A. Living costs (price trends on consumer goods).
1. General average (as compared with other cities).
2. Commodity prices, for food, clothing, etc. (average for year).
3. Trends (comparisons with previous years sufficient to reveal general trends in local living costs).
4. Other data on living costs (indicative of specific local advantages or limitations—fresh fruits and vegetables at reasonable cost, etc.).

B. Family expenditures.
1. Kinds of goods and services used (by consumers of the various income classes).
2. Average expenditure for major purposes (estimated annual expenditures), by income classes.
3. Comparisons (such comparisons with other years and other cities as are possible).
4. Other data of special local significance.

C. Housing (considered together with XI).
1. Number of occupied dwellings.
   a. Percentage occupied by the owners.
   b. Distribution of home owners, by lower, middle, and upper income levels (estimates).
   c. Average value of owner-occupied dwellings; recent trends.
2. Rent (average for year per dwelling unit) as compared with other cities; estimates, if possible, as to average rents paid by families of the various income classes, and as to trends in average yearly rent in relation to changes in family income.
3. Light, heat, water, and telephone service.

a. Rates charged for home use; comparisons with other cities.
b. Recent local changes in rates or service.
c. Other (additional points of special interest).

4. Modern improvements in homes, situation with respect to—
   a. General adequacy of plumbing and heating facilities.
   b. Number of installed air-conditioning units.
   c. Other.
5. Other data (such other data as may be essential to a true picture of the local housing situation).

D. Related indexes of consumer use of buying power.

1. Number of residence telephones.
2. Number of registered automobiles.
3. Number of domestic electric meters.
4. Number of families having radios.
5. Circulation of newspapers.
6. Circulation of magazines.
7. Trends (items D-1 to D-6, inclusive).
8. Other indexes (pertinent data are also included under "Banking," "Retail Trade," "Construction and Real Estate").
9. Comparisons (1–8) with other cities; and with previous years (sufficient to indicate trends in total increase or decrease, also trends in terms of relationship between increased or decreased population).

E. Other data pertinent to the local situation.

XI. Construction and Real Estate.

A. Construction (by kinds of business, Census classification: general contractors for building, highway, and heavy construction; special trade contractors, such as carpentering, concreting, electrical, excavating, heating and plumbing, roofing, and sheet metal, etc.).

1. Number of contracting establishments.
2. Value of work performed (dollar volume for year).
3. Number of active proprietors and firm members.
4. Number of employees (average for year).
5. Pay rolls (dollar volume).
6. Cost of materials, brick, cement, lumber, steel, and all other materials (actual or estimated dollar volume for year).
7. Comparison of local construction costs with other cities, where comparable information is available.

B. Building permits (total number for year and dollar value).

1. Residences.
   a. One-, two-, and three-family units (by cost groups).
   b. Apartments.
   c. Hotels.
2. Office buildings, retail stores, etc.
3. Industrial structures (factory buildings, etc.).
4. School buildings, churches, theaters, municipal buildings, etc.
5. Other (comparisons with other cities, etc.).

C. Demolition and conversion permits (with such breakdown as is important).

D. Number of vacant buildings by areas in the city (classified as to residences of the various types, office buildings, manufacturing plants,

etc. Data on general condition of vacant buildings sufficient to indicate relationship to construction needs).

E. Building regulations, local and state, affecting local building activity (zoning, etc.).

F. Trends in local construction (as indicated by such statistics for preceding years as are available).

G. Number of real estate firms, average number of employees for the year, actual or estimated total wages and salaries paid.

H. Real estate sales, mortgages, etc. (for latest year of record; comparisons with preceding years sufficient to indicate general trends).
1. Number of sales and dollar value of property sold.
2. Number of mortgages recorded and total amount of mortgage debts.
3. Number of foreclosures.

I. Real estate taxes (details to supplement general data under "Municipal Administration," XXVII).

J. Rent (considered under "Housing," *X*-C).

K. Real estate prices (general trend during recent years).

L. Supply of improved and unimproved properties (for home building, industrial and commercial expansion; available plant site close to railroad, river crossing, and such other special features as should prove of interest to industries of the types for which the city is suited).

M. Other (such additional facts on construction and real estate as are of special importance to the community).

XII. Wholesale Trade.

A. Number of establishments.
1. Total (all establishments operated by "wholesalers" as defined in the latest census of business).
2. By types of operation.
   a. Full-service and limited-function.
   b. Manufacturers' sales offices.
   c. Bulk-tank stations (independents and chains engaged primarily in the storing and wholesale distribution of petroleum products).
   d. Agents and brokers ("middlemen" such as auction companies, merchandise brokers, commission merchants, etc.).
   e. Assemblers (cream stations, cooperative marketing associations, grain elevators, and other buyers and sellers of farm products on a wholesale basis).
3. By kinds of business (grocery, drug, etc.).
   a. Independent.
   b. Chain.

B. Employment.
1. Total number employees (average for year).
2. Number employees in each kind of business (average for year).
3. Seasonal tendencies.

C. Pay rolls (total for year).
1. Part-time.
2. Full-time.

D. Sales.
1. Total (dollar volume for year).
2. By kinds of business (dollar volume for year).

3. Seasonal tendencies in sales volume.

E. Local regulations.

F. Number and kind of new establishments between the period covered by the last census of business and that covered by the preceding census, or during some other recent period for which comparable records may be available.

G. Business failures within the period referred to under XII-*F*.

H. Other facts (special characteristics affecting wholesale distribution through local establishments, such as recent trends toward cooperative wholesaling or other developments of importance).

I. Importance of the city as a wholesale trading center (if the city is not itself an important wholesale center, relate this subject to significant facts under "Location" with respect to wholesale markets, item II-2-*b*).

XIII. Retail Trade and Metropolitan Shopping Area.

A. Retail trade.

1. Number of establishments.
   - *a.* Total (all types and kinds of retail outlets in the city).
     - i. Independent.
     - ii. Chain (national, sectional, and local).
     - iii. Other.
   - *b.* By kinds of business (grocery, drug, etc.).
2. Employment.
   - *a.* Total number employees (average for year).
   - *b.* Number employees in each kind of business.
   - *c.* Seasonal tendencies.
3. Pay rolls (total for year).
   - *a.* Part-time.
   - *b.* Full-time.
4. Sales.
   - *a.* Total (dollar volume for year).
   - *b.* By kinds of business (dollar volume for year).
   - *c.* Per capita sales.
   - *d.* Comparisons with comparable data.
     - i. In preceding censuses.
     - ii. For other cities.
   - *e.* Seasonal tendencies in sales volume.
5. Approximate capitalization, or size of leading stores (if a particularly important local feature).
6. Store operating expenses (such comparisons with other cities as may be possible).
7. Uniformity of store policies (with respect to credit, returned-goods, advertising, frequency of deliveries, etc.).
8. Attitude of principal stores toward each other (generally cooperative or otherwise, as indicated by membership in local organizations, such as chamber of commerce, retail credit association, or other credit agency, etc.).
9. Local regulations.
10. Store locations in general with respect to shopping areas (downtown, suburban, etc.) in relation to trends in population within the city and convenience to consumers.

11. Number and kind of new establishments between the period covered by the last census of business and that covered by the preceding census or during some other recent period for which comparable records may be available.
12. Number of business failures within the period referred to under item XIII-11.
13. Other facts (such as principal wholesale sources of locally retailed foods, clothing, and furniture, if the local situation seems to warrant a direct inquiry on this point).

B. Metropolitan shopping area.

1. Consumer buying "radius" in square miles as indicated by such measures as density of population in suburban areas (census method), extent of the city's newspaper distribution, retail store deliveries, use of city's internal transportation facilities, etc.
2. Suburban-customer patronage of downtown stores (extent of).
3. Patronage of local stores by consumers from other cities (extent of).
4. Other important considerations.

XIV. Buying Habits of Local Consumers.

A. Proportion of cash purchases to total purchases; comparisons with other communities; apparent trends with respect to instalment buying.

B. Apparent general preferences as to shopping hours of the day, days of the week, and time of the month (refer to IX-*J*).

C. Buying in other cities by local consumers.

1. Estimated percentage (of total purchases) bought in other cities.
2. Classes of goods most frequently bought in other cities.
3. Names of cities from which goods are frequently purchased.
4. Recent trends in connection with extent of outside buying.

D. Other important characteristics (applying to an analysis of why trade is lost to or gained from competing communities).

XV. Service Establishments, Hotels, Places of Amusement.

A. Service establishments, by kind of service (Census classification—Personal Services: Barber shops, beauty parlors, cleaning shops, etc.; Business Services: Adjustment and credit bureaus, collection agencies, dental laboratories, window cleaning services, etc.; Repair Services, and Custom Industries: Automotive repairs and services, parking lots, radio repair shops, printing and publishing shops, etc.; Miscellaneous Services: Typewriter and rental shops, designers' establishments, and such other services as can not properly be classified under any of the other service groups).

1. Number of establishments (recognized places of business).
2. Number of employees (average for year).
3. Pay rolls (dollar volume for year).
   a. Full-time.
   b. Part-time.
4. Receipts for the year (in cities covered by Census reports, at least).
5. Seasonal tendencies of importance.
6. General trends within recent years (in number of employees and in other respects suggested by the local situation).

B. Hotels.

1. Number of hotels; date of establishment, size, and other general description of principal hotels.

2 to 5. (As suggested for *A*-2 to 5 inclusive.)

6. Rates.
    - *a.* Current rates (in principal hotels).
    - *b.* Recent trends.
    - *c.* Comparisons with rates in comparable cities.
7. Recent general trends in volume of business.
8. Number of first and second class hotels closed within recent years.
9. Other facts of significance (convention facilities, see XXI-*A*; etc.).

C. Places of amusement (amusement parks, athletic fields, bathing beaches, billiard and pool parlors, dance halls, skating rinks, theaters, other important amusement places).

1. Description of principal places.

2 to 5. (As suggested for *A*-2 to 5, inclusive.)

6. Recent general trends in volume of business.
7. Other facts of importance to the community.

XVI. Transportation Facilities.

A. Railway.

1. Number and names of railways serving the city.
2. Passenger traffic.
    - *a.* Number of trains daily (convenience as to time schedule).
    - *b.* Outgoing.
        - i. Average number passengers (per day, week or month).
        - ii. Important destinations (points to which largest number of tickets are sold).
        - iii. Seasonal tendencies.
    - *c.* Incoming.
        - i. Average number passengers (estimated).
        - ii. Seasonal tendencies.
3. Freight traffic.
    - *a.* Total number of trains daily (convenience as to time schedule).
    - *b.* Number of through trains daily (direct routes to markets).
    - *c.* Number of freight yards.
        - i. Yard-track miles.
        - ii. Industry-track miles (sidings adjoining plants).
    - *d.* Volume of freight handled (in carloads or tons).
        - i. Comparison between estimated volume of incoming and outgoing freight.
        - ii. Other facts bearing on economical accessibility to markets (freight comparisons—preferential and non-preferential rates to principal points, etc.).
    - *e.* Number of stations for each railway.
    - *f.* Extent of "pick up and delivery" freight service.
4. Freight warehouses.
    - *a.* Number.
    - *b.* Capacity.
    - *c.* Ownership (railway-owned, etc.).
5. Additional data indicating special or unusual facilities.

6. Important developments or changes within recent years.
7. General trends in volume of business (passenger and freight).
8. Other facts significant of the local situation (travel time to important points, etc.).

B. Water.

1. Number and names of lines.
2. Markets served and frequency of service (by ocean, lake, river, and canal).
    - *a*. Domestic.
        - i. Name of markets.
        - ii. Number of sailings to each port (during week, month, or other specified period).
    - *b*. Foreign (name of markets—number of sailings to each port).
        - i. Direct to foreign markets.
            - (*a*) Under American flag.
            - (*b*) Under foreign flags.
        - ii. Indirect to foreign markets (by transshipment at other ports, domestic or foreign).
3. Volume of trade.
    - *a*. Outgoing.
        - i. Domestic.
        - ii. Foreign.
    - *b*. Incoming.
        - i. Domestic.
        - ii. Foreign.
    - *c*. Terminal facilities and service.
        - i. Number of piers—number of berths, dry docks, etc.
        - ii. Warehouse facilities.
            - (*a*) Number of bonded warehouses.
            - (*b*) Special data (such as capacity, ownership, etc.).
        - iii. Lighterage facilities.
        - iv. Port development, not elsewhere specified (data such as depth of water, bunker-coal supply, fuel-oil supply, fresh water supply, anchorage facilities, and other local developments of importance to shipping companies).
        - v. Customs house.
        - vi. Foreign consul service.
4. Other data of importance.
5. Important developments or changes within recent years.
6. General trends in water transportation (volume of business, etc.).

C. Air service.

1. Ownership or control.
2. Localities served.
    - *a*. Directly (without change of planes).
    - *b*. Travel time to important points.
3. Frequency of flights.
    - *a*. Mail, express, and passenger.
    - *b*. Approximate volume of traffic.
        - i. Outgoing.
        - ii. Incoming.

4. Airport (city's accessibility to) and other special facilities (such as lighting, radio communication, weather reporting, public conveniences, etc.).
5. Important developments or changes within recent years.
6. General trends (in volume of business, etc.).
7. Other important facts.

D. Highway transportation.
1. Principal highways (entering and leaving the city—U. S. and state highways by number).
   a. Primary (main interstate routes).
   b. Secondary and county.
   c. Condition of road surfaces.
   d. Extent of road improvement within recent years.
   e. Other features of importance.
2. Motor bus and truck service (intercity, interstate, etc.).
   a. Number and names of companies.
      i. Number of units in operation (total or by each company, number of routes covered, mileage, etc.).
      ii. Cities served by each line (points on routes of particular importance to the city that is being surveyed).
      iii. Number of daily or weekly trips.
      iv. Approximate volume of freight handled (tons).
         (*a*) Outgoing.
         (*b*) Incoming.
      v. Average number of passengers carried (estimated, per week or month).
         (*a*) Outgoing.
         (*b*) Incoming.
   b. Special terminal facilities.
   c. Recent trends (volume of passenger and freight business to and from the city, etc.).
3. Important developments or changes (equipment, service) within recent years.
4. Other facts of importance.

E. Electric car service (interurban).
1. Number of lines (passenger, freight, or both).
   a. Cities serviced by each line.
   b. Number of daily or weekly trips.
2. Average volume of business (per day, week, or month).
   a. Passenger.
   b. Freight.
3. Recent trends.
4. Other data of local importance.

F. Pipe-line facilities (for transporting of oil, gas, etc., to or from the city).
1. Number and names of lines.
2. Cities served by each line (points of particular importance to the city that is being surveyed).
   a. Gathering (lines).
   b. Trunk.
3. Capacity.

4. Products carried (fuel oil, gas, etc.).
5. Volume of shipments (to and from the city)
6. Developments within recent years.

G. Comparisons (if pertinent to local problems) between types of transportation facilities serving the city, as to—

1. Rates (passenger and freight).
2. Convenience.
3. Trends toward increased or decreased business within recent years.

H. City's internal transportation facilities.

1. Street-car service.
    a. Ownership (public or private).
    b. Average daily number of passengers.
    c. Number miles of track or route.
    d. Routes, from the point of view of best service to the city.
    e. Rates (of fares).
    f. Important developments or changes within recent years in—
        i. Service.
        ii. Rates.
        iii. Type of equipment.
        iv. Average daily number of passengers (general trend toward increase or decrease).
    g. City regulations.
    h. Other (special features of the local situation).
2. Bus lines (same points as for street-car service).
3. Cooperative delivery service (or other special merchandise delivery facilities for serving the city).
4. Other facilities of importance (taxicab service, etc.).

I. Other (all other significant facts concerning transportation facilities in relation to present and potential requirements).

XVII. Public Warehousing.

A. Statistics by kind of warehousing establishments (household goods, general merchandise, farm products, cold storage, other).

1. Number of warehouses.
2. Storage rates for principal commodities.
3. Number of employees (average for year).
4. Pay rolls (dollar volume).
5. Revenue (estimated total for year, all warehousing establishments).
6. Total local revenue and employment as compared with totals for the region in which the city is located (New England, Middle Atlantic, etc.) or such other helpful comparisons as may be possible.
7. Recent changes in number of warehouse establishments through firms going out of business or new warehouses established.
8. Adequacy or inadequacy of present public warehousing facilities.

B. Seasonal tendencies in warehousing (month or months with highest dollar volume of business, lowest dollar volume; largest number of employees, smallest number of employees, etc.).

C. General trends in volume of business within recent years.

D. Other facts of importance.

XVIII. Banking and Finance, Insurance.

A. Banks.

1. Names of principal banks, dates of establishment, and description of each as to kind: national, state, private, mutual savings, industrial (including Morris Plan), Federal Reserve, Joint Stock, Land, and foreign banking agencies; also classified as to "unit" and "branch banks" (Census definition).
2. Total number of banks.
3. Employment and pay rolls (for year).
   a. Average number of executives (including presidents, vice-presidents, cashiers, assistant cashiers, assistant treasurers, and chairmen of boards of directors if actively engaged on a salary basis).
   b. Average number of other employees.
   c. Total salaries and wages, bonuses, and all other payments to officers and employees (dollar volume).
4. Rank of the largest bank in relation to other banks in the Federal Reserve district, state, etc. (or other helpful comparisons).
5. Bank deposits (number of accounts and dollar volume for year).
   a. Demand.
   b. Time.
6. Total loans and discounts (dollar volume for year); total assets.
7. Interest rates.

B. Financial institutions other than banks (security brokers and dealers, Federal savings and loan associations, state building and loan associations, instalment finance companies, personal finance companies, mortgage and farm companies, and miscellaneous, such as commodity exchange, bank clearing house, etc.)

1. Description of important institutions.
2. Employment and volume of business for the year of all such institutions.

C. Insurance and brokerage offices (identifiable as business establishments in the city).

1, 2. (Same information as suggested under "Financial institutions other than banks.")

D. General.

1. Total loans through all locally established financial institutions (dollar volume for year); or sufficient data to indicate the trend toward increase or decrease.
   a. Loans to business firms or other business organizations.
   b. Loans to individuals or families.
2. Comparisons, if possible, to show the general trend in interest rates, employment, volume of business and business failures, in local banking, finance, and insurance institutions.
3. Other (such additional information as may be of significance in considering the local situation, particularly with respect to abilities and inclinations of banks to extend credit to industrial and commercial enterprises. Trends in total dollar volume of savings accounts).

XIX. Professional Service (physicians, dentists, lawyers, etc.).

A. Number engaged in professional work (other than teaching) by—

1. Kind of profession.
2. Sex.

B. Revenues (for year) estimated, or known income from professional work.

C. Employment and pay rolls (for year).

1. Number of employees (office assistants, etc.).
2. Salaries and wages (dollar volume)

D. Comparisons with other years of record, sufficient to indicate trends in average fees charged and trends in the relation between available service and local requirements in each field.

E. Other important data.

XX. Newspapers, Radio Stations.

A. Newspapers.

1. Names of papers.
   - *a.* Dailies (classified as to morning, evening, other and as to Democratic, Republican, Independent, etc.).
   - *b.* Weeklies.
2. Revenues from—
   - *a.* Subscriptions.
   - *b.* Newsstand sales.
   - *c.* Advertising.
3. Employment and pay rolls (for year).
   - *a.* Number executives, contributors (average).
   - *b.* Number other employers (average).
   - *c.* Salaries and wages (dollar volume).
4. Largest circulation area (square miles covered by routes).
5. Important changes or trends within recent years.
   - *a.* Amount of increase or decrease in largest circulation area.
   - *b.* Number and kind of papers added or discontinued.
   - *c.* In equipment.
   - *d.* Advertising rates and volume.
   - *e.* Activities and influence (cooperation on local surveys, charity drives, etc.).
6. Other significant features (such dates of establishment as are of interest, etc.).

B. Radio stations (essential details as to names of stations, ownership and employment, revenues and wattage; and significant trends as to importance as an advertising medium, etc. Any additional details suggested for newspapers that apply to the community's radio facilities).

XXI. Expositions, Fairs, and Conventions.

A. Local facilities (consider convenience and adequacy of assembly places and equipment; all other local features that may appreciably influence patronage: see XV-*B*-9).

B. Nature of assemblies (annual, occasional or permanent expositions, fairs, trade shows, etc. of importance to the community considered from the angle of employment, revenue, type, and extent of out-of-town patronage, advantages to business and industry, etc.).

C. Other (allocation of responsibility for attracting conventions; extent of general cooperation within the community, etc.; see, also, XVI-*A*-2 and *H*).

XXII. Industrial and Commercial Associations (chambers of commerce, commodity and technical associations, grain exchanges, etc.).

A. Names of all associations and institutes having within the city an established place of business, classified as to national, regional, state, or local, and by nature of service.

1. Local membership (generally representative of the local business or industry served, or limited in representation).
2. Number employed (average for year).
3. Pay rolls (dollar volume for year).

B. Effectiveness (of organizations concerned with local conditions).

1. As aid to established local business and industry.
2. In advancing harmonious and cooperative relations with neighboring communities.
3. As factual authorities concerning the kinds of new industries and business establishments desirable from the point of view of successful development of each industry and of the community as a whole.
4. In promoting the location of suitable industries and commercial establishments and in discouraging potential industries under conditions of limited opportunity.

XXIII. Universities and Other Institutions (state, county, etc., having appreciable enrollments from other communities; other schools are covered under XXVIII).

A. Universities, colleges, academies, and all other educational institutions drawing a transient population.

1. Description of each important institution; date of establishment; special features attracting a student population, etc.
2. Enrollment.
   a. Total for year.
      i. From local population.
      ii. From other communities.
      iii. By age groups (sufficient to indicate predominating types of consumers brought in through these institutions).
   b. Total for month of highest enrollment.
   c. Total for month of lowest enrollment.
3. Revenue (dollar volume for year).
   a. Tuition fees.
   b. Other (classified as to sources).
4. Recent trends in enrollments and revenue.
5. Economic importance of these educational institutions.
   a. Employment and pay rolls (for year).
      i. Number of teachers and administrators (average).
      ii. Number of other employees by types of employment (average).
      iii. Pay rolls (total dollar volume).
   b. Estimated expenditures within the metropolitan area for supplies, service, and materials necessary for operation (dollar volume for year).
   c. Estimated total expenditures within the metropolitan area (exclusive of expenditures going to the institutions) of the enrolled individuals indrawn from other communities.

6. Other significant data (capacity in relation to trends in enrollments, etc.).

B. Federal, state, or county hospitals, asylums, etc. (all other than educational institutions largely subject to enrollments from outside the city).

1, 2. (Same type of data suggested under *A*-1 and *A*-2.)

3. Employment and pay rolls (for year).
   a. Number of administrative officials and professionals (average).
   b. Number of other employees, by types of employment (average).
   c. Pay rolls (total dollar volume).
4. Revenue (for year) classified by sources.
5. Estimated total expenditures within the metropolitan area for supplies, services, and materials necessary for operation (dollar volume for year).
6. Other significant data (capacity in relation to trends in requirements).

XXIV. Federal, State, or County Government Establishments (for a state capital or county seat, such special establishments and facilities as contribute important activities to this type of city).

A. Description of each important activity and establishment; significant details as to extent of employment and pay rolls for year, extent of permanent and transient population brought to the community through these activities; consumer characteristics of this population; architectural features of buildings; etc.

B. Recent changes or trends of importance.

XXV. Other (all other types of establishments and special features that tend to contribute to the community's employment, purchasing power, and consumption) description of each, and significant details as suggested under XXIV.

XXVI. Summarization of Principal State and Local Laws and Regulations (affecting industry and commerce and consumer purchasing power).

A. Taxation.

1. Enumeration of state and local laws and regulations providing for the levying of fees; franchise taxes, business privilege taxes, and other direct taxes, against commercial and industrial corporations, companies, etc., located in the city; also taxes and special assessments on real and personal business property.
2. Enumeration of state and local laws and regulations providing for the levying of direct taxes against individuals and families residing in the city; also taxes on real and personal property (other than business property).
3. State and local excises and other indirect taxes.
4. Recent trends in tax legislation.

B. Incorporation, and registration of individuals using firm name (requirements and cost).

C. Labor laws and regulations with respect to—

1. Hours and wages.
2. Unions.
3. Other labor factors.
4. Factory laws.
5. Recent trends in labor legislation.

- *D*. Banking and credit.
- *E*. Insurance (restrictions, rates, etc.).
- *F*. Zoning.
- *G*. Traffic (intra-city motor-truck regulations, etc.).
- *H*. Local laws regulating competition; other important regulations.
- *I*. All important recent trends in legislation not covered above under individual subjects or included under item XXVII.
- *J*. Recent court decisions of particular importance to the local situation.
- *K*. History of local taxation with respect to efforts to avoid excessive taxing of industry as compared with taxes in other communities within the geographic region (such as efforts toward self-sustaining improvements where feasible, consultation with large tax-paying industries preliminary to planning improvements involving heavier taxes, etc.).

XXVII. Municipal Administration and Related Data.

- *A*. Form of city government (mayor-council, commissions, council-manager, town-meeting); comparison with form in comparable cities.
  1. Internal organization and personnel.
     - *a*. Departments, boards, and commissions (organization chart or other enumeration of data showing the government set-up; departments; number and purpose of all boards and commissions and how members are selected; what chief municipal officials are elected by the people, what officials appointed by the mayor or city manager).
     - *b*. Number local government employees (total for year; estimated number part-time employees) classified as to administrative officials and others.
     - *c*. Salaries and wages paid from city funds (total for year).
     - *d*. Trends in pay rolls; personnel "lay-offs" and salary cuts, etc.
  2. Recent changes of importance in municipal administration (such as changes in administrative organization; personnel administration; financial planning; municipal budgeting; municipal accounting; public purchasing policies; municipal powers—recent legislative enactments limiting or increasing municipal powers; city planning and zoning provisions; ordinances relating to retail trade; provisions relating to playgrounds and recreation, and public welfare; police administration, public works, schools and education, public libraries, judicial administration, etc.).
- *B*. Financial statistics.
  1. Revenue receipts for the year, classified as to source (general property taxes; special property and other special taxes; poll taxes; license taxes; special assessments; fines, forfeits, and escheats; grants in aid; donations; pension assessments; highway privileges; rents and interest; earnings of general departments; earnings of public-service enterprises, etc.).
     - *a*. Total.
     - *b*. Per capita.
  2. Payments for operation, for the year (including salaries and wages of personnel) classified as to object of payment (such as general government; protection to person and property; conservation of health; sanitation; highways; charities, hospitals, and corrections; education; recreation; miscellaneous).

a. Total.
b. Per capita.

3. Total gross debt (at year-end of accounting—classified as to character of obligations, such as funded or fixed, special assessment bonds and certificates, and revenue notes); "net indebtedness"; or such other classification of debt obligations as will adequately describe the city government financial situation.
4. Difference between total revenue receipts and total payments for operation in terms of excess or deficit (for the latest year of record).
5. Trends in municipal debt; comparison with other cities.
6. Financing relief and recovery.
   a. Methods used.
   b. Trends in number receiving relief from public funds (comparisons for recent years of record).
7. Comparisons with other cities.
8. Recent economies effected in local government.

C. Taxation (by kind of taxes: real estate, poll, income, license, etc.).

1. Current rates, also comparisons with previous years sufficient to indicate trends; comparisons with comparable cities.
2. Total assessed value of all taxable real estate.
3. Total delinquent taxes for the year (by kind of taxes); total accumulated delinquent taxes.
4. Other facts of importance.

D. Utilities owned and operated (data as to kind and importance of each activity).

E. Number of registered voters (comparisons with number registered during preceding years of record).

F. Other facts of significance.

XXVIII. Civic, Social, and Related Facilities and Activities.

A. Clubs, lodges, social service agencies, etc.—general data as to—

1. Principal organizations.
2. Nature and importance of influence in the community.
3. Clubhouses and other facilities.

B. Churches (number, classified by denominations of membership).

C. Schools (exclusive of "Universities and other institutions" under XXIII).

1. Number, classified as to type (grade, high schools, public, private, parochial, etc.).
2. Recent trends in enrollment.
3. Adequacy of buildings and other facilities.
4. Recent trends in public school development (general situation; salary scale of teaching force in public schools; such comparisons with other cities as may be helpful).
5. Other facts indicative of educational developments.

D. Library facilities (number and kind—public, private, special; adequacy).

E. Theaters, motion-picture houses, etc.

F. Playgrounds and parks (description of existing facilities, as to location, size, accessibility, recent improvements, adequacy or inadequacy).

1. Play lots, for children of pre-school age.
2. Neighborhood playgrounds, for children up to fourteen years.

3. District playfields, for active play of people over fourteen years.
4. Athletic fields, for organized sports.
5. Small neighborhood parks, for passive recreation.
6. Large parks, for passive recreation.
7. Reservations (large parks left in natural state).

G. Other recreational facilities (golf courses, fishing and hunting facilities, etc.).

H. Special facilities (central garage, community market, etc.).

I. Relief facilities.
1. Community chest or other local provisions.
2. Recent trends in the local relief situation.

J. Health and safety facilities and conditions.
1. Hospitalization and medical care.
   a. Description of hospital facilities (other than state, county, etc. under XXVIII).
   b. Provision for free clinics, etc.
   c. Recent trends in rates to patients, developments in service and facilities.
2. Vital statistics (birth and death rates for year; facts sufficient to indicate trends over a period of years; comparisons with other cities).
3. Water for domestic use (source, adequacy of supply, and quality; recent trends in rate scale to consumers).
4. Sewage and garbage disposal.
5. Fire protection (adequacy, details as to water pressure, placement of hydrants, etc.).
6. Police protection (adequacy, details as to measures taken to safeguard the community).
7. Traffic and parking regulations (recent trends toward increase or decrease in traffic accidents); essential details not included under XXVI.
8. Sanitary promotion (measures taken to safeguard the public from unsanitary conditions in public eating places, beauty parlors, barber shops; requirements as to care in the wholesaling and retailing of perishable goods, etc.).

K. Other important civic and social features.

L. Recent civic and social changes or trends of outstanding importance.

XXIX. Important Physical Facilities and Special Features not elsewhere described (number miles or blocks of pavement, lighted streets, etc.; boulevards, water front, bridges, etc.).

SECTION II.—SOURCES OF DATA

The listings given in this section are numbered and lettered to indicate at what point each of the various informational items apply to major subjects and sub-headings enumerated in the outline under Section I.[1]

I. Historical Sketch.

A-1 and 2. American Guide Series. List of published guides available from Federal Writers' Project, Works Progress Administration, Washington, D. C. (As of early 1938, 150 city, state, and regional guides had been printed by private publishing houses. Other reports in process.)

Local authorities[2] including oldest inhabitants, encyclopedias, and historical references in local libraries, newspapers, city, county, state, and Federal governmental records.

B-1. American Guide Series.

State Maps, Geological Survey, U. S. Department of Interior. (Includes details of topography.) Available from U. S. Department of Interior, Washington, D. C. Prices from 25 cents to $1.

"Climatic Summary of the United States" (by areas). Weather Bureau, U. S. Department of Agriculture. Superintendent of Documents, Washington, D. C. Price $5. (Reports for individual areas, price 10 cents.)

"Atlas of American Agriculture," 1936, Bureau of Agricultural Economics, U. S. Department of Agriculture. Superintendent of Documents. Price $17.

2. American Guide Series.

"Map of the Principal Waterways of the United States." Army Engineer Corps, U. S. War Department. Available from U. S. War Department, Washington, D. C. Free.

Port Series and Lake Series. Army Engineer Corps, U. S. War Department. (Separate report for each port.) Superintendent of Documents. Prices vary from 25 cents to $2.25.

Local authorities (keepers of township and county historical records, newspaper files, etc.).

3, 4, 5, 6, and 7. American Guide Series.

Local authorities (oldest inhabitants, local historical records, etc.).

C. American Guide Series.

Local authorities (city officials in charge of departments, surveyor's personal check and analysis).

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[1] The purchase price and source are indicated the first time reference is made to a publication. Prices listed refer to prices on publications available as of July 1938. Prices on later editions treating the same subjects may vary in price and not be identical in title.

[2] This reference and subsequent references to local authorities are given as possible sources of information when published material applicable to the subject is not known to exist or may be inadequate for local purposes. If information for the individual city is not available from outside publications and has not been collected by some local organization, the investigator will necessarily choose between not including the subject in his study or covering it through a field survey.

II. Location.

A. State Maps, Geological Survey.
Port Series and Lake Series, Army Engineers Corps.
"Map of the Principal Waterways of the United States." Army Engineers Corps.
"Atlas of American Agriculture."
United States atlases issued by private publishers. Available for reference in public libraries.

B-1. State Maps, Geological Survey.
"Atlas of American Agriculture."

2. "Climatic Summary of the United States."
"Atlas of American Agriculture."

3. American Guide Series.
Local authorities (personal observers, managers of tourist offices, offices of city departments, etc.).

C-1-*a*. *Census of Agriculture*, 1935. Bureau of the Census, U. S. Department of Commerce. (Statistics by Counties.) Superintendent of Documents. Three volumes, price $6.75.
"Atlas of American Agriculture" (soil formations).
Soil Survey Reports. Bureau of Chemistry and Soils, U. S. Department of Agriculture. (Separate reports for individual counties.) Superintendent of Documents. Prices from 10 cents to 75 cents.
Minerals Yearbook. Bureau of mines, U. S. Department of Interior. Superintendent of Documents. Price 1937 edition $2.25.
"The Lumber Industry, Including a Selected List of Trade Associations, 1936." Bureau of Foreign and Domestic Commerce, U. S. Department of Commerce. Available only from Bureau of Foreign and Domestic Commerce, Washington, D. C. Price 10 cents.
Lumber, Lath and Shingles, 1936. Bureau of the Census. Available from the Bureau of the Census, Washington, D. C. Free.
"Forest Regions of the United States, 1933." Forest Service, U. S. Department of Agriculture. (Map.) Available from Forest Service, U. S. Department of Agriculture, Washington, D. C. Free.
"Present and Potential Timber Resources, 1933." (Reprinted from a "National Plan for American Forestry," Senate Document No. 12, 72d Congress.) Forest Service, U. S. Department of Agriculture. Free.

b. *Minerals Yearbook*.

b and *c*. "National Power Survey, 1935. Interim report Power Series No. 1," Federal Power Commission. Superintendent of Documents. Price 75 cents.
Local authorities.

C-2. Sources of information in the city and nearby cities.

C-3-*a*. "Industrial Market Data Handbook of the United States, 1938." Bureau of Foreign and Domestic Commerce. (Statistics by counties and states.) Available from Superintendent of Documents. In process.
Biennial Census of Manufactures, 1935. Bureau of the Census. (Statistics for counties and cities above 10,000 population.) Superintendent of Documents. Price $1.75.

Minerals Yearbook.

b. "Atlas of Wholesale Grocery Trading Areas." Bureau of Foreign and Domestic Commerce. Available only from Bureau of Foreign and Domestic Commerce. Price $1.00.

Census of Business, 1935. "Wholesale Distribution, Vol. III." Bureau of the Census. (Statistics for counties and cities above 5,000 population.) Available from Bureau of the Census. Free.

c. "Consumer Market Data Handbook, 1938." Bureau of Foreign and Domestic Commerce. (Statistics by counties and cities above 2,500 population.) Superintendent of Documents. In process.

"Patterns of Stores, Sales, and Population in the United States." Bureau of Foreign and Domestic Commerce. (Concentration of stores by kinds of business.) Available only from Bureau of Foreign and Domestic Commerce. Price 10 cents.

Domestic Regional Surveys. Bureau of Foreign and Domestic Commerce. (Separate volume for each area covered.) Superintendent of Documents. Prices range from 60 cents to $1.85.

Census of Business, 1935. "Retail Distribution, Vol. III." Bureau of the Census. (Statistics by counties and cities above 2500 population by kinds of business.) Available from Bureau of the Census. Free.

"Consumer Trading Areas of the United States, 1935." (Map.) Hearst Magazines, Inc., 57th Street at 8th Avenue, New York City.

"Hagstrom's 4-color Retail Trading Area Map of the United States, 1936." (Trading areas of principal sales centers.) Hagstrom Co., Inc., 22 Vesey Street, New York City.

d. Port Series and Lake Series, Army Engineers Corps.

Foreign Commerce and Navigation of the United States. (Annual.) Bureau of Foreign and Domestic Commerce. Superintendent of Documents. Price of 1936 edition (2 vols.) $3.00.

Monthly Summary of Foreign Commerce of the United States. Bureau of Foreign and Domestic Commerce. Superintendent of Documents. Subscription price, $1 a year; single copies, 10 cents. (Includes export statistics by country of destination.)

III. Population.

A-1 and 2. Census of Population, 1930, Vol. III. Bureau of the Census. (States, counties, cities of more than 1,000 population and minor civil divisions.) Superintendent of Documents. Two Parts. Price $6.50. (New Census every ten years.)

"Metropolitan Districts," 1930. Bureau of the Census. (96 metropolitan districts with population of 100,000 or more.) Superintendent of Documents. Price 85 cents.

B. Census of Population, 1930, Vol. III.

C. Census of Population, 1930, Vol. VI. Bureau of the Census. (States, counties, and cities above 2,500 population.) Superintendent of Documents. Price $3.

D-1. Census of Population, 1930, Vol. III.

2. Annual midyear estimates of population by states. Bureau of the Census. (Cities above 10,000 also covered in 1933 estimates.) Available from Bureau of the Census. Free.

State and Special Censuses since 1930. (Multilithed.) Bureau of the Census. (Cities above 10,000 in the six states covered, also Chicago and Cincinnati.) Available from the Bureau of the Census. Free.

3. Local authorities.

E and *F*. Census of Population, 1930, Vol. III.

G. Surveyor's analysis.

IV. Materials Required by Industry (availability and cost factors).

A and *B*. Local sources of information.

V. Power.

A. "National Power Survey, Interim report, Power series No. 1." Federal Power Commission, Washington, D. C. (Power resources and requirements of the United States including maps and statistics.) Superintendent of Documents. Price 75 cents.

"National Power Survey, Principal Electric Utility Systems in the United States, Power Series No. 2." Federal Power Commission. Superintendent of Documents. Price $1 (paper); $1.50 (buckram).

Local and state authorities, such as government departments, local supplier, and individual users.

B. "Rates for Electric Service to Commercial and Industrial Customers." Federal Power Commission. Superintendent of Documents. Price 50 cents.

Local municipal administrative officials, records at source of supply, oldest industries, such state or regional authorities as may be concerned.

C, *D*, *E*, and *F*. Local sources of information.

VI. Industry.

A and *B*-1 and 2. *Biennial Census of Manufactures* 1935, (Statistics by industries and states). NOTE: Multigraphed releases were issued in 1937–1938 by the Bureau of the Census covering statistics by industries in 33 major industrial areas and showing in total (not by industries) production figures for counties and for cities above 10,000. In the report of the 1929 census and in preceding censuses of manufactures the data were broken down by industries in cities above 100,000. A complete understanding as to available census reports and the price of each report can best be obtained through reference to the current edition of "List of Publications of the Department of Commerce" available, without charge, from the Department of Commerce or Superintendent of Documents; for information as to latest processed census reports write direct to the Bureau of the Census.

"Industrial Market Data Handbook" (statistics by counties), 1938.

Editor and Publisher Market Guide (annual) includes lists of principal industries by cities in a form convenient for comparisons between cities of similar size, etc. Available from Editor and Publisher, Suite 1700 Times Building, New York. Subscrip-

tion price $4.00. (Probably available for reference in local newspaper offices.)

Local and state authorities: Manufacturers' association, chamber of commerce, state officials concerned with industry, executives of individual industries, etc. (Since individual industries fill out schedules for the biennial census it may be possible through general local cooperation to consult individual industries' file copies of present and past reports for a direct survey covering details not included in the published census reports.)

B-3. Local authorities and executives of industries moved to other cities.

4. Local authorities.

C. Local authorities.

VII. Labor Situation.

A. "Consumer Market Data Handbook, 1938."

Biennial Census of Manufactures, 1935.

Census of Population, 1930. Vol. III.

Census of Population, 1930. Vol. IV. (Number of gainfully employed by occupation.) Superintendent of Documents. Price $3.50.

Census of Unemployment, 1930. Vol. I. (Number of unemployed by industries and by sex.) Superintendent of Documents. Price $1.75.

"Census of Partial Employment, Unemployment, and Occupations: 1937." (Separate report for each state.) Bureau of the Census. Free.

Local authorities.

B. "Union Scales of Wages and Hours in the Building Trades," 1936. Bureau of Labor Statistics, United States Department of Labor. Superintendent of Documents. Price 10 cents. (This is an example of the type of bulletin available for principal industries.)

Note:—The Bureau of Labor Statistics has issued a number of bulletins by industries showing hours and wages in various cities. See Price list 33. Superintendent of Documents. Also write direct to the Department of Labor for latest information.

Local authorities.

C. "Workmen's Compensation Legislation of the United States and Canada, as of January 1, 1929." Bureau of Labor Statistics. (Texts of laws of the various states. Supplements available upon request.) Superintendent of Documents. Price 55 cents.

"Handbook of Labor Statistics," 1936. Bureau of Labor Statistics. (Contains sections on workmen's compensation, old-age pensions and unemployment insurance under Social Security, minimum wages and hours, etc.) Superintendent of Documents. Price $1.25.

"Occupational-Disease Legislation in the United States," 1936. Bureau of Labor Statistics. (Texts of the laws of the various states.) Superintendent of Documents. Price 10 cents.

Monthly Labor Review. Bureau of Labor Statistics. (Keeps pace with new and amendatory actions of state legislatures in the field of work-

men's compensation legislation, etc.) Superintendent of Documents. Subscription \$3.50; single copy 30 cents.

State and local authorities.

D. Local trade unions.

E and *F*. Local industrial and labor leaders, chambers of commerce, and other local industrial associations.

G. Local authorities.

VIII. Employment and Unemployment.

A. "Consumer Market Data Handbook, 1938." (Total gainfully employed in 1930 in counties and all cities above 2,500 population.)

Census of Population, 1930. Vol. III. (Condensed tabulation for principal industries only, by counties and cities above 25,000.)

Census of Population, 1930, Vol. IV. (Detailed break-down by all occupations, by sex, by color and nativity, by marital condition, and by age for states and cities above 25,000.) Superintendent of Documents. Price \$3.50.

B. Census of Population, 1930, Vol. IV.

C. Local authorities.

D. Census of Unemployment, 1930. Vol. I. (Number of unemployed by industries and by sex for cities of 50,000 and over; by sex only for cities above 10,000.) Superintendent of Documents. Price \$1.75.

"Census of Partial Employment, Unemployment and Occupations; 1937." Special Census under the administration of John D. Biggers. (Statistics on unemployed by age, sex, color, occupations, skilled and unskilled, etc., for counties and for cities above 10,000.) Final reports by states available from Census of Unemployment. Free.

Private and governmental social agencies and local authorities. (Records of local registrations and placements may be obtainable from the nearest office of the U. S. Employment Service of the Department of Labor; local government departments in some cities, such as Philadelphia, Baltimore, Buffalo, Columbus, and Syracuse, and possibly many other cities have assembled considerable data on the local situation; special unemployment data of interest to cities within the state have been assembled in some states such as Pennsylvania, Michigan, Washington, and Massachusetts; state departments of labor and industry in most states have some definitely helpful material; and state unemployment compensation boards, in compliance with the Social Security Act, have individual and plant pay roll records by months, quarters, and years.)

E. Local authorities.

IX. Consumer Purchasing Power (estimated income in dollars of individuals and families considered in relation to Cost of Living).

NOTE:—State and local income tax records, where income tax is paid and records are available for reference, are invaluable sources of material in lieu of or supplementary to a direct local survey among a representative sampling of the city's family population. Where a local field survey is not feasible and local income-tax records are not available, estimates derived through the application of all available published material is a valuable substitute. It is suggested that, in addition to considering such material as is referred to in the following list, a careful check be

made to ascertain what information for the city is available from offices of state departments in the state in which the city is located.[1]

A-1. *Census of Agriculture,* 1935. (Value of crops and livestock by counties.)

Receipts from the Sale of Principal Farm Products. Bureau of Agricultural Economics. (Monthly by states.) Available from Bureau of Agricultural Economics, U. S. Department of Agriculture. Free.

2. "Consumer Market Data Handbook, 1938." (Mining pay rolls by counties.)

"Personnel and Pay Roll in Industry and Business, and Farm Personnel, by Counties," 1935. Bureau of the Census. Available from the Bureau of the Census. Free.

3. Officials of local utility companies.

4. *Biennial Census of Manufactures,* 1935.

"Consumer Market Data Handbook, 1938."

"Personnel and Pay Roll in Industry and Business, and Farm Personnel, by Counties," 1935.

5. Construction Industry, 1935. Vol. I. Bureau of the Census. (States and Cities of 500,000 population or more.) Available from the Bureau of the Census. Free.

"Personnel and Pay Roll in Industry and Business, and Farm Personnel, by Counties," 1935.

6. "Motor Bus Transportation," 1935. Bureau of the Census. (Geographical regions only.) Available from the Bureau of the Census. Free.

"Motor Trucking for Hire," 1935. Bureau of the Census. (Geographical regions only.) Free.

Local railroad officials, for such information as may be available on income from employment in railroad activities.

7. "Radio Broadcasting," 1935. Bureau of the Census. (States, and cities above 500,000.) Available from the Bureau of the Census. Free.

8. "Personnel and Pay Roll in Industry and Business, and Farm Personnel, by Counties," 1935.

Census of Business, 1935. Retail Distribution, Vol. II. Bureau of the Census. (Pay rolls by counties and cities above 2,500.) Available from Bureau of the Census. Free.

Census of Business, 1935. Wholesale Distribution, Vol. III. (Pay rolls by counties and cities above 5,000.)

"Consumer Market Data Handbook, 1938."

9. "Personnel and Pay Roll in Industry and Business, and Farm Personnel, by Counties," 1935.

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[1] "There is a dearth of basic data from which to compute reasonably satisfactory income estimates. Local income is sometimes roughly estimated by applying population figures to per capita income as computed from estimates of national income; by a combination of projections from data on income-tax returns, together with existing results of sample studies on family-income distribution and by other methods suggested by material under "Consumer Purchasing Power" in the Basic Outline. Factual data for any community would necessarily come from a specific field survey. From "Suggestions for Use in Making a City Survey," see footnote on p. 671.

"Banks," 1935. Bureau of the Census. (States only.) Available from the Bureau of the Census. Free.

"Financial Institutions Other than Banks," 1935. Bureau of the Census. (States only.) Available from the Bureau of the Census. Free.

10. Governmental authorities in the community or area surveyed.

11. Census of Business, 1935. Service Establishments, Vol. II. Bureau of the Census. (Statistics for states, counties, and cities of 2,500 population or more.) Available from the Bureau of the Census. Free.

"Personnel and Pay Roll in Industry and Business, and Farm Personnel, by Counties," 1935.

12. "Personnel and Pay Roll in Industry and Business, and Farm Personnel, by Counties," 1935.

B-1. *Assets and Liabilities of Operating Insured Banks*, December 31, 1937. Federal Deposit Insurance Corp. (Time deposits of all insured banks by states.) Available from Federal Deposit Insurance Corporation, Washington, D. C. Free.

Financial statements of banks in local newspapers.

2 and 3. Local authorities. (Data usually not available except through field survey.)

C and *D*. NOTE:—The data on national income by sources such as agriculture, manufacturing, income from investments, etc., prepared annually by the Economic Research Division, Bureau of Foreign and Domestic Commerce, are pertinent to these and all other sub-headings under "Consumer Purchasing Power." Estimates by states will be released late in 1938.

E. *Sales Management Survey of Buying Power*. (Published annually in April. Estimated "effective buying income" by counties.) Sales Management, Inc., 420 Lexington Avenue, New York City.

Local estimates.

F. "Consumer Use of Selected Goods and Services, by Income Classes." Bureau of Foreign and Domestic Commerce. (50 representative cities.) Available only from the Bureau of Foreign and Domestic Commerce. Price, 12 vols., $1.65.

"Distribution of Families by Income and Family Type, 1935–1936." Bureau of Home Economics, United States Department of Agriculture, and Bureau of Labor Statistics, United States Department of Labor. (Preliminary reports covering native white families including both husband and wife in 51 cities, 140 villages, and 64 counties.) City reports available from Bureau of Labor Statistics, United States Department of Labor, Washington, D. C. Free. County and village reports available from Bureau of Home Economics, United States Department of Agriculture. Free.

"Consumer Incomes in the United States, Their Distribution in 1935–1936." National Resources Committee. (Includes estimates on average and aggregate incomes of nonrelief families by types of communities.) Superintendent of Documents. Price 30 cents.

"Financial Survey of Urban Housing," Bureau of Foreign and Domestic Commerce. (Detailed statistics for 22 cities on incomes of families in owner-occupied and rented dwelling units.) Superintendent

of Documents. Price $3.25. Available for reference in some public libraries.

(For many communities very little information is available except through special field surveys among representative samplings of the local family population.)

Statistics of Income Supplement, Section 1, "Number of Individual Income Tax Returns for 1934 Classified by States, Counties, and Cities of 25,000 and Over Population, by Net·Income Classes." Bureau of Internal Revenue, United States Treasury Department. Superintendent of Documents. Price 15 cents.

G and *H*. *Individual Income Tax Returns.* Bureau of Internal Revenue, United States Treasury Department. (Number of returns by states, by counties, and by cities and towns published annually.) Available from Bureau of Internal Revenue, United States Treasury Department, Washington, D. C. Free.

I, *J* and *K*. Local authorities.

X. Living Costs, Standards, and Conditions (and Related Indexes of Consumer Use of Buying Power).

A. "Retail Prices of Food, 1923–1936." Bureau of Labor Statistics. (51 cities.) Superintendent of Documents, Price 20 cents. Note: Other retail prices are collected and can be obtained upon request to the Bureau of Labor Statistics.

Changes in Cost of Living. Bureau of Labor Statistics. (Figures on 51 cities published quarterly.) Available from Bureau of Labor Statistics. Free.

"Intercity Differences in 59 Cities." Works Progress Administration, Washington, D. C. (Costs of Living in 59 cities.) Available from Works Progress Administration, Washington, D. C. Free.

State and local cost of living studies such as studies made by universities, chambers of commerce, and trade and economic groups. (The list is too long for specific mention here. See references in *Market Research Sources*, 1938 ed., Bureau of Foreign and Domestic Commerce. Superintendent of Documents. Price 30 cents.)

"Cost of Living in the United States, 1914–1936." National Industrial Conference Board. (Indexes of retail prices of foods, clothing, rents in 145 cities, coal in 56 cities, gas and electricity in 131 cities.) National Industrial Conference Board, 247 Park Avenue, New York City. Local authorities (intensive study usually involves a local field survey).

B. "Consumer Use of Selected Goods and Services, by Income Classes." Bureau of Foreign and Domestic Commerce. "Urban Study of Consumer Purchases," 1935–1936." Bureau of Labor Statistics. (Preliminary reports.) Available from Bureau of Labor Statistics. Free.

Family Expenditure Data, 1935–1936. Bureau of Home Economics. (19 small cities, 140 villages—preliminary reports.) Available from Bureau of Home Economics. Free.

"Money Disbursements of Wage Earners and Lower-Salaried Clerical Workers," 1934. Bureau of Labor Statistics. (Preliminary reports on 55 cities.) Available from Bureau of Labor Statistics. Free.

NOTE:—See *Market Research Sources*, 1938 ed., Bureau of Foreign and Domestic Commerce, for a list of various local studies on cost of living. Superintendent of Documents. Price 30 cents.

Local authorities.

C-1 and 2. Census of Population, 1930, Vol. VI. (Owned and rented dwellings by cities and counties, value and rental classes by cities.)

"Real Property Inventory," 1934. Bureau of Foreign and Domestic Commerce. (Separate report on each of 64 cities includes data on number of owner-occupant and tenant dwellings, values, rents, number of families with electricity, gas, installed bathing facilities, etc.) Available only from Bureau of Foreign and Domestic Commerce. Price 10 cents each, $5 per set.

NOTE:—Real Property Inventories have been made in more than 200 cities. A list of cities covered can be obtained from the Federal Housing Administration, Washington, D. C.

"Consumer Use of Selected Goods and Services, by Income Classes."

"Financial Survey of Urban Housing."

"Changes in Cost of Living."

"Intercity Differences in 59 Cities."

3. "Typical Net Monthly Bills for Electric Service in Effect January 1, 1937, in Cities of 50,000 Population or More." Federal Power Commission. Superintendent of Documents. Price 15 cents.

"Typical Net Monthly Bills for Electric Service in Effect January 1, 1937, in the State of (the state)." Federal Power Commission. (Separate report for each state covering all cities and towns within the state.) Superintendent of Documents. Prices 5 cents and 10 cents according to size.

"Rate Service," American Gas Association. (Complete gas-rate schedules for all communities in the United States.) 420 Lexington Avenue, New York City.

"Changes in Cost of Living." Bureau of Labor Statistics. (Includes coal prices in 51 cities.)

"Water Supply Statistics of American Municipalities, 1929," (rates and consumption) American City (Magazine), 470 Fourth Avenue, New York City.

Local public utilities.

"Real Property Inventory," 1934.

"Markets for Plumbing and Heating Facilities in Residences, an Analysis of the Real Property Inventory, 64 Cities." Bureau of Foreign and Domestic Commerce. Available only from Bureau of Foreign and Domestic Commerce. Price 10 cents.

"Markets for Electrical and Gas Facilities in Residences, an Analysis of the Real Property Inventory, 64 Cities." Bureau of Foreign and Domestic Commerce. Available only from Bureau of Foreign and Domestic Commerce. Price 10 cents.

"Effect of City Water and Sewerage Facilities on Industrial Markets." Bureau of Foreign and Domestic Commerce.

(Number of installations of air-conditioning equipment in 434 cities of 20,000 or more population.) Available only from Bureau of Foreign and Domestic Commerce. Price 10 cents.

5. Local sources of information.

D-1. "Consumer Market Data Handbook, 1938." (Number of telephones by counties.)
Local telephone company.

2. "Consumer Market Data Handbook, 1938." (Number of automobiles by counties.)
County or state officials.

3. "Consumer Market Data Handbook, 1938." (Number of meters by counties and cities of 2,500 or more.)
"Basic Industrial Markets in the United States, Gas and Electric Utilities." Bureau of Foreign and Domestic Commerce. (Number of gas and electric customers by counties.) Available only from Bureau of Foreign and Domestic Commerce. Price 10 cents.
Local power company.

4. "Consumer Market Data Handbook, 1938." Estimated number of families with radios by counties for 1938—*Source:* Joint Committee on Radio Research.
Census of Population, 1930, Vol. VI. (Actual number of families reporting radio sets for counties and cities of 2,500 or more.)

5. "Market Data Handbook of the United States," 1929. Bureau of Foreign and Domestic Commerce. (Newspaper circulation by counties as of 1927.) Superintendent of Documents. Price $2.50.
"Directory of Newspapers and Periodicals." N. W. Ayer & Son Co. (Circulation of individual newspapers.) N. W. Ayer & Son Co., West Washington Square, Philadelphia, Pennsylvania.
Local newspaper publishers.

6. "Consumer Market Data Handbook, 1938." (Circulation of 12 national magazines by counties and cities of 2,500 or more.)

7, 8, and 9. Local sources of information for special data applying to the local situation.

E. Local authorities.

XI. Construction and Real Estate.

A. Census of Business, 1935. Construction Industry, Vol. I. Bureau of the Census. (States and cities of 500,000 or over.) Available from Bureau of the Census. Free.

B and *C*. Building Construction. Bureau of Labor Statistics. (Monthly releases on about 2,000 cities of 2,500 population and over.) Available from Bureau of Labor Statistics. Free.
Local building permit officer.

D. "Real Property Inventory," 1934. (Total number of vacant residential units.)

E. Local building inspector, zoning board, etc. (Local study in most cities would require a special field survey.)

F. Local authorities.

G. Census of Business, 1935. Real Estate Agencies (cities of 25,000 and over). Available from the Bureau of the Census. Free.

H. Local recorder's office.

I. *Financial Statistics of Cities*, 1935. Bureau of the Census. (Assessed valuation of taxable property and amount of taxes levied for cities of 100,000 and over. Published annually.) Superintendent of Documents. Price $1.

"Financial Statistics of State and Local Governments," 1932. Bureau of the Census. (Assessed valuation and tax levies.) Superintendent of Documents. Price $2.50.

"Digest of State Laws Relating to Taxation and Revenue," 1922. Bureau of the Census. (Available for reference in public libraries. 1932 Digest separates are available on 12 states only.) Superintendent of Documents. Price 5 cents each.

Local and state tax authorities.

J, *K*, and *L*. Local real estate board, or association, chamber of commerce, and other local authorities.

XII. Wholesale Trade.

A, *B*, *C*, and *D*. Census of Business, 1935. Wholesale Distribution, Vol. III. Bureau of the Census. (Data for counties and cities of 5,000 or more; analysis by type of operation for cities of 50,000 or more; by kind of business for cities of 20,000 or more.) Available from Bureau of the Census. Free.

E. State and local authorities.

F. Census of American Business, 1933. Wholesale Distribution, Vol. VIII. Bureau of the Census. (Number of new establishments for a few selected states.) Available for reference in most large public libraries.

Local wholesalers.

G and *H*. Court records, Chamber of Commerce, etc.

I. "Atlas of Wholesale Grocery Trading Areas."

Census of Business, 1935. Wholesale Distribution, Vol. III.

Local sources of information.

XIII. Retail Trade and Metropolitan Shopping Areas.

A-1-*a*. Census of Business, 1935. Retail Trade, Vol. IV. Bureau of the Census. (States and Cities of 500,000 and over.) Available from Bureau of the Census. Free.

Local field survey.

b. Census of Business, 1935. Retail Trade, Vol. III. (Counties and cities of 2,500 or more.) Available from the Bureau of the Census. Free.

"Consumer Market Data Handbook, 1938."

Census of Business, 1935. Vol. II. (Detailed breakdown by kinds of business for cities of 50,000 and over.) Available from Bureau of the Census. Free.

2 and 3. Census of Business, 1935. Retail Trade, Vol. II. (Total employees and pay rolls for cities of 2,500 or more; kinds of business breakdowns for cities of 10,000 or more.)

Census of Business, 1935. Retail Trade, Vol. V. (Monthly employment data by kinds of business, for states and cities of 500,000 or more.) Available from Bureau of the Census. Free.

4-*a* and *b*. Census of Business, 1935. Retail Trade, Vol. III. (Cities of 2,500 or more.)

c. "Consumer Market Data Handbook, 1938." (Per capita sales for counties and cities of 2,500 and over.)

d. Analysis by the individual making the study.

e. *Retail Sales, Independent Stores.* Bureau of Foreign and Domestic Commerce. (Monthly releases showing percentage changes in sales in most cities in 26 states. Percentage changes in state figures are shown by types of business.) Available only from Bureau of Foreign and Domestic Commerce. Free. Local retailers.

5. Local retail trade association and chamber of commerce. Local retailers.

6. Local store executives and proprietors.
Census of Business, 1935. Retail Operating Expenses. (Rent and total operating expense for all kinds of business combined, by states; total operating expense by kinds of business in selected states. References to the more detailed operating expense data in previous censuses are contained in the foreword of the Census publication.) Available from the Bureau of the Census. Free.
Cost of doing business studies for particular kinds of business in particular localities made by the Bureau of Foreign and Domestic Commerce, other governmental departments, university schools of business administration, etc.
NOTE:—The list is too long for specific mention here. See references in *Market Research Sources*, 1938 edition, or make specific inquiry to the Marketing Research Division, Bureau of Foreign and Domestic Commerce.

7 and 8. Local merchants' associations, store executives, and proprietors.

9. Municipal and state authorities.

10. Local survey for detailed study. (In some cities, chain store organizations, or other authorities have made special investigations.)

11 and 12. Census of American Business, 1933. Retail Distribution, Vol. IV. (Statistics by states on the number of stores opened by kinds of business by quarters during 1933 and for the preceding three-year period.) Available for reference in most large public libraries.
Court records.

13. Local retailers.

B. "Metropolitan Districts, Population and Area," 1930. Bureau of the Census. (For all metropolitan districts over 100,000 population.) Superintendent of Documents. Price 85 cents.
"Consumer Trading Areas of the United States, 1935." (Map) Hearst Magazines, Inc.
"Hagstrom's 4-color Retail Trading Area Map of the United States, 1936." Hagstrom Company.
Local newspapers and chamber of commerce. Other local authorities. (The local situation may suggest a special field study.)

XIV. Buying Habits of Local Consumers.

A. "Retail Credit Survey," 1937. Bureau of Foreign and Domestic Commerce. (Based on reports from 86 cities on some of which

statistics will be presented separately showing open account, deferred-payment, and cash sales.) Superintendent of Documents. Price 20 cents.

B, *C*, and *D*. "Market Records, Buying Habits and Brand Preferences of Consumers in Sixteen Cities," 1938. Scripps-Howard Newspapers, National Advertising Department, 230 Park Ave., New York City, and other local studies on consumer buying habits made by newspapers, university bureaus of business research, etc.

NOTE:—The reports are too numerous to mention specifically in this list but references may be found in the 1938 edition of *Market Research Sources*. (Adequate information on the local situation may involve a special field study.)

XV. Service Establishments, Hotels, Places of Amusement.

A. Census of Business, 1935. Service Establishments, Vol. II. (States, counties, and cities of 2,500 and over.) Available from the Bureau of the Census. Free.

Census of Business, 1935. Service Establishments, Vol. III. (Monthly employment by states and cities of 500,000 and over.) Available from Bureau of the Census. Free.

Local service establishments.

B. Census of Business, 1935. Hotels. (By states, counties, and selected cities of 100,000 or more.) Available from the Bureau of the Census. Free.

Hotel Redbook and Directory. (Annual) American Hotel Association Directory Corporation. (Number of rooms, rates, and facilities.) Available from American Hotel Association Directory Corporation, 221 West 57th Street, New York City.

Local hotel managers, and state, county, and municipal records.

C. Census of Business, 1935. Places of Amusement. (By states.) Available from the Bureau of the Census. Free.

Proprietors of local places of amusement.

XVI. Transportation Facilities.

A-1 and 2. *Official Guide of the Railways*, National Publishing Co., 424 West 33rd Street, New York, New York.

Local station master.

Local passenger agents of the various railroads.

3 and 4. "Freight Commodity Statistics," Interstate Commerce Commission. (By regions and districts.) Superintendent of Documents. Price 75 cents.

"Transcontinental and Intercoastal Trade of the Pacific Southwest in 1926." Bureau of Foreign and Domestic Commerce. Superintendent of Documents. Price 25 cents.

5, 6, 7, and 8. Local authorities. (Lack of available information on the local situation may indicate the need of a special study.)

B. *Green's Marine Directory of the Great Lakes*. Green's Marine Directory of the Great Lakes Company, Rockefeller Bldg., Cleveland, Ohio.

Pacific Shipper. 311 California St., San Francisco, California.

Shipping Digest. 420 Lexington Ave., New York, New York.

Official Steamship and Airway Guide. Transportation Guides, Inc., 420 Lexington Ave., New York, New York.

Export Trade and Shipper. Business Publishers' International Corporation, 330 West 42nd St., New York, New York.

Port Series and Lake Series. Corps of Engineers, War Department.

"Inland-waterway Freight Transportation Lines in the United States." Bureau of Foreign and Domestic Commerce. Superintendent of Documents. Price 30 cents.

"Great Lakes-to-Ocean Waterways." Bureau of Foreign and Domestic Commerce. Superintendent of Documents. Price 25 cents.

United States Maritime Commission, Washington, D. C.

American Steamship Owners' Association, 11 Broadway, New York, New York.

Officer in charge of local station (if any) of the Army and Navy Corps of Engineers.

Local officials of shipping lines.

Foreign Commerce and Navigation of the United States. Bureau of Foreign and Domestic Commerce.

C-1. *Official Aviation Guide.* Official Aviation Guide Co., Inc., 608 South Dearborn St., Chicago, Illinois.

Airway Map of the United States. Bureau of Air Commerce, United States Department of Commerce. Available from Civil Aeronautics Authority, Washington, D. C. Free.

"Scheduled Airline Operators" (mimeographed). Bureau of Air Commerce. Available from Civil Aeronautics Authority. Free.

2 and 3. *Official Aviation Guide.*

Local authorities, such as information office of airport, airline ticket office in city or railway station, and airport managers.

"Description of Airports and Landing Fields in the United States." Bureau of Air Commerce. Available from Civil Aeronautics Authority.

"Tabulation of Air Navigation Radio Aids." Bureau of Air Commerce. Available from Civil Aeronautics Authority. Free.

Aeronautical Charts. Coast and Geodetic Survey, United States Department of Commerce. (Shows airports, trolley lines, railroads, highways, beacon lights, radio stations, etc.) Available from Coast and Geodetic Survey, Washington, D. C. Prices vary from 40 cents to $1.20.

Aeronautical Communications Stations. (Map) Bureau of Air Commerce. Available from Civil Aeronautics Authority. Free.

"Communications Manual." Aeronautical Radio, Inc., National Press Building, Washington, D. C.

"National Airport Plan." Bureau of Air Commerce. (Recommended facilities and selected data for individual cities.) Available from Civil Aeronautics Authority. Free. (Distribution limited to planning agencies.)

4, 5, 6, and 7. Local authorities.

D-1. Transportation Maps. Bureau of Public Roads, U. S. Department of Agriculture. Superintendent of Documents. Prices vary

according to the number of sheets per state. 20 cents per sheet.

United States System of Highways. (Map.) Bureau of Public Roads. (Shows the number system of United States highways.) Superintendent of Documents. Price 15 cents.

Annual reports of the state highway commission and records of the highway department of the particular state studied.

Local sources such as filling station operators, etc.

2. "Russel's National Motor Coach Guide." 817 Second Avenue, S. E., Cedar Rapids, Iowa.

"Official Motor Freight Guide." Universal Guide Corporation. (Principal truck and trailer companies by cities, schedules, local and long distance maps of each line.) Universal Guide Corporation, 732–38 West Van Buren Street, Chicago, Illinois.

Public service or public utility commission in the state involved.

Interstate Commerce Commission, Washington, D. C.

Records and officials of operating companies.

Highway planning survey service of the Bureau of Public Roads.

3 and 4. Local authorities.

E. Operating company records.

Public service or public utility commission of the particular state involved.

Interstate Commerce Commission.

F. "The Petroleum Industry of the Gulf Southwest." Bureau of Foreign and Domestic Commerce. Superintendent of Documents. Price 65 cents.

"Transportation of Gasoline by Pipe Lines." Bureau of Mines. Superintendent of Documents Price 10 cents.

Operating company records.

Public utility or public service commission of the state involved.

Interstate Commerce Commission.

G. Comparisons from other figures under XVI.

H. American Transit Association, 292 Madison Avenue, New York, New York.

Operating company records.

Municipal public utilities commission.

I. Local authorities.

XVII. Public Warehousing.

A. Census of Business, 1935. Public Warehousing. (By regions and states.) Available from Bureau of the Census. Free.

Municipal and state records, managers of local warehouses, and chamber of commerce.

B, *C*, and *D*. Managers of local warehouses.

XVIII. Banking and Finance, Insurance.

A. *Bankers' Directory*. Rand-McNally and Company, Chicago, Illinois.

Census of Business, 1935. Banks. (By states.) Available from Bureau of the Census. Free.

Assets and Liabilities of Operating Insured Banks, December 31, 1937. Federal Deposit Insurance Corp.

Officers of local banks, and financial statements of banks in local newspapers.

B. Census of Business, 1935. Financial Institutions other than Banks. (By states.) Available from Bureau of the Census. Free.
Officers of local financial institutions.

C. Census of Business, 1935. Insurance. (By states and cities of 10,000 and over.) Available from Bureau of the Census. Free.
Local officers and agents of insurance companies.

D. Local authorities. (The local situation may suggest the need for a special field survey.)

XIX. Professional Service (physicians, dentists, lawyers, etc.).
Census of Population, 1930, Vol. IV, Occupations. (Number engaged in professional pursuits for cities of 25,000 and over.) Superintendent of Documents. Price $3.50.
Telephone and city directories, and local professional men.

XX. Newspapers, Radio Stations.

A. *N. W. Ayer & Son's Directory of Newspapers and Periodicals.* (Names and circulations of newspapers arranged by states and cities.) N. W. Ayer & Son, West Washington Square, Philadelphia, Pennsylvania.
Advertising rate cards of local newspapers, and editors, officers, and files of local papers.

B. Census of Business, 1935. Radio Broadcasting. (By states and largest cities.) Available from Bureau of the Census. Free.
Managers of local stations.

XXI. Expositions, Fairs, and Conventions.
World Convention Dates (monthly magazine). Hendrickson Publishing Co., 330 West 42nd Street, New York, New York.
Local chamber of commerce and other local authorities.

XXII. Industrial and Commercial Associations (chambers of commerce, commodity and technical associations, grain exchanges, etc.).
"Selected Trade Associations of the United States, 1937 Edition." Bureau of Foreign and Domestic Commerce. (Directory of 2,400 national and interstate associations.) Available only from Bureau of Foreign and Domestic Commerce. Price 10 cents.
"List of State and Local Trade Associations in [*name of state*]." Bureau of Foreign and Domestic Commerce. (Separate directory for each state.) Available only from Bureau of Foreign and Domestic Commerce. Free.
Local industrial and commercial association executives.

XXIII. Universities and Other Institutions (state, county, etc., having appreciable enrollments from other communities).

A. "Educational Directory, Part 3." Office of Education, United States Department of Interior. (Covers colleges and universities.) Available from the Department of Interior, Washington, D. C. Free.
Registrars and other officers of local universities and institutions.

B. Officers of local, Federal state and municipal institutions.

XXIV. Federal, State, or County Government Establishments (for a state capital or county seat, such special establishments and facilities as contribute important activities to this type of city).
Local officials of Federal, state or county governments.

XXV. Other (all other types of establishments and special features that tend to contribute to the community's employment, purchasing power, and consumption).

Local authorities.

XXVI. Summarization of Principal State and Local Laws and Regulations affecting industry and commerce and consumer purchasing power (The supervision of this section by a local attorney or attorneys is desirable.)

"United States Code Annotated."

State codes and statutes, municipal ordinances. State secretary of state.

A. "Financial Statistics of State and Local Governments," 1932. Bureau of the Census.

"Financial Statistics of Cities," 1935. Bureau of the Census.

"Digests of State Laws Relating to Taxation and Revenue," 1922. Bureau of the Census. (For use as a possible guide as to how a current digest can be made.)

B. State codes.

C. "Workmen's Compensation Legislation in the United States and Canada, as of January 1, 1929." Bureau of Labor Statistics.

"Handbook of Labor Statistics, 1936." Bureau of Labor Statistics.

"Occupational Disease Legislation of the United States," 1936. Bureau of Labor Statistics.

Monthly Labor Review. Bureau of Labor Statistics.

D, *E*, *F*, and *G*. State codes and statutes, municipal ordinances.

H. "List of Unfair Trade Practices." Marketing Research Division, Bureau of Foreign and Domestic Commerce. (Free as long as the limited supply lasts.)

I, *J*, and *K*. Local authorities.

XXVII. Municipal Administration and Related Data.

A. Municipal Yearbook (annual). The International City Managers' Association, 1313 East 60th Street, Chicago, Illinois.

Local municipal authorities.

B and *C*. "Financial Statistics of Cities," 1935. Bureau of the Census.

Municipal Yearbook. (This includes data from authoritative sources on each phase of municipal administration.)

Local municipal authorities.

D, *E*, and *F*. Local authorities.

XXVIII. Civic, Social, and Related Facilities and Activities.

A to *L*. Local authorities, including Chamber of Commerce, telephone directory, officials of the public school system, city health officials, etc.

B. "Census of Religious Bodies," Bureau of the Census, 1936. (Cities of 25,000 or more.) In preparation.

C. "Education Directory," Office of Education, United States Department of Interior. (In four parts.) Available from Department of Interior. Free.

"Statistics of City School Systems, 1933–34." Office of Education, United States Department of Interior. (Survey of Education in the United States.) Superintendent of Documents. 30 cents.

J. "Directory of Hospitals," American Medical Association, 535 South Dearborn Street, Chicago, Illinois.

"Vital Statistics Special Reports." Division of Vital Statistics, Bureau of the Census. (Statistics on births, deaths, and infant mortality issued in various forms from time to time. The Division of Vital Statistics' Weekly Health Index, covering 86 cities, and Weekly Accident Bulletin, for 130 cities, can also be obtained from this Division.)

"Water Supply Statistics for American Municipalities," 1929. American City Magazine, 470 Fourth Avenue, New York, New York.

XXIX. Important Physical Facilities and Special Features (not elsewhere described). Local authorities.

Sources of Related Material

This material supplements specific references in Section II applicable to points of inquiry enumerated in the Basic Outline. (Printing-space limitations require that publications referred to in this supplementary list be selected for their potential helpfulness as guides to additional material from other sources, as well as for more direct considerations.)

Federal Agencies

"Price Lists" by General Subjects.—These give the names of publications under the classification and the agency of the government issuing each publication, and are currently available. Such lists as the following may be of interest: Price List No. 10. "Laws"; 15, "Geological Survey"; 21, "Fishes"; 28, "Finance"; 31, "Education"; 33, "Labor"; 35, "Geography and Explorations"; 36, "Government Periodicals"; 37, "Tariff"; 38, "Animal Industry"; 42, "Irrigation, Drainage, Water-power"; 43, "Forestry"; 45, "Roads"; 46, "Agricultural Chemistry, and Soils and Fertilizers"; 48, "Weather, Astronomy and Meterology"; 51, "Health"; 53, "Maps": Government maps and directions for obtaining them; 58, "Mines": Mineral resources, fuel-testing, coal, gas, gasoline, explosives; 59, "Interstate Commerce Commission Publications"; 62, "Commerce and Manufactures"; 69, "Pacific States"; 70, "Census"; and 72, "Publications of interest to suburbanites and home builders." (Library references.) Price lists are obtainable from the Superintendent of Documents, Washington, D. C. No charge.

"List of Publications of the U. S. Department of Commerce."—Superintendent of Documents. No charge.

"Market Research Sources."—A guide to information on domestic marketing emanating from offices of the Federal Government, state governments, colleges, universities, and foundations, publishers of books and trade directories, and from commercial organizations. Bureau of Foreign and Domestic Commerce. Superintendent of Documents. 1938 ed., 30 cents.

Domestic Commerce.—A periodical appearing on the 10th, 20th, and 30th of each month, provides a current review of developments in the field of marketing and new studies in the field of distribution by government agencies and non-governmental authorities. Bureau of Foreign and Domestic Commerce. Yearly subscription, $1.00.

"Selected Trade Associations of the United States."—A directory of approximately 2,400 national and interstate associations. Available only from the Bureau of Foreign and Domestic Commerce. 10 cents.

State and Local Trade Association Directories.—Directories for each state are available, covering trade associations in cities of more than 3,000 population. Available only from Bureau of Foreign and Domestic Commerce. No charge.

Circular 10, Status of City and County Planning (subject to current revision), and other leaflets on planning activities in various communities. National Resources Committee, Washington, D. C. No charge.

"Planning Our Resources," 1938, includes description of the work of regional planning agencies in various sections of the country. National Resources Committee. Superintendent of Documents. 10 cents.

Associations, Commercial Research Agencies, and Other Sources

"The Location of Industry—A Select Bibliography."—Includes references to community outlines and related material prepared by authorities in the United States as well as in Canada. 1937. McGill University, Social Science Offices, 3466 University Street, Montreal, Canada.

Among the survey outlines referred to in the above-named bibliography are: "Industrial Development for a Community" available (without charge) from the Policyholders Service Bureau, Metropolitan Life Insurance Company, New York; "Industrial Surveys, an Outline for Communities and Manufacturers," and other material by George C. Smith, Engineer, Missouri-Kansas-Texas Railroad Company, Railway Exchange Building, St. Louis, Missouri; "Preparation and Use of Industrial Surveys" and "The Bid for New Industries" by Charles P. Wood, Lockwood Greene Engineers, Inc., New York,; "Survey of Retail Trading Area," University of Illinois, Business Research Bulletin No. 44, and "A Community Labor Survey," Business Research Bulletin, No. 34, University of Illinois, Urbana, Illinois.

Similarly available are "Industrial Plant Location Study," and other material by Harold V. Coes, Industrial Department, Ford, Bacon and Davis, Inc., Engineers, New York; "Traffic and Trade" by Paver and McClintock, 1935 (Book 3), McGraw-Hill Publishing Company, New York; "How to Reduce Municipal Expenditures" (15 cents), International City Managers' Association, Chicago, Illinois; "The Codification and Drafting of Ordinances for Small Towns" (50 cents), Public Administration Service, Chicago, Illinois; "America's Capacity to Consume" ($3.00), The Brookings Institution, Washington, D. C.; "The Economic Importance of Boston University to the Community" (25 cents), Boston University College of Business Administration, Boston, Massachusetts; "Community Planning" (25 cents) and other publications of the Regional Plan Association, New York; "Is Industry Decentralizing?" by Daniel B. Creamer ($1.00), University of Pennsylvania Press, Philadelphia, Pennsylvania, 1935; "Migration and Economic Opportunities" by Carter Goodrich and others ($5.00), University of Pennsylvania Press, Philadelphia, Pennsylvania, 1936.

"List of Publications of the Chamber of Commerce of the United States," includes references to "Balanced Rebuilding of Cities," 1937; "Zoning, A Statement of Principles, and Procedure," 1936; "New Opportunities for City Planning"; "Outline of a Master Community"; "Special Inducements to Industries"; "Reducing State and Local Taxes" (10 cents); "State Sales Taxes," a digest of current state tax laws (10 cents); "State and Local Budgetary Methods" (10 cents), and many other publications available from the Chamber of Commerce of the United States, Washington, D. C.

"Urban Markets and Retail Sales."—This important new type of marketing atlas was published in the fall of 1938 by Outdoor Advertising, Inc., 60 East 42d St., New York. (Published too late for review and listing under specific points in the Basic Outline. Price not given.)

"The Municipal Year Book."—The 1938 edition provides a résumé of activities and statistical data of American cities and a description of services of associations and other organizations devoted to city planning and to special subjects of community interest such as taxes, traffic, zoning, education, housing, etc., also an extensive source list of information on practically every type of major community problem. Available for reference in most large libraries. Published by the International City Managers' Association, Chicago, Illinois. (Price varies for different years.)

"The Industrial Arts Index."—Lists current material on industry, engineering, transportation, trade, business, finance, and distribution. Available for reference in most large libraries. Published by H. W. Wilson Company, New York.

APPENDIX V

PREPARATION OF STATISTICAL REPORTS

After the data in any particular survey have been gathered and presented in such tabular and graphic forms that they may be readily analyzed, one is ordinarily ready to prepare a report on the investigation. In the present discussion, which aims to furnish assistance in the preparation of statistical reports, the points made are offered as suggestions rather than rules. These suggestions, however, are based upon many years of practical experience in preparing statistical reports for the use of business men.

In the preparation of statistical reports, it is important to remember that it is not necessary or generally advisable to follow the rules of journalistic writing. The attitude of the reader is entirely different. The newspaper must catch the eye with the first sentence, and hence it generally places the most important fact or idea first. The statistical report aims to convince only after a complete presentation or analysis has been given. The final conviction is built on a foundation of statistical facts presented in an orderly manner. With a proper progression, the conclusion will be reached without the necessity of any elaborate or arbitrary assertions.

It must be understood, however, that in business practice, frequently decisions are made based largely on confidence in the judgment of subordinates or experts who have conducted the investigation. Where such a relationship exists, the report should ordinarily be introduced by a brief summary of the main facts and analyses for the purpose of rapid review. The body of the report, however, should give all the data on which the conclusions are based, arranged in progressive order, laying the foundation first and building to the final conclusion.

Use of the Third Person.—The point of view in business reports is ordinarily that of the third person, and when once decided upon it should be maintained consistently. In writing statistical reports, it is easy to shift unintentionally from the third to the first or second persons; care should be taken to guard against this error. Though the final determination of the point of view depends upon the nature of the subject and the purpose of the report, the use of the third person is usually the most effective.

Use of Illustrations.—Charts, diagrams, tables, pictures, and other exhibits are valuable means of bringing out clearly the significant facts, but care should be taken to illustrate only the worth-while points if there is to be a logical development of ideas. When using such exhibits, it is usually best to give them titles, and these titles should be selected with reference to the particular points which it is desired to illustrate or emphasize.

Length of Report.—The number of words, phrases, sentences, or paragraphs given to any thought should be the minimum necessary to express it clearly and forcefully. Often a 50-page report may be improved by cutting it to 25 or less. On the whole, the report should be just long enough to express in specific concrete statements the thought of the writer, so that the reader will gain a vivid impression of the significant points and a thorough understanding of the subject matter.

The Outline.—At the beginning of an investigation, when the general object is determined, the problem is ordinarily broken up into its component elements, but when one is ready to write the report, it usually is possible to regroup the material

gathered in a more practical manner for the purpose at hand. When this has been done, the proposed report should be carefully thought through and a detailed outline prepared, which not only lists the topics to be covered, but shows the position of all tables, charts, and important items throughout the report. The point cannot be too strongly emphasized that carefully thinking through a proposed report is necessary in preparing a good outline, and that a good outline is essential in systematically and logically developing the presentation and analysis of the material gathered.

The First Draft.—One is ready to write only after the outline has been carefully thought out and the material has been thoroughly assimilated. One should think over the problem until it has been fully determined what is to be said, and then write the first draft rapidly with the mind fully occupied with the subject of the report, giving choice of words or turn of phrases but little consideration. Then, after the subject matter has been written down, the report should be carefully revised and restated in order to make it more effective.

The Final Revised Copy.—To bring out the important points prominently, leaving the minor points in the background, is the chief function of revision. Every unnecessary word should be eliminated and every weak place should be improved to make the report concise and convincing. The writer should put his thoughts on paper in such a way as to tell others in the fewest words possible exactly what he has in mind; then the important points will stand out clearly. Clearness and forcefulness require the careful observation and application of the principles of *unity*, *coherence*, and *emphasis* in the arrangement of words, phrases, sentences, and paragraphs.

Appearance of the Report.—In addition to the principles previously outlined in this discussion, it is important to consider the general appearance of the report. Slovenly appearance gives an impression of slovenly work. Such formal parts of a report as cover, title page, letter of submittal, table of contents, topical headings, subtitles, footnotes, and references should be carefully and completely presented.

Formal Parts of a Report.—The formal parts of a report, in the order in which they are ordinarily arranged are:

1. Cover title.
2. Title page.
3. Letter of submittal (if included as part of the report).
4. Table of contents.
5. List or lists of exhibits (tables, charts, etc.).
6. Main body of the report.
7. Appendix.

Cover Title.—On the cover should be stated the title of the report, and ordinarily the name of the author and person or firm for whom the report was made, with any other information of value in filing. On small reports, the title may ordinarily be typed on the cover or typed on a label and pasted on, but on more formal reports bound in leather or fabrikoid, the title should ordinarily be stamped on in gold-leaf lettering (see Exhibit 250).

Title Page.—The title page duplicates the information given on the cover and, in addition, often gives a more complete title with such information as the date and place of writing.

Letter of Submittal.—A report is usually accompanied by a letter of submittal. Ordinarily the introductory paragraph of this letter should present the subject of the report, the authority or reasons for writing it, and its purpose, while the body of the letter should point out any other important considerations regarding the report. Sometimes the letter is bound in the report, immediately following the title page, but usually it is submitted separately. Circumstances will determine the plan to be followed.

Table of Contents.—The table of contents, placed at the beginning of the report, lists the chapter titles and topical headings with their page numbers. It is not only used for reference purposes, but it is also a skeleton outline which shows at a glance the significant items and their interrelations.

Summary.—If it is desirable to include a summary, it should as a rule present in brief sentence paragraphs the outstanding facts, the conclusions reached, and the action recommended, or other outstanding features of the investigation. The summary is a great convenience to the business man in making a rapid review of the report. In most cases, the best position for the summary of a business survey is just preceding the main body of the report.

Section Titles, Subtitles and Topical Headings.—Considerable care should be taken to choose titles that will be significant of what is covered in the chapters or other

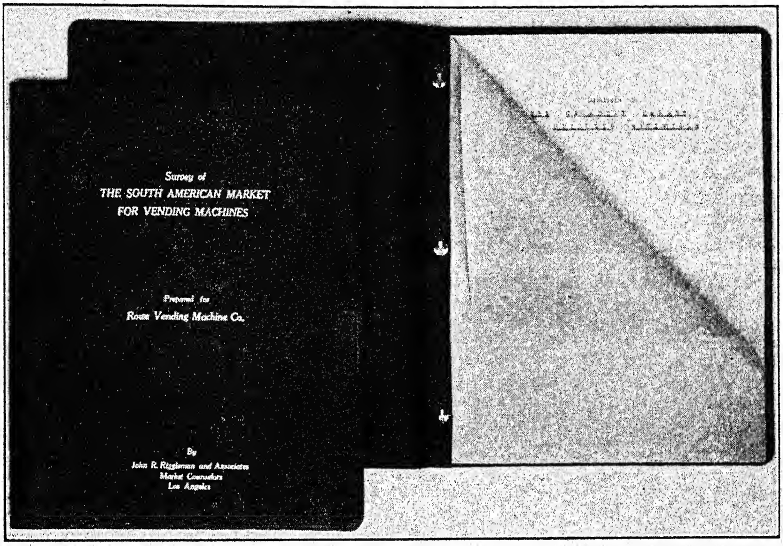

EXHIBIT 250.—Flexible leather and fabrikoid covers for formal reports of from 25 to 100 pages. The report on the left shows the position suggested for title lettering, etc. The one on the right shows the fly leaf, title page, and screw binding.

sections of a report. From the title, one should be able to understand clearly what subject or field is covered. Subtitles are given to distinct parts of a major section, and (as an aid to unity, coherence, and emphasis) topical headings are given to paragraphs or to related series of paragraphs.

Footnotes.—Such information as sources of data should be given in footnotes. "The use of the asterisk is proper . . . though . . . in modern usage footnotes are usually referred to by means of small figures, technically called superiors."[1] If the source is a publication, the name of the author, title of the work, page number, the name and address of the publisher, and the date of publication should be given, in the order mentioned. If the source is a personal interview, the name, position, and address of the person may be given with the date and place of the interview. These references to sources are very important if the information is to be verified or supplemented.

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[1] Wooley, Edwin C., "Mechanics of Writing," p. 18, D. C. Heath & Company, Boston, 1909.

Position of Illustrations and Exhibits.—When charts, diagrams, tables, pictures. and other exhibits are used to supplement the written material, they should be placed so that they can be used with the minimum of effort on the part of the reader. All illustrations should be as small as they can be made without losing their effectiveness. It is often possible to make a chart small enough so that it can be placed within the paragraph that it illustrates. In this position it can be easily observed when the written matter is read. Such illustrations may be drawn directly on the sheet or they may be taken from other sources and pasted on the pages of the report. In many instances, as a matter of expediency, it is necessary to have such exhibits as tables and charts on separate sheets by themselves. In position, such an exhibit should ordinarily follow the page on which its analysis is presented. Large exhibits may be folded and placed in the back of the report, but their use should be avoided when small exhibits would be equally effective.

Typing the Report.—The final copy of a business statistics report should be carefully typewritten, using only one side of the paper.[1] The lines should ordinarily be double spaced, though usually it is best to single space quotations. Topical headings should be emphasized by underlining or by placing them in a special position. Once decided upon, the same relative position and emphasis should be maintained for topical headings throughout the report. Care should be taken to see that a sufficient number of carbon copies are made.

Reproduction of Illustrations.—If several copies of charts are to be made they can be reproduced by blue printing or by photostating. Other methods of reproduction are available for special purposes.

Paper.—The paper used in reports should be of good quality, and $8\frac{1}{2}$ by 11 inches in size unless otherwise specified. The covers for small reports may be heavy paper such as that commonly used as covers for pamphlets and published reports. The subject of covers will be discussed further in connection with binding.

Spacing.—A margin of 1 to $1\frac{1}{4}$ inches should be left at the right and bottom of the page, and $1\frac{1}{2}$ inches should be left at the top. On the left, a wider margin of $1\frac{1}{2}$ to 2 inches should be left to allow for binding. The foregoing applies, of course, to reports bound on the side. Sometimes it is desirable to bind them at the top, when care should be taken to allow a top binding margin.

Paging.—A report should be paged with Arabic numerals either in the upper right corner or at the center of the bottom of the page. Full page charts, diagrams, exhibits, and other illustrations should ordinarily be paged along with the rest of the report. All illustrations should ordinarily be given a number for reference. In many reports, the tables are designated "Table 1," "Table 2," etc., and the charts are designated "Chart I," "Chart II," etc. In other reports, the charts and tables are all referred to as "exhibits" with simply one series of Arabic numerals. Other systems are also followed, but these two methods will cover most requirements.

Binding.—As far as appearance is concerned, no part of the report is more important than its binding. While, strictly speaking, the kind of binding should have perhaps nothing to do with the quality of the report, the psychological effect of having a good report in a good binding should not be overlooked. Just as a merchant places his high-quality merchandise in a high-class package, so also is there an advantage in placing a good report in an appropriate cover. The point should be emphasized, however, that the cover should be *appropriate* for the purpose. It is extremely bad taste to place an insignificant report in an elaborate binding. An appropriate cover may not necessarily be an expensive cover, but it should be neat and expertly attached.

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[1] If a report is being prepared for publication, the publisher's special instructions as tc final preparation should be followed.

Small reports of a few pages may be fastened together with two or three staples or eyelets and placed in special paper covers which are provided for the purpose by any well-stocked stationery store. Larger reports of, say, from 25 to 100 pages, may also be placed in paper bindings, but if they are of a more formal nature, it is ordinarily desirable to place them in leather or fabrikoid bindings, such as those illustrated in Exhibit 250 on page 715. Reports of 100 or more pages should ordinarily be bound by a professional binder, and the binding should be selected in accordance with what is appropriate for the particular report. Reports of a few hundred pages can be stapled for certain types of use, but if they are important enough, they should be sewed and bound like books.

APPENDIX VI

INDEXES OF BUSINESS CYCLES AND BUILDING CYCLES

This brief description of the business cycle and the building cycle indexes that were presented in Exhibit 158 on page 331[1] is included, not only for the reason that it may be helpful in making more detailed studies than are discussed in Chapters XVI and XXII, but also for the reason that it may be helpful in indicating methods of making systematic approaches to similar problems.

THE CLEVELAND TRUST COMPANY'S INDEX OF GENERAL BUSINESS ACTIVITY

The Cleveland Trust Company's Index of General Business Activity, presented in Exhibit 158, page 331, for the period from 1830 to 1937, inclusive, is available for the period from 1790 to date.[2] This index "is composed of one set of 10 series of annual data from 1790 to 1885, and of another set of 10 series of annual data from 1855 to 1901. The fluctuations above and below normal were computed for each series separately, and the 10 were then combined in one. Normal values for each series were means between one set of lines running from one prosperity peak to the next, and another similar set of lines running from each depression bottom to the next. The annual figures from 1901 to 1919 are those of the Thomas index of manufacturing production with mineral production added, and from 1919 to date the monthly figures of the Federal Reserve index of industrial production have been used. All the data were reduced to a per capita basis.

"The computations of the fluctuations of each of the 10 series constituting the index from 1855 to 1901 were carried through to include 1930, and the coefficient of correlation between each of them and the production series running from 1901 through 1930 was computed. Their deviations were then multiplied through by constants so as to equate their amplitudes of cyclical fluctuation. Each of the 10 series was then given a weight based on its degree of correlation with the production series, and with these weightings they were combined into a single index. The 10 series with these weights are pig iron consumption 15, railroad freight ton miles 15, cotton consumption 14, canal freight (New York and Sault Ste. Marie) 12, coal production 12, construction of miles of new railroads 12, blast furnace activity 10, rail production 6, locomotive production 2, and ship construction 2. The 10 series combined give results closely similar to those of the production series for the overlap period from 1901 through 1930. The heights of prosperities, and the depths of depressions, are closely alike in the two series. The coefficient of correlation for the period is .95. Their average deviations for the period are equal.

"In a similar way the computations of the fluctuations of each of the 10 series constituting the index from 1790 to 1855 were carried through to include 1882, and the coefficient of correlation between each of them and the first 28 years of the index

[1] The building cycle index is also included in Exhibit 207, on p. 533.

[2] This index was developed by Colonel Leonard P. Ayres, Vice President of the Cleveland Trust Company.

running from 1855 to 1901 was computed. Their deviations were then multiplied through by constants so as to equate their amplitudes of cyclical fluctuation. Each of the 10 series was then given a weight based on its degree of correlation with the first 28 years of the index from 1855 to 1901, and with these weightings they were combined into a single index. The 10 series with these weights are commodity prices 20, imports 18, imports retained for consumption 16, government receipts 14, ship construction 12, government expenditures 6, coal production 6, exports 5, iron exports 2, and tons of registered shipping in service 1. The 10 series combined give results closely similar to those of the other index for the overlap period from 1855 through 1882. The heights of prosperities, and the depths of depressions, are closely alike in the two series. The coefficient of correlation for the period is .90. Their average deviations for the period are equal.

"When the annual data were determined the monthly data were fitted to them. These monthly data were based on the figures of the business index of the American Telephone and Telegraph Company and on data for blast furnace activity from 1877 to 1919. Monthly data for bank clearings and for stock prices were used from 1861 to 1877, and those for security and commodity prices from 1815 to 1861. From 1790 to 1815 the monthly data are based on commodity prices."[1] The secular trend of this index for the period from 1920 to 1937 has been revised for the chart in Exhibit 158, and, consequently, the position of the curve differs somewhat from that of the curve as originally published.

INDEX OF BUILDING CYCLES, 1830–1937

The Index of Building Cycles presented in Exhibit 158, on page 331, and in Exhibit 207, on page 533, is designed to show the great swings in building activity in the United States for the period since 1830.[2] This index is based upon per capita building values for an increasing number of cities from 1830 to 1900 and a constant number since that date. The number is built up, as the data become available, from three cities in 1830 to 65 by 1900, and the 65 cities are carried through from 1900 to 1937, inclusive.

Building permit data were used as far back as they could be obtained, but for the period from 1830 to about 1850 the index is based principally upon other types of data. In many instances different sources reported substantially different figures on building permits for an individual city. In such instances, what seemed to be the most reliable data were selected after a careful study of the various sources and series. In some cases, fragments of series had to be adjusted before they could be spliced together into a homogeneous series. For recent years, only value data are used. For the earlier years, only data on number of permits issued are available for several cities. These were converted to a value basis for inclusion in the index.

Beginning with New York (Manhattan), Boston, and Washington in 1830, cities were added to the index as follows: Milwaukee in 1849; Philadelphia in 1855; Indianapolis in 1858; Baltimore in 1864; Kansas City (Missouri) in 1865; St. Paul in 1867; St. Louis in 1868; Denver in 1871; Chicago, Colorado Springs, Bronx, Brooklyn, Detroit, and San Francisco in 1874; Louisville, Minneapolis, New Orleans, Portland (Oregon), and Providence in 1875; Newark and Pittsburgh in 1881; Bridgeport and Fall River in 1884; Omaha in 1885; Cincinnati in 1887; Salt Lake City and Worcester in 1888; Cleveland and Seattle in 1889; Duluth and Los Angeles in 1890; Albany,

[1] From chart "American Business Activity since 1790" published by Cleveland Trust Company, Cleveland, Ohio.

[2] This index was developed by John R. Riggleman.

Buffalo, and Syracuse in 1892; Memphis in 1893; Galveston in 1895; Atlanta and Hartford in 1896; Tampa in 1897; Des Moines, Rochester, and Toledo in 1898; Richmond and Queens (New York) in 1899; and Birmingham, Cambridge, Columbus, Dayton, Grand Rapids, Houston, Jacksonville, Jersey City, Kansas City (Kansas), Lowell, Nashville, New Bedford, New Haven, Oakland, Peoria, Reading, Richmond, St. Joseph, San Diego, Scranton, Spokane, and Tacoma in 1900.

In order to simplify the problem of trend determination and of using an increasing number of cities from 1830 to 1900, the permit values and estimates were placed on a per capita basis. The populations of the cities included were estimated as of July 1 of each year on the basis of a smoothed logarithmic curve drawn through the population figures of each city for the different decennial census dates. While special adjustments for territorial annexations were made only in the more important instances, which is believed to be a satisfactory procedure for the purposes of this national study, it is considered important in analyses of local conditions to make careful adjustments for all annexations or changes in boundaries which occur between census dates.

The building permit figures and other estimates, stated in terms of *value*, were converted to the equivalent of 1913 dollars, in order that they might more nearly indicate *volume*. Otherwise the significance of the fluctuations might be obscured, owing to the changes in price conditions, especially during the Civil War and World War inflation periods. For this purpose, a special index of building costs was devised, which was based upon a new compilation for the period from 1830 to 1851, the American Appraisal Company's index numbers of construction costs for the period from 1852 to 1903, and the construction cost index of the *Engineering News Record*, with a few modifications, for the period from 1904 to 1937.

In using the 1830–1937 index of building costs just described, it is recognized that its basis is limited to building-material prices and labor rates and that it may not represent any of the other changes that take place in building costs, such as in contractors' overhead and profits, financing charges, architects' fees, technical improvements in machinery, economy in design, greater prefabrication of materials, and cyclical and secular variations in the efficiency of labor and management. These factors may vary considerably from time to time, and the changes may not correspond to the changes in building-material prices and in total construction costs. If a synthetic index of this type does not properly reflect the changes in the efficiency of labor and management and improvements in the use of materials, its use may cause an appreciable bias in the secular trend, as discussed later.[1]

When the data had been expressed as building values per capita, in terms of the 1913 dollar, a line representing the long-time trend was computed by the method of least squares for the period from 1830 to 1933 and projected to 1937. This trend increased from \$28.72 per capita in 1830 to \$31.93 per capita in 1937, which is an increase of \$3.21 per capita for the entire period or 3 cents per capita per year. Apparently the declining rate of growth of cities has not yet caused a downward long-time trend in per capita building activity, although it is possible that a fundamental change, not yet determinable, has taken place during the last quarter, or even the last half, of the period under consideration. However, too much significance should not be attached

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[1] See also James S. Taylor, "Construction Cost Statistics," *Journal of the American Statistical Association*, Supplement, Vol. XXIX, No. 185A, pp. 38–42, March 1934; and Lowell J. Chawner, "Construction Cost Indexes as Influenced by Technological Change and Other Factors," *Journal of the American Statistical Association*, Supplement, Vol. 30, pp. 561–576, September 1935.

to the slope of this trend line, since it is probable that it would be changed somewhat by an extension of the period, either backward or forward, or by the addition of cities in the earlier parts of the period. Furthermore, the elimination of price changes by means of a synthetic cost index, based upon fixed amounts of labor and materials, might affect the trend slope. It is possible that the cumulative upward bias in such an index may become very considerable over a period of twenty years or more.[1] If a cumulative upward bias does exist in the cost index used, it means that the per capita trend of actual building since 1830 should have a correspondingly greater upward slope.

The above qualifications apply to any interpretations that might be made of the secular trend. The objective of this particular study, however, is to show cycles, and the secular trend has been eliminated. Since any trend that is indirectly eliminated by a steadily accumulating bias in the cost index simply reduces the slope of the trend that is directly eliminated, the cycles in as erratic a series as the one under consideration would not be affected to any practical extent by a secular bias in the cost index.

When the building cycle curve is stated in terms of percentage deviations from the normal trend, it appears as shown in Exhibit 158, on page 331, and Exhibit 207, on page 533.

While many questions may be raised and many criticisms may be made, it is believed, nevertheless, that the index discussed above is a reasonable indicator of the nation's building cycles.[2] However, no interpretation should be made without due consideration of the limitations and qualifications of the data. Although the cities used in making up the index are believed to be generally representative, it is possible that they may not be representative enough for some purposes. For an area with a higher rate of growth than the average of the cities included, the index may not be sensitive enough and the fluctuations may not be wide enough, while for an area with a lower rate of growth the fluctuations may be too extreme. It is also possible that the inaccuracies of the original data may throw the index curve somewhat out of line, especially in regard to the minor cycles. The estimates for the early part of the period are not rigid and may not be accurate enough for certain purposes, especially in regard to the amplitude of the resulting cycles. Furthermore, the construction cost index may not be as representative as it should be for the particular group of cities used. In spite of all of the qualifications pointed out, however, it is believed that the building cycle index presents a reasonably good picture of the fluctuations in building activity in the United States during the past 108 years.

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[1] See opinion of James S. Taylor, formerly Chief of the Division of Building and Housing, United States Department of Commerce, and now Associate Director, Division of Research, Federal Housing Administration, in "Building Cycles in the United States" by John R. Riggleman, *Journal of the American Statistical Association*, Vol. XXVIII, No. 182, June 1933, p. 79.

[2] The building cycle index described above has been compared with a number of other indexes of building activity for periods since 1875 in an independent study by George F. Warren and Frank A. Pearson. A remarkably close correspondence was found in the general movements of the cycles as indicated by the different indexes. See Warren and Pearson, "World Prices and the Building Industry," John Wiley & Sons, New York, 1937, Chap. VI, particularly p. 108.

APPENDIX VII

HOW TO USE LOGARITHMS—TABLE OF LOGARITHMS

The processes of multiplication, division, raising to powers, and extracting roots are greatly facilitated by the use of logarithms. By merely substituting the logarithms for the numbers, the problem of multiplication becomes one of addition, that of division becomes one of subtraction, that of raising to a power becomes one of simple multiplication, and that of extracting a root becomes one of simple division. The use of logarithms is often looked upon as something strange and complicated by those who are not mathematically trained. As a matter of fact, however, it is much easier to use logarithmic tables than it is to use many of the interest and other tables commonly used by business men.

Common or Briggs Logarithms.—The logarithms referred to in this discussion are the Common or Briggs Logarithms, which are logarithms to the base 10. That is, if natural numbers are regarded as powers of 10, the exponents of the powers are the logarithms of the numbers.[1]

Logarithms Briefly Explained.—The logarithms of 1, 10, 100, 1000, etc., and of 0.1, 0.01, 0.001, 0.0001, etc., are integral numbers. The logarithms of all other numbers are fractions. That is:

$1 = 10^0$ hence $\log 1 = 0$;	$0.1 = 10^{-1}$ hence $\log 0.1 = -1$;
$10 = 10^1$ hence $\log 10 = 1$;	$0.01 = 10^{-2}$ hence $\log 0.01 = -2$;
$100 = 10^2$ hence $\log 100 = 2$;	$0.001 = 10^{-3}$ hence $\log 0.001 = -3$;
$1{,}000 = 10^3$ hence $\log 1{,}000 = 3$;	and so on.[2]

If the number is between	1	and	10,	the logarithm is between	0	and	1.
" " "	10	"	100,	" "	1	"	2.
" " "	100	"	1,000,	" "	2	"	3.
" " "	1	"	0.1,	" "	0	"	−1.
" " "	0.1	"	0.01,	" "	−1	"	−2.

And so on.

If the number is less than 1, the logarithm is negative, but it is written in such a form that the fractional part is always positive. A logarithm, therefore, consists of two parts—a *positive or negative* integral number, which is called the **characteristic,** and a *positive* proper fraction which is called the **mantissa.** Thus, in the logarithm 5.66296, the integral number 5 is the characteristic, and the fraction .66296 is the mantissa.

In determining and using the characteristics of logarithms, the following rules are helpful:

To determine the characteristic when given a certain number:

If the number is greater than 1, the characteristic of the logarithm is one less than the number of digits to the left of the decimal point.

If the number is less than 1, the characteristic (which is negative) is one greater than the number of zeros between the decimal point and the first significant figure of the given number.

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[1] In the more advanced mathematical work, it is sometimes necessary to use the Natural or Napierian Logarithms, in which the base is $e = 2.71828$.

[2] The expression "log 10 = 1" means that the logarithm of 10 is 1. For the sake of brevity "logarithm of" is stated simply as "log."

To point off a certain number when given its logarithm:

If the characteristic is positive, the number of digits in the integral part of the corresponding number is one greater than the characteristic.

If the characteristic is negative, the number of zeros between the decimal point and the first significant figure of the corresponding number is one less than the characteristic.

From the above it follows that:

The characteristic of all numbers from	1 to 9	inclusive is	0;
" " "	10 to 99	"	1;
" " "	100 to 999	"	2;
" " "	1,000 to 9,999	"	3;

and so on; and that

the characteristic of all numbers from	0.1 to 0.9	inclusive is	−1;
" " "	0.01 to 0.09	"	−2;
" " "	0.001 to 0.009	"	−3;

and so on.

When stating a logarithm which has a negative characteristic, the minus sign is placed above the characteristic to show that it alone is negative. Thus the logarithm of 0.0009283 is $\bar{4}.96769$, which means $-4 + 0.96769$, or $0.96769 - 4$. It should not be written -4.96769 because the minus sign would then indicate that both characteristic and mantissa were negative, when, as previously pointed out, the mantissa should always be considered as positive.

In order to simplify computations involving negative characteristics, 10 is usually added to the characteristic and subtracted at the right of the mantissa. Thus $\bar{3}.96886$ is expressed $7.96886 - 10$. (Computations involving negative characteristics are illustrated on pages 643 and 644.)

The characteristic of a number is ordinarily determined by inspection, but the mantissa is found by referring to the tables. Strictly speaking a "table of logarithms of natural numbers" is a table of mantissas.[1] Thus if we wish to know the logarithm of 892.1, we know by inspection that the characteristic is 2, and we find by referring to the tables that the mantissa is 95041. (The first three places of the number are read in the column designated "No." and the fourth is read in the row at the top, the desired mantissa being found on page 663 in the row opposite 892 and in the column headed "1.") The logarithm, therefore, of 892.1 is 2.95041. Conversely, if we have the logarithm 2.95041 and we wish to know the natural number (or antilogarithm) that it represents, we locate the mantissa 95041 and find the number 8921. We know from the logarithm that this number should be pointed off with a characteristic of 2 (with 3 digits to the left of the decimal point) and that, therefore, the desired number is 892.1.

Logarithms of the numbers that can be derived one from the other by multiplying or dividing by an integral power of 10 all have the same mantissa. Only the characteristics differ. Thus:

The logarithm of	6,742	is 3.82879;
The logarithm of	674.2	is 2.82879;
The logarithm of	67.42	is 1.82879;
The logarithm of	6.742	is 0.82879;
The logarithm of	0.6742	is $\bar{1}.82879$;

and so on.

[1] Sometimes a summary table of characteristics is included, but it is not ordinarily used in actual practice.

Interpolation.—The logarithmic tables in this book (pages 729 to 747) provide for numbers of four digits. If the number under consideration has more than four significant figures, a process called interpolation is required. Interpolation is based upon the assumption that the change between two consecutive mantissas in the table is directly proportional to the change in the number. Suppose, for instance, that we require the logarithm of 34053. The required mantissa is the same as the mantissa for 3405.3. Hence it can be found by adding to the mantissa of 3405, three-tenths of the difference between the mantissas for 3405 and 3406. These mantissas are 53212 and 53224, respectively, and the difference is 12. Therefore, the mantissa for 3405.3 is $53212 + (0.3 \times 12)$, or 532156, and the required logarithm of 34053 is 4.532156.

If, when looking up antilogarithms, a given mantissa cannot be found in the table, find the two mantissas between which the given mantissa lies. The four figures of the number corresponding to the smaller of these two mantissas will be the first four figures of the required number. The additional digits are determined (when necessary) by interpolation as follows: Suppose that we wish to find the number corresponding to the logarithm 2.48508. The two mantissas in the table adjacent to the given mantissa are .48501 and .48515. The corresponding numbers are 3055 and 3056. The smaller of these contains the first four digits of the required number. The difference between the numbers (3055 and 3056) is 1, and the difference between the corresponding mantissas (48501 and 48515) is 14. The difference between the smaller of these adjacent mantissas (48501) and the given mantissa of the problem (48508) is 7. Hence, the amount to be added to 3055 is $\frac{7}{14}$ of 1 or 0.5, and the fifth digit of the required number is 5. Therefore, the number corresponding to the logarithm 2.48508 is 305.55.[1]

Computation by Logarithms.—The following simple examples are presented to indicate the processes to be followed in multiplication, division, raising to powers, and extracting roots:

Multiplication.—To find the product of two or more numbers, find the sum of their logarithms and then look up the number of which this sum is the logarithm. (The sum of the logarithms of a series of numbers is the logarithm of their product.) Thus:

(1) Multiply 37.49×4.299.

$$\begin{aligned} \log 37.49 &= 1.57392 \\ \log 4.299 &= 0.63337 \\ \hline \text{log of desired product} &= 2.20729 \\ \therefore \text{product (natural number)} &= 161.2. \end{aligned}$$

(2) Multiply 34053×0.08142.

$$\begin{aligned} \log 34053 &= 4.53216 \\ \log 0.08142 = \bar{2}.91073 &= 8.91073 - 10 \\ \hline \text{log of desired product} &= 13.44289 - 10 \\ \text{or} \quad &= 3.44289 \\ \therefore \text{product (natural number)} &= 2773. \end{aligned}$$

Division.—To find the quotient of two numbers, subtract the logarithm of the divisor from the logarithm of the dividend. The difference is the logarithm of the quotient. Thus:

[1] In some tables of logarithms, tables of proportional parts are included, but, with a little practice, the additional figures can be determined about as rapidly without them. Their omission leaves more space for other purposes.

(1) Solve 42.721 ÷ 6.732.

$$\begin{aligned} \log 42.721 &= 1.630631 \\ \log \ \ 6.732 &= 0.828144 \\ \hline \text{log of quotient} &= 0.802487 \\ \therefore \text{quotient} &= 6.346. \end{aligned}$$

Raising to Powers.—To raise a number to a certain power, multiply the logarithm of the number by the exponent of the power; the resulting product is the logarithm of the required power. Thus:

(1) Find the square of 428.7.

$$\begin{aligned} \log 428.7 &= 2.63215 \\ \text{multiply by 2} & \qquad\quad\ 2 \\ \hline \text{log of square} &= 5.26430 \\ \therefore \text{square} &= 183{,}784. \end{aligned}$$

(2) Find the cube of 0.004287.

$$\begin{aligned} \log 0.004287 &= 7.63215 - 10 \\ \text{multiply by 3} & \qquad\qquad\quad 3 \\ \hline \text{log of cube} &= 22.89645 - 30 \\ &= \bar{8}.89645 \\ \therefore \text{cube} &= 0.00000007879. \end{aligned}$$

Extracting Root.—To find a certain root of a number, divide the logarithm of the number by the index of the root, the quotient obtained is the logarithm of the required root. Thus:

(1) Find the square root of 6436.

$$\begin{aligned} \log 6436 &= \quad 3.80862 \\ \text{divide by 2} & \quad 2)\underline{3.80862} \\ \text{log of square root} &= \quad 1.90431 \\ \therefore \text{square root} &= \quad 80.225. \end{aligned}$$

(2) Find the 17th root of 78240.

$$\begin{aligned} \log 78240 &= \quad 4.89343 \\ \text{divide by 17} & \quad 17)\underline{4.89343} \\ \text{log of 17th root} &= \quad 0.28785 \\ \therefore \text{17th root} &= \quad 1.9403 \end{aligned}$$

Geometric Mean.—To find the geometric mean extract the nth root of the product of n items. Thus:

Determine the geometric mean of 209, 3199, 42940, 485, 594, and 6494. (Multiply the six items together and extract the 6th root.)

$$\begin{aligned} \log \ \ 209 &= \quad 2.32015 \\ \log \ \ 3199 &= \quad 3.50501 \\ \log 42940 &= \quad 4.63286 \\ \log \ \ 485 &= \quad 2.68574 \\ \log \ \ 594 &= \quad 2.77379 \\ \log \ \ 6494 &= \quad 3.81251 \\ \hline \text{log of product} &= \quad 19.73006 \\ \text{divide by 6} & \quad 6)\underline{19.73006} \\ \text{log of geometric mean} & \quad\ \ 3.28834 \\ \therefore \text{geometric mean} &= 1942.4. \end{aligned}$$

Exercises

1. State the characteristic for each of the following numbers:

934	764.53	0.7248	5083217
4623	3.7892	0.0006	0.74683
24	12.364	1000.1	0.00001
7	7582.8	0.0072	0.00232

2. If log 25 = 1.39794, give the logarithms of the following:

250	250000	2.5	0.0025
2500	2500000	0.25	0.00025
25000	25000000	0.025	0.000025

3. Given the following numbers, check the accompanying logarithms and determine which are correct:

Numbers	Logarithms	Numbers	Logarithms
2754	3.43996	0.8391	1.92381
3053	3.48473	76.49	1.85309
345.9	2.33895	628.1	3.79803
514	2.71096	10.01	1.00043

4. Given the following logarithms check the accompanying numbers and determine which are correct:

Logarithms	Numbers	Logarithms	Numbers
2.63789	434.4	3.93847	8769
0.70689	0.5092	1.24279	14.36
3.80963	6451	1.28870	19.44
2.85425	71.49	2.49066	309.5

5. State the logarithms of the following numbers:

2947	7.823	0.8462	76832
86.53	496.8	0.9864	41697
2184	9682	0.0026	82348
765.1	3.127	0.0007	17646

NOTE:—The last column above requires interpolation.

6. State the numbers corresponding to the following logarithms:

1.33062	0.85058	0.86314	1.86073
3.46864	$\bar{1}.58535$	$\bar{2}.59006$	0.58108
5.73584	0.98941	$\bar{3}.45163$	2.44065
2.79204	1.60108	1.52466	3.59643

NOTE:—The last column above requires interpolation.

7. Compute the following:

134×183	7446×8371	1.6225×0.7214
264×321	66.56×28.31	0.1652×0.6215
567×184	4.527×897.6	0.0071×0.0042

8. Compute the following:

144 ÷ 12	8721 ÷ 7634	4762 ÷ 8216
143 ÷ 13	68.33 ÷ 29.38	27.34 ÷ 0.062
783 ÷ 19	4.631 ÷ 2.715	0.0062 ÷ 0.0034

9. Find the following:

Square of 8797	4th power of 7368
Cube of 9688	9th power of 6979

10. Find the following:

Square root of 7968	5th root of 8697
Cube root of 9879	19th root of 6799

TABLE OF

THE COMMON OR BRIGGS

LOGARITHMS

OF THE NATURAL NUMBERS

FROM 1 TO 1000

1–100

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
0	———	**20**	1.30 103	**40**	1.60 206	**60**	1.77 815	**80**	1.90 309
1	0.00 000	21	1.32 222	41	1.61 278	61	1.78 533	81	1.90 849
2	0.30 103	22	1.34 242	42	1.62 325	62	1.79 239	82	1.91 381
3	0.47 712	23	1.36 173	43	1.63 347	63	1.79 934	83	1.91 908
4	0.60 206	24	1.38 021	44	1.64 345	64	1.80 618	84	1.92 428
5	0.69 897	25	1.39 794	45	1.65 321	65	1.81 291	85	1.92 942
6	0.77 815	26	1.41 497	46	1.66 276	66	1.81 954	86	1.93 450
7	0.84 510	27	1.43 136	47	1.67 210	67	1.82 607	87	1.93 952
8	0.90 309	28	1.44 716	48	1.68 124	68	1.83 251	88	1.94 448
9	0.95 424	29	1.46 240	49	1.69 020	69	1.83 885	89	1.94 939
10	1.00 000	**30**	1.47 712	**50**	1.69 897	**70**	1.84 510	**90**	1.95 424
11	1.04 139	31	1.49 136	51	1.70 757	71	1.85 126	91	1.95 904
12	1.07 918	32	1.50 515	52	1.71 600	72	1.85 733	92	1.96 379
13	1.11 394	33	1.51 851	53	1.72 428	73	1.86 332	93	1.96 848
14	1.14 613	34	1.53 148	54	1.73 239	74	1.86 923	94	1.97 313
15	1.17 609	35	1.54 407	55	1.74 036	75	1.87 506	95	1.97 772
16	1.20 412	36	1.55 630	56	1.74 819	76	1.88 081	96	1.98 227
17	1.23 045	37	1.56 820	57	1.75 587	77	1.88 649	97	1.98 677
18	1.25 527	38	1.57 978	58	1.76 343	78	1.89 209	98	1.99 123
19	1.27 875	39	1.59 106	59	1.77 085	79	1.89 763	99	1.99 564
20	1.30 103	**40**	1.60 206	**60**	1.77 815	**80**	1.90 309	**100**	2.00 000

1–100

100–149

No.	0	1	2	3	4	5	6	7	8	9
100	00 000	00 043	00 087	00 130	00 173	00 217	00 260	00 303	00 346	00 389
101	00 432	00 475	00 518	00 561	00 604	00 647	00 689	00 732	00 775	00 817
102	00 860	00 903	00 945	00 988	01 030	01 072	01 115	01 157	01 199	01 242
103	01 284	01 326	01 368	01 410	01 452	01 494	01 536	01 578	01 620	01 662
104	01 703	01 745	01 787	01 828	01 870	01 912	01 953	01 995	02 036	02 078
105	02 119	02 160	02 202	02 243	02 284	02 325	02 366	02 407	02 449	02 490
106	02 531	02 572	02 612	02 653	02 694	02 735	02 776	02 816	02 857	02 898
107	02 938	02 979	03 019	03 060	03 100	03 141	03 181	03 222	03 262	03 302
108	03 342	03 383	03 423	03 463	03 503	03 543	03 583	03 623	03 663	03 703
109	03 743	03 782	03 822	03 862	03 902	03 941	03 981	04 021	04 060	04 100
110	04 139	04 179	04 218	04 258	04 297	04 336	04 376	04 415	04 454	04 493
111	04 532	04 571	04 610	04 650	04 689	04 727	04 766	04 805	04 844	04 883
112	04 922	04 961	04 999	05 038	05 077	05 115	05 154	05 192	05 231	05 269
113	05 308	05 346	05 385	05 423	05 461	05 500	05 538	05 576	05 614	05 652
114	05 690	05 729	05 767	05 805	05 843	05 881	05 918	05 956	05 994	06 032
115	06 070	06 108	06 145	06 183	06 221	06 258	06 296	06 333	06 371	06 408
116	06 446	06 483	06 521	06 558	06 595	06 633	06 670	06 707	06 744	06 781
117	06 819	06 856	06 893	06 930	06 967	07 004	07 041	07 078	07 115	07 151
118	07 188	07 225	07 262	07 298	07 335	07 372	07 408	07 445	07 482	07 518
119	07 555	07 591	07 628	07 664	07 700	07 737	07 773	07 809	07 846	07 882
120	07 918	07 954	07 990	08 027	08 063	08 099	08 135	08 171	08 207	08 243
121	08 279	08 314	08 350	08 386	08 422	08 458	08 493	08 529	08 565	08 600
122	08 636	08 672	08 707	08 743	08 778	08 814	08 849	08 884	08 920	08 955
123	08 991	09 026	09 061	09 096	09 132	09 167	09 202	09 237	09 272	09 307
124	09 342	09 377	09 412	09 447	09 482	09 517	09 552	09 587	09 621	09 656
125	09 691	09 726	09 760	09 795	09 830	09 864	09 899	09 934	09 968	10 003
126	10 037	10 072	10 106	10 140	10 175	10 209	10 243	10 278	10 312	10 346
127	10 380	10 415	10 449	10 483	10 517	10 551	10 585	10 619	10 653	10 687
128	10 721	10 755	10 789	10 823	10 857	10 890	10 924	10 958	10 992	11 025
129	11 059	11 093	11 126	11 160	11 193	11 227	11 261	11 294	11 327	11 361
130	11 394	11 428	11 461	11 494	11 528	11 561	11 594	11 628	11 661	11 694
131	11 727	11 760	11 793	11 826	11 860	11 893	11 926	11 959	11 992	12 024
132	12 057	12 090	12 123	12 156	12 189	12 222	12 254	12 287	12 320	12 352
133	12 385	12 418	12 450	12 483	12 516	12 548	12 581	12 613	12 646	12 678
134	12 710	12 743	12 775	12 808	12 840	12 872	12 905	12 937	12 969	13 001
135	13 033	13 066	13 098	13 130	13 162	13 194	13 226	13 258	13 290	13 322
136	13 354	13 386	13 418	13 450	13 481	13 513	13 545	13 577	13 609	13 640
137	13 672	13 704	13 735	13 767	13 799	13 830	13 862	13 893	13 925	13 956
138	13 988	14 019	14 051	14 082	14 114	14 145	14 176	14 208	14 239	14 270
139	14 301	14 333	14 364	14 395	14 426	14 457	14 489	14 520	14 551	14 582
140	14 613	14 644	14 675	14 706	14 737	14 768	14 799	14 829	14 860	14 891
141	14 922	14 953	14 983	15 014	15 045	15 076	15 106	15 137	15 168	15 198
142	15 229	15 259	15 290	15 320	15 351	15 381	15 412	15 442	15 473	15 503
143	15 534	15 564	15 594	15 625	15 655	15 685	15 715	15 746	15 776	15 806
144	15 836	15 866	15 897	15 927	15 957	15 987	16 017	16 047	16 077	16 107
145	16 137	16 167	16 197	16 227	16 256	16 286	16 316	16 346	16 376	16 406
146	16 435	16 465	16 495	16 524	16 554	16 584	16 613	16 643	16 673	16 702
147	16 732	16 761	16 791	16 820	16 850	16 879	16 909	16 938	16 967	16 997
148	17 026	17 056	17 085	17 114	17 143	17 173	17 202	17 231	17 260	17 289
149	17 319	17 348	17 377	17 406	17 435	17 464	17 493	17 522	17 551	17 580
No.	0	1	2	3	4	5	6	7	8	9

150–199

No.	0	1	2	3	4	5	6	7	8	9
150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869
151	17 898	17 926	17 955	17 984	18 013	18 041	18 070	18 099	18 127	18 156
152	18 184	18 213	18 241	18 270	18 298	18 327	18 355	18 384	18 412	18 441
153	18 469	18 498	18 526	18 554	18 583	18 611	18 639	18 667	18 696	18 724
154	18 752	18 780	18 808	18 837	18 865	18 893	18 921	18 949	18 977	19 005
155	19 033	19 061	19 089	19 117	19 145	19 173	19 201	19 229	19 257	19 285
156	19 312	19 340	19 368	19 396	19 424	19 451	19 479	19 507	19 535	19 562
157	19 590	19 618	19 645	19 673	19 700	19 728	19 756	19 783	19 811	19 838
158	19 866	19 893	19 921	19 948	19 976	20 003	20 030	20 058	20 085	20 112
159	20 140	20 167	20 194	20 222	20 249	20 276	20 303	20 330	20 358	20 385
160	20 412	20 439	20 466	20 493	20 520	20 548	20 575	20 602	20 629	20 656
161	20 683	20 710	20 737	20 763	20 790	20 817	20 844	20 871	20 898	20 925
162	20 952	20 978	21 005	21 032	21 059	21 085	21 112	21 139	21 165	21 192
163	21 219	21 245	21 272	21 299	21 325	21 352	21 378	21 405	21 431	21 458
164	21 484	21 511	21 537	21 564	21 590	21 617	21 643	21 669	21 696	21 722
165	21 748	21 775	21 801	21 827	21 854	21 880	21 906	21 932	21 958	21 985
166	22 011	22 037	22 063	22 089	22 115	22 141	22 167	22 194	22 220	22 246
167	22 272	22 298	22 324	22 350	22 376	22 401	22 427	22 453	22 479	22 505
168	22 531	22 557	22 583	22 608	22 634	22 660	22 686	22 712	22 737	22 763
169	22 789	22 814	22 840	22 866	22 891	22 917	22 943	22 968	22 994	23 019
170	23 045	23 070	23 096	23 121	23 147	23 172	23 198	23 223	23 249	23 274
171	23 300	23 325	23 350	23 376	23 401	23 426	23 452	23 477	23 502	23 528
172	23 553	23 578	23 603	23 629	23 654	23 679	23 704	23 729	23 754	23 779
173	23 805	23 830	23 855	23 880	23 905	23 930	23 955	23 980	24 005	24 030
174	24 055	24 080	24 105	24 130	24 155	24 180	24 204	24 229	24 254	24 279
175	24 304	24 329	24 353	24 378	24 403	24 428	24 452	24 477	24 502	24 527
176	24 551	24 576	24 601	24 625	24 650	24 674	24 699	24 724	24 748	24 773
177	24 797	24 822	24 846	24 871	24 895	24 920	24 944	24 969	24 993	25 018
178	25 042	25 066	25 091	25 115	25 139	25 164	25 188	25 212	25 237	25 261
179	25 285	25 310	25 334	25 358	25 382	25 406	25 431	25 455	25 479	25 503
180	25 527	25 551	25 575	25 600	25 624	25 648	25 672	25 696	25 720	25 744
181	25 768	25 792	25 816	25 840	25 864	25 888	25 912	25 935	25 959	25 983
182	26 007	26 031	26 055	26 079	26 102	26 126	26 150	26 174	26 198	26 221
183	26 245	26 269	26 293	26 316	26 340	26 364	26 387	26 411	26 435	26 458
184	26 482	26 505	26 529	26 553	26 576	26 600	26 623	26 647	26 670	26 694
185	26 717	26 741	26 764	26 788	26 811	26 834	26 858	26 881	26 905	26 928
186	26 951	26 975	26 998	27 021	27 045	27 068	27 091	27 114	27 138	27 161
187	27 184	27 207	27 231	27 254	27 277	27 300	27 323	27 346	27 370	27 393
188	27 416	27 439	27 462	27 485	27 508	27 531	27 554	27 577	27 600	27 623
189	27 646	27 669	27 692	27 715	27 738	27 761	27 784	27 807	27 830	27 852
190	27 875	27 898	27 921	27 944	27 967	27 989	28 012	28 035	28 058	28 081
191	28 103	28 126	28 149	28 171	28 194	28 217	28 240	28 262	28 285	28 307
192	28 330	28 353	28 375	28 398	28 421	28 443	28 466	28 488	28 511	28 533
193	28 556	28 578	28 601	28 623	28 646	28 668	28 691	28 713	28 735	28 758
194	28 780	28 803	28 825	28 847	28 870	28 892	28 914	28 937	28 959	28 981
195	29 003	29 026	29 048	29 070	29 092	29 115	29 137	29 159	29 181	29 203
196	29 226	29 248	29 270	29 292	29 314	29 336	29 358	29 380	29 403	29 425
197	29 447	29 469	29 491	29 513	29 535	29 557	29 579	29 601	29 623	29 645
198	29 667	29 688	29 710	29 732	29 754	29 776	29 798	29 820	29 842	29 863
199	29 885	29 907	29 929	29 951	29 973	29 994	30 016	30 038	30 060	30 081
No.	0	1	2	3	4	5	6	7	8	9

200–249

No.	0	1	2	3	4	5	6	7	8	9
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298
201	30 320	30 341	30 363	30 384	30 406	30 428	30 449	30 471	30 492	30 514
202	30 535	30 557	30 578	30 600	30 621	30 643	30 664	30 685	30 707	30 728
203	30 750	30 771	30 792	30 814	30 835	30 856	30 878	30 899	30 920	30 942
204	30 963	30 984	31 006	31 027	31 048	31 069	31 091	31 112	31 133	31 154
205	31 175	31 197	31 218	31 239	31 260	31 281	31 302	31 323	31 345	31 366
206	31 387	31 408	31 429	31 450	31 471	31 492	31 513	31 534	31 555	31 576
207	31 597	31 618	31 639	31 660	31 681	31 702	31 723	31 744	31 765	31 785
208	31 806	31 827	31 848	31 869	31 890	31 911	31 931	31 952	31 973	31 994
209	32 015	32 035	32 056	32 077	32 098	32 118	32 139	32 160	32 181	32 201
210	32 222	32 243	32 263	32 284	32 305	32 325	32 346	32 366	32 387	32 408
211	32 428	32 449	32 469	32 490	32 510	32 531	32 552	32 572	32 593	32 613
212	32 634	32 654	32 675	32 695	32 715	32 736	32 756	32 777	32 797	32 818
213	32 838	32 858	32 879	32 899	32 919	32 940	32 960	32 980	33 001	33 021
214	33 041	33 062	33 082	33 102	33 122	33 143	33 163	33 183	33 203	33 224
215	33 244	33 264	33 284	33 304	33 325	33 345	33 365	33 385	33 405	33 425
216	33 445	33 465	33 486	33 506	33 526	33 546	33 566	33 586	33 606	33 626
217	33 646	33 666	33 686	33 706	33 726	33 746	33 766	33 786	33 806	33 826
218	33 846	33 866	33 885	33 905	33 925	33 945	33 965	33 985	34 005	34 025
219	34 044	34 064	34 084	34 104	34 124	34 143	34 163	34 183	34 203	34 223
220	34 242	34 262	34 282	34 301	34 321	34 341	34 361	34 380	34 400	34 420
221	34 439	34 459	34 479	34 498	34 518	34 537	34 557	34 577	34 596	34 616
222	34 635	34 655	34 674	34 694	34 713	34 733	34 753	34 772	34 792	34 811
223	34 830	34 850	34 869	34 889	34 908	34 928	34 947	34 967	34 986	35 005
224	35 025	35 044	35 064	35 083	35 102	35 122	35 141	35 160	35 180	35 199
225	35 218	35 238	35 257	35 276	35 295	35 315	35 334	35 353	35 372	35 392
226	35 411	35 430	35 449	35 468	35 488	35 507	35 526	35 545	35 564	35 583
227	35 603	35 622	35 641	35 660	35 679	35 698	35 717	35 736	35 755	35 774
228	35 793	35 813	35 832	35 851	35 870	35 889	35 908	35 927	35 946	35 965
229	35 984	36 003	36 021	36 040	36 059	36 078	36 097	36 116	36 135	36 154
230	36 173	36 192	36 211	36 229	36 248	36 267	36 286	36 305	36 324	36 342
231	36 361	36 380	36 399	36 418	36 436	36 455	36 474	36 493	36 511	36 530
232	36 549	36 568	36 586	36 605	36 624	36 642	36 661	36 680	36 698	36 717
233	36 736	36 754	36 773	36 791	36 810	36 829	36 847	36 866	36 884	36 903
234	36 922	36 940	36 959	36 977	36 996	37 014	37 033	37 051	37 070	37 088
235	37 107	37 125	37 144	37 162	37 181	37 199	37 218	37 236	37 254	37 273
236	37 291	37 310	37 328	37 346	37 365	37 383	37 401	37 420	37 438	37 457
237	37 475	37 493	37 511	37 530	37 548	37 566	37 585	37 603	37 621	37 639
238	37 658	37 676	37 694	37 712	37 731	37 749	37 767	37 785	37 803	37 822
239	37 840	37 858	37 876	37 894	37 912	37 931	37 949	37 967	37 985	38 003
240	38 021	38 039	38 057	38 075	38 093	38 112	38 130	38 148	38 166	38 184
241	38 202	38 220	38 238	38 256	38 274	38 292	38 310	38 328	38 346	38 364
242	38 382	38 399	38 417	38 435	38 453	38 471	38 489	38 507	38 525	38 543
243	38 561	38 578	38 596	38 614	38 632	38 650	38 668	38 686	38 703	38 721
244	38 739	38 757	38 775	38 792	38 810	38 828	38 846	38 863	38 881	38 899
245	38 917	38 934	38 952	38 970	38 987	39 005	39 023	39 041	39 058	39 076
246	39 094	39 111	39 129	39 146	39 164	39 182	39 199	39 217	39 235	39 252
247	39 270	39 287	39 305	39 322	39 340	39 358	39 375	39 393	39 410	39 428
248	39 445	39 463	39 480	39 498	39 515	39 533	39 550	39 568	39 585	39 602
249	39 620	39 637	39 655	39 672	39 690	39 707	39 724	39 742	39 759	39 777
No.	0	1	2	3	4	5	6	7	8	9

250–299

No.	0	1	2	3	4	5	6	7	8	9
250	39 794	39 811	39 829	39 846	39 863	39 881	39 898	39 915	39 933	39 950
251	39 967	39 985	40 002	40 019	40 037	40 054	40 071	40 088	40 106	40 123
252	40 140	40 157	40 175	40 192	40 209	40 226	40 243	40 261	40 278	40 295
253	40 312	40 329	40 346	40 364	40 381	40 398	40 415	40 432	40 449	40 466
254	40 483	40 500	40 518	40 535	40 552	40 569	40 586	40 603	40 620	40 637
255	40 654	40 671	40 688	40 705	40 722	40 739	40 756	40 773	40 790	40 807
256	40 824	40 841	40 858	40 875	40 892	40 909	40 926	40 943	40 960	40 976
257	40 993	41 010	41 027	41 044	41 061	41 078	41 095	41 111	41 128	41 145
258	41 162	41 179	41 196	41 212	41 229	41 246	41 263	41 280	41 296	41 313
259	41 330	41 347	41 363	41 380	41 397	41 414	41 430	41 447	41 464	41 481
260	41 497	41 514	41 531	41 547	41 564	41 581	41 597	41 614	41 631	41 647
261	41 664	41 681	41 697	41 714	41 731	41 747	41 764	41 780	41 797	41 814
262	41 830	41 847	41 863	41 880	41 896	41 913	41 929	41 946	41 963	41 979
263	41 996	42 012	42 029	42 045	42 062	42 078	42 095	42 111	42 127	42 144
264	42 160	42 177	42 193	42 210	42 226	42 243	42 259	42 275	42 292	42 308
265	42 325	42 341	42 357	42 374	42 390	42 406	42 423	42 439	42 455	42 472
266	42 488	42 504	42 521	42 537	42 553	42 570	42 586	42 602	42 619	42 635
267	42 651	42 667	42 684	42 700	42 716	42 732	42 749	42 765	42 781	42 797
268	42 813	42 830	42 846	42 862	42 878	42 894	42 911	42 927	42 943	42 959
269	42 975	42 991	43 008	43 024	43 040	43 056	43 072	43 088	43 104	43 120
270	43 136	43 152	43 169	43 185	43 201	43 217	43 233	43 249	43 265	43 281
271	43 297	43 313	43 329	43 345	43 361	43 377	43 393	43 409	43 425	43 441
272	43 457	43 473	43 489	43 505	43 521	43 537	43 553	43 569	43 584	43 600
273	43 616	43 632	43 648	43 664	43 680	43 696	43 712	43 727	43 743	43 759
274	43 775	43 791	43 807	43 823	43 838	43 854	43 870	43 886	43 902	43 917
275	43 933	43 949	43 965	43 981	43 996	44 012	44 028	44 044	44 059	44 075
276	44 091	44 107	43 122	44 138	44 154	44 170	44 185	44 201	44 217	44 232
277	44 248	44 264	44 279	44 295	44 311	44 326	44 342	44 358	44 373	44 389
278	44 404	44 420	44 436	44 451	44 467	44 483	44 498	44 514	44 529	44 545
279	44 560	44 576	44 592	44 607	44 623	44 638	44 654	44 669	44 685	44 700
280	44 716	44 731	44 747	44 762	44 778	44 793	44 809	44 824	44 840	44 855
281	44 871	44 886	44 902	44 917	44 932	44 948	44 963	44 979	44 994	45 010
282	45 025	45 040	45 056	45 071	45 086	45 102	45 117	45 133	45 148	45 163
283	45 179	45 194	45 209	45 225	45 240	45 255	45 271	45 286	45 301	45 317
284	45 332	45 347	45 362	45 378	45 393	45 408	45 423	45 439	45 454	45 469
285	45 484	45 500	45 515	45 530	45 545	45 561	45 576	45 591	45 606	45 621
286	45 637	45 652	45 667	45 682	45 697	45 712	45 728	45 743	45 758	45 773
287	45 788	45 803	45 818	45 834	45 849	45 864	45 879	45 894	45 909	45 924
288	45 939	45 954	45 969	45 984	46 000	46 015	46 030	46 045	46 060	46 075
289	46 090	46 105	46 120	46 135	46 150	46 165	46 180	46 195	46 210	46 225
290	46 240	46 255	46 270	46 285	46 300	46 315	46 330	46 345	46 359	46 374
291	46 389	46 404	46 419	46 434	46 449	46 464	46 479	46 494	46 509	46 523
292	46 538	46 553	46 568	46 583	46 598	46 613	46 627	46 642	46 657	46 672
293	46 687	46 702	46 716	46 731	46 746	46 761	46 776	46 790	46 805	46 820
294	46 835	46 850	46 864	46 879	46 894	46 909	46 923	46 938	46 953	46 967
295	46 982	46 997	47 012	47 026	47 041	47 056	47 070	47 085	47 100	47 114
296	47 129	47 144	47 159	47 173	47 188	47 202	47 217	47 232	47 246	47 261
297	47 276	47 290	47 305	47 319	47 334	47 349	47 363	47 378	47 392	47 407
298	47 422	47 436	47 451	47 465	47 480	47 494	47 509	47 524	47 538	47 553
299	47 567	47 582	47 596	47 611	47 625	47 640	47 654	47 669	47 683	47 698
No.	0	1	2	3	4	5	6	7	8	9

300–349

No.	0	1	2	3	4	5	6	7	8	9
300	47 712	47 727	47 741	47 756	47 770	47 784	47 799	47 813	47 828	47 842
301	47 857	47 871	47 885	47 900	47 914	47 929	47 943	47 958	47 972	47 986
302	48 001	48 015	48 029	48 044	48 058	48 073	48 087	48 101	48 116	48 130
303	48 144	48 159	48 173	48 187	48 202	48 216	48 230	48 244	48 259	48 273
304	48 287	48 302	48 316	48 330	48 344	48 359	48 373	48 387	48 401	48 416
305	48 430	48 444	48 458	48 473	48 487	48 501	48 515	48 530	48 544	48 558
306	48 572	48 586	48 601	48 615	48 629	48 643	48 657	48 671	48 686	48 700
307	48 714	48 728	48 742	48 756	48 770	48 785	48 799	48 813	48 827	48 841
308	48 855	48 869	48 883	48 897	48 911	48 926	48 940	48 954	48 968	48 982
309	48 996	49 010	49 024	49 038	49 052	49 066	49 080	49 094	49 108	49 122
310	49 136	49 150	49 164	49 178	49 192	49 206	49 220	49 234	49 248	49 262
311	49 276	49 290	49 304	49 318	49 332	49 346	49 360	49 374	49 388	49 402
312	49 415	49 429	49 443	49 457	49 471	49 485	49 499	49 513	49 527	49 541
313	49 554	49 568	49 582	49 596	49 610	49 624	49 638	49 651	49 665	49 679
314	49 693	49 707	49 721	49 734	49 748	49 762	49 776	49 790	49 803	49 817
315	49 831	49 845	49 859	49 872	49 886	49 900	49 914	49 927	49 941	49 955
316	49 969	49 982	49 996	50 010	50 024	50 037	50 051	50 065	50 079	50 092
317	50 106	50 120	50 133	50 147	50 161	50 174	50 188	50 202	50 215	50 229
318	50 243	50 256	50 270	50 284	50 297	50 311	50 325	50 338	50 352	50 365
319	50 379	50 393	50 406	50 420	50 433	50 447	50 461	50 474	50 488	50 501
320	50 515	50 529	50 542	50 556	50 569	50 583	50 596	50 610	50 623	50 637
321	50 651	50 664	50 678	50 691	50 705	50 718	50 732	50 745	50 759	50 772
322	50 786	50 799	50 813	50 826	50 840	50 853	50 866	50 880	50 893	50 907
323	50 920	50 934	50 947	50 961	50 974	50 987	51 001	51 014	51 028	51 041
324	51 055	51 068	51 081	51 095	51 108	51 121	51 135	51 148	51 162	51 175
325	51 188	51 202	51 215	51 228	51 242	51 255	51 268	51 282	51 295	51 308
326	51 322	51 335	51 348	51 362	51 375	51 388	51 402	51 415	51 428	51 441
327	51 455	51 468	51 481	51 495	51 508	51 521	51 534	51 548	51 561	51 574
328	51 587	51 601	51 614	51 627	51 640	51 654	51 667	51 680	51 693	51 706
329	51 720	51 733	51 746	51 759	51 772	51 786	51 799	51 812	51 825	51 838
330	51 851	51 865	51 878	51 891	51 904	51 917	51 930	51 943	51 957	51 970
331	51 983	51 996	52 009	52 022	52 035	52 048	52 061	52 075	52 088	52 101
332	52 114	52 127	52 140	52 153	52 166	52 179	52 192	52 205	52 218	52 231
333	52 244	52 257	52 270	52 284	52 297	52 310	52 323	52 336	52 349	52 362
334	52 375	52 388	52 401	52 414	52 427	52 440	52 453	52 466	52 479	52 492
335	52 504	52 517	52 530	52 543	52 556	52 569	52 582	52 595	52 608	52 621
336	52 634	52 647	52 660	52 673	52 686	52 699	52 711	52 724	52 737	52 750
337	52 763	52 776	52 789	52 802	52 815	52 827	52 840	52 853	52 866	52 879
338	52 892	52 905	52 917	52 930	52 943	52 956	52 969	52 982	52 994	53 007
339	53 020	53 033	53 046	53 058	53 071	53 084	53 097	53 110	53 122	53 135
340	53 148	53 161	53 173	53 186	53 199	53 212	53 224	53 237	53 250	53 263
341	53 275	53 288	53 301	53 314	53 326	53 339	53 352	53 364	53 377	53 390
342	53 403	53 415	53 428	53 441	53 453	53 466	53 479	53 491	53 504	53 517
343	53 529	53 542	53 555	53 567	53 580	53 593	53 605	53 618	53 631	53 643
344	53 656	53 668	53 681	53 694	53 706	53 719	53 732	53 744	53 757	53 769
345	53 782	53 794	53 807	53 820	53 832	53 845	53 857	53 870	53 882	53 895
346	53 908	53 920	53 933	53 945	53 958	53 970	53 983	53 995	54 008	54 020
347	54 033	54 045	54 058	54 070	54 083	54 095	54 108	54 120	54 133	54 145
348	54 158	54 170	54 183	54 195	54 208	54 220	54 233	54 245	54 258	54 270
349	54 283	54 295	54 307	54 320	54 332	54 345	54 357	54 370	54 382	54 394
No.	0	1	2	3	4	5	6	7	8	9

350–399

No.	0	1	2	3	4	5	6	7	8	9
350	54 407	54 419	54 432	54 444	54 456	54 469	54 481	54 494	54 506	54 518
351	54 531	54 543	54 555	54 568	54 580	54 593	54 605	54 617	54 630	54 642
352	54 654	54 667	54 679	54 691	54 704	54 716	54 728	54 741	54 753	54 765
353	54 777	54 790	54 802	54 814	54 827	54 839	54 851	54 864	54 876	54 888
354	54 900	54 913	54 925	54 937	54 949	54 962	54 974	54 986	54 998	55 011
355	55 023	55 035	55 047	55 060	55 072	55 084	55 096	55 108	55 121	55 133
356	55 145	55 157	55 169	55 182	55 194	55 206	55 218	55 230	55 242	55 255
357	55 267	55 279	55 291	55 303	55 315	55 328	55 340	55 352	55 364	55 376
358	55 388	55 400	55 413	55 425	55 437	55 449	55 461	55 473	55 485	55 497
359	55 509	55 522	55 534	55 546	55 558	55 570	55 582	55 594	55 606	55 618
360	55 630	55 642	55 654	55 666	55 678	55 691	55 703	55 715	55 727	55 739
361	55 751	55 763	55 775	55 787	55 799	55 811	55 823	55 835	55 847	55 859
362	55 871	55 883	55 895	55 907	55 919	55 931	55 943	55 955	55 967	55 979
363	55 991	56 003	56 015	56 027	56 038	56 050	56 062	56 074	56 086	56 098
364	56 110	56 122	56 134	56 146	56 158	56 170	56 182	56 194	56 205	56 217
365	56 229	56 241	56 253	56 265	56 277	56 289	56 301	56 312	56 324	56 336
366	56 348	56 360	56 372	56 384	56 396	56 407	56 419	56 431	56 443	56 455
367	56 467	56 478	56 490	56 502	56 514	56 526	56 538	56 549	56 561	56 573
368	56 585	56 597	56 608	56 620	56 632	56 644	56 656	56 667	56 679	56 691
369	56 703	56 714	56 726	56 738	56 750	56 761	56 773	56 785	56 797	56 808
370	56 820	56 832	56 844	56 855	56 867	56 879	56 891	56 902	56 914	56 926
371	56 937	56 949	56 961	56 972	56 984	56 996	57 008	57 019	57 031	57 043
372	57 054	57 066	57 078	57 089	57 101	57 113	57 124	57 136	57 148	57 159
373	57 171	57 183	57 194	57 206	57 217	57 229	57 241	57 252	57 264	57 276
374	57 287	57 299	57 310	57 322	57 334	57 345	57 357	57 368	57 380	57 392
375	57 403	57 415	57 426	57 438	57 449	57 461	57 473	57 484	57 496	57 507
376	57 519	57 530	57 542	57 553	57 565	57 576	57 588	57 600	57 611	57 623
377	57 634	57 646	57 657	57 669	57 680	57 692	57 703	57 715	57 726	57 738
378	57 749	57 761	57 772	57 784	57 795	57 807	57 818	57 830	57 841	57 852
379	57 864	57 875	57 887	57 898	57 910	57 921	57 933	57 944	57 955	57 967
380	57 978	57 990	58 001	58 013	58 024	58 035	58 047	58 058	58 070	58 081
381	58 092	58 104	58 115	58 127	58 138	58 149	58 161	58 172	58 184	58 195
382	58 206	58 218	58 229	58 240	58 252	58 263	58 274	58 286	58 297	58 309
383	58 320	58 331	58 343	58 354	58 365	58 377	58 388	58 399	58 410	58 422
384	58 433	58 444	58 456	58 467	58 478	58 490	58 501	58 512	58 524	58 535
385	58 546	58 557	58 569	58 580	58 591	58 602	58 614	58 625	58 636	58 647
386	58 659	58 670	58 681	58 692	58 704	58 715	58 726	58 737	58 749	58 760
387	58 771	58 782	58 794	58 805	58 816	58 827	58 838	58 850	58 861	58 872
388	58 883	58 894	58 906	58 917	58 928	58 939	58 950	58 961	58 973	58 984
389	58 995	59 006	59 017	59 028	59 040	59 051	59 062	59 073	59 084	59 095
390	59 106	59 118	59 129	59 140	59 151	59 162	59 173	59 184	59 195	59 207
391	59 218	59 229	59 240	59 251	59 262	59 273	59 284	59 295	59 306	59 318
392	59 329	59 340	59 351	59 362	59 373	59 384	59 395	59 406	59 417	59 428
393	59 439	59 450	59 461	59 472	59 483	59 494	59 506	59 517	59 528	59 539
394	59 550	59 561	59 572	59 583	59 594	59 605	59 616	59 627	59 638	59 649
395	59 660	59 671	59 682	59 693	59 704	59 715	59 726	59 737	59 748	59 759
396	59 770	59 780	59 791	59 802	59 813	59 824	59 835	59 846	59 857	59 868
397	59 879	59 890	59 901	59 912	59 923	59 934	59 945	59 956	59 966	59 977
398	59 988	59 999	60 010	60 021	60 032	60 043	60 054	60 065	60 076	60 086
399	60 097	60 108	60 119	60 130	60 141	60 152	60 163	60 173	60 184	60 195
No.	0	1	2	3	4	5	6	7	8	9

350–399

400–449

No.	0	1	2	3	4	5	6	7	8	9
400	60 206	60 217	60 228	60 239	60 249	60 260	60 271	60 282	60 293	60 304
401	60 314	60 325	60 336	60 347	60 358	60 369	60 379	60 390	60 401	60 412
402	60 423	60 433	60 444	60 455	60 466	60 477	60 487	60 498	60 509	60 520
403	60 531	60 541	60 552	60 563	60 574	60 584	60 595	60 606	60 617	60 627
404	60 638	60 649	60 660	60 670	60 681	60 692	60 703	60 713	60 724	60 735
405	60 746	60 756	60 767	60 778	60 788	60 799	60 810	60 821	60 831	60 842
406	60 853	60 863	60 874	60 885	60 895	60 906	60 917	60 927	60 938	60 949
407	60 959	60 970	60 981	60 991	61 002	61 013	61 023	61 034	61 045	61 055
408	61 066	61 077	61 087	61 098	61 109	61 119	61 130	61 140	61 151	61 162
409	61 172	61 183	61 194	61 204	61 215	61 225	61 236	61 247	61 257	61 268
410	61 278	61 289	61 300	61 310	61 321	61 331	61 342	61 352	61 363	61 374
411	61 384	61 395	61 405	61 416	61 426	61 437	61 448	61 458	61 469	61 479
412	61 490	61 500	61 511	61 521	61 532	61 542	61 553	61 563	61 574	61 584
413	61 595	61 606	61 616	61 627	61 637	61 648	61 658	61 669	61 679	61 690
414	61 700	61 711	61 721	61 731	61 742	61 752	61 763	61 773	61 784	61 794
415	61 805	61 815	61 826	61 836	61 847	61 857	61 868	61 878	61 888	61 899
416	61 909	61 920	61 930	61 941	61 951	61 962	61 972	61 982	61 993	62 003
417	62 014	62 024	62 034	62 045	62 055	62 066	62 076	62 086	62 097	62 107
418	62 118	62 128	62 138	62 149	62 159	62 170	62 180	62 190	62 201	62 211
419	62 221	62 232	62 242	62 252	62 263	62 273	62 284	62 294	62 304	62 315
420	62 325	62 335	62 346	62 356	62 366	62 377	62 387	62 397	62 408	62 418
421	62 428	62 439	62 449	62 459	62 469	62 480	62 490	62 500	62 511	62 521
422	62 531	62 542	62 552	62 562	62 572	62 583	62 593	62 603	62 613	62 624
423	62 634	62 644	62 655	62 665	62 675	62 685	62 696	62 706	62 716	62 726
424	62 737	62 747	62 757	62 767	62 778	62 788	62 798	62 808	62 818	62 829
425	62 839	62 849	62 859	62 870	62 880	62 890	62 900	62 910	62 921	62 931
426	62 941	62 951	62 961	62 972	62 982	62 992	63 002	63 012	63 022	63 033
427	63 043	63 053	63 063	63 073	63 083	63 094	63 104	63 114	63 124	63 134
428	63 144	63 155	63 165	63 175	63 185	63 195	63 205	63 215	63 225	63 236
429	63 246	63 256	63 266	63 276	63 286	63 296	63 306	63 317	63 327	63 337
430	63 347	63 357	63 367	63 377	63 387	63 397	63 407	63 417	63 428	63 438
431	63 448	63 458	63 468	63 478	63 488	63 498	63 508	63 518	63 528	63 538
432	63 548	63 558	63 568	63 579	63 589	63 599	63 609	63 619	63 629	63 639
433	63 649	63 659	63 669	63 679	63 689	63 699	63 709	63 719	63 729	63 739
434	63 749	63 759	63 769	63 779	63 789	63 799	63 809	63 819	63 829	63 839
435	63 849	63 859	63 869	63 879	63 889	63 899	63 909	63 919	63 929	63 939
436	63 949	63 959	63 969	63 979	63 988	63 998	64 008	64 018	64 028	64 038
437	64 048	64 058	64 068	64 078	64 088	64 098	64 108	64 118	64 128	64 137
438	64 147	64 157	64 167	64 177	64 187	64 197	64 207	64 217	64 227	64 237
439	64 246	64 256	64 266	64 276	64 286	64 296	64 306	64 316	64 326	64 335
440	64 345	64 355	64 365	64 375	64 385	64 395	64 404	64 414	64 424	64 434
441	64 444	64 454	64 464	64 473	64 483	64 493	64 503	64 513	64 523	64 532
442	64 542	64 552	64 562	64 572	64 582	64 591	64 601	64 611	64 621	64 631
443	64 640	64 650	64 660	64 670	64 680	64 689	64 699	64 709	64 719	64 729
444	64 738	64 748	64 758	64 768	64 777	64 787	64 797	64 807	64 816	64 826
445	64 836	64 846	64 856	64 865	64 875	64 885	64 895	64 904	64 914	64 924
446	64 933	64 943	64 953	64 963	64 972	64 982	64 992	65 002	65 011	65 021
447	65 031	65 040	65 050	65 060	65 070	65 079	65 089	65 099	65 108	65 118
448	65 128	65 137	65 147	65 157	65 167	65 176	65 186	65 196	65 205	65 215
449	65 225	65 234	65 244	65 254	65 263	65 273	65 283	65 292	65 302	65 312
No.	0	1	2	3	4	5	6	7	8	9

400–449

450–499

No.	0	1	2	3	4	5	6	7	8	9
450	65 321	65 331	65 341	65 350	65 360	65 369	65 379	65 389	65 398	65 408
451	65 418	65 427	65 437	65 447	65 456	65 466	65 475	65 485	65 495	65 504
452	65 514	65 523	65 533	65 543	65 552	65 562	65 571	65 581	65 591	65 600
453	65 610	65 619	65 629	65 639	65 648	65 658	65 667	65 677	65 686	65 696
454	65 706	65 715	65 725	65 734	65 744	65 753	65 763	65 772	65 782	65 792
455	65 801	65 811	65 820	65 830	65 839	65 849	65 858	65 868	65 877	65 887
456	65 896	65 906	65 916	65 925	65 935	65 944	65 954	65 963	65 973	65 982
457	65 992	66 001	66 011	66 020	66 030	66 039	66 049	66 058	66 068	66 077
458	66 087	66 096	66 106	66 115	66 124	66 134	66 143	66 153	66 162	66 172
459	66 181	66 191	66 200	66 210	66 219	66 229	66 238	66 247	66 257	66 266
460	66 276	66 285	66 295	66 304	66 314	66 323	66 332	66 342	66 351	66 361
461	66 370	66 380	66 389	66 398	66 408	66 417	66 427	66 436	66 445	66 455
462	66 464	66 474	66 483	66 492	66 502	66 511	66 521	66 530	66 539	66 549
463	66 558	66 567	66 577	66 586	66 596	66 605	66 614	66 624	66 633	66 642
464	66 652	66 661	66 671	66 680	66 689	66 699	66 708	66 717	66 727	66 736
465	66 745	66 755	66 764	66 773	66 783	66 792	66 801	66 811	66 820	66 829
466	66 839	66 848	66 857	66 867	66 876	66 885	66 894	66 904	66 913	66 922
467	66 932	66 941	66 950	66 960	66 969	66 978	66 987	66 997	67 006	67 015
468	67 025	67 034	67 043	67 052	67 062	67 071	67 080	67 089	67 099	67 108
469	67 117	67 127	67 136	67 145	67 154	67 164	67 173	67 182	67 191	67 201
470	67 210	67 219	67 228	67 237	67 247	67 256	67 265	67 274	67 284	67 293
471	67 302	67 311	67 321	67 330	67 339	67 348	67 357	67 367	67 376	67 385
472	67 394	67 403	67 413	67 422	67 431	67 440	67 449	67 459	67 468	67 477
473	67 486	67 495	67 504	67 514	67 523	67 532	67 541	67 550	67 560	67 569
474	67 578	67 587	67 596	67 605	67 614	67 624	67 633	67 642	67 651	67 660
475	67 669	67 679	67 688	67 697	67 706	67 715	67 724	67 733	67 742	67 752
476	67 761	67 770	67 779	67 788	67 797	67 806	67 815	67 825	67 834	67 843
477	67 852	67 861	67 870	67 879	67 888	67 897	67 906	67 916	67 925	67 934
478	67 943	67 952	67 961	67 970	67 979	67 988	67 997	68 006	68 015	68 024
479	68 034	68 043	68 052	68 061	68 070	68 079	68 088	68 097	68 106	68 115
480	68 124	68 133	68 142	68 151	68 160	68 169	68 178	68 187	68 196	68 205
481	68 215	68 224	68 233	68 242	68 251	68 260	68 269	68 278	68 287	68 296
482	68 305	68 314	68 323	68 332	68 341	68 350	68 359	68 368	68 377	68 386
483	68 395	68 404	68 413	68 422	68 431	68 440	68 449	68 458	68 467	68 476
484	68 485	68 494	68 502	68 511	68 520	68 529	68 538	68 547	68 556	68 565
485	68 574	68 583	68 592	68 601	68 610	68 619	68 628	68 637	68 646	68 655
486	68 664	68 673	68 681	68 690	68 699	68 708	68 717	68 726	68 735	68 744
487	68 753	68 762	68 771	68 780	68 789	68 797	68 806	68 815	68 824	68 833
488	68 842	68 851	68 860	68 869	68 878	68 886	68 895	68 904	68 913	68 922
489	68 931	68 940	68 949	68 958	68 966	68 975	68 984	68 993	69 002	69 011
490	69 020	69 028	69 037	69 046	69 055	69 064	69 073	69 082	69 090	69 099
491	69 108	69 117	69 126	69 135	69 144	69 152	69 161	69 170	69 179	69 188
492	69 197	69 205	69 214	69 223	69 232	69 241	69 249	69 258	69 267	69 276
493	69 285	69 294	69 302	69 311	69 320	69 329	69 338	69 346	69 355	69 364
494	69 373	69 381	69 390	69 399	69 408	69 417	69 425	69 434	69 443	69 452
495	69 461	69 469	69 478	69 487	69 496	69 504	69 513	69 522	69 531	69 539
496	69 548	69 557	69 566	69 574	69 583	69 592	69 601	69 609	69 618	69 627
497	69 636	69 644	69 653	69 662	69 671	69 679	69 688	69 697	69 705	69 714
498	69 723	69 732	69 740	69 749	69 758	69 767	69 775	69 784	69 793	69 801
499	69 810	69 819	69 827	69 836	69 845	69 854	69 862	69 871	69 880	69 888
No.	0	1	2	3	4	5	6	7	8	9

450–499

500–549

No.	0	1	2	3	4	5	6	7	8	9
500	69 897	69 906	69 914	69 923	69 932	69 940	69 949	69 958	69 966	69 975
501	69 984	69 992	70 001	70 010	70 018	70 027	70 036	70 044	70 053	70 062
502	70 070	70 079	70 088	70 096	70 105	70 114	70 122	70 131	70 140	70 148
503	70 157	70 165	70 174	70 183	70 191	70 200	70 209	70 217	70 226	70 234
504	70 243	70 252	70 260	70 269	70 278	70 286	70 295	70 303	70 312	70 321
505	70 329	70 338	70 346	70 355	70 364	70 372	70 381	70 389	70 398	70 406
506	70 415	70 424	70 432	70 441	70 449	70 458	70 467	70 475	70 484	70 492
507	70 501	70 509	70 518	70 526	70 535	70 544	70 552	70 561	70 569	70 578
508	70 586	70 595	70 603	70 612	70 621	70 629	70 638	70 646	70 655	70 663
509	70 672	70 680	70 689	70 697	70 706	70 714	70 723	70 731	70 740	70 749
510	70 757	70 766	70 774	70 783	70 791	70 800	70 808	70 817	70 825	70 834
511	70 842	70 851	70 859	70 868	70 876	70 885	70 893	70 902	70 910	70 919
512	70 927	70 935	70 944	70 952	70 961	70 969	70 978	70 986	70 995	71 003
513	71 012	71 020	71 029	71 037	71 046	71 054	71 063	71 071	71 079	71 088
514	71 096	71 105	71 113	71 122	71 130	71 139	71 147	71 155	71 164	71 172
515	71 181	71 189	71 198	71 206	71 214	71 223	71 231	71 240	71 248	71 257
516	71 265	71 273	71 282	71 290	71 299	71 307	71 315	71 324	71 332	71 341
517	71 349	71 357	71 366	71 374	71 383	71 391	71 399	71 408	71 416	71 425
518	71 433	71 441	71 450	71 458	71 466	71 475	71 483	71 492	71 500	71 508
519	71 517	71 525	71 533	71 542	71 550	71 559	71 567	71 575	71 584	71 592
520	71 600	71 609	71 617	71 625	71 634	71 642	71 650	71 659	71 667	71 675
521	71 684	71 692	71 700	71 709	71 717	71 725	71 734	71 742	71 750	71 759
522	71 767	71 775	71 784	71 792	71 800	71 809	71 817	71 825	71 834	71 842
523	71 850	71 858	71 867	71 875	71 883	71 892	71 900	71 908	71 917	71 925
524	71 933	71 941	71 950	71 958	71 966	71 975	71 983	71 991	71 999	72 008
525	72 016	72 024	72 032	72 041	72 049	72 057	72 066	72 074	72 082	72 090
526	72 099	72 107	72 115	72 123	72 132	72 140	72 148	72 156	72 165	72 173
527	72 181	72 189	72 198	72 206	72 214	72 222	72 230	72 239	72 247	72 255
528	72 263	72 272	72 280	72 288	72 296	72 304	72 313	72 321	72 329	72 337
529	72 346	72 354	72 362	72 370	72 378	72 387	72 395	72 403	72 411	72 419
530	72 428	72 436	72 444	72 452	72 460	72 469	72 477	72 485	72 493	72 501
531	72 509	72 518	72 526	72 534	72 542	72 550	72 558	72 567	72 575	72 583
532	72 591	72 599	72 607	72 616	72 624	72 632	72 640	72 648	72 656	72 665
533	72 673	72 681	72 689	72 697	72 705	72 713	72 722	72 730	72 738	72 746
534	72 754	72 762	72 770	72 779	72 787	72 795	72 803	72 811	72 819	72 827
535	72 835	72 843	72 852	72 860	72 868	72 876	72 884	72 892	72 900	72 908
536	72 916	72 925	72 933	72 941	72 949	72 957	72 965	72 973	72 981	72 989
537	72 997	73 006	73 014	73 022	73 030	73 038	73 046	73 054	73 062	73 070
538	73 078	73 086	73 094	73 102	73 111	73 119	73 127	73 135	73 143	73 151
539	73 159	73 167	73 175	73 183	73 191	73 199	73 207	73 215	73 223	73 231
540	73 239	73 247	73 255	73 263	73 272	73 280	73 288	73 296	73 304	73 312
541	73 320	73 328	73 336	73 344	73 352	73 360	73 368	73 376	73 384	73 392
542	73 400	73 408	73 416	73 424	73 432	73 440	73 448	73 456	73 464	73 472
543	73 480	73 488	73 496	73 504	73 512	73 520	73 528	73 536	73 544	73 552
544	73 560	73 568	73 576	73 584	73 592	73 600	73 608	73 616	73 624	73 632
545	73 640	73 648	73 656	73 664	73 672	73 679	73 687	73 695	73 703	73 711
546	73 719	73 727	73 735	73 743	73 751	73 759	73 767	73 775	73 783	73 791
547	73 799	73 807	73 815	73 823	73 830	73 838	73 846	73 854	73 862	73 870
548	73 878	73 886	73 894	73 902	73 910	73 918	73 926	73 933	73 941	73 949
549	73 957	73 965	73 973	73 981	73 989	73 997	74 005	74 013	74 020	74 028
No.	0	1	2	3	4	5	6	7	8	9

550–599

No.	0	1	2	3	4	5	6	7	8	9
550	74 036	74 044	74 052	74 060	74 068	74 076	74 084	74 092	74 099	74 107
551	74 115	74 123	74 131	74 139	74 147	74 155	74 162	74 170	74 178	74 186
552	74 194	74 202	74 210	74 218	74 225	74 233	74 241	74 249	74 257	74 265
553	74 273	74 280	74 288	74 296	74 304	74 312	74 320	74 327	74 335	74 343
554	74 351	74 359	74 367	74 374	74 382	74 390	74 398	74 406	74 414	74 421
555	74 429	74 437	74 445	74 453	74 461	74 468	74 476	74 484	74 492	74 500
556	74 507	74 515	74 523	74 531	74 539	74 547	74 554	74 562	74 570	74 578
557	74 586	74 593	74 601	74 609	74 617	74 624	74 632	74 640	74 648	74 656
558	74 663	74 671	74 679	74 687	74 695	74 702	74 710	74 718	74 726	74 733
559	74 741	74 749	74 757	74 764	74 772	74 780	74 788	74 796	74 803	74 811
560	74 819	74 827	74 834	74 842	74 850	74 858	74 865	74 873	74 881	74 889
561	74 896	74 904	74 912	74 920	74 927	74 935	74 943	74 950	74 958	74 966
562	74 974	74 981	74 989	74 997	75 005	75 012	75 020	75 028	75 035	75 043
563	75 051	75 059	75 066	75 074	75 082	75 089	75 097	75 105	75 113	75 120
564	75 128	75 136	75 143	75 151	75 159	75 166	75 174	75 182	75 189	75 197
565	75 205	75 213	75 220	75 228	75 236	75 243	75 251	75 259	75 266	75 274
566	75 282	75 289	75 297	75 305	75 312	75 320	75 328	75 335	75 343	75 351
567	75 358	75 366	75 374	75 381	75 389	75 397	75 404	75 412	75 420	75 427
568	75 435	75 442	75 450	75 458	75 465	75 473	75 481	75 488	75 496	75 504
569	75 511	75 519	75 526	75 534	75 542	75 549	75 557	75 565	75 572	75 580
570	75 587	75 595	75 603	75 610	75 618	75 626	75 633	75 641	75 648	75 656
571	75 664	75 671	75 679	75 686	75 694	75 702	75 709	75 717	75 724	75 732
572	75 740	75 747	75 755	75 762	75 770	75 778	75 785	75 793	75 800	75 808
573	75 815	75 823	75 831	75 838	75 846	75 853	75 861	75 868	75 876	75 884
574	75 891	75 899	75 906	75 914	75 921	75 929	75 937	75 944	75 952	75 959
575	75 967	75 974	75 982	75 989	75 997	76 005	76 012	76 020	76 027	76 035
576	76 042	76 050	76 057	76 065	76 072	76 080	76 087	76 095	76 103	76 110
577	76 118	76 125	76 133	76 140	76 148	76 155	76 163	76 170	76 178	76 185
578	76 193	76 200	76 208	76 215	76 223	76 230	76 238	76 245	76 253	76 260
579	76 268	76 275	76 283	76 290	76 298	76 305	76 313	76 320	76 328	76 335
580	76 343	76 350	76 358	76 365	76 373	76 380	76 388	76 395	76 403	76 410
581	76 418	76 425	76 433	76 440	76 448	76 455	76 462	76 470	76 477	76 485
582	76 492	76 500	76 507	76 515	76 522	76 530	76 537	76 545	76 552	76 559
583	76 567	76 574	76 582	76 589	76 597	76 604	76 612	76 619	76 626	76 634
584	76 641	76 649	76 656	76 664	76 671	76 678	76 686	76 693	76 701	76 708
585	76 716	76 723	76 730	76 738	76 745	76 753	76 760	76 768	76 775	76 782
586	76 790	76 797	76 805	76 812	76 819	76 827	76 834	76 842	76 849	76 856
587	76 864	76 871	76 879	76 886	76 893	76 901	76 908	76 916	76 923	76 930
588	76 938	76 945	76 953	76 960	76 967	76 975	76 982	76 989	76 997	77 004
589	77 012	77 019	77 026	77 034	77 041	77 048	77 056	77 063	77 070	77 078
590	77 085	77 093	77 100	77 107	77 115	77 122	77 129	77 137	77 144	77 151
591	77 159	77 166	77 173	77 181	77 188	77 195	77 203	77 210	77 217	77 225
592	77 232	77 240	77 247	77 254	77 262	77 269	77 276	77 283	77 291	77 298
593	77 305	77 313	77 320	77 327	77 335	77 342	77 349	77 357	77 364	77 371
594	77 379	77 386	77 393	77 401	77 408	77 415	77 422	77 430	77 437	77 444
595	77 452	77 459	77 466	77 474	77 481	77 488	77 495	77 503	77 510	77 517
596	77 525	77 532	77 539	77 546	77 554	77 561	77 568	77 576	77 583	77 590
597	77 597	77 605	77 612	77 619	77 627	77 634	77 641	77 648	77 656	77 663
598	77 670	77 677	77 685	77 692	77 699	77 706	77 714	77 721	77 728	77 735
599	77 743	77 750	77 757	77 764	77 772	77 779	77 786	77 793	77 801	77 808
No.	0	1	2	3	4	5	6	7	8	9

550–599

600–649

No.	0	1	2	3	4	5	6	7	8	9
600	77 815	77 822	77 830	77 837	77 844	77 851	77 859	77 866	77 873	77 880
601	77 887	77 895	77 902	77 909	77 916	77 924	77 931	77 938	77 945	77 952
602	77 960	77 967	77 974	77 981	77 988	77 996	78 003	78 010	78 017	78 025
603	78 032	78 039	78 046	78 053	78 061	78 068	78 075	78 082	78 089	78 097
604	78 104	78 111	78 118	78 125	78 132	78 140	78 147	78 154	78 161	78 168
605	78 176	78 183	78 190	78 197	78 204	78 211	78 219	78 226	78 233	78 240
606	78 247	78 254	78 262	78 269	78 276	78 283	78 290	78 297	78 305	78 312
607	78 319	78 326	78 333	78 340	78 347	78 355	78 362	78 369	78 376	78 383
608	78 390	78 398	78 405	78 412	78 419	78 426	78 433	78 440	78 447	78 455
609	78 462	78 469	78 476	78 483	78 490	78 497	78 504	78 512	78 519	78 526
610	78 533	78 540	78 547	78 554	78 561	78 569	78 576	78 583	78 590	78 597
611	78 604	78 611	78 618	78 625	78 633	78 640	78 647	78 654	78 661	78 668
612	78 675	78 682	78 689	78 696	78 704	78 711	78 718	78 725	78 732	78 739
613	78 746	78 753	78 760	78 767	78 774	78 781	78 789	78 796	78 803	78 810
614	78 817	78 824	78 831	78 838	78 845	78 852	78 859	78 866	78 873	78 880
615	78 888	78 895	78 902	78 909	78 916	78 923	78 930	78 937	78 944	78 951
616	78 958	78 965	78 972	78 979	78 986	78 993	79 000	79 007	79 014	79 021
617	79 029	79 036	79 043	79 050	79 057	79 064	79 071	79 078	79 085	79 092
618	79 099	79 106	79 113	79 120	79 127	79 134	79 141	79 148	79 155	79 162
619	79 169	79 176	79 183	79 190	79 197	79 204	79 211	79 218	79 225	79 232
620	79 239	79 246	79 253	79 260	79 267	79 274	79 281	79 288	79 295	79 302
621	79 309	79 316	79 323	79 330	79 337	79 344	79 351	78 358	79 365	79 372
622	79 379	79 386	79 393	79 400	79 407	79 414	79 421	79 428	79 435	79 442
623	79 449	79 456	79 463	79 470	79 477	79 484	79 491	79 498	79 505	79 511
624	79 518	79 525	79 532	79 539	79 546	79 553	79 560	79 567	79 574	79 581
625	79 588	79 595	79 602	79 609	79 616	79 623	79 630	79 637	79 644	79 650
626	79 657	79 664	79 671	79 678	79 685	79 692	79 699	79 706	79 713	79 720
627	79 727	79 734	79 741	79 748	79 754	79 761	79 768	79 775	79 782	79 789
628	79 796	79 803	79 810	79 817	79 824	79 831	79 837	79 844	79 851	79 858
629	79 865	79 872	79 879	79 886	79 893	79 900	79 906	79 913	79 920	79 927
630	79 934	79 941	79 948	79 955	79 962	79 969	79 975	79 982	79 989	79 996
631	80 003	80 010	80 017	80 024	80 030	80 037	80 044	80 051	80 058	80 065
632	80 072	80 079	80 085	80 092	80 099	80 106	80 113	80 120	80 127	80 134
633	80 140	80 147	80 154	80 161	80 168	80 175	80 182	80 188	80 195	80 202
634	80 209	80 216	80 223	80 229	80 236	80 243	80 250	80 257	80 264	80 271
635	80 277	80 284	80 291	80 298	80 305	80 312	80 318	80 325	80 332	80 339
636	80 346	80 353	80 359	80 366	80 373	80 380	80 387	80 393	80 400	80 407
637	80 414	80 421	80 428	80 434	80 441	80 448	80 455	80 462	80 468	80 475
638	80 482	80 489	80 496	80 502	80 509	80 516	80 523	80 530	80 536	80 543
639	80 550	80 557	80 564	80 570	80 577	80 584	80 591	80 598	80 604	80 611
640	80 618	80 625	80 632	80 638	80 645	80 652	80 659	80 665	80 672	80 679
641	80 686	80 693	80 699	80 706	80 713	80 720	80 726	80 733	80 740	80 747
642	80 754	80 760	80 767	80 774	80 781	80 787	80 794	80 801	80 808	80 814
643	80 821	80 828	80 835	80 841	80 848	80 855	80 862	80 868	80 875	80 882
644	80 889	80 895	80 902	80 909	80 916	80 922	80 929	80 936	80 943	80 949
645	80 956	80 963	80 969	80 976	80 983	80 990	80 996	81 003	81 010	81 017
646	81 023	81 030	81 037	81 043	81 050	81 057	81 064	81 070	81 077	81 084
647	81 090	81 097	81 104	81 111	81 117	81 124	81 131	81 137	81 144	81 151
648	81 158	81 164	81 171	81 178	81 184	81 191	81 198	81 204	81 211	81 218
649	81 224	81 231	81 238	81 245	81 251	81 258	81 265	81 271	81 278	81 285
No.	0	1	2	3	4	5	6	7	8	9

600–649

650–699

No.	0	1	2	3	4	5	6	7	8	9
650	81 291	81 298	81 305	81 311	81 318	81 325	81 331	81 338	81 345	81 351
651	81 358	81 365	81 371	81 378	81 385	81 391	81 398	81 405	81 411	81 418
652	81 425	81 431	81 438	81 445	81 451	81 458	81 465	81 471	81 478	81 485
653	81 491	81 498	81 505	81 511	81 518	81 525	81 531	81 538	81 544	81 551
654	81 558	81 564	81 571	81 578	81 584	81 591	81 598	81 604	81 611	81 617
655	81 624	81 631	81 637	81 644	81 651	81 657	81 664	81 671	81 677	81 684
656	81 690	81 697	81 704	81 710	81 717	81 723	81 730	81 737	81 743	81 750
657	81 757	81 763	81 770	81 776	81 783	81 790	81 796	81 803	81 809	81 816
658	81 823	81 829	81 836	81 842	81 849	81 856	81 862	81 869	81 875	81 882
659	81 889	81 895	81 902	81 908	81 915	81 921	81 928	81 935	81 941	81 948
660	81 954	81 961	81 968	81 974	81 981	81 987	81 994	82 000	82 007	82 014
661	82 020	82 027	82 033	82 040	82 046	82 053	82 060	82 066	82 073	82 079
662	82 086	82 092	82 099	82 105	82 112	82 119	82 125	82 132	82 138	82 145
663	82 151	82 158	82 164	82 171	82 178	82 184	82 191	82 197	82 204	82 210
664	82 217	82 223	82 230	82 236	82 243	82 249	82 256	82 263	82 269	82 276
665	82 282	82 289	82 295	82 302	82 308	82 315	82 321	82 328	82 334	82 341
666	82 347	82 354	82 360	82 367	82 373	82 380	82 387	82 393	82 400	82 406
667	82 413	82 419	82 426	82 432	82 439	82 445	82 452	82 458	82 465	82 471
668	82 478	82 484	82 491	82 497	82 504	82 510	82 517	82 523	82 530	82 536
669	82 543	82 549	82 556	82 562	82 569	82 575	82 582	82 588	82 595	82 601
670	82 607	82 614	82 620	82 627	82 633	82 640	82 646	82 653	82 659	82 666
671	82 672	82 679	82 685	82 692	82 698	82 705	82 711	82 718	82 724	82 730
672	82 737	82 743	82 750	82 756	82 763	82 769	82 776	82 782	82 789	82 795
673	82 802	82 808	82 814	82 821	82 827	82 834	82 840	82 847	82 853	82 860
674	82 866	82 872	82 879	82 885	82 892	82 898	82 905	82 911	82 918	82 924
675	82 930	82 937	82 943	82 950	82 956	82 963	82 969	82 975	82 982	82 988
676	82 995	83 001	83 008	83 014	83 020	83 027	83 033	83 040	83 046	83 052
677	83 059	83 065	83 072	83 078	83 085	83 091	83 097	83 104	83 110	83 117
678	83 123	83 129	83 136	83 142	83 149	83 155	83 161	83 168	83 174	83 181
679	83 187	83 193	83 200	83 206	83 213	83 219	83 225	83 232	83 238	83 245
680	83 251	83 257	83 264	83 270	83 276	83 283	83 289	83 296	83 302	83 308
681	83 315	83 321	83 327	83 334	83 340	83 347	83 353	83 359	83 366	83 372
682	83 378	83 385	83 391	83 398	83 404	83 410	83 417	83 423	83 429	83 436
683	83 442	83 448	83 455	83 461	83 467	83 474	83 480	83 487	83 493	83 499
684	83 506	83 512	83 518	83 525	83 531	83 537	83 544	83 550	83 556	83 563
685	83 569	83 575	83 582	83 588	83 594	83 601	83 607	83 613	83 620	83 626
686	83 632	83 639	83 645	83 651	83 658	83 664	83 670	83 677	83 683	83 689
687	83 696	83 702	83 708	83 715	83 721	83 727	83 734	83 740	83 746	83 753
688	83 759	83 765	83 771	83 778	83 784	83 790	83 797	83 803	83 809	83 816
689	83 822	83 828	83 835	83 841	83 847	83 853	83 860	83 866	83 872	83 879
690	83 885	83 891	83 897	83 904	83 910	83 916	83 923	83 929	83 935	83 942
691	83 948	83 954	83 960	83 967	83 973	83 979	83 985	83 992	83 998	84 004
692	84 011	84 017	84 023	84 029	84 036	84 042	84 048	84 055	84 061	84 067
693	84 073	84 080	84 086	84 092	84 098	84 105	84 111	84 117	84 123	84 130
694	84 136	84 142	84 148	84 155	84 161	84 167	84 173	84 180	84 186	84 192
695	84 198	84 205	84 211	84 217	84 223	84 230	84 236	84 242	84 248	84 255
696	84 261	84 267	84 273	84 280	84 286	84 292	84 298	84 305	84 311	84 317
697	84 323	84 330	84 336	84 342	84 348	84 354	84 361	84 367	84 373	84 379
698	84 386	84 392	84 398	84 404	84 410	84 417	84 423	84 429	84 435	84 442
699	84 448	84 454	84 460	84 466	84 473	84 479	84 485	84 491	84 497	84 504
No.	0	1	2	3	4	5	6	7	8	9

700–749

No.	0	1	2	3	4	5	6	7	8	9
700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
701	84 572	84 578	84 584	84 590	84 597	84 603	84 609	84 615	84 621	84 628
702	84 634	84 640	84 646	84 652	84 658	84 665	84 671	84 677	84 683	84 689
703	84 696	84 702	84 708	84 714	84 720	84 726	84 733	84 739	84 745	84 751
704	84 757	84 763	84 770	84 776	84 782	84 788	84 794	84 800	84 807	84 813
705	84 819	84 825	84 831	84 837	84 844	84 850	84 856	84 862	84 868	84 874
706	84 880	84 887	84 893	84 899	84 905	84 911	84 917	84 924	84 930	84 936
707	84 942	84 948	84 954	84 960	84 967	84 973	84 979	84 985	84 991	84 997
708	85 003	85 009	85 016	85 022	85 028	85 034	85 040	85 046	85 052	85 058
709	85 065	85 071	85 077	85 083	85 089	85 095	85 101	85 107	85 114	85 120
710	85 126	85 132	85 138	85 144	85 150	85 156	85 163	85 169	85 175	85 181
711	85 187	85 193	85 199	85 205	85 211	85 217	85 224	85 230	85 236	85 242
712	85 248	85 254	85 260	85 266	85 272	85 278	85 285	85 291	85 297	85 303
713	85 309	85 315	85 321	85 327	85 333	85 339	85 345	85 352	85 358	85 364
714	85 370	85 376	85 382	85 388	85 394	85 400	85 406	85 412	85 418	85 425
715	85 431	85 437	85 443	85 449	85 455	85 461	85 467	85 473	85 479	85 485
716	85 491	85 497	85 503	85 509	85 516	85 522	85 528	85 534	85 540	85 546
717	85 552	85 558	85 564	85 570	85 576	85 582	85 588	85 594	85 600	85 606
718	85 612	85 618	85 625	85 631	85 637	85 643	85 649	85 655	85 661	85 667
719	85 673	85 679	85 685	85 691	85 697	85 703	85 709	85 715	85 721	85 727
720	85 733	85 739	85 745	85 751	85 757	85 763	85 769	85 775	85 781	85 788
721	85 794	85 800	85 806	85 812	85 818	85 824	85 830	85 836	85 842	85 848
722	85 854	85 860	85 866	85 872	85 878	85 884	85 890	85 896	85 902	85 908
723	85 914	85 920	85 926	85 932	85 938	85 944	85 950	85 956	85 962	85 968
724	85 974	85 980	85 986	85 992	85 998	86 004	86 010	86 016	86 022	86 028
725	86 034	86 040	86 046	86 052	86 058	86 064	86 070	86 076	86 082	86 088
726	86 094	86 100	86 106	86 112	86 118	86 124	86 130	86 136	86 141	86 147
727	86 153	86 159	86 165	86 171	86 177	86 183	86 189	86 195	86 201	86 207
728	86 213	86 219	86 225	86 231	86 237	86 243	86 249	86 255	86 261	86 267
729	86 273	86 279	86 285	86 291	86 297	86 303	86 308	86 314	86 320	86 326
730	86 332	86 338	86 344	86 350	86 356	86 362	86 368	86 374	86 380	86 386
731	86 392	86 398	86 404	86 410	86 415	86 421	86 427	86 433	86 439	86 445
732	86 451	86 457	86 463	86 469	86 475	86 481	86 487	86 493	86 499	86 504
733	86 510	86 516	86 522	86 528	86 534	86 540	86 546	86 552	86 558	86 564
734	86 570	86 576	86 581	86 587	86 593	86 599	86 605	86 611	86 617	86 623
735	86 629	86 635	86 641	86 646	86 652	86 658	86 664	86 670	86 676	86 682
736	86 688	86 694	86 700	86 705	86 711	86 717	86 723	86 729	86 735	86 741
737	86 747	86 753	86 759	86 764	86 770	86 776	86 782	86 788	86 794	86 800
738	86 806	86 812	86 817	86 823	86 829	86 835	86 841	86 847	86 853	86 859
739	86 864	86 870	86 876	86 882	86 888	86 894	86 900	86 906	86 911	86 917
740	86 923	86 929	86 935	86 941	86 947	86 953	86 958	86 964	86 970	86 976
741	86 982	86 988	86 994	86 999	87 005	87 011	87 017	87 023	87 029	87 035
742	87 040	87 046	87 052	87 058	87 064	87 070	87 075	87 081	87 087	87 093
743	87 099	87 105	87 111	87 116	87 122	87 128	87 134	87 140	87 146	87 151
744	87 157	87 163	87 169	87 175	87 181	87 186	87 192	87 198	87 204	87 210
745	87 216	87 221	87 227	87 233	87 239	87 245	87 251	87 256	87 262	87 268
746	87 274	87 280	87 286	87 291	87 297	87 303	87 309	87 315	87 320	87 326
747	87 332	87 338	87 344	87 349	87 355	87 361	87 367	87 373	87 379	87 384
748	87 390	87 396	87 402	87 408	87 413	87 419	87 425	87 431	87 437	87 442
749	87 448	87 454	87 460	87 466	87 471	87 477	87 483	87 489	87 495	87 500

No. 0 1 2 3 4 5 6 7 8 9

700–749

750–799

No.	0	1	2	3	4	5	6	7	8	9
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
751	87 564	87 570	87 576	87 581	87 587	87 593	87 599	87 604	87 610	87 616
752	87 622	87 628	87 633	87 639	87 645	87 651	87 656	87 662	87 668	87 674
753	87 679	87 685	87 691	87 697	87 703	87 708	87 714	87 720	87 726	87 731
754	87 737	87 743	87 749	87 754	87 760	87 766	87 772	87 777	87 783	87 789
755	87 795	87 800	87 806	87 812	87 818	87 823	87 829	87 835	87 841	87 846
756	87 852	87 858	87 864	87 869	87 875	87 881	87 887	87 892	87 898	87 904
757	87 910	87 915	87 921	87 927	87 933	87 938	87 944	87 950	87 955	87 961
758	87 967	87 973	87 978	87 984	87 990	87 996	88 001	88 007	88 013	88 018
759	88 024	88 030	88 036	88 041	88 047	88 053	88 058	88 064	88 070	88 076
760	88 081	88 087	88 093	88 098	88 104	88 110	88 116	88 121	88 127	88 133
761	88 138	88 144	88 150	88 156	88 161	88 167	88 173	88 178	88 184	88 190
762	88 195	88 201	88 207	88 213	88 218	88 224	88 230	88 235	88 241	88 247
763	88 252	88 258	88 264	88 270	88 275	88 281	88 287	88 292	88 298	88 304
764	88 309	88 315	88 321	88 326	88 332	88 338	88 343	88 349	88 355	88 360
765	88 366	88 372	88 377	88 383	88 389	88 395	88 400	88 406	88 412	88 417
766	88 423	88 429	88 434	88 440	88 446	88 451	88 457	88 463	88 468	88 474
767	88 480	88 485	88 491	88 497	88 502	88 508	88 513	88 519	88 525	88 530
768	88 536	88 542	88 547	88 553	88 559	88 564	88 570	88 576	88 581	88 587
769	88 593	88 598	88 604	88 610	88 615	88 621	88 627	88 632	88 638	88 643
770	88 649	88 655	88 660	88 666	88 672	88 677	88 683	88 689	88 694	88 700
771	88 705	88 711	88 717	88 722	88 728	88 734	88 739	88 745	88 750	88 756
772	88 762	88 767	88 773	88 779	88 784	88 790	88 795	88 801	88 807	88 812
773	88 818	88 824	88 829	88 835	88 840	88 846	88 852	88 857	88 863	88 868
774	88 874	88 880	88 885	88 891	88 897	88 902	88 908	88 913	88 919	88 925
775	88 930	88 936	88 941	88 947	88 953	88 958	88 964	88 969	88 975	88 981
776	88 986	88 992	88 997	89 003	89 009	89 014	89 020	89 025	89 031	89 037
777	89 042	89 048	89 053	89 059	89 064	89 070	89 076	89 081	89 087	89 092
778	89 098	89 104	89 109	89 115	89 120	89 126	89 131	89 137	89 143	89 148
779	89 154	89 159	89 165	89 170	89 176	89 182	89 187	89 193	89 198	89 204
780	89 209	89 215	89 221	89 226	89 232	89 237	89 243	89 248	89 254	89 260
781	89 265	89 271	89 276	89 282	89 287	89 293	89 298	89 304	89 310	89 315
782	89 321	89 326	89 332	89 337	89 343	89 348	89 354	89 360	89 365	89 371
783	89 376	89 382	89 387	89 393	89 398	89 404	89 409	89 415	89 421	89 426
784	89 432	89 437	89 443	89 448	89 454	89 459	89 465	89 470	89 476	89 481
785	89 487	89 492	89 498	89 504	89 509	89 515	89 520	89 526	89 531	89 537
786	89 542	89 548	89 553	89 559	89 564	89 570	89 575	89 581	89 586	89 592
787	89 597	89 603	89 609	89 614	89 620	89 625	89 631	89 636	89 642	89 647
788	89 653	89 658	89 664	89 669	89 675	89 680	89 686	89 691	89 697	89 702
789	89 708	89 713	89 719	89 724	89 730	89 735	89 741	89 746	89 752	89 757
790	89 763	89 768	89 774	89 779	89 785	89 790	89 796	89 801	89 807	89 812
791	89 818	89 823	89 829	89 834	89 840	89 845	89 851	89 856	89 862	89 867
792	89 873	89 878	89 883	89 889	89 894	89 900	89 905	89 911	89 916	89 922
793	89 927	89 933	89 938	89 944	89 949	89 955	89 960	89 966	89 971	89 977
794	89 982	89 988	89 993	89 998	90 004	90 009	90 015	90 020	90 026	90 031
795	90 037	90 042	90 048	90 053	90 059	90 064	90 069	90 075	90 080	90 086
796	90 091	90 097	90 102	90 108	90 113	90 119	90 124	90 129	90 135	90 140
797	90 146	90 151	90 157	90 162	90 168	90 173	90 179	90 184	90 189	90 195
798	90 200	90 206	90 211	90 217	90 222	90 227	90 233	90 238	90 244	90 249
799	90 255	90 260	90 266	90 271	90 276	90 282	90 287	90 293	90 298	90 304
No.	0	1	2	3	4	5	6	7	8	9

800–849

No.	0	1	2	3	4	5	6	7	8	9
800	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358
801	90 363	90 369	90 374	90 380	90 385	90 390	90 396	90 401	90 407	90 412
802	90 417	90 423	90 428	90 434	90 439	90 445	90 450	90 455	90 461	90 466
803	90 472	90 477	90 482	90 488	90 493	90 499	90 504	90 509	90 515	90 520
804	90 526	90 531	90 536	90 542	90 547	90 553	90 558	90 563	90 569	90 574
805	90 580	90 585	90 590	90 596	90 601	90 607	90 612	90 617	90 623	90 628
806	90 634	90 639	90 644	90 650	90 655	90 660	90 666	90 671	90 677	90 682
807	90 687	90 693	90 698	90 703	90 709	90 714	90 720	90 725	90 730	90 736
808	90 741	90 747	90 752	90 757	90 763	90 768	90 773	90 779	90 784	90 789
809	90 795	90 800	90 806	90 811	90 816	90 822	90 827	90 832	90 838	90 843
810	90 849	90 854	90 859	90 865	90 870	90 875	90 881	90 886	90 891	90 897
811	90 902	90 907	90 913	90 918	90 924	90 929	90 934	90 940	90 945	90 950
812	90 956	90 961	90 966	90 972	90 977	90 982	90 988	90 993	90 998	91 004
813	91 009	91 014	91 020	91 025	91 030	91 036	91 041	91 046	91 052	91 057
814	91 062	91 068	91 073	91 078	91 084	91 089	91 094	91 100	91 105	91 110
815	91 116	91 121	91 126	91 132	91 137	91 142	91 148	91 153	91 158	91 164
816	91 169	91 174	91 180	91 185	91 190	91 196	91 201	91 206	91 212	91 217
817	91 222	91 228	91 233	91 238	91 243	91 249	91 254	91 259	91 265	91 270
818	91 275	91 281	91 286	91 291	91 297	91 302	91 307	91 312	91 318	91 323
819	91 328	91 334	91 339	91 344	91 350	91 355	91 360	91 365	91 371	91 376
820	91 381	91 387	91 392	91 397	91 403	91 408	91 413	91 418	91 424	91 429
821	91 434	91 440	91 445	91 450	91 455	91 461	91 466	91 471	91 477	91 482
822	91 487	91 492	91 498	91 503	91 508	91 514	91 519	91 524	91 529	91 535
823	91 540	91 545	91 551	91 556	91 561	91 566	91 572	91 577	91 582	91 587
824	91 593	91 598	91 603	91 609	91 614	91 619	91 624	91 630	91 635	91 640
825	91 645	91 651	91 656	91 661	91 666	91 672	91 677	91 682	91 687	91 693
826	91 698	91 703	91 709	91 714	91 719	91 724	91 730	91 735	91 740	91 745
827	91 751	91 756	91 761	91 766	91 772	91 777	91 782	91 787	91 793	91 798
828	91 803	91 808	91 814	91 819	91 824	91 829	91 834	91 840	91 845	91 850
829	91 855	91 861	91 866	91 871	91 876	91 882	91 887	91 892	91 897	91 903
830	91 908	91 913	91 918	91 924	91 929	91 934	91 939	91 944	91 950	91 955
831	91 960	91 965	91 971	91 976	91 981	91 986	91 991	91 997	92 002	92 007
832	92 012	92 018	92 023	92 028	92 033	92 038	92 044	92 049	92 054	92 059
833	92 065	92 070	92 075	92 080	92 085	92 091	92 096	92 101	92 106	92 111
834	92 117	92 122	92 127	92 132	92 137	92 143	92 148	92 153	92 158	92 163
835	92 169	92 174	92 179	92 184	92 189	92 195	92 200	92 205	92 210	92 215
836	92 221	92 226	92 231	92 236	92 241	92 247	92 252	92 257	92 262	92 267
837	92 273	92 278	92 283	92 288	92 293	92 298	92 304	92 309	92 314	92 319
838	92 324	92 330	92 335	92 340	92 345	92 350	92 355	92 361	92 366	92 371
839	92 376	92 381	92 387	92 392	92 397	92 402	92 407	92 412	92 418	92 423
840	92 428	92 433	92 438	92 443	92 449	92 454	92 459	92 464	92 469	92 474
841	92 480	92 485	92 490	92 495	92 500	92 505	92 511	92 516	92 521	92 526
842	92 531	92 536	92 542	92 547	92 552	92 557	92 562	92 567	92 572	92 578
843	92 583	92 588	92 593	92 598	92 603	92 609	92 614	92 619	92 624	92 629
844	92 634	92 639	92 645	92 650	92 655	92 660	92 665	92 670	92 675	92 681
845	92 686	92 691	92 696	92 701	92 706	92 711	92 716	92 722	92 727	92 732
846	92 737	92 742	92 747	92 752	92 758	92 763	92 768	92 773	92 778	92 783
847	92 788	92 793	92 799	92 804	92 809	92 814	92 819	92 824	92 829	92 834
848	92 840	92 845	92 850	92 855	92 860	92 865	92 870	92 875	92 881	92 886
849	92 891	92 896	92 901	92 906	92 911	92 916	92 921	92 927	92 932	92 937
No.	0	1	2	3	4	5	6	7	8	9

850–899

No.	0	1	2	3	4	5	6	7	8	9
850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988
851	92 993	92 998	93 003	93 008	93 013	93 018	93 024	93 029	93 034	93 039
852	93 044	93 049	93 054	93 059	93 064	93 069	93 075	93 080	93 085	93 090
853	93 095	93 100	93 105	93 110	93 115	93 120	93 125	93 131	93 136	93 141
854	93 146	93 151	93 156	93 161	93 166	93 171	93 176	93 181	93 186	93 192
855	93 197	93 202	93 207	93 212	93 217	93 222	93 227	93 232	93 237	93 242
856	93 247	93 252	93 258	93 263	93 268	93 273	93 278	93 283	93 288	93 293
857	93 298	93 303	93 308	93 313	93 318	93 323	93 328	93 334	93 339	93 344
858	93 349	93 354	93 359	93 364	93 369	93 374	93 379	93 384	93 389	93 394
859	93 399	93 404	93 409	93 414	93 420	93 425	93 430	93 435	93 440	93 445
860	93 450	93 455	93 460	93 465	93 470	93 475	93 480	93 485	93 490	93 495
861	93 500	93 505	93 510	94 515	93 520	93 526	93 531	93 536	93 541	93 546
862	93 551	93 556	93 561	93 566	93 571	93 576	93 581	93 586	93 591	93 596
863	93 601	93 606	93 611	93 616	93 621	93 626	93 631	93 636	93 641	93 646
864	93 651	93 656	93 661	93 666	93 671	93 676	93 682	93 687	93 692	93 697
865	93 702	93 707	93 712	93 717	93 722	93 727	93 732	93 737	93 742	93 747
866	93 752	93 757	93 762	93 767	93 772	93 777	93 782	93 787	93 792	93 797
867	93 802	93 807	93 812	93 817	93 822	93 827	93 832	93 837	93 842	93 847
868	93 852	93 857	93 862	93 867	93 872	93 877	93 882	93 887	93 892	93 897
869	93 902	93 907	93 912	93 917	93 922	93 927	93 932	93 937	93 942	93 947
870	93 952	93 957	93 962	93 967	93 972	93 977	93 982	93 987	93 992	93 997
871	94 002	94 007	94 012	94 017	94 022	94 027	94 032	94 037	94 042	94 047
872	94 052	94 057	94 062	94 067	94 072	94 077	94 082	94 086	94 091	94 096
873	94 101	94 106	94 111	94 116	94 121	94 126	94 131	94 136	94 141	94 146
874	94 151	94 156	94 161	94 166	94 171	94 176	94 181	94 186	94 191	94 196
875	94 201	94 206	94 211	94 216	94 221	94 226	94 231	94 236	94 240	94 245
876	94 250	94 255	94 260	94 265	94 270	94 275	94 280	94 285	94 290	94 295
877	94 300	94 305	94 310	94 315	94 320	94 325	94 330	94 335	94 340	94 345
878	94 349	94 354	94 359	94 364	94 369	94 374	94 379	94 384	94 389	94 394
879	94 399	94 404	94 409	94 414	94 419	94 424	94 429	94 433	94 438	94 443
880	94 448	94 453	94 458	94 463	94 468	94 473	94 478	94 483	94 488	94 493
881	94 498	94 503	94 507	94 512	94 517	94 522	94 527	94 532	94 537	94 542
882	94 547	94 552	94 557	94 562	94 567	94 571	94 576	94 581	94 586	94 591
883	94 596	94 601	94 606	94 611	94 616	94 621	94 626	94 630	94 635	94 640
884	94 645	94 650	94 655	94 660	94 665	94 670	94 675	94 680	94 685	94 689
885	94 694	94 699	94 704	94 709	94 714	94 719	94 724	94 729	94 734	94 738
886	94 743	94 748	94 753	94 758	94 763	94 768	94 773	94 778	94 783	94 787
887	94 792	94 797	94 802	94 807	94 812	94 817	94 822	94 827	94 832	94 836
888	94 841	94 846	94 851	94 856	94 861	94 866	94 871	94 876	94 880	94 885
889	94 890	94 895	94 900	94 905	94 910	94 915	94 919	94 924	94 929	94 934
890	94 939	94 944	94 949	94 954	94 959	94 963	94 968	94 973	94 978	94 983
891	94 988	94 993	94 998	95 002	95 007	95 012	95 017	95 022	95 027	95 032
892	95 036	95 041	95 046	95 051	95 056	95 061	95 066	95 071	95 075	95 080
893	95 085	95 090	95 095	95 100	95 105	95 109	95 114	95 119	95 124	95 129
894	95 134	95 139	95 143	95 148	95 153	95 158	95 163	95 168	95 173	95 177
895	95 182	95 187	95 192	95 197	95 202	95 207	95 211	95 216	95 221	95 226
896	95 231	95 236	95 240	95 245	95 250	95 255	95 260	95 265	95 270	95 274
897	95 279	95 284	95 289	95 294	95 299	95 303	95 308	95 313	95 318	95 323
898	95 328	95 332	95 337	95 342	95 347	95 352	95 357	95 361	95 366	95 371
899	95 376	95 381	95 386	95 390	95 395	95 400	95 405	95 410	95 415	95 419
No.	0	1	2	3	4	5	6	7	8	9

900–949

No.	0	1	2	3	4	5	6	7	8	9
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
901	95 472	95 477	95 482	95 487	95 492	95 497	95 501	95 506	95 511	95 516
902	95 521	95 525	95 530	95 535	95 540	95 545	95 550	95 554	95 559	95 564
903	95 569	95 574	95 578	95 583	95 588	95 593	95 598	95 602	95 607	95 612
904	95 617	95 622	95 626	95 631	95 636	95 641	95 646	95 650	95 655	95 660
905	95 665	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	95 713	95 718	95 722	95 727	95 732	95 737	95 742	95 746	95 751	95 756
907	95 761	95 766	95 770	95 775	95 780	95 785	95 789	95 794	95 799	95 804
908	95 809	95 813	95 818	95 823	95 828	95 832	95 837	95 842	95 847	95 852
909	95 856	95 861	95 866	95 871	95 875	95 880	95 885	95 890	95 895	95 899
910	95 904	95 909	95 914	95 918	95 923	95 928	95 933	95 938	95 942	95 947
911	95 952	95 957	95 961	95 966	95 971	95 976	95 980	95 985	95 990	95 995
912	95 999	96 004	96 009	96 014	96 019	96 023	96 028	96 033	96 038	96 042
913	96 047	96 052	96 057	96 061	96 066	96 071	96 076	96 080	96 085	96 090
914	96 095	96 099	96 104	96 109	96 114	96 118	96 123	96 128	96 133	96 137
915	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185
916	96 190	96 194	96 199	96 204	96 209	96 213	96 218	96 223	96 227	96 232
917	96 237	96 242	96 246	96 251	96 256	96 261	96 265	96 270	96 275	96 280
918	96 284	96 289	96 294	96 298	96 303	96 308	96 313	96 317	96 322	96 327
919	96 332	96 336	96 341	96 346	96 350	96 355	96 360	96 365	96 369	96 374
920	96 379	96 384	96 388	96 393	96 398	96 402	96 407	96 412	96 417	96 421
921	96 426	96 431	96 435	96 440	96 445	96 450	96 454	96 459	96 464	96 468
922	96 473	96 478	96 483	96 487	96 492	96 497	96 501	96 506	96 511	96 515
923	96 520	96 525	96 530	96 534	96 539	96 544	96 548	96 553	96 558	96 562
924	96 567	96 572	96 577	96 581	96 586	96 591	96 595	96 600	96 605	96 609
925	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	96 661	96 666	96 670	96 675	96 680	96 685	96 689	96 694	96 699	96 703
927	96 708	96 713	96 717	96 722	96 727	96 731	96 736	96 741	96 745	96 750
928	96 755	96 759	96 764	96 769	96 774	96 778	96 783	96 788	96 792	96 797
929	96 802	96 806	96 811	96 816	96 820	96 825	96 830	96 834	96 839	96 844
930	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
931	96 895	96 900	96 904	96 909	96 914	96 918	96 923	96 928	96 932	96 937
932	96 942	96 946	96 951	96 956	96 960	96 965	96 970	96 974	96 979	96 984
933	96 988	96 993	96 997	97 002	97 007	97 011	97 016	97 021	97 025	97 030
934	97 035	97 039	97 044	97 049	97 053	97 058	97 063	97 067	97 072	97 077
935	97 081	97 086	97 090	97 095	97 100	97 104	97 109	97 114	97 118	97 123
936	97 128	97 132	97 137	97 142	97 146	97 151	97 155	97 160	97 165	97 169
937	97 174	97 179	97 183	97 188	97 192	97 197	97 202	97 206	97 211	97 216
938	97 220	97 225	97 230	97 234	97 239	97 243	97 248	97 253	97 257	97 262
939	97 267	97 271	97 276	97 280	97 285	97 290	97 294	97 299	97 304	97 308
940	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 350	97 354
941	97 359	97 364	97 368	97 373	97 377	97 382	97 387	97 391	97 396	97 400
942	97 405	97 410	97 414	97 419	97 424	97 428	97 433	97 437	97 442	97 447
943	97 451	97 456	97 460	97 465	97 470	97 474	97 479	97 483	97 488	97 493
944	97 497	97 502	97 506	97 511	97 516	97 520	97 525	97 529	97 534	97 539
945	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585
946	97 589	97 594	97 598	97 603	97 607	97 612	97 617	97 621	97 626	97 630
947	97 635	97 640	97 644	97 649	97 653	97 658	97 663	97 667	97 672	97 676
948	97 681	97 685	97 690	97 695	97 699	97 704	97 708	97 713	97 717	97 722
949	97 727	97 731	97 736	97 740	97 745	97 749	97 754	97 759	97 763	97 768

No. 0 1 2 3 4 5 6 7 8 9

900–949

950–1000

No.	0	1	2	3	4	5	6	7	8	9
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
951	97 818	97 823	97 827	97 832	97 836	97 841	97 845	97 850	97 855	97 859
952	97 864	97 868	97 873	97 877	97 882	97 886	97 891	97 896	97 900	97 905
953	97 909	97 914	97 918	97 923	97 928	97 932	97 937	97 941	97 946	97 950
954	97 955	97 959	97 964	97 968	97 973	97 978	97 982	97 987	97 991	97 996
955	98 000	98 005	98 009	98 014	98 019	98 023	98 028	98 032	98 037	98 041
956	98 046	98 050	98 055	98 059	98 064	98 068	98 073	98 078	98 082	98 087
957	98 091	98 096	98 100	98 105	98 109	98 114	98 118	98 123	98 127	98 132
958	98 137	98 141	98 146	98 150	98 155	98 159	98 164	98 168	98 173	98 177
959	98 182	98 186	98 191	98 195	98 200	98 204	98 209	98 214	98 218	98 223
960	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	98 272	98 277	98 281	98 286	98 290	98 295	98 299	98 304	98 308	98 313
962	98 318	98 322	98 327	98 331	98 336	98 340	98 345	98 349	98 354	98 358
963	98 363	98 367	98 372	98 376	98 381	98 385	98 390	98 394	98 399	98 403
964	98 408	98 412	98 417	98 421	98 426	98 430	98 435	98 439	98 444	98 448
965	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	98 498	98 502	98 507	98 511	98 516	98 520	98 525	98 529	98 534	98 538
967	98 543	98 547	98 552	98 556	98 561	98 565	98 570	98 574	98 579	98 583
968	98 588	98 592	98 597	98 601	98 605	98 610	98 614	98 619	98 623	98 628
969	98 632	98 637	98 641	98 646	98 650	98 655	98 659	98 664	98 668	98 673
970	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	98 722	98 726	98 731	98 735	98 740	98 744	98 749	98 753	98 758	98 762
972	98 767	98 771	98 776	98 780	98 784	98 789	98 793	98 798	98 802	98 807
973	98 811	98 816	98 820	98 825	98 829	98 834	98 838	98 843	98 847	98 851
974	98 856	98 860	98 865	98 869	98 874	98 878	98 883	98 887	98 892	98 896
975	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	98 945	98 949	98 954	98 958	98 963	98 967	98 972	98 976	98 981	98 985
977	98 989	98 994	98 998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	99 047	99 052	99 056	99 061	99 065	99 069	99 074
979	99 078	99 083	99 087	99 092	99 096	99 100	99 105	99 109	99 114	99 118
980	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	99 167	99 171	99 176	99 180	99 185	99 189	99 193	99 198	99 202	99 207
982	99 211	99 216	99 220	99 224	99 229	99 233	99 238	99 242	99 247	99 251
983	99 255	99 260	99 264	99 269	99 273	99 277	99 282	99 286	99 291	99 295
984	99 300	99 304	99 308	99 313	99 317	99 322	99 326	99 330	99 335	99 339
985	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	99 388	99 392	99 396	99 401	99 405	99 410	99 414	99 419	99 423	99 427
987	99 432	99 436	99 441	99 445	99 449	99 454	99 458	99 463	99 467	99 471
988	99 476	99 480	99 484	99 489	99 493	99 498	99 502	99 506	99 511	99 515
989	99 520	99 524	99 528	99 533	99 537	99 542	99 546	99 550	99 555	99 559
990	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	99 607	99 612	99 616	99 621	99 625	99 629	99 634	99 638	99 642	99 647
992	99 651	99 656	99 660	99 664	99 669	99 673	99 677	99 682	99 686	99 691
993	99 695	99 699	99 704	99 708	99 712	99 717	99 721	99 726	99 730	99 734
994	99 739	99 743	99 747	99 752	99 756	99 760	99 765	99 769	99 774	99 778
995	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	99 822
996	99 826	99 830	99 835	99 839	99 843	99 848	99 852	99 856	99 861	99 865
997	99 870	99 874	99 878	99 883	99 887	99 891	99 896	99 900	99 904	99 909
998	99 913	99 917	99 922	99 926	99 930	99 935	99 939	99 944	99 948	99 952
999	99 957	99 961	99 965	99 970	99 974	99 978	99 983	99 987	99 991	99 996
1000	00 000	00 004	00 009	00 013	00 017	00 022	00 026	00 030	00 035	00 039
No.	0	1	2	3	4	5	6	7	8	9

900–949

No.	0	1	2	3	4	5	6	7	8	9
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
901	95 472	95 477	95 482	95 487	95 492	95 497	95 501	95 506	95 511	95 516
902	95 521	95 525	95 530	95 535	95 540	95 545	95 550	95 554	95 559	95 564
903	95 569	95 574	95 578	95 583	95 588	95 593	95 598	95 602	95 607	95 612
904	95 617	95 622	95 626	95 631	95 636	95 641	95 646	95 650	95 655	95 660
905	95 665	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	95 713	95 718	95 722	95 727	95 732	95 737	95 742	95 746	95 751	95 756
907	95 761	95 766	95 770	95 775	95 780	95 785	95 789	95 794	95 799	95 804
908	95 809	95 813	95 818	95 823	95 828	95 832	95 837	95 842	95 847	95 852
909	95 856	95 861	95 866	95 871	95 875	95 880	95 885	95 890	95 895	95 899
910	95 904	95 909	95 914	95 918	95 923	95 928	95 933	95 938	95 942	95 947
911	95 952	95 957	95 961	95 966	95 971	95 976	95 980	95 985	95 990	95 995
912	95 999	96 004	96 009	96 014	96 019	96 023	96 028	96 033	96 038	96 042
913	96 047	96 052	96 057	96 061	96 066	96 071	96 076	96 080	96 085	96 090
914	96 095	96 099	96 104	96 109	96 114	96 118	96 123	96 128	96 133	96 137
915	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185
916	96 190	96 194	96 199	96 204	96 209	96 213	96 218	96 223	96 227	96 232
917	96 237	96 242	96 246	96 251	96 256	96 261	96 265	96 270	96 275	96 280
918	96 284	96 289	96 294	96 298	96 303	96 308	96 313	96 317	96 322	96 327
919	96 332	96 336	96 341	96 346	96 350	96 355	96 360	96 365	96 369	96 374
920	96 379	96 384	96 388	96 393	96 398	96 402	96 407	96 412	96 417	96 421
921	96 426	96 431	96 435	96 440	96 445	96 450	96 454	96 459	96 464	96 468
922	96 473	96 478	96 483	96 487	96 492	96 497	96 501	96 506	96 511	96 515
923	96 520	96 525	96 530	96 534	96 539	96 544	96 548	96 553	96 558	96 562
924	96 567	96 572	96 577	96 581	96 586	96 591	96 595	96 600	96 605	96 609
925	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	96 661	96 666	96 670	96 675	96 680	96 685	96 689	96 694	96 699	96 703
927	96 708	96 713	96 717	96 722	96 727	96 731	96 736	96 741	96 745	96 750
928	96 755	96 759	96 764	96 769	96 774	96 778	96 783	96 788	96 792	96 797
929	96 802	96 806	96 811	96 816	96 820	96 825	96 830	96 834	96 839	96 844
930	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
931	96 895	96 900	96 904	96 909	96 914	96 918	96 923	96 928	96 932	96 937
932	96 942	96 946	96 951	96 956	96 960	96 965	96 970	96 974	96 979	96 984
933	96 988	96 993	96 997	97 002	97 007	97 011	97 016	97 021	97 025	97 030
934	97 035	97 039	97 044	97 049	97 053	97 058	97 063	97 067	97 072	97 077
935	97 081	97 086	97 090	97 095	97 100	97 104	97 109	97 114	97 118	97 123
936	97 128	97 132	97 137	97 142	97 146	97 151	97 155	97 160	97 165	97 169
937	97 174	97 179	97 183	97 188	97 192	97 197	97 202	97 206	97 211	97 216
938	97 220	97 225	97 230	97 234	97 239	97 243	97 248	97 253	97 257	97 262
939	97 267	97 271	97 276	97 280	97 285	97 290	97 294	97 299	97 304	97 308
940	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 350	97 354
941	97 359	97 364	97 368	97 373	97 377	97 382	97 387	97 391	97 396	97 400
942	97 405	97 410	97 414	97 419	97 424	97 428	97 433	97 437	97 442	97 447
943	97 451	97 456	97 460	97 465	97 470	97 474	97 479	97 483	97 488	97 493
944	97 497	97 502	97 506	97 511	97 516	97 520	97 525	97 529	97 534	97 539
945	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585
946	97 589	97 594	97 598	97 603	97 607	97 612	97 617	97 621	97 626	97 630
947	97 635	97 640	97 644	97 649	97 653	97 658	97 663	97 667	97 672	97 676
948	97 681	97 685	97 690	97 695	97 699	97 704	97 708	97 713	97 717	97 722
949	97 727	97 731	97 736	97 740	97 745	97 749	97 754	97 759	97 763	97 768
No.	0	1	2	3	4	5	6	7	8	9

900–949

950–1000

No.	0	1	2	3	4	5	6	7	8	9
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
951	97 818	97 823	97 827	97 832	97 836	97 841	97 845	97 850	97 855	97 859
952	97 864	97 868	97 873	97 877	97 882	97 886	97 891	97 896	97 900	97 905
953	97 909	97 914	97 918	97 923	97 928	97 932	97 937	97 941	97 946	97 950
954	97 955	97 959	97 964	97 968	97 973	97 978	97 982	97 987	97 991	97 996
955	98 000	98 005	98 009	98 014	98 019	98 023	98 028	98 032	98 037	98 041
956	98 046	98 050	98 055	98 059	98 064	98 068	98 073	98 078	98 082	98 087
957	98 091	98 096	98 100	98 105	98 109	98 114	98 118	98 123	98 127	98 132
958	98 137	98 141	98 146	98 150	98 155	98 159	98 164	98 168	98 173	98 177
959	98 182	98 186	98 191	98 195	98 200	98 204	98 209	98 214	98 218	98 223
960	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	98 272	98 277	98 281	98 286	98 290	98 295	98 299	98 304	98 308	98 313
962	98 318	98 322	98 327	98 331	98 336	98 340	98 345	98 349	98 354	98 358
963	98 363	98 367	98 372	98 376	98 381	98 385	98 390	98 394	98 399	98 403
964	98 408	98 412	98 417	98 421	98 426	98 430	98 435	98 439	98 444	98 448
965	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	98 498	98 502	98 507	98 511	98 516	98 520	98 525	98 529	98 534	98 538
967	98 543	98 547	98 552	98 556	98 561	98 565	98 570	98 574	98 579	98 583
968	98 588	98 592	98 597	98 601	98 605	98 610	98 614	98 619	98 623	98 628
969	98 632	98 637	98 641	98 646	98 650	98 655	98 659	98 664	98 668	98 673
970	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	98 722	98 726	98 731	98 735	98 740	98 744	98 749	98 753	98 758	98 762
972	98 767	98 771	98 776	98 780	98 784	98 789	98 793	98 798	98 802	98 807
973	98 811	98 816	98 820	98 825	98 829	98 834	98 838	98 843	98 847	98 851
974	98 856	98 860	98 865	98 869	98 874	98 878	98 883	98 887	98 892	98 896
975	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	98 945	98 949	98 954	98 958	98 963	98 967	98 972	98 976	98 981	98 985
977	98 989	98 994	98 998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	99 047	99 052	99 056	99 061	99 065	99 069	99 074
979	99 078	99 083	99 087	99 092	99 096	99 100	99 105	99 109	99 114	99 118
980	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	99 167	99 171	99 176	99 180	99 185	99 189	99 193	99 198	99 202	99 207
982	99 211	99 216	99 220	99 224	99 229	99 233	99 238	99 242	99 247	99 251
983	99 255	99 260	99 264	99 269	99 273	99 277	99 282	99 286	99 291	99 295
984	99 300	99 304	99 308	99 313	99 317	99 322	99 326	99 330	99 335	99 339
985	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	99 388	99 392	99 396	99 401	99 405	99 410	99 414	99 419	99 423	99 427
987	99 432	99 436	99 441	99 445	99 449	99 454	99 458	99 463	99 467	99 471
988	99 476	99 480	99 484	99 489	99 493	99 498	99 502	99 506	99 511	99 515
989	99 520	99 524	99 528	99 533	99 537	99 542	99 546	99 550	99 555	99 559
990	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	99 607	99 612	99 616	99 621	99 625	99 629	99 634	99 638	99 642	99 647
992	99 651	99 656	99 660	99 664	99 669	99 673	99 677	99 682	99 686	99 691
993	99 695	99 699	99 704	99 708	99 712	99 717	99 721	99 726	99 730	99 734
994	99 739	99 743	99 747	99 752	99 756	99 760	99 765	99 769	99 774	99 778
995	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	99 822
996	99 826	99 830	99 835	99 839	99 843	99 848	99 852	99 856	99 861	99 865
997	99 870	99 874	99 878	99 883	99 887	99 891	99 896	99 900	99 904	99 909
998	99 913	99 917	99 922	99 926	99 930	99 935	99 939	99 944	99 948	99 952
999	99 957	99 961	99 965	99 970	99 974	99 978	99 983	99 987	99 991	99 996
1000	00 000	00 004	00 009	00 013	00 017	00 022	00 026	00 030	00 035	00 039
No.	0	1	2	3	4	5	6	7	8	9

APPENDIX VIII

TABLE OF SQUARES, SQUARE ROOTS, AND RECIPROCALS—1 TO 1000

The accompanying table presents the numbers from 1 to 1000 in **bold-face** type, and for each of these numbers the corresponding square, square root, and reciprocal are presented in light-face type.

In statistical work, tables of squares and square roots are valuable aids in such problems as determining standard deviations, correlation coefficients, or least squares trend lines.

The use of reciprocals greatly facilitates division. This is true whether the work is done by hand or by machine. The process is simple. Instead of dividing by a number we multiply by its reciprocal. Suppose, for instance, that we wish to divide 79,326 by 893. By computation or by referring to the tables (see page 758) we find that the reciprocal of 893 is 0.00112. By multiplying the dividend by this reciprocal (79326×0.00112) we get 88.84512. Therefore, $79326 \div 893 = 88.84512$.

When many numbers are to be divided by the same number, the use of the reciprocal is particularly advantageous. In computing percentage relatives, for instance, it is much easier to multiply the original figures in the series by the reciprocal of the base than it is to divide each figure in the series by the actual base figure.

TABLE OF SQUARES, SQUARE ROOTS, AND RECIPROCALS—1 TO 1000

Numbers	Squares	Square roots	Reciprocals	Numbers	Squares	Square roots	Reciprocals
1	1	1.0000	1.000000	**51**	2601	7.1414	.019608
2	4	1.4142	.500000	**52**	2704	7.2111	.019231
3	9	1.7321	.333333	**53**	2809	7.2801	.018868
4	16	2.0000	.250000	**54**	2916	7.3485	.018519
5	25	2.2361	.200000	**55**	3025	7.4162	.018182
6	36	2.4495	.166667	**56**	3136	7.4833	.017857
7	49	2.6458	.142857	**57**	3249	7.5498	.017544
8	64	2.8284	.125000	**58**	3364	7.6158	.017241
9	81	3.0000	.111111	**59**	3481	7.6811	.016949
10	100	3.1623	.100000	**60**	3600	7.7460	.016667
11	121	3.3166	.090909	**61**	3721	7.8102	.016393
12	144	3.4641	.083333	**62**	3844	7.8740	.016129
13	169	3.6056	.076923	**63**	3969	7.9373	.015873
14	196	3.7417	.071429	**64**	4096	8.0000	.015625
15	225	3.8730	.066667	**65**	4225	8.0623	.015385
16	256	4.0000	.062500	**66**	4356	8.1240	.015152
17	289	4.1231	.058824	**67**	4489	8.1854	.014925
18	324	4.2426	.055556	**68**	4624	8.2462	.014706
19	361	4.3589	.052632	**69**	4761	8.3066	.014493
20	400	4.4721	.050000	**70**	4900	8.3666	.014286
21	441	4.5826	.047619	**71**	5041	8.4261	.014085
22	484	4.6904	.045455	**72**	5184	8.4853	.013889
23	529	4.7958	.043478	**73**	5329	8.5440	.013699
24	576	4.8990	.041667	**74**	5476	8.6023	.013514
25	625	5.0000	.040000	**75**	5625	8.6603	.013333
26	676	5.0990	.038462	**76**	5776	8.7178	.013158
27	729	5.1962	.037037	**77**	5929	8.7750	.012987
28	784	5.2915	.035714	**78**	6084	8.8318	.012821
29	841	5.3852	.034483	**79**	6241	8.8882	.012658
30	900	5.4772	.033333	**80**	6400	8.9443	.012500
31	961	5.5678	.032258	**81**	6561	9.0000	.012346
32	1024	5.6569	.031250	**82**	6724	9.0554	.012195
33	1089	5.7446	.030303	**83**	6889	9.1104	.012048
34	1156	5.8310	.029412	**84**	7056	9.1652	.011905
35	1225	5.9161	.028571	**85**	7225	9.2195	.011765
36	1296	6.0000	.027778	**86**	7396	9.2736	.011628
37	1369	6.0828	.027027	**87**	7569	9.3274	.011494
38	1444	6.1644	.026316	**88**	7744	9.3808	.011364
39	1521	6.2450	.025641	**89**	7921	9.4340	.011236
40	1600	6.3246	.025000	**90**	8100	9.4868	.011111
41	1681	6.4031	.024390	**91**	8281	9.5394	.010989
42	1764	6.4807	.023810	**92**	8464	9.5917	.010870
43	1849	6.5574	.023256	**93**	8649	9.6437	.010753
44	1936	6.6332	.022727	**94**	8836	9.6954	.010638
45	2025	6.7082	.022222	**95**	9025	9.7468	.010526
46	2116	6.7823	.021739	**96**	9216	9.7980	.010417
47	2209	6.8557	.021277	**97**	9409	9.8489	.010309
48	2304	6.9282	.020833	**98**	9604	9.8995	.010204
49	2401	7.0000	.020408	**99**	9801	9.9499	.010101
50	2500	7.0711	.020000	**100**	10000	10.0000	.010000

TABLE OF SQUARES, SQUARE ROOTS, AND RECIPROCALS—1 TO 1000.—(*Continued*)

Numbers	Squares	Square roots	Reciprocals	Numbers	Squares	Square roots	Reciprocals
101	10201	10.0499	.009901	**151**	22801	12.2882	.006623
102	10404	10.0995	.009804	**152**	23104	12.3288	.006579
103	10609	10.1489	.009709	**153**	23409	12.3693	.006536
104	10816	10.1980	.009615	**154**	23716	12.4097	.006494
105	11025	10.2470	.009524	**155**	24025	12.4499	.006452
106	11236	10.2956	.009434	**156**	24336	12.4900	.006410
107	11449	10.3441	.009346	**157**	24649	12.5300	.006369
108	11664	10.3923	.009259	**158**	24964	12.5698	.006329
109	11881	10.4403	.009174	**159**	25281	12.6095	.006289
110	12100	10.4881	.009091	**160**	25600	12.6491	.006250
111	12321	10.5357	.009009	**161**	25921	12.6886	.006211
112	12544	10.5830	.008929	**162**	26244	12.7279	.006173
113	12769	10.6301	.008850	**163**	26569	12.7671	.006135
114	12996	10.6771	.008772	**164**	26896	12.8062	.006098
115	13225	10.7238	.008696	**165**	27225	12.8452	.006061
116	13456	10.7703	.008621	**166**	27556	12.8841	.006024
117	13689	10.8167	.008547	**167**	27889	12.9228	.005988
118	13924	10.8628	.008475	**168**	28224	12.9615	.005952
119	14161	10.9087	.008403	**169**	28561	13.0000	.005917
120	14400	10.9545	.008333	**170**	28900	13.0384	.005882
121	14641	11.0000	.008264	**171**	29241	13.0767	.005848
122	14884	11.0454	.008197	**172**	29584	13.1149	.005814
123	15129	11.0905	.008130	**173**	29929	13.1529	.005780
124	15376	11.1355	.008065	**174**	30276	13.1909	.005747
125	15625	11.1803	.008000	**175**	30625	13.2288	.005714
126	15876	11.2250	.007937	**176**	30976	13.2665	.005682
127	16129	11.2694	.007874	**177**	31329	13.3041	.005650
128	16384	11.3137	.007813	**178**	31684	13.3417	.005618
129	16641	11.3578	.007752	**179**	32041	13.3791	.005587
130	16900	11.4018	.007692	**180**	32400	13.4164	.005556
131	17161	11.4455	.007634	**181**	32761	13.4536	.005525
132	17424	11.4891	.007576	**182**	33124	13.4907	.005495
133	17689	11.5326	.007519	**183**	33489	13.5277	.005464
134	17956	11.5758	.007463	**184**	33856	13.5647	.005435
135	18225	11.6190	.007407	**185**	34225	13.6015	.005405
136	18496	11.6619	.007353	**186**	34596	13.6382	.005376
137	18769	11.7047	.007299	**187**	34969	13.6748	.005348
138	19044	11.7473	.007246	**188**	35344	13.7113	.005319
139	19321	11.7898	.007194	**189**	35721	13.7477	.005291
140	19600	11.8322	.007143	**190**	36100	13.7840	.005263
141	19881	11.8743	.007092	**191**	36481	13.8203	.005236
142	20164	11.9164	.007042	**192**	36864	13.8564	.005208
143	20449	11.9583	.006993	**193**	37249	13.8924	.005181
144	20736	12.0000	.006944	**194**	37636	13.9284	.005155
145	21025	12.0416	.006897	**195**	38025	13.9642	.005128
146	21316	12.0830	.006849	**196**	38416	14.0000	.005102
147	21609	12.1244	.006803	**197**	38809	14.0357	.005076
148	21904	12.1655	.006757	**198**	39204	14.0712	.005051
149	22201	12.2066	.006711	**199**	39601	14.1067	.005025
150	22500	12.2474	.006667	**200**	40000	14.1421	.005000

TABLE OF SQUARES, SQUARE ROOTS, AND RECIPROCALS—1 TO 1000.—(*Continued*)

Numbers	Squares	Square roots	Reciprocals	Numbers	Squares	Square roots	Reciprocals
201	40401	14.1774	.004975	**251**	63001	15.8430	.003984
202	40804	14.2127	.004950	**252**	63504	15.8745	.003968
203	41209	14.2478	.004926	**253**	64009	15.9060	.003953
204	41616	14.2829	.004902	**254**	64516	15.9374	.003937
205	42025	14.3178	.004878	**255**	65025	15.9687	.003922
206	42436	14.3527	.004854	**256**	65536	16.0000	.003906
207	42849	14.3875	.004831	**257**	66049	16.0312	.003891
208	43264	14.4222	.004808	**258**	66564	16.0624	.003876
209	43681	14.4568	.004785	**259**	67081	16.0935	.003861
210	44100	14.4914	.004762	**260**	67600	16.1245	.003846
211	44521	14.5258	.004739	**261**	68121	16.1555	.003831
212	44944	14.5602	.004717	**262**	68644	16.1864	.003817
213	45369	14.5945	.004695	**263**	69169	16.2173	.003802
214	45796	14.6287	.004673	**264**	69696	16.2481	.003788
215	46225	14.6629	.004651	**265**	70225	16.2788	.003774
216	46656	14.6969	.004630	**266**	70756	16.3095	.003759
217	47089	14.7309	.004608	**267**	71289	16.3401	.003745
218	47524	14.7648	.004587	**268**	71824	16.3707	.003731
219	47961	14.7986	.004566	**269**	72361	16.4012	.003717
220	48400	14.8324	.004545	**270**	72900	16.4317	.003704
221	48841	14.8661	.004525	**271**	73441	16.4621	.003690
222	49284	14.8997	.004505	**272**	73984	16.4924	.003676
223	49729	14.9332	.004484	**273**	74529	16.5227	.003663
224	50176	14.9666	.004464	**274**	75076	16.5529	.003650
225	50625	15.0000	.004444	**275**	75625	16.5831	.003636
226	51076	15.0333	.004425	**276**	76176	16.6132	.003623
227	51529	15.0665	.004405	**277**	76729	16.6433	.003610
228	51984	15.0997	.004386	**278**	77284	16.6733	.003597
229	52441	15.1327	.004367	**279**	77841	16.7033	.003584
230	52900	15.1658	.004348	**280**	78400	16.7332	.003571
231	53361	15.1987	.004329	**281**	78961	16.7631	.003559
232	53824	15.2315	.004310	**282**	79524	16.7929	.003546
233	54289	15.2643	.004292	**283**	80089	16.8226	.003534
234	54756	15.2971	.004274	**284**	80656	16.8523	.003521
235	55225	15.3297	.004255	**285**	81225	16.8819	.003509
236	55696	15.3623	.004237	**286**	81796	16.9115	.003497
237	56169	15.3948	.004219	**287**	82369	16.9411	.003484
238	56644	15.4272	.004202	**288**	82944	16.9706	.003472
239	57121	15.4596	.004184	**289**	83521	17.0000	.003460
240	57600	15.4919	.004167	**290**	84100	17.0294	.003448
241	58081	15.5242	.004149	**291**	84681	17.0587	.003436
242	58564	15.5563	.004132	**292**	85264	17.0880	.003425
243	59049	15.5885	.004115	**293**	85849	17.1172	.003413
244	59536	15.6205	.004098	**294**	86436	17.1464	.003401
245	60025	15.6525	.004082	**295**	87025	17.1756	.003390
246	60516	15.6844	.004065	**296**	87616	17.2047	.003378
247	61009	15.7162	.004049	**297**	88209	17.2337	.003367
248	61504	15.7480	.004032	**298**	88804	17.2627	.003356
249	62001	15.7797	.004016	**299**	89401	17.2916	.003344
250	62500	15.8114	.004000	**300**	90000	17.3205	.003333

TABLE OF SQUARES, SQUARE ROOTS, AND RECIPROCALS—1 TO 1000.—(*Continued*)

Numbers	Squares	Square roots	Reciprocals	Numbers	Squares	Square roots	Reciprocals
301	90601	17.3494	.003322	**351**	123201	18.7350	.002849
302	91204	17.3781	.003311	**352**	123904	18.7617	.002841
303	91809	17.4069	.003300	**353**	124609	18.7883	.002833
304	92416	17.4356	.003289	**354**	125316	18.8149	.002825
305	93025	17.4642	.003279	**355**	126025	18.8414	.002817
306	93636	17.4929	.003268	**356**	126736	18.8680	.002809
307	94249	17.5214	.003257	**357**	127449	18.8944	.002801
308	94864	17.5499	.003247	**358**	128164	18.9209	.002793
309	95481	17.5784	.003236	**359**	128881	18.9473	.002786
310	96100	17.6068	.003226	**360**	129600	18.9737	.002778
311	96721	17.6352	.003215	**361**	130321	19.0000	.002770
312	97344	17.6635	.003205	**362**	131044	19.0263	.002762
313	97969	17.6918	.003195	**363**	131769	19.0526	.002755
314	98596	17.7200	.003185	**364**	132496	19.0788	.002747
315	99225	17.7482	.003175	**365**	133225	19.1050	.002740
316	99856	17.7764	.003165	**366**	133956	19.1311	.002732
317	100489	17.8045	.003155	**367**	134689	19.1572	.002725
318	101124	17.8326	.003145	**368**	135424	19.1833	.002717
319	101761	17.8606	.003135	**369**	136161	19.2094	.002710
320	102400	17.8885	.003125	**370**	136900	19.2354	.002703
321	103041	17.9165	.003115	**371**	137641	19.2614	.002695
322	103684	17.9444	.003106	**372**	138384	19.2873	.002688
323	104329	17.9722	.003096	**373**	139129	19.3132	.002681
324	104976	18.0000	.003086	**374**	139876	19.3391	.002674
325	105625	18.0278	.003077	**375**	140625	19.3649	.002667
326	106276	18.0555	.003067	**376**	141376	19.3907	.002660
327	106929	18.0831	.003058	**377**	142129	19.4165	.002653
328	107584	18.1108	.003049	**378**	142884	19.4422	.002646
329	108241	18.1384	.003040	**379**	143641	19.4679	.002639
330	108900	18.1659	.003030	**380**	144400	19.4936	.002632
331	109561	18.1934	.003021	**381**	145161	19.5192	.002625
332	110224	18.2209	.003012	**382**	145924	19.5448	.002618
333	110889	18.2483	.003003	**383**	146689	19.5704	.002611
334	111556	18.2757	.002994	**384**	147456	19.5959	.002604
335	112225	18.3030	.002985	**385**	148225	19.6214	.002597
336	112896	18.3303	.002976	**386**	148996	19.6469	.002591
337	113569	18.3576	.002967	**387**	149769	19.6723	.002584
338	114244	18.3848	.002959	**388**	150544	19.6977	.002577
339	114921	18.4120	.002950	**389**	151321	19.7231	.002571
340	115600	18.4391	.002941	**390**	152100	19.7484	.002564
341	116281	18.4662	.002933	**391**	152881	19.7737	.002558
342	116964	18.4932	.002924	**392**	153664	19.7990	.002551
343	117649	18.5203	.002915	**393**	154449	19.8242	.002545
344	118336	18.5472	.002907	**394**	155236	19.8494	.002538
345	119025	18.5742	.002899	**395**	156025	19.8746	.002532
346	119716	18.6011	.002890	**396**	156816	19.8997	.002525
347	120409	18.6279	.002882	**397**	157609	19.9249	.002519
348	121104	18.6548	.002874	**398**	158404	19.9499	.002513
349	121801	18.6815	.002865	**399**	159201	19.9750	.002506
350	122500	18.7083	.002857	**400**	160000	20.0000	.002500

TABLE OF SQUARES, SQUARE ROOTS, AND RECIPROCALS—1 TO 1000.—(*Continued*)

Numbers	Squares	Square roots	Reciprocals	Numbers	Squares	Square roots	Reciprocals
401	160801	20.0250	.002494	**451**	203401	21.2368	.002217
402	161604	20.0499	.002488	**452**	204304	21.2603	.002212
403	162409	20.0749	.002481	**453**	205209	21.2838	.002208
404	163216	20.0998	.002475	**454**	206116	21.3073	.002203
405	164025	20.1246	.002469	**455**	207025	21.3307	.002198
406	164836	20.1494	.002463	**456**	207936	21.3542	.002193
407	165649	20.1742	.002457	**457**	208849	21.3776	.002188
408	166464	20.1990	.002451	**458**	209764	21.4009	.002183
409	167281	20.2237	.002445	**459**	210681	21.4243	.002179
410	168100	20.2485	.002439	**460**	211600	21.4476	.002174
411	168921	20.2731	.002433	**461**	212521	21.4709	.002169
412	169744	20.2978	.002427	**462**	213444	21.4942	.002165
413	170569	20.3224	.002421	**463**	214369	21.5174	.002160
414	171396	20.3470	.002415	**464**	215296	21.5407	.002155
415	172225	20.3715	.002410	**465**	216225	21.5639	.002151
416	173056	20.3961	.002404	**466**	217156	21.5870	.002146
417	173889	20.4206	.002398	**467**	218089	21.6102	.002141
418	174724	20.4450	.002392	**468**	219024	21.6333	.002137
419	175561	20.4695	.002387	**469**	219961	21.6564	.002132
420	176400	20.4939	.002381	**470**	220900	21.6795	.002128
421	177241	20.5183	.002375	**471**	221841	21.7025	.002123
422	178084	20.5426	.002370	**472**	222784	21.7256	.002119
423	178929	20.5670	.002364	**473**	223729	21.7486	.002114
424	179776	20.5913	.002358	**474**	224676	21.7715	.002110
425	180625	20.6155	.002353	**475**	225625	21.7945	.002105
426	181476	20.6398	.002347	**476**	226576	21.8174	.002101
427	182329	20.6640	.002342	**477**	227529	21.8403	.002096
428	183184	20.6882	.002336	**478**	228484	21.8632	.002092
429	184041	20.7123	.002331	**479**	229441	21.8861	.002088
430	184900	20.7364	.002326	**480**	230400	21.9089	.002083
431	185761	20.7605	.002320	**481**	231361	21.9317	.002079
432	186624	20.7846	.002315	**482**	232324	21.9545	.002075
433	187489	20.8087	.002309	**483**	233289	21.9773	.002070
434	188356	20.8327	.002304	**484**	234256	22.0000	.002066
435	189225	20.8567	.002299	**485**	235225	22.0227	.002062
436	190096	20.8806	.002294	**486**	236196	22.0454	.002058
437	190969	20.9045	.002288	**487**	237169	22.0681	.002053
438	191844	20.9284	.002283	**488**	238144	22.0907	.002049
439	192721	20.9523	.002278	**489**	239121	22.1133	.002045
440	193600	20.9762	.002273	**490**	240100	22.1359	.002041
441	194481	21.0000	.002268	**491**	241081	22.1585	.002037
442	195364	21.0238	.002262	**492**	242064	22.1811	.002033
443	196249	21.0476	.002257	**493**	243049	22.2036	.002028
444	197136	21.0713	.002252	**494**	244036	22.2261	.002024
445	198025	21.0950	.002247	**495**	245025	22.2486	.002020
446	198916	21.1187	.002242	**496**	246016	22.2711	.002016
447	199809	21.1424	.002237	**497**	247009	22.2935	.002012
448	200704	21.1660	.002232	**498**	248004	22.3159	.002008
449	201601	21.1896	.002227	**499**	249001	22.3383	.002004
450	202500	21.2132	.002222	**500**	250000	22.3607	.002000

TABLE OF SQUARES, SQUARE ROOTS, AND RECIPROCALS—1 TO 1000.—(*Continued*)

Numbers	Squares	Square roots	Recipro-cals	Numbers	Squares	Square roots	Recipro-cals
501	251001	22.3830	.001996	**551**	303601	23.4734	.001815
502	252004	22.4054	.001992	**552**	304704	23.4947	.001812
503	253009	22.4277	.001988	**553**	305809	23.5160	.001808
504	254016	22.4499	.001984	**554**	306916	23.5372	.001805
505	255025	22.4722	.001980	**555**	308025	23.5584	.001802
506	256036	22.4944	.001976	**556**	309136	23.5797	.001799
507	257049	22.5167	.001972	**557**	310249	23.6008	.001795
508	258064	22.5389	.001969	**558**	311364	23.6220	.001792
509	259081	22.5610	.001965	**559**	312481	23.6432	.001789
510	260100	22.5832	.001961	**560**	313600	23.6643	.001786
511	261121	22.6053	.001957	**561**	314721	23.6854	.001783
512	262144	22.6274	.001953	**562**	315844	23.7065	.001779
513	263169	22.6495	.001949	**563**	316969	23.7276	.001776
514	264196	22.6716	.001946	**564**	318096	23.7487	.001773
515	265225	22.6936	.001942	**565**	319225	23.7697	.001770
516	266256	22.7156	.001938	**566**	320356	23.7908	.001767
517	267289	22.7376	.001934	**567**	321489	23.8118	.001764
518	268324	22.7596	.001931	**568**	322624	23.8328	.001761
519	269361	22.7816	.001927	**569**	323761	23.8537	.001757
520	270400	22.8035	.001923	**570**	324900	23.8747	.001754
521	271441	22.8254	.001919	**571**	326041	23.8956	.001751
522	272484	22.8473	.001916	**572**	327184	23.9160	.001748
523	273529	22.8692	.001912	**573**	328329	23.9374	.001745
524	274576	22.8910	.001908	**574**	329476	23.9583	.001742
525	275625	22.9129	.001905	**575**	330625	23.9792	.001739
526	276676	22.9347	.001901	**576**	331776	24.0000	.001736
527	277729	22.9565	.001898	**577**	332929	24.0208	.001733
528	278784	22.9783	.001894	**578**	334084	24.0416	.001730
529	279841	23.0000	.001890	**579**	335241	24.0624	.001727
530	280900	23.0217	.001887	**580**	336400	24.0832	.001724
531	281961	23.0434	.001883	**581**	337561	24.1039	.001721
532	283024	23.0651	.001880	**582**	338724	24.1247	.001718
533	284089	23.0868	.001876	**583**	339889	24.1454	.001715
534	285156	23.1084	.001873	**584**	341056	24.1661	.001712
535	286225	23.1301	.001869	**585**	342225	24.1868	.001709
536	287296	23.1517	.001866	**586**	343396	24.2074	.001706
537	288369	23.1733	.001862	**587**	344569	24.2281	.001704
538	289444	23.1948	.001859	**588**	345744	24.2487	.001701
539	290521	23.2164	.001855	**589**	346921	24.2693	.001698
540	291600	23.2379	.001852	**590**	348100	24.2899	.001695
541	292681	23.2594	.001848	**591**	349281	24.3105	.001692
542	293764	23.2809	.001845	**592**	350464	24.3311	.001689
543	294849	23.3024	.001842	**593**	351649	24.3516	.001686
544	295936	23.3238	.001838	**594**	352836	24.3721	.001684
545	297025	23.3452	.001835	**595**	354025	24.3926	.001681
546	298116	23.3666	.001832	**596**	355216	24.4131	.001678
547	299209	23.3880	.001828	**597**	356409	24.4336	.001675
548	300304	23.4094	.001825	**598**	357604	24.4540	.001672
549	301401	23.4307	.001821	**599**	358801	24.4745	.001669
550	302500	23.4521	.001818	**600**	360000	24.4949	.001667

Table of Squares, Square Roots, and Reciprocals—1 to 1000.—(*Continued*)

Numbers	Squares	Square roots	Reciprocals	Numbers	Squares	Square roots	Reciprocals
601	361201	24.5153	.001664	**651**	423801	25.5147	.001536
602	362404	24.5357	.001661	**652**	425104	25.5343	.001534
603	363609	24.5561	.001658	**653**	426409	25.5539	.001531
604	364816	24.5764	.001656	**654**	427716	25.5734	.001529
605	366025	24.5967	.001653	**655**	429025	25.5930	.001527
606	367236	24.6171	.001650	**656**	430336	25.6125	.001524
607	368449	24.6374	.001647	**657**	431649	25.6320	.001522
608	369664	24.6577	.001645	**658**	432964	25.6515	.001520
609	370881	24.6779	.001642	**659**	434281	25.6710	.001517
610	372100	24.6982	.001639	**660**	435600	25.6905	.001515
611	373321	24.7184	.001637	**661**	436921	25.7099	.001513
612	374544	24.7386	.001634	**662**	438244	25.7294	.001511
613	375769	24.7588	.001631	**663**	439569	25.7488	.001508
614	376996	24.7790	.001629	**664**	440896	25.7682	.001506
615	378225	24.7992	.001626	**665**	442225	25.7876	.001504
616	379456	24.8193	.001623	**666**	443556	25.8070	.001502
617	380689	24.8395	.001621	**667**	444889	25.8263	.001499
618	381924	24.8596	.001618	**668**	446224	25.8457	.001497
619	383161	24.8797	.001616	**669**	447561	25.8650	.001495
620	384400	24.8998	.001613	**670**	448900	25.8844	.001493
621	385641	24.9199	.001610	**671**	450241	25.9037	.001490
622	386884	24.9399	.001608	**672**	451584	25.9230	.001488
623	388129	24.9600	.001605	**673**	452929	25.9422	.001486
624	389376	24.9800	.001603	**674**	454276	25.9615	.001484
625	390625	25.0000	.001600	**675**	455625	25.9808	.001481
626	391876	25.0200	.001597	**676**	456976	26.0000	.001479
627	393129	25.0400	.001595	**677**	458329	26.0192	.001477
628	394384	25.0599	.001592	**678**	459684	26.0384	.001475
629	395641	25.0799	.001590	**679**	461041	26.0576	.001473
630	396900	25.0998	.001587	**680**	462400	26.0768	.001471
631	398161	25.1197	.001585	**681**	463761	26.0960	.001468
632	399424	25.1396	.001582	**682**	465124	26.1151	.001466
633	400689	25.1595	.001580	**683**	466489	26.1343	.001464
634	401956	25.1794	.001577	**684**	467856	26.1534	.001462
635	403225	25.1992	.001575	**685**	469225	26.1725	.001460
636	404496	25.2190	.001572	**686**	470596	26.1916	.001458
637	405769	25.2389	.001570	**687**	471969	26.2107	.001456
638	407044	25.2587	.001567	**688**	473344	26.2298	.001453
639	408321	25.2784	.001565	**689**	474721	26.2488	.001451
640	409600	25.2982	.001563	**690**	476100	26.2679	.001449
641	410881	25.3180	.001560	**691**	477481	26.2869	.001447
642	412164	25.3377	.001558	**692**	478864	26.3059	.001445
643	413449	25.3574	.001555	**693**	480249	26.3249	.001443
644	414736	25.3772	.001553	**694**	481636	26.3439	.001441
645	416025	25.3969	.001550	**695**	483025	26.3629	.001439
646	417316	25.4165	.001548	**696**	484416	26.3818	.001437
647	418609	25.4362	.001546	**697**	485809	26.4008	.001435
648	419904	25.4558	.001543	**698**	487204	26.4197	.001433
649	421201	25.4755	.001541	**699**	488601	26.4386	.001431
650	422500	25.4951	.001538	**700**	490000	26.4575	.001429

TABLE OF SQUARES, SQUARE ROOTS, AND RECIPROCALS—1 TO 1000.—(*Continued*)

Numbers	Squares	Square roots	Reciprocals	Numbers	Squares	Square roots	Reciprocals
701	491401	26.4764	.001427	**751**	564001	27.4044	.001332
702	492804	26.4953	.001425	**752**	565504	27.4226	.001330
703	494209	26.5141	.001422	**753**	567009	27.4408	.001328
704	495616	26.5330	.001420	**754**	568516	27.4591	.001326
705	497025	26.5518	.001418	**755**	570025	27.4773	.001325
706	498436	26.5707	.001416	**756**	571536	27.4955	.001323
707	499849	26.5895	.001414	**757**	573049	27.5136	.001321
708	501264	26.6083	.001412	**758**	574564	27.5318	.001319
709	502681	26.6271	.001410	**759**	576081	27.5500	.001318
710	504100	26.6458	.001408	**760**	577600	27.5681	.001316
711	505521	26.6646	.001406	**761**	579121	27.5862	.001314
712	506944	26.6833	.001404	**762**	580644	27.6043	.001312
713	508369	26.7021	.001403	**763**	582169	27.6225	.001311
714	509796	26.7208	.001401	**764**	583696	27.6405	.001309
715	511225	26.7395	.001399	**765**	585225	27.6586	.001307
716	512656	26.7582	.001397	**766**	586756	27.6767	.001305
717	514089	26.7769	.001395	**767**	588289	27.6948	.001304
718	515524	26.7955	.001393	**768**	589824	27.7128	.001302
719	516961	26.8142	.001391	**769**	591361	27.7308	.001300
720	518400	26.8328	.001389	**770**	592900	27.7489	.001299
721	519841	26.8514	.001387	**771**	594441	27.7669	.001297
722	521284	26.8701	.001385	**772**	595984	27.7849	.001295
723	522729	26.8887	.001383	**773**	597529	27.8029	.001294
724	524176	26.9072	.001381	**774**	599076	27.8209	.001292
725	525625	26.9258	.001379	**775**	600625	27.8388	.001290
726	527076	26.9444	.001377	**776**	602176	27.8568	.001289
727	528529	26.9629	.001376	**777**	603729	27.8747	.001287
728	529984	26.9815	.001374	**778**	605284	27.8927	.001285
729	531441	27.0000	.001372	**779**	606841	27.9106	.001284
730	532900	27.0185	.001370	**780**	608400	27.9285	.001282
731	534361	27.0370	.001368	**781**	609961	27.9464	.001280
732	535824	27.0555	.001366	**782**	611524	27.9643	.001279
733	537289	27.0740	.001364	**783**	613089	27.9821	.001277
734	538756	27.0924	.001362	**784**	614656	28.0000	.001276
735	540225	27.1109	.001361	**785**	616225	28.0179	.001274
736	541696	27.1293	.001359	**786**	617796	28.0357	.001272
737	543169	27.1477	.001357	**787**	619369	28.0535	.001271
738	544644	27.1662	.001355	**788**	620944	28.0713	.001269
739	546121	27.1846	.001353	**789**	622521	28.0891	.001267
740	547600	27.2029	.001351	**790**	624100	28.1069	.001266
741	549081	27.2213	.001350	**791**	625681	28.1247	.001264
742	550564	27.2397	.001348	**792**	627264	28.1425	.001263
743	552049	27.2580	.001346	**793**	628849	28.1603	.001261
744	553536	27.2764	.001344	**794**	630436	28.1780	.001259
745	555025	27.2947	.001342	**795**	632025	28.1957	.001258
746	556516	27.3130	.001340	**796**	633616	28.2135	.001256
747	558009	27.3313	.001339	**797**	635209	28.2312	.001255
748	559504	27.3496	.001337	**798**	636804	28.2489	.001253
749	561001	27.3679	.001335	**799**	638401	28.2666	.001252
750	562500	27.3861	.001333	**800**	640000	28.2843	.001250

TABLE OF SQUARES, SQUARE ROOTS, AND RECIPROCALS—1 TO 1000.—(*Continued*)

Numbers	Squares	Square roots	Reciprocals	Numbers	Squares	Square roots	Reciprocals
801	641601	28.3019	.001248	**851**	724201	29.1719	.001175
802	643204	28.3196	.001247	**852**	725904	29.1890	.001174
803	644809	28.3373	.001245	**853**	727609	29.2062	.001172
804	646416	28.3549	.001244	**854**	729316	29.2233	.001171
805	648025	28.3725	.001242	**855**	731025	29.2404	.001170
806	649636	28.3901	.001241	**856**	732736	29.2575	.001168
807	651249	28.4077	.001239	**857**	734449	29.2746	.001167
808	652864	28.4253	.001238	**858**	736164	29.2916	.001166
809	654481	28.4429	.001236	**859**	737881	29.3087	.001164
810	656100	28.4605	.001235	**860**	739600	29.3258	.001163
811	657721	28.4781	.001233	**861**	741321	29.3428	.001161
812	659344	28.4956	.001232	**862**	743044	29.3598	.001160
813	660969	28.5132	.001230	**863**	744769	29.3769	.001159
814	662596	28.5307	.001229	**864**	746496	29.3939	.001157
815	664225	28.5482	.001227	**865**	748225	29.4109	.001156
816	665856	28.5657	.001225	**866**	749956	29.4279	.001155
817	667489	28.5832	.001224	**867**	751689	29.4449	.001153
818	669124	28.6007	.001222	**868**	753424	29.4618	.001152
819	670761	28.6182	.001221	**869**	755161	29.4788	.001151
820	672400	28.6356	.001220	**870**	756900	29.4958	.001149
821	674041	28.6531	.001218	**871**	758641	29.5127	.001148
822	675684	28.6705	.001217	**872**	760384	29.5296	.001147
823	677329	28.6880	.001215	**873**	762129	29.5466	.001145
824	678976	28.7054	.001214	**874**	763876	29.5635	.001144
825	680625	28.7228	.001212	**875**	765625	29.5804	.001143
826	682276	28.7402	.001211	**876**	767376	29.5973	.001142
827	683929	28.7576	.001209	**877**	769129	29.6142	.001140
828	685584	28.7750	.001208	**878**	770884	29.6311	.001139
829	687241	28.7924	.001206	**879**	772641	29.6479	.001138
830	688900	28.8097	.001205	**880**	774400	29.6648	.001136
831	690561	28.8271	.001203	**881**	776161	29.6816	.001135
832	692224	28.8444	.001202	**882**	777924	29.6985	.001134
833	693889	28.8617	.001200	**883**	779689	29.7153	.001133
834	695556	28.8791	.001199	**884**	781456	29.7321	.001131
835	697225	28.8964	.001198	**885**	783225	29.7489	.001130
836	698896	28.9137	.001196	**886**	784996	29.7658	.001129
837	700569	28.9310	.001195	**887**	786769	29.7825	.001127
838	702244	28.9482	.001193	**888**	788544	29.7993	.001126
839	703921	28.9655	.001192	**889**	790321	29.8161	.001125
840	705600	28.9828	.001190	**890**	792100	29.8329	.001124
841	707281	29.0000	.001189	**891**	793881	29.8496	.001122
842	708964	29.0172	.001188	**892**	795664	29.8664	.001121
843	710649	29.0345	.001186	**893**	797449	29.8831	.001120
844	712336	29.0517	.001185	**894**	799236	29.8998	.001119
845	714025	29.0689	.001183	**895**	801025	29.9166	.001117
846	715716	29.0861	.001182	**896**	802816	29.9323	.001116
847	717409	29.1033	.001181	**897**	804609	29.9500	.001115
848	719104	29.1204	.001179	**898**	806404	29.9666	.001114
849	720801	29.1376	.001178	**899**	808201	29.9833	.001112
850	722500	29.1548	.001176	**900**	810000	30.0000	.001111

Table of Squares, Square Roots, and Reciprocals—1 to 1000.—(*Concluded*)

Numbers	Squares	Square roots	Reciprocals	Numbers	Squares	Square roots	Reciprocals
901	811801	30.0167	.001110	**951**	904401	30.8383	.001052
902	813604	30.0333	.001109	**952**	906304	30.8545	.001050
903	815409	30.0500	.001107	**953**	908209	30.8707	.001049
904	817216	30.0666	.001106	**954**	910116	30.8869	.001048
905	819025	30.0832	.001105	**955**	912025	30.9031	.001047
906	820836	30.0998	.001104	**956**	913936	30.9192	.001046
907	822649	30.1164	.001103	**957**	915849	30.9354	.001045
908	824464	30.1330	.001101	**958**	917764	30.9516	.001044
909	826281	30.1496	.001100	**959**	919681	30.9677	.001043
910	828100	30.1662	.001099	**960**	921600	30.9839	.001042
911	829921	30.1828	.001098	**961**	923521	31.0000	.001041
912	831744	30.1993	.001096	**962**	925444	31.0161	.001040
913	833569	30.2159	.001095	**963**	927369	31.0322	.001038
914	835396	30.2324	.001094	**964**	929296	31.0483	.001037
915	837225	30.2490	.001093	**965**	931225	31.0644	.001036
916	839056	30.2655	.001092	**966**	933156	31.0805	.001035
917	840889	30.2820	.001091	**967**	935089	31.0966	.001034
918	842724	30.2985	.001089	**968**	937024	31.1127	.001033
919	844561	30.3150	.001088	**969**	938961	31.1288	.001032
920	846400	30.3315	.001087	**970**	940900	31.1448	.001031
921	848241	30.3480	.001086	**971**	942841	31.1609	.001030
922	850084	30.3645	.001085	**972**	944784	31.1769	.001029
923	851929	30.3809	.001083	**973**	946729	31.1929	.001028
924	853776	30.3974	.001082	**974**	948676	31.2090	.001027
925	855625	30.4138	.001081	**975**	950625	31.2250	.001026
926	857476	30.4302	.001080	**976**	952576	31.2410	.001025
927	859329	30.4467	.001079	**977**	954529	31.2570	.001024
928	861184	30.4631	.001078	**978**	956484	31.2730	.001022
929	863041	30.4795	.001076	**979**	958441	31.2890	.001021
930	864900	30.4959	.001075	**980**	960400	31.3050	.001020
931	866761	30.5123	.001074	**981**	962361	31.3209	.001019
932	868624	30.5287	.001073	**982**	964324	31.3369	.001018
933	870489	30.5450	.001072	**983**	966289	31.3528	.001017
934	872356	30.5614	.001071	**984**	968256	31.3688	.001016
935	874225	30.5778	.001070	**985**	970225	31.3847	.001015
936	876096	30.5941	.001068	**986**	972196	31.4006	.001014
937	877969	30.6105	.001067	**987**	974169	31.4166	.001013
938	879844	30.6268	.001066	**988**	976144	31.4325	.001012
939	881721	30.6431	.001065	**989**	978121	31.4484	.001011
940	883600	30.6594	.001064	**990**	980100	31.4643	.001010
941	885481	30.6757	.001063	**991**	982081	31.4802	.001009
942	887364	30.6920	.001062	**992**	984064	31.4960	.001008
943	889249	30.7083	.001060	**993**	986049	31.5119	.001007
944	891136	30.7246	.001059	**994**	988036	31.5278	.001006
945	893025	30.7409	.001058	**995**	990025	31.5436	.001005
946	894916	30.7571	.001057	**996**	992016	31.5595	.001004
947	896809	30.7734	.001056	**997**	994009	31.5753	.001003
948	898704	30.7896	.001055	**998**	996004	31.5911	.001002
949	900601	30.8058	.001054	**999**	998001	31.6070	.001001
950	902500	30.8221	.001053	**1000**	1000000	31.6228	.001000

APPENDIX IX

GENERAL LIST OF REFERENCES

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